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Math for Machine Learning

Linear algebra - Week 4

Bases

Span

Orthogonal and orthonormal bases

Orthogonal and orthonormal matrices

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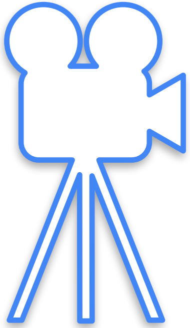
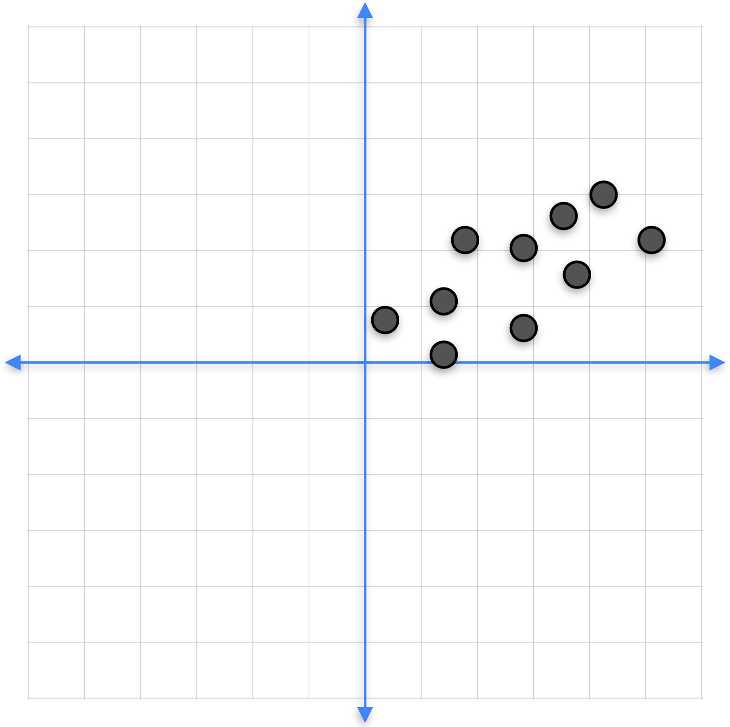
Determinants and Eigenvectors

Machine learning motivation

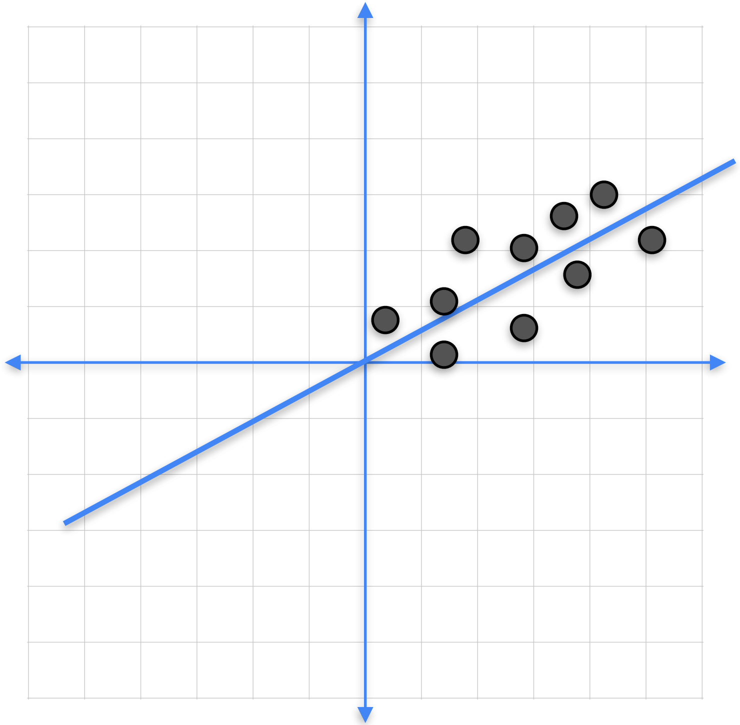
PCA



Principal Component Analysis



Principal Component Analysis



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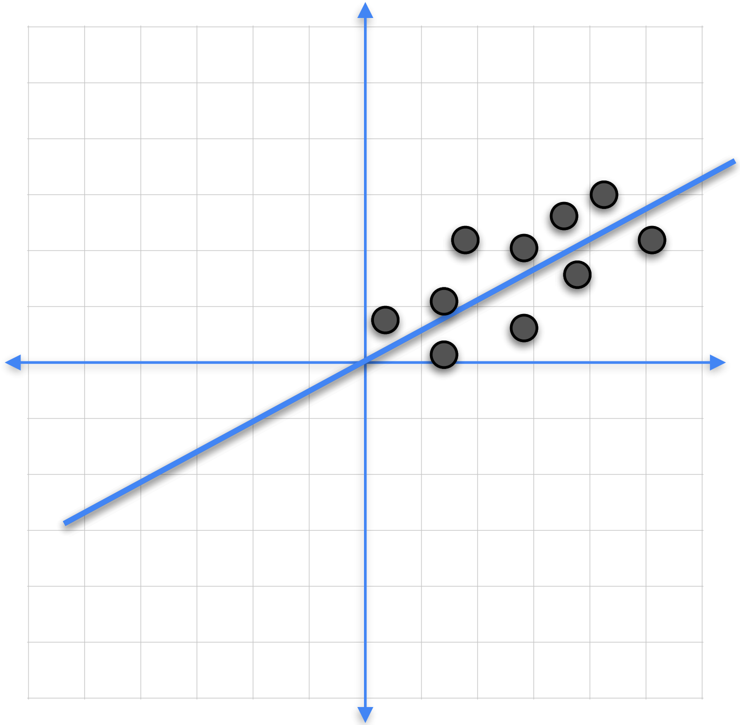
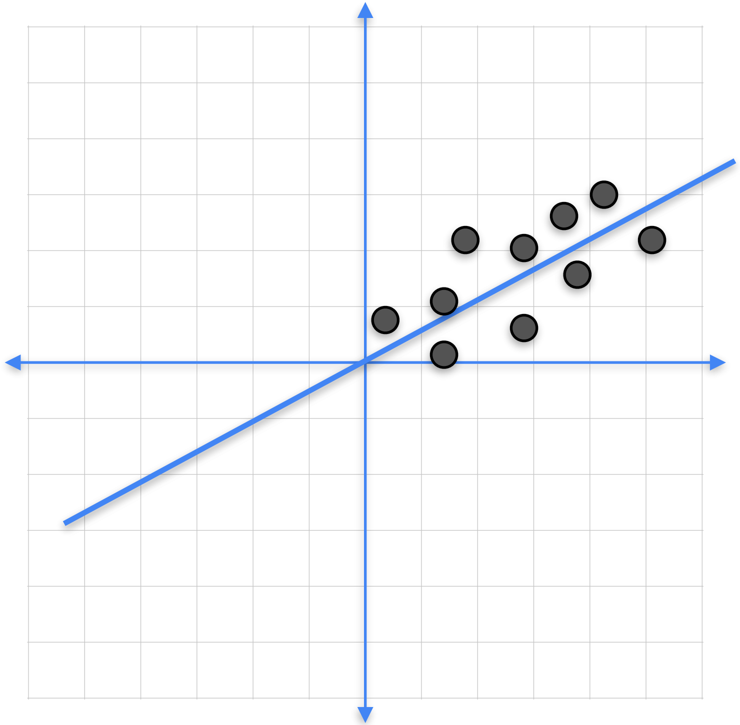
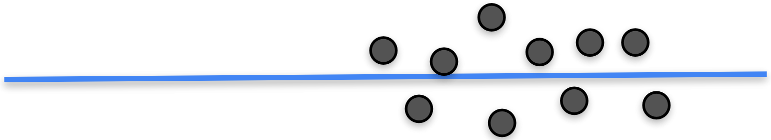


Principal Component Analysis





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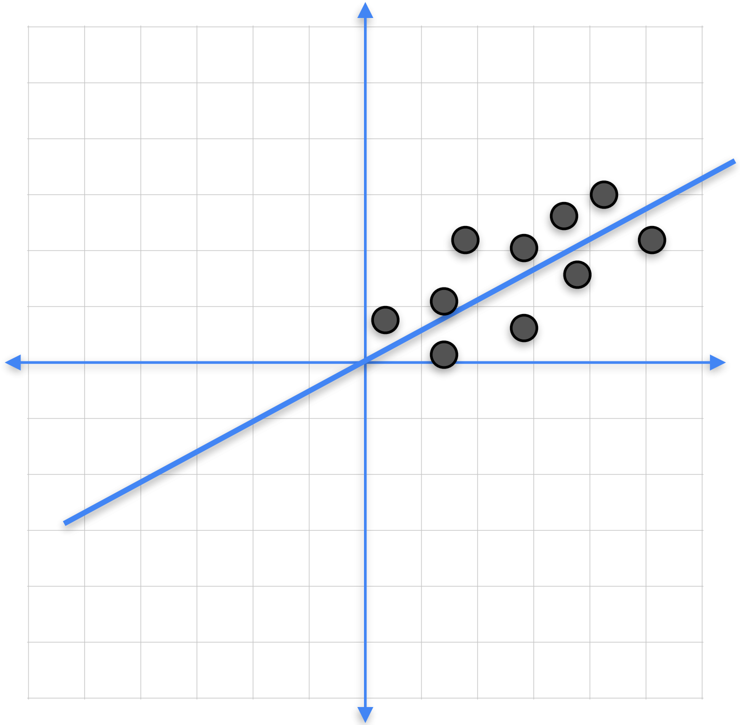

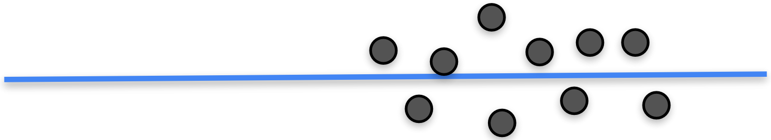

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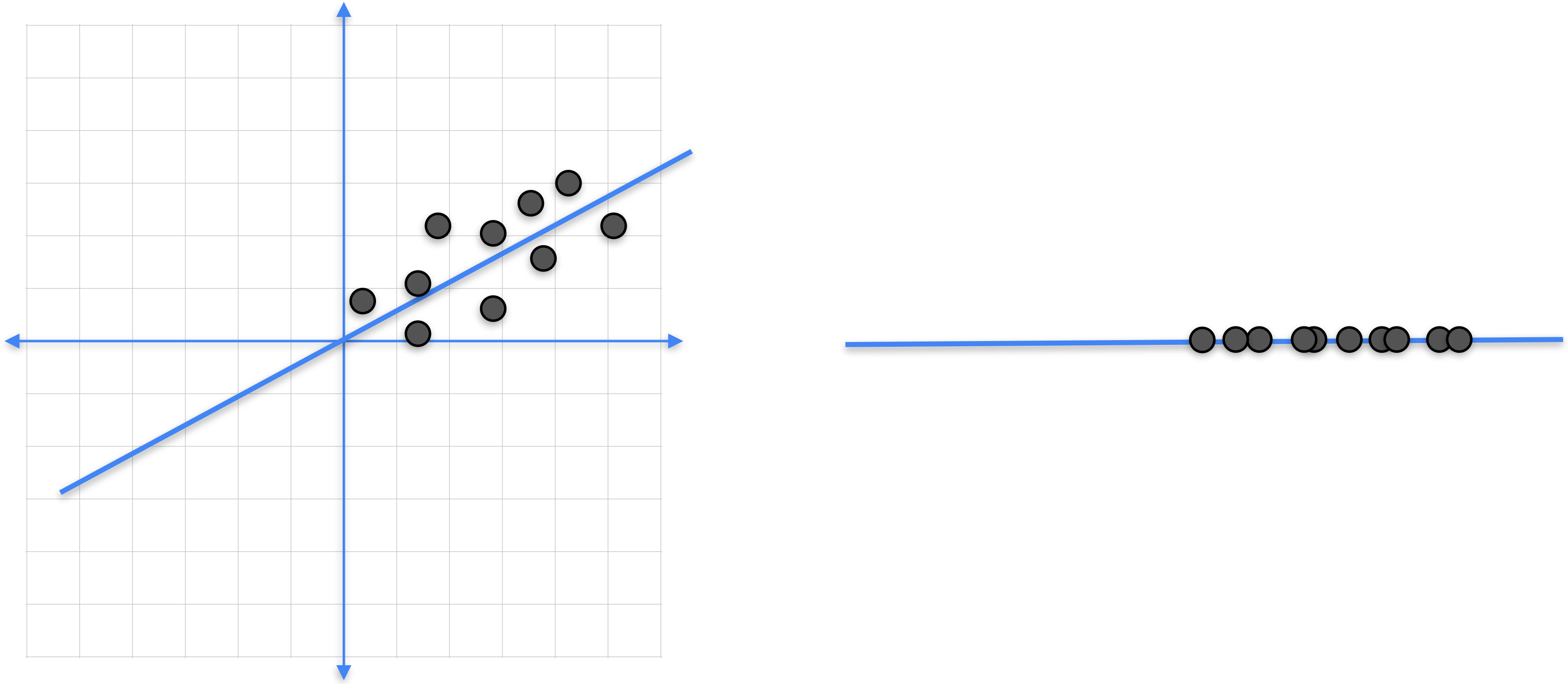

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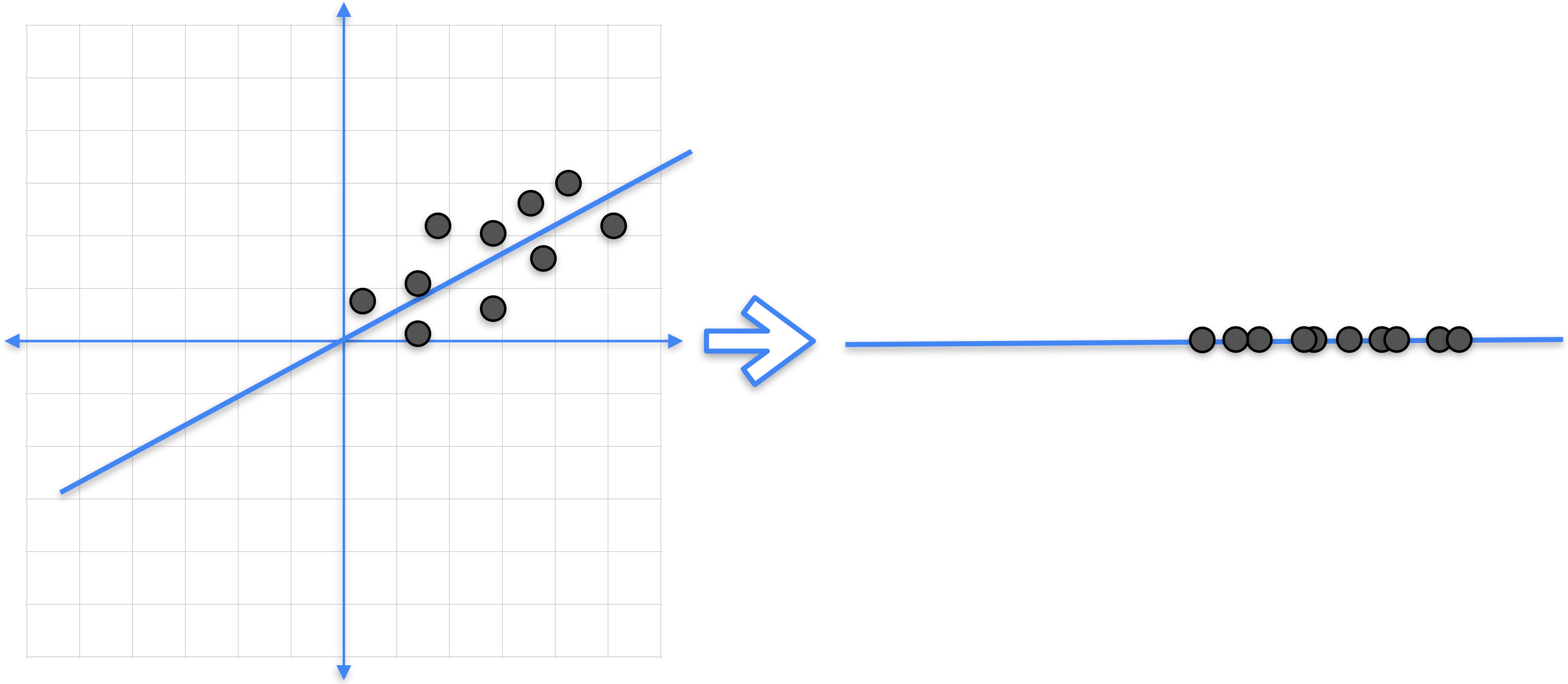
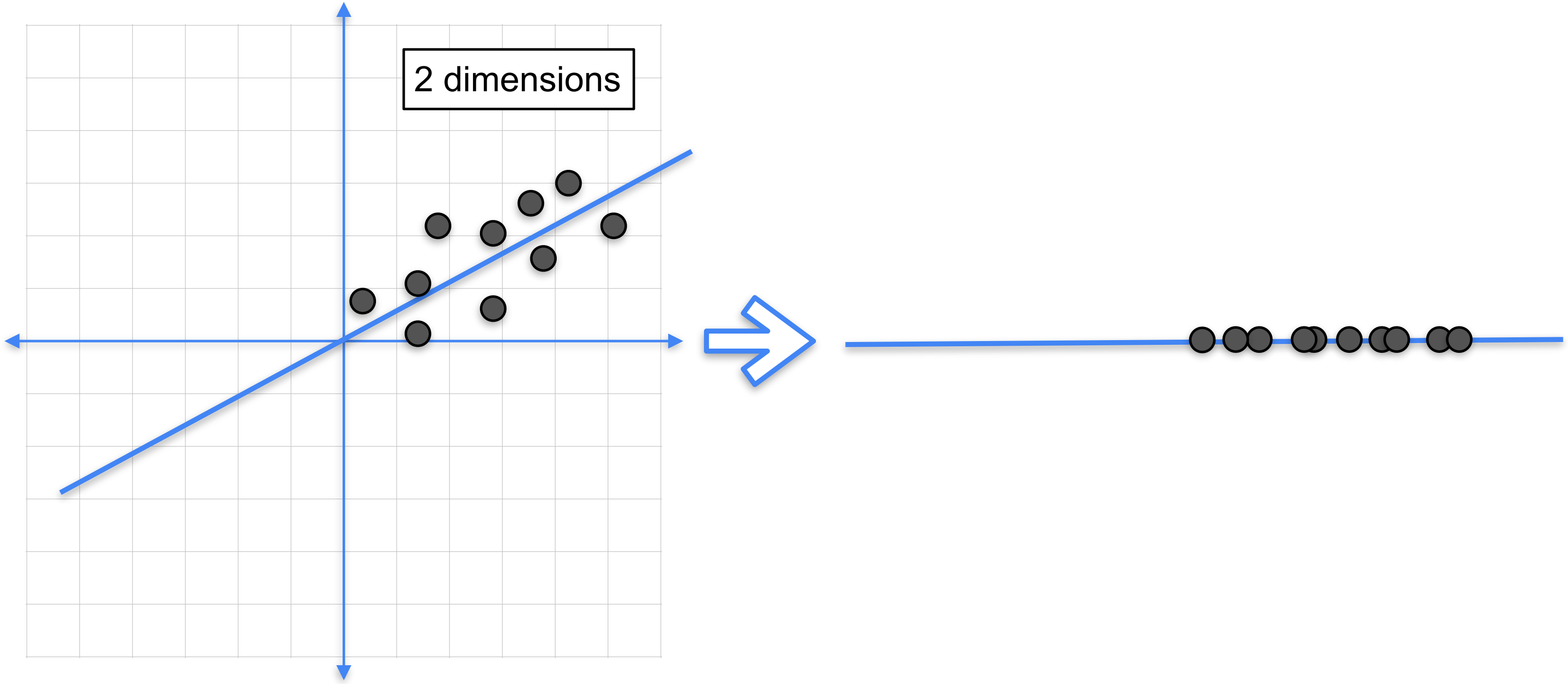


Principal Component Analysis



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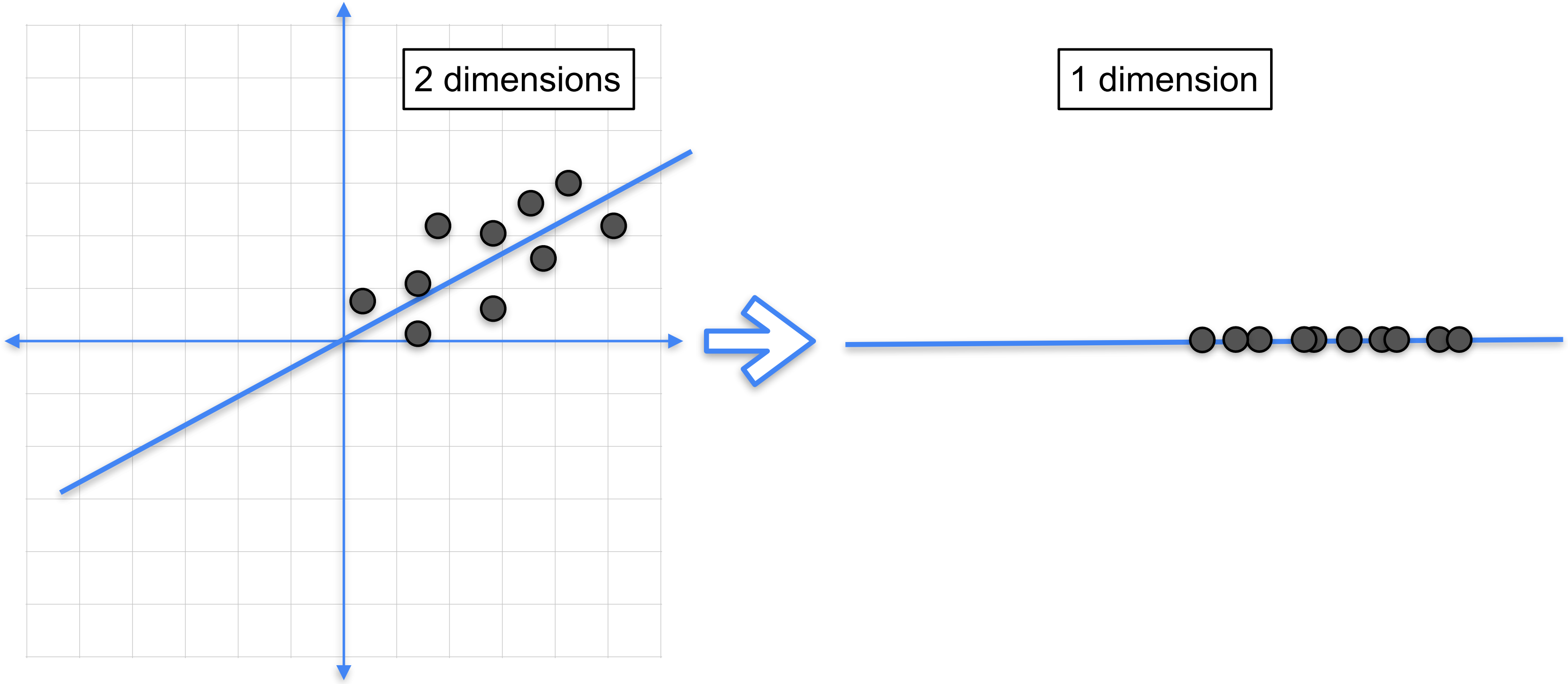
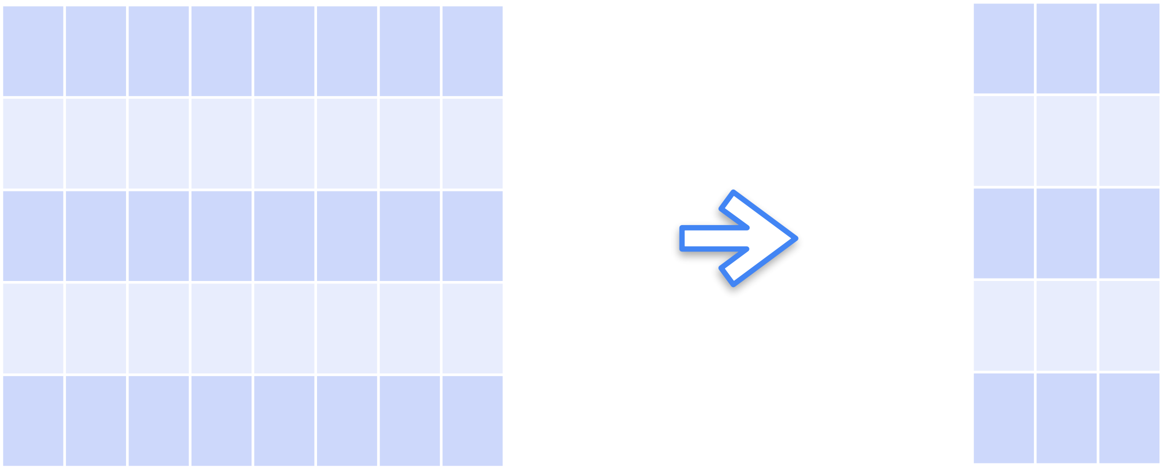
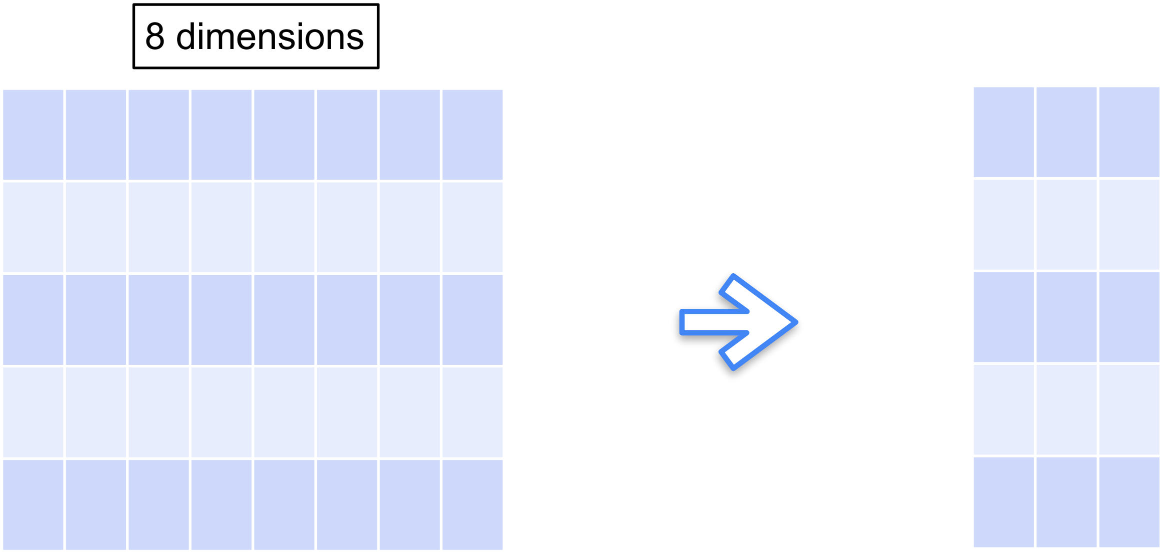


Principal Component Analysis

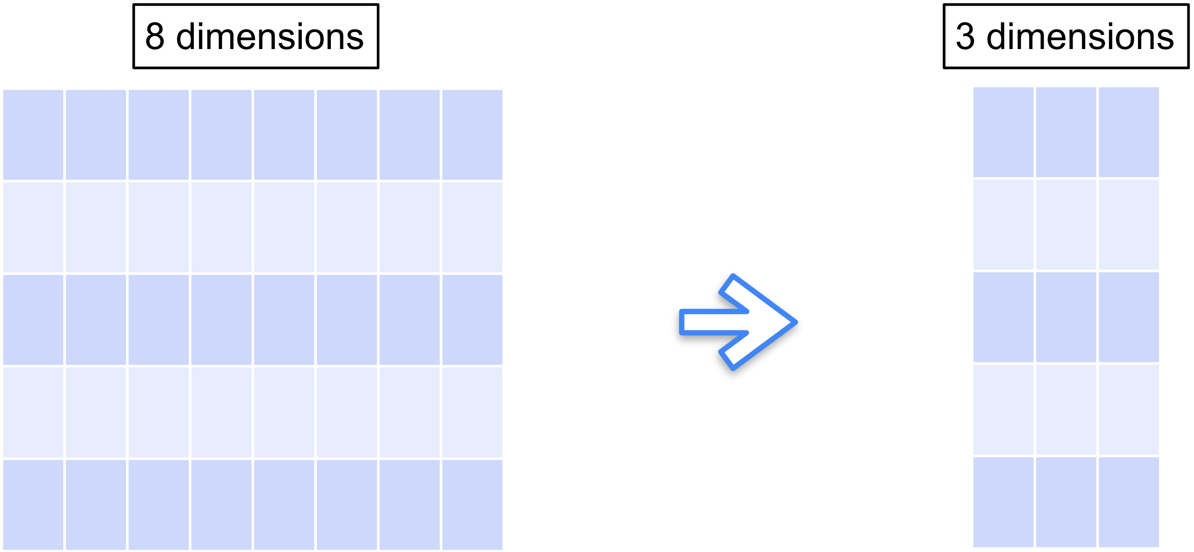
Principal Component Analysis



Principal Component Analysis



Principal Component Analysis



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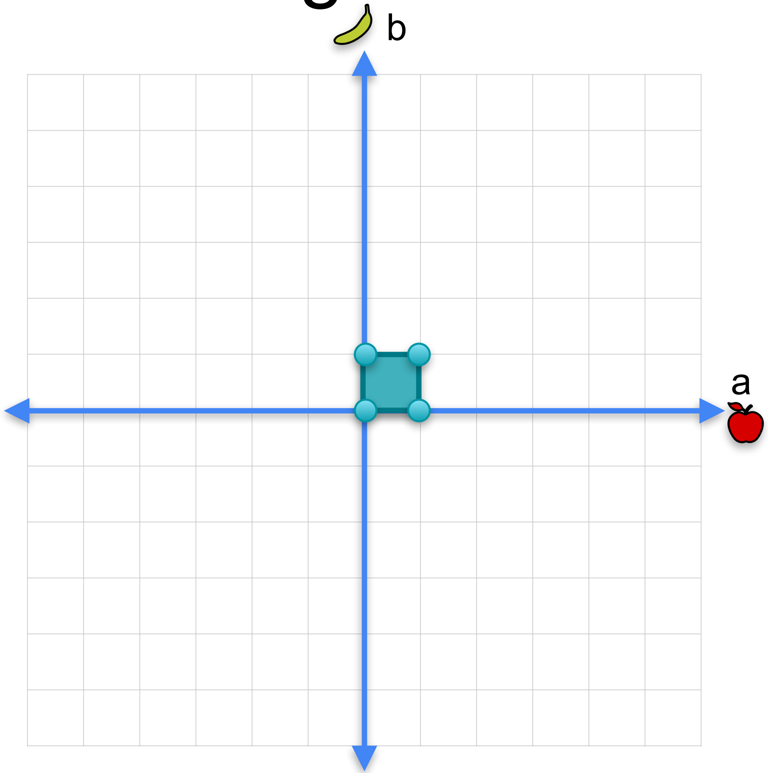
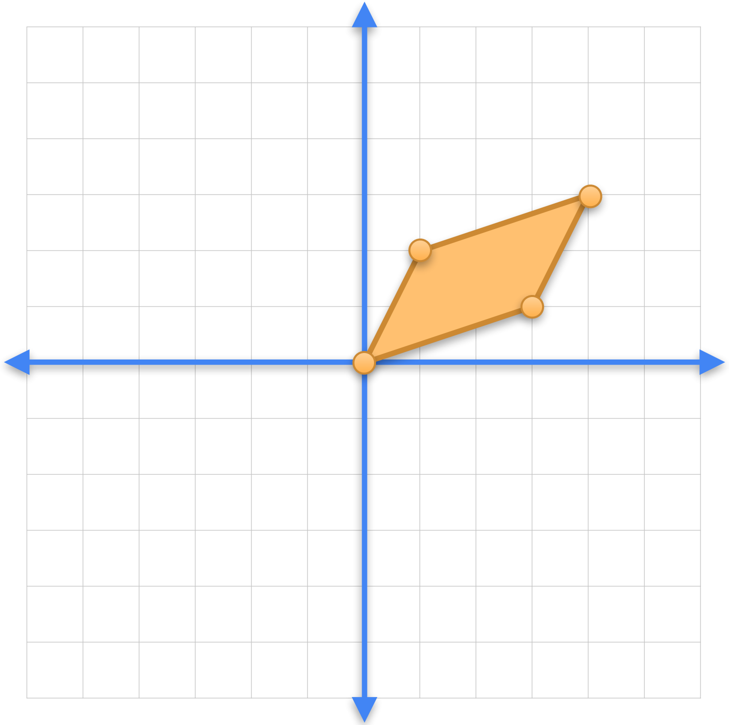
Determinants and Eigenvectors

Singularity and rank of linear transformations

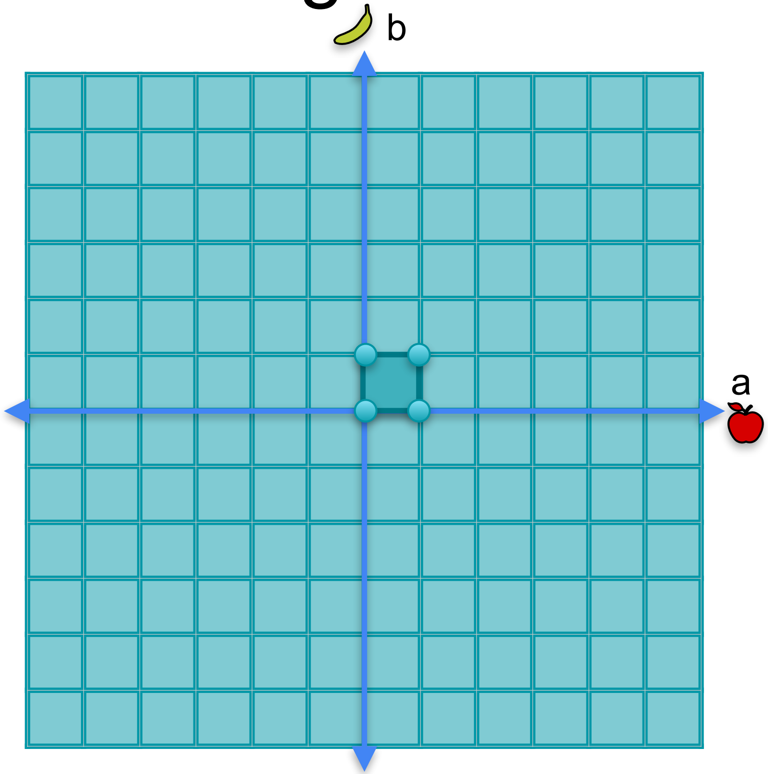
Non-singular transformation



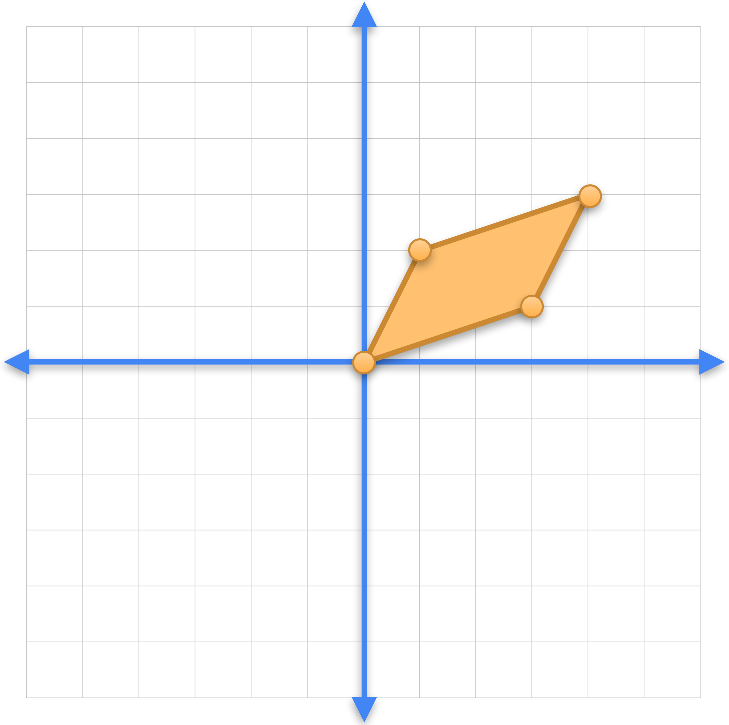
🍎	🍌
3	1
1	2



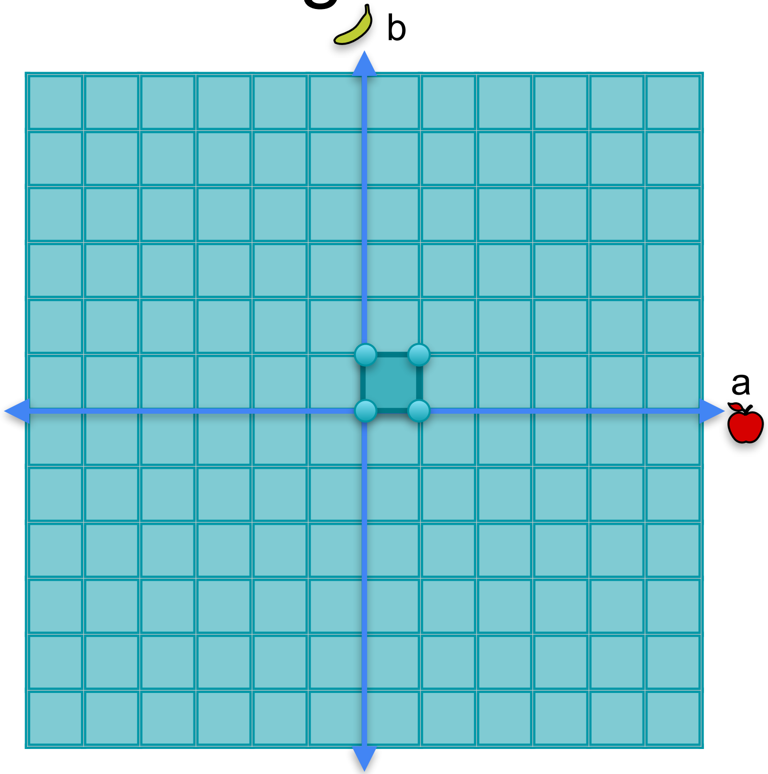
Non-singular transformation



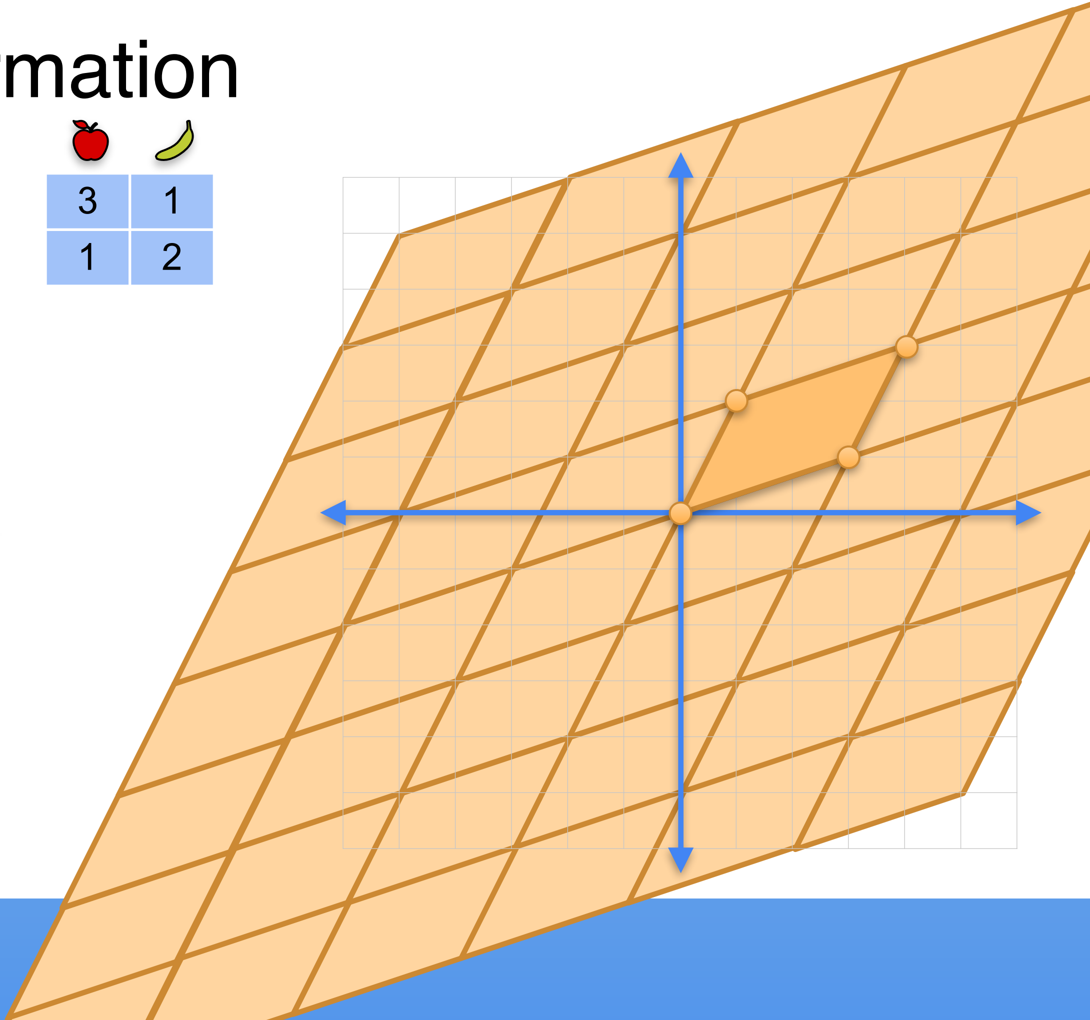
3	1
1	2



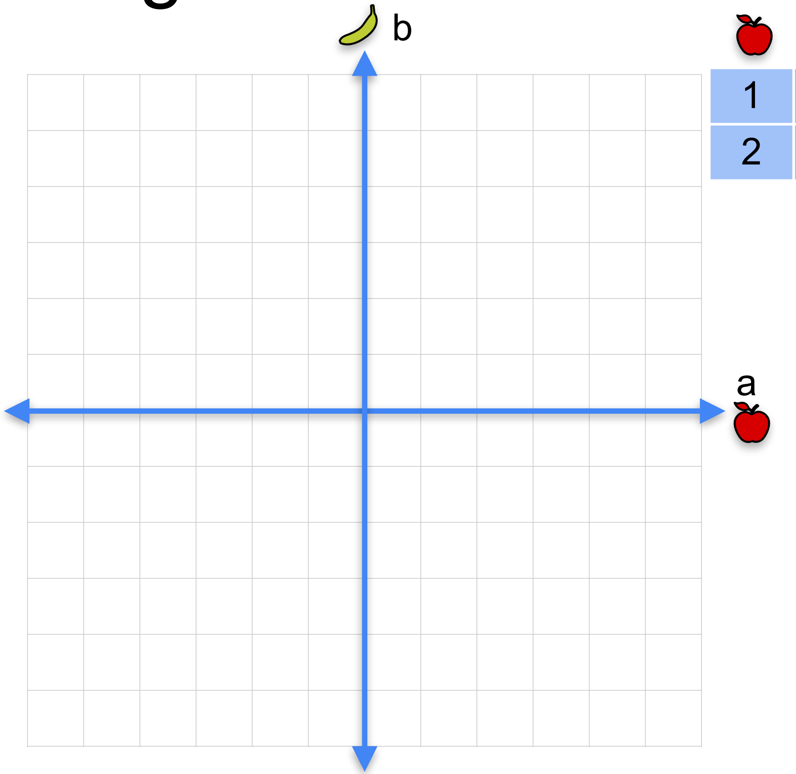
Non-singular transformation



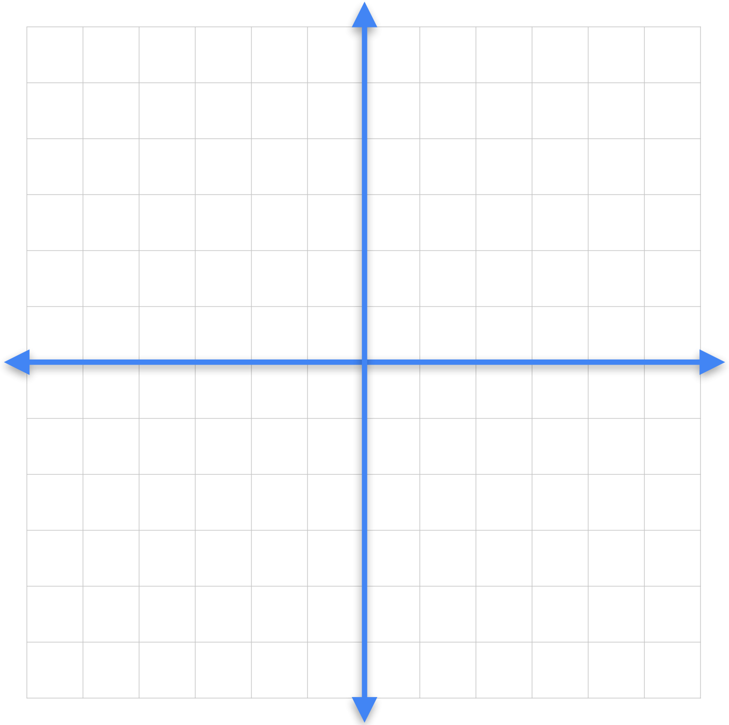
🍎	🍌
3	1
1	2



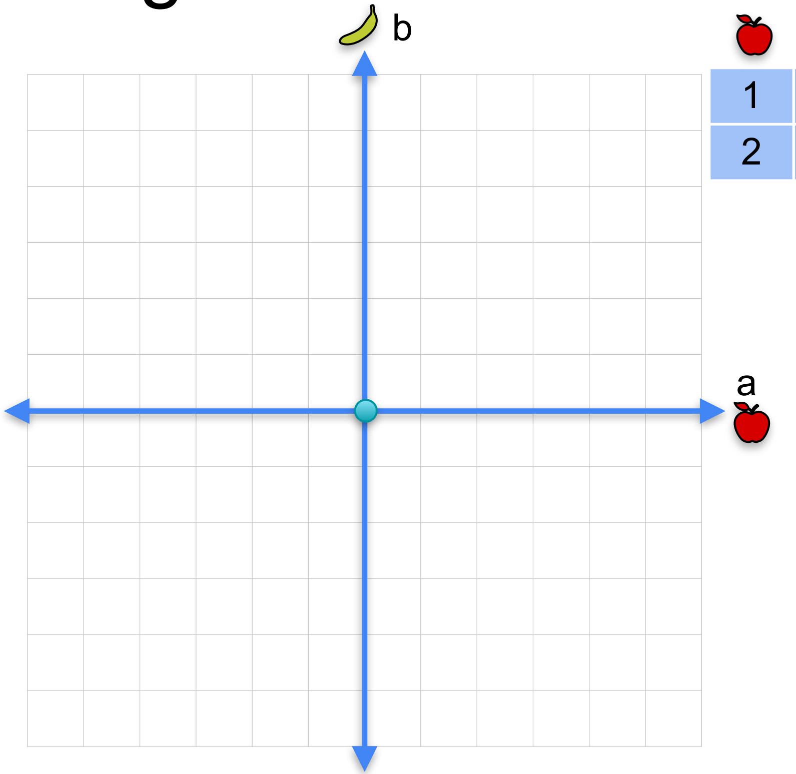
Singular transformation



🍎	🍌
1	1
2	2



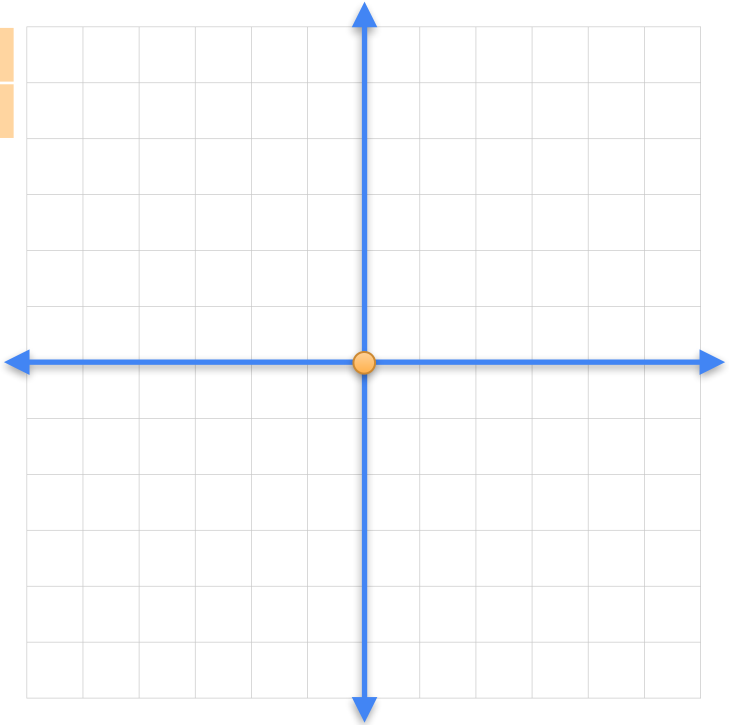
Singular transformation



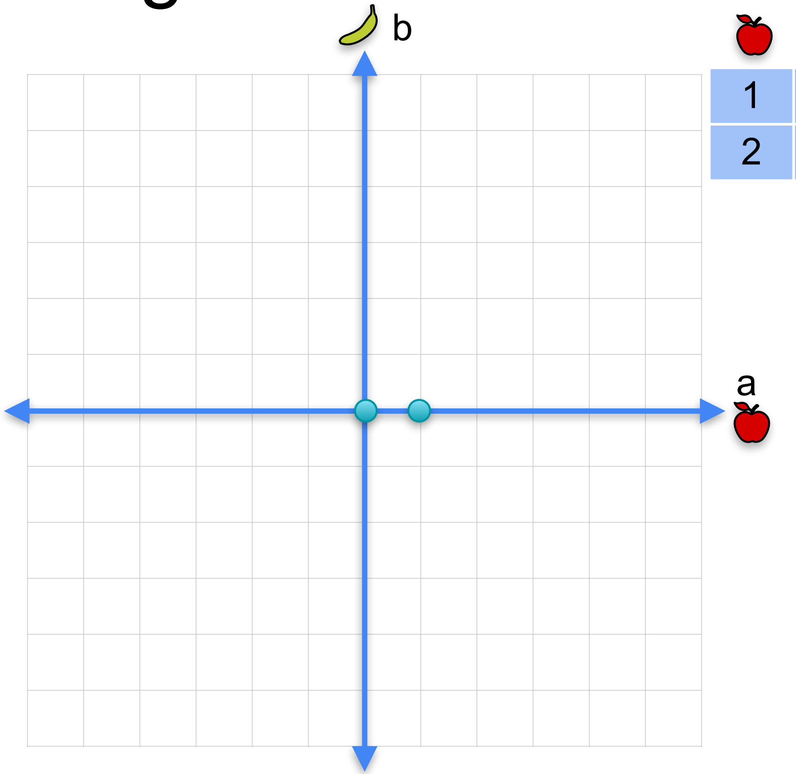
$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)



Singular transformation

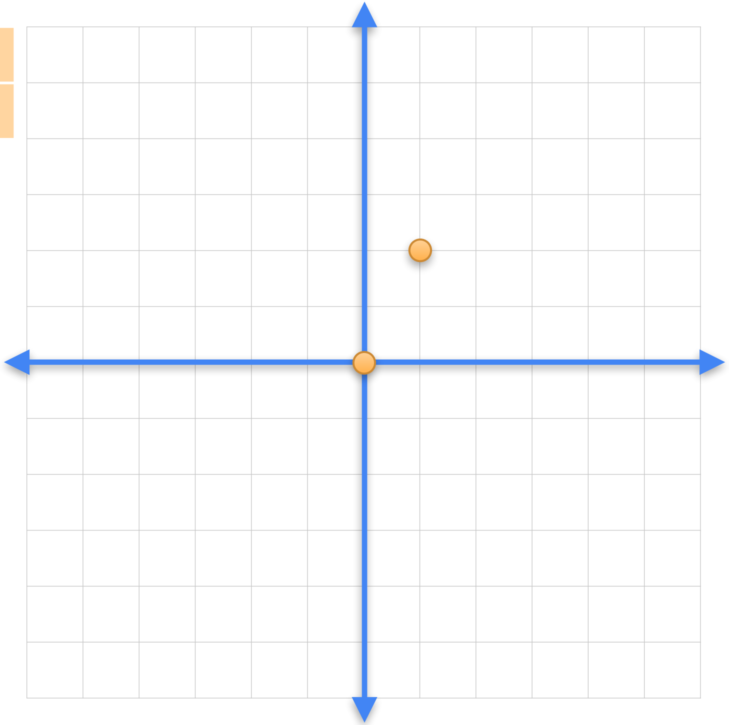


$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$

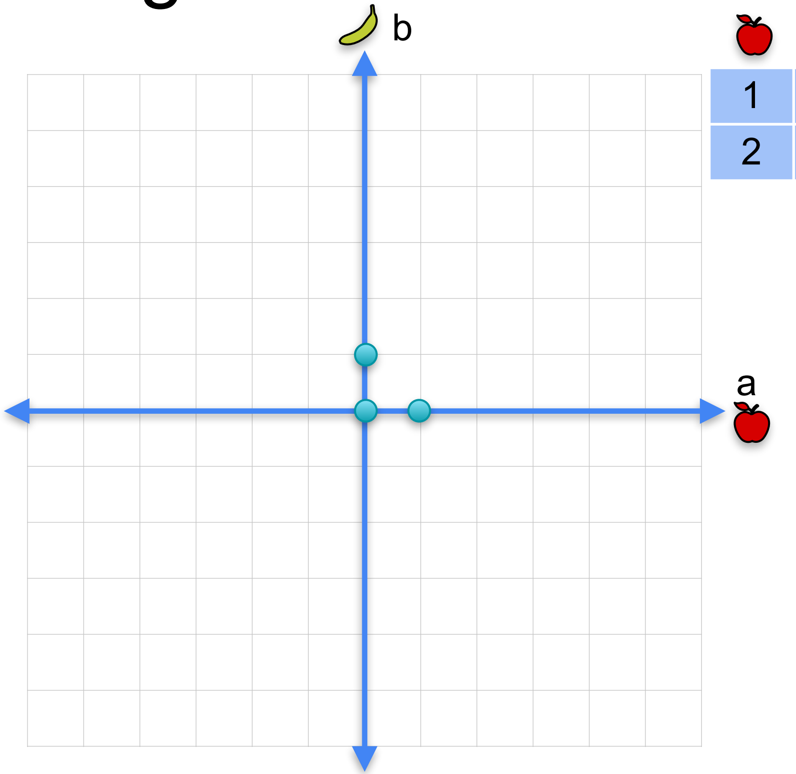
$$\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 2\end{bmatrix}$$

$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (1,2)$



Singular transformation

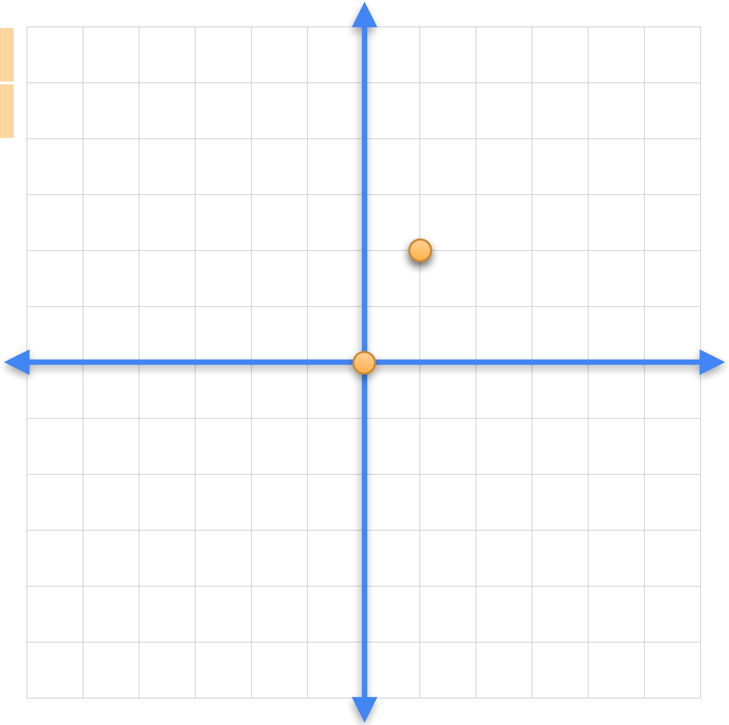


$$\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}$$

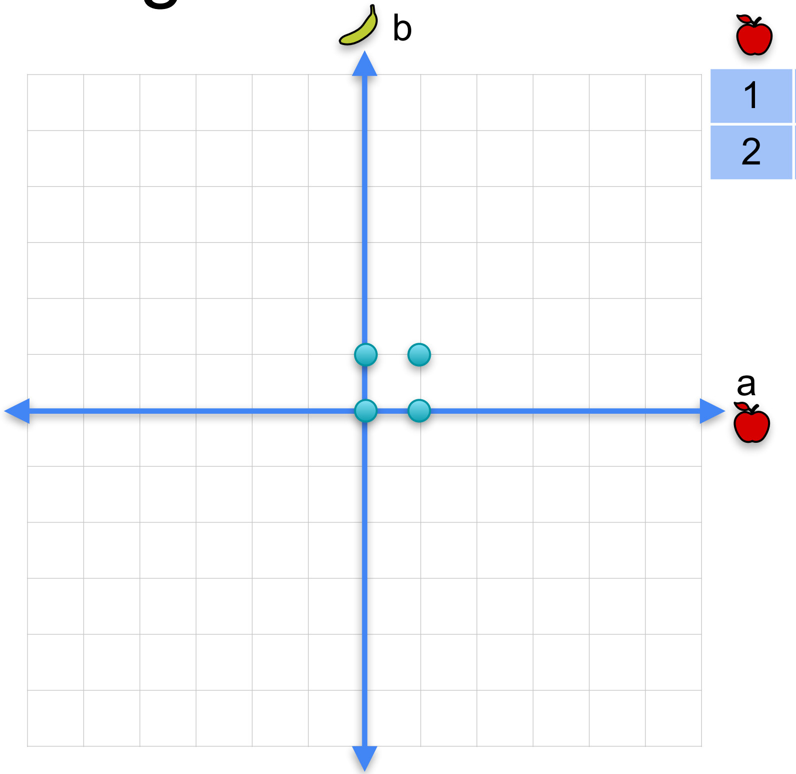
(0,0) → (0,0)

(1,0) → (1,2)

(0,1) → (1,2)



Singular transformation



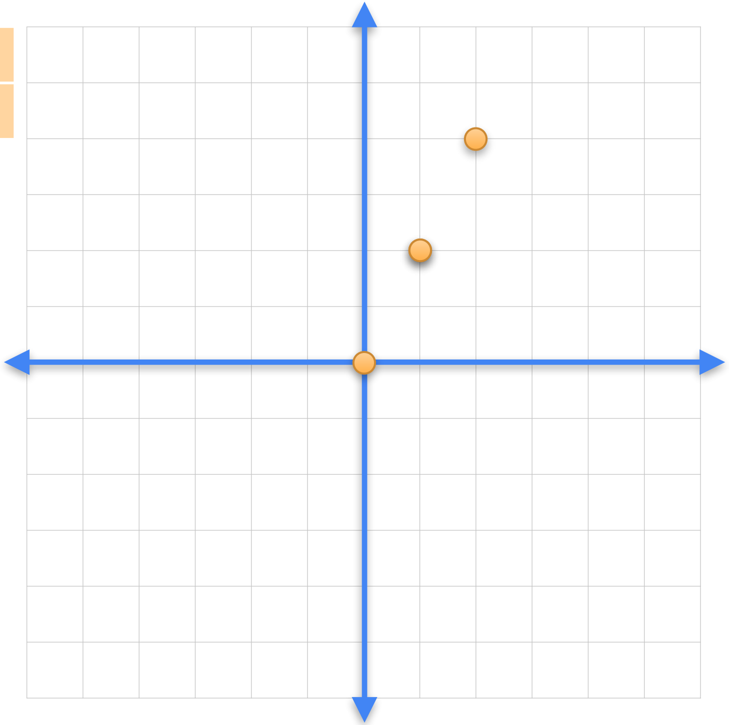
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

(0,0) → (0,0)

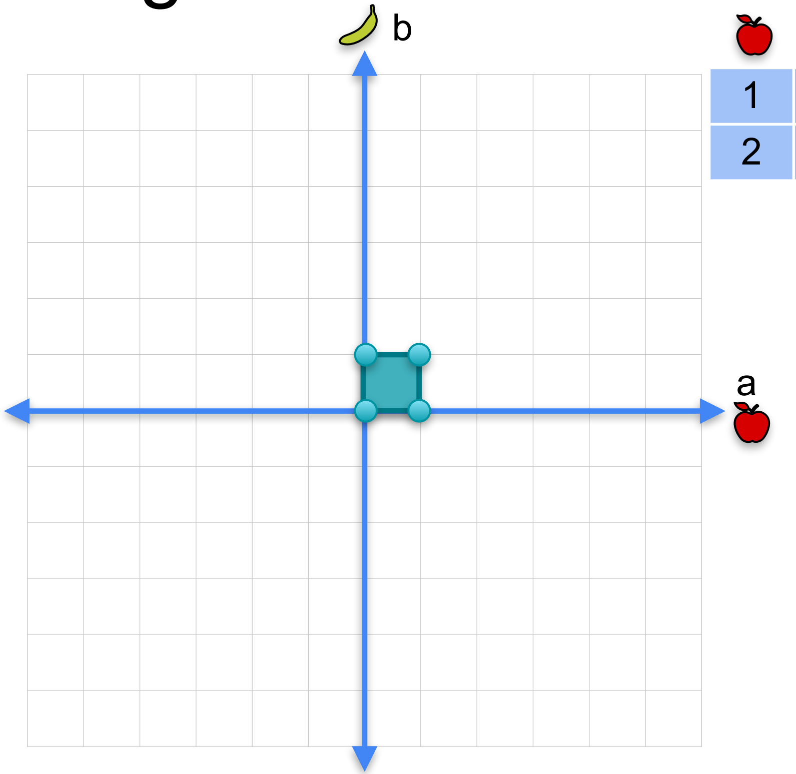
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)



Singular transformation



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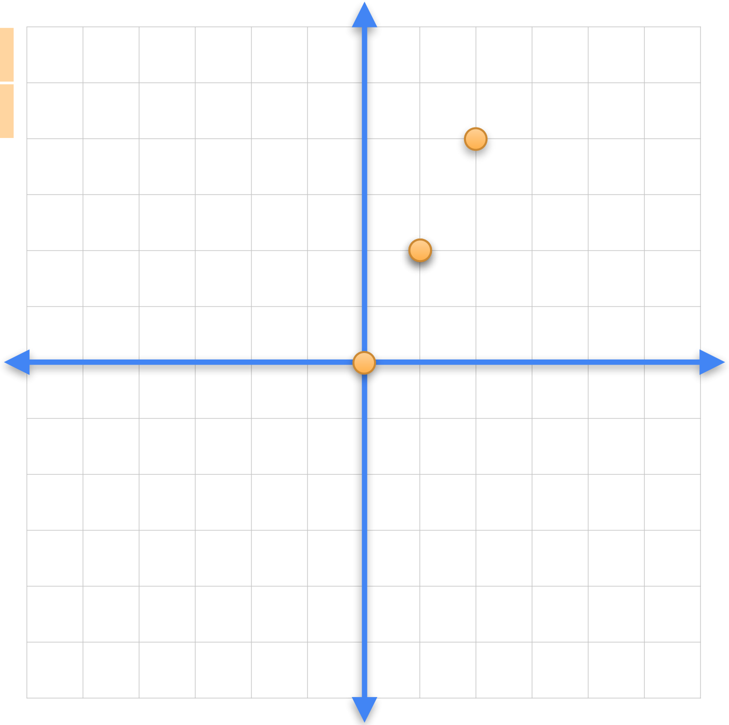
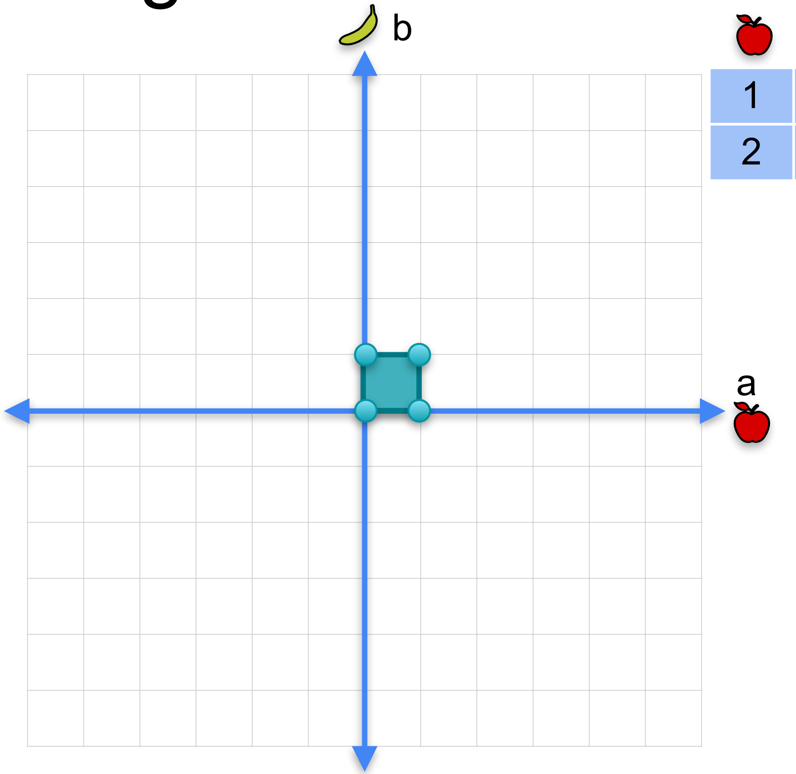
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)

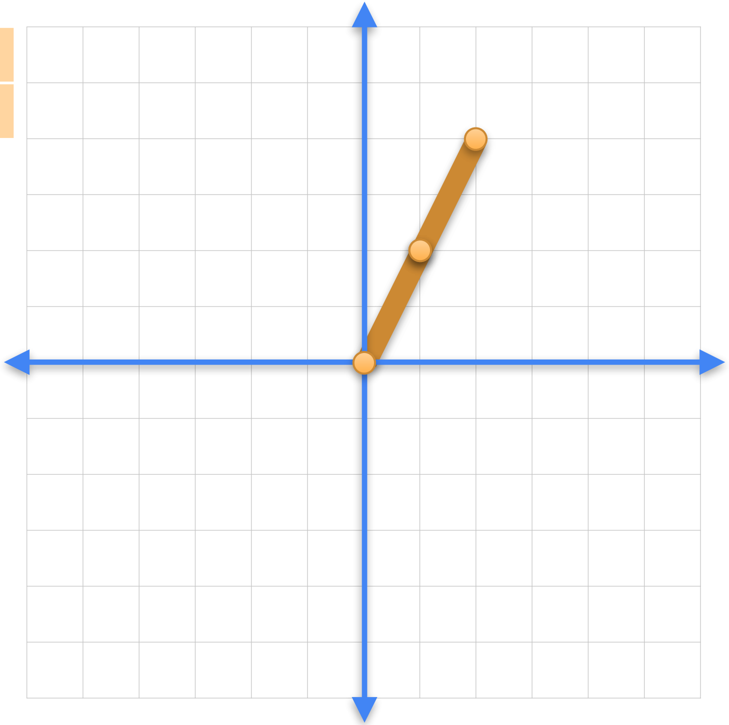


Singular transformation

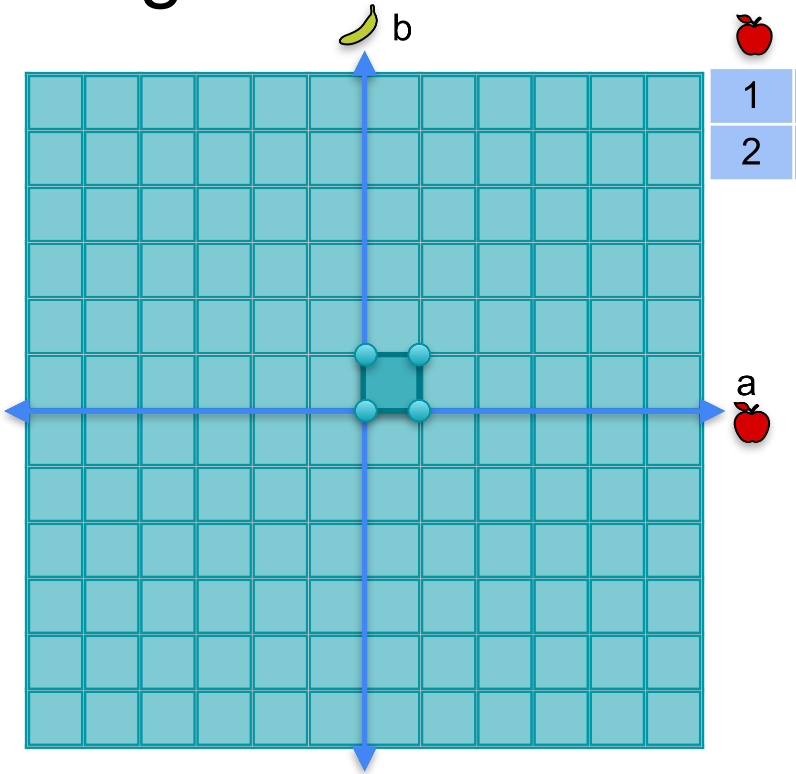


$$\begin{bmatrix}1 & 1\\ 2 & 2\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}2\\ 4\end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (1,2)
(0,1) → (1,2)
(1,1) → (2,4)



Singular transformation



$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

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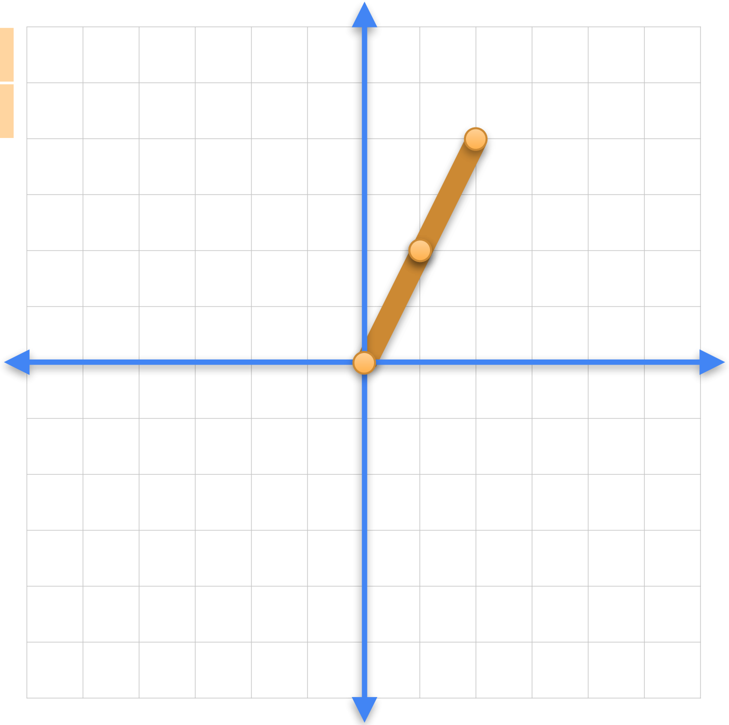
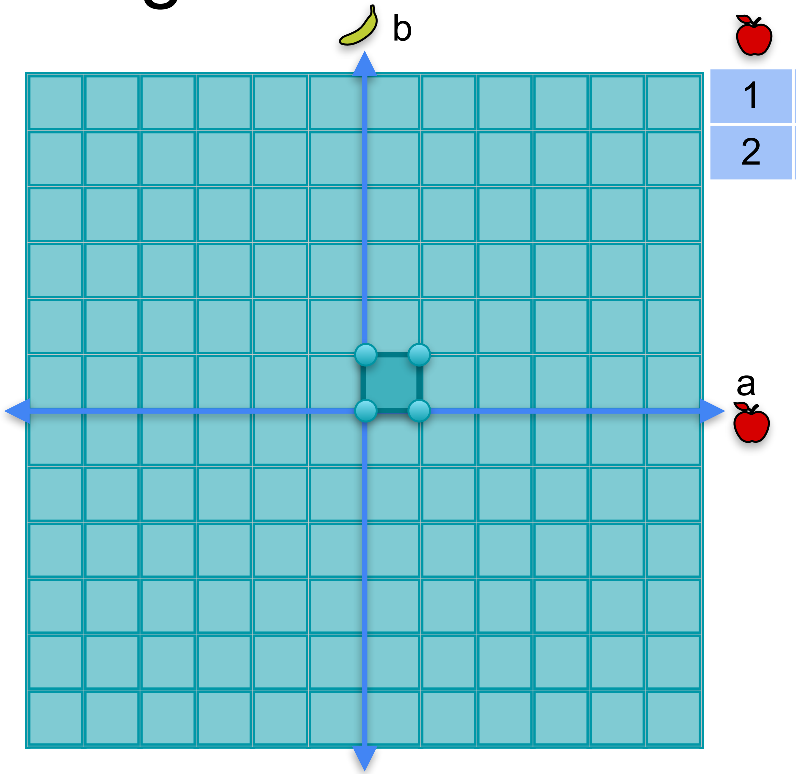


Singular transformation



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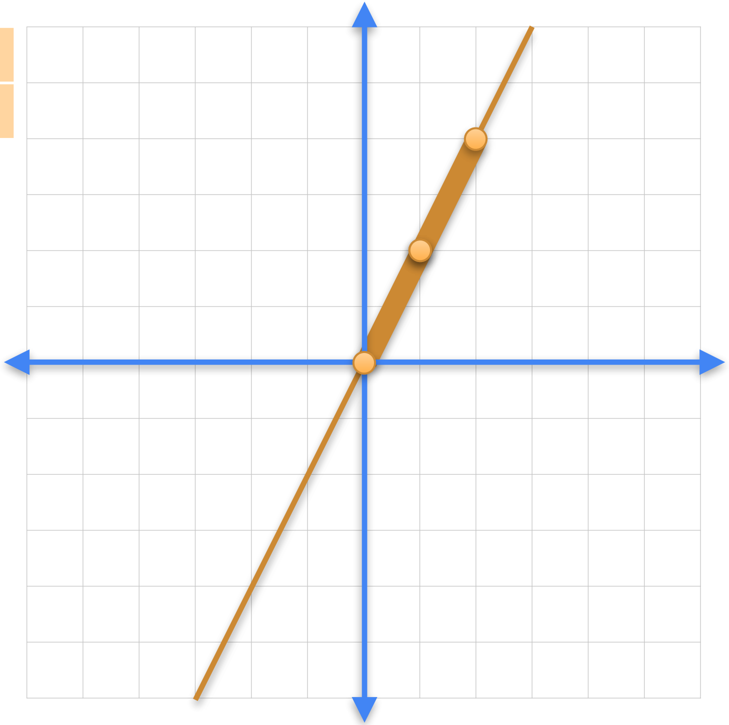
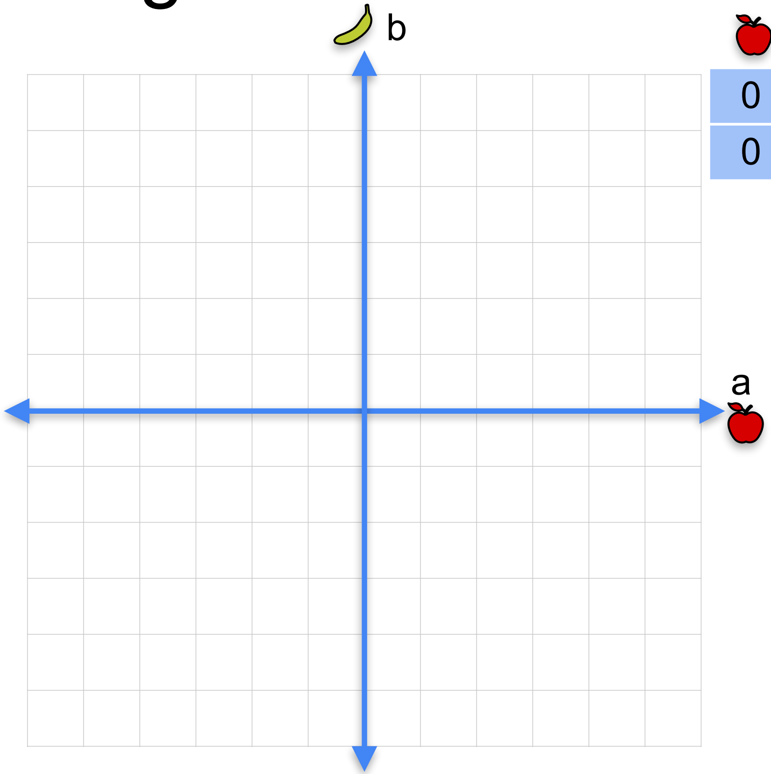
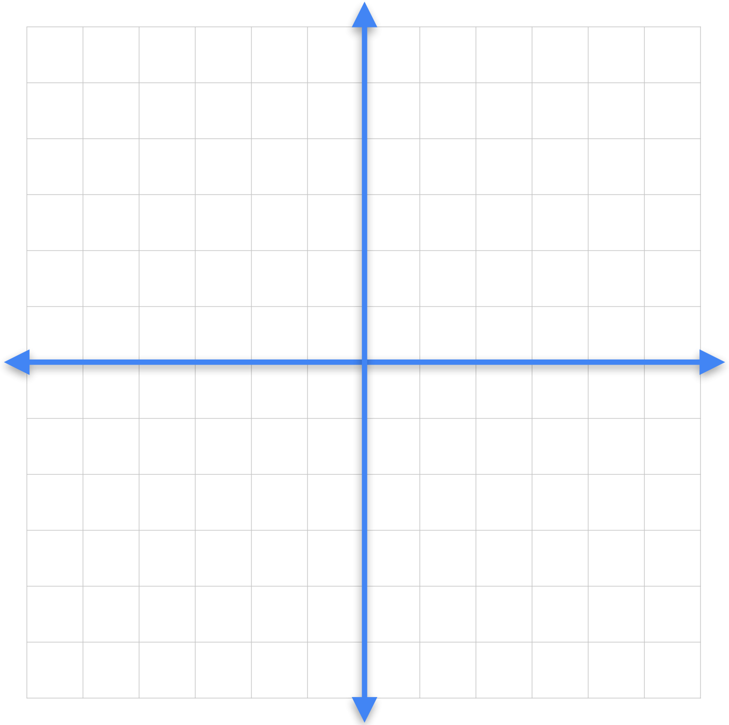


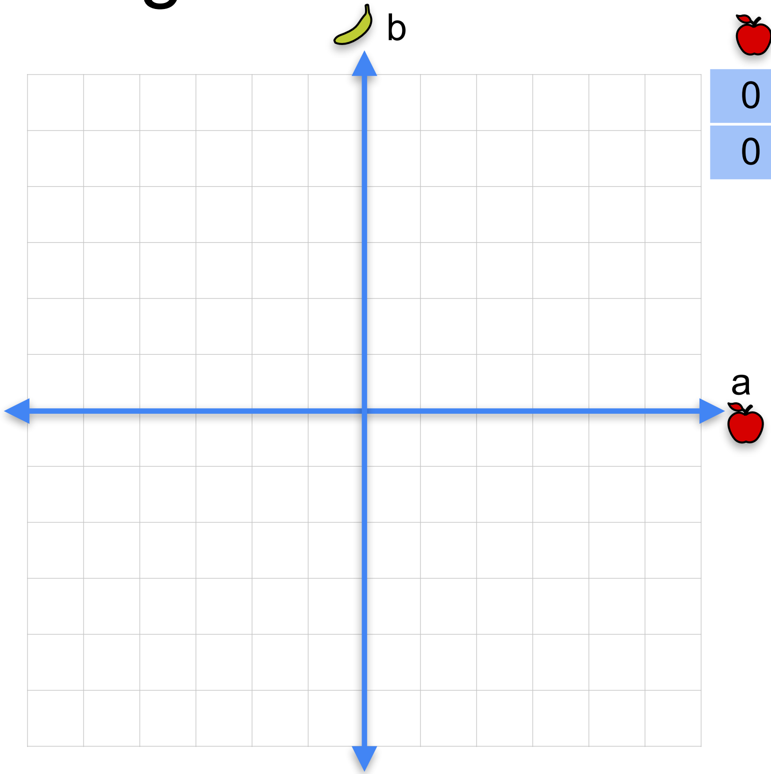
Singular transformation



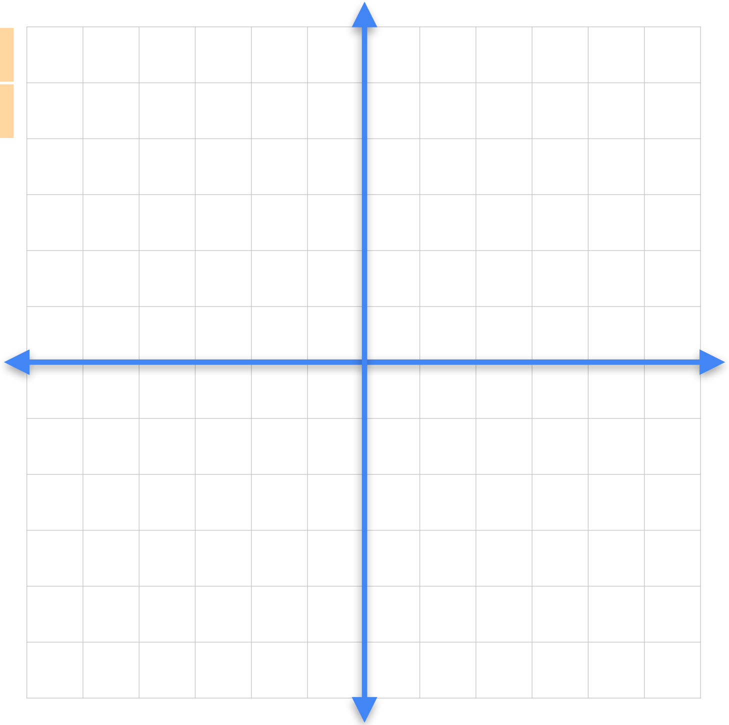
0	0
0	0



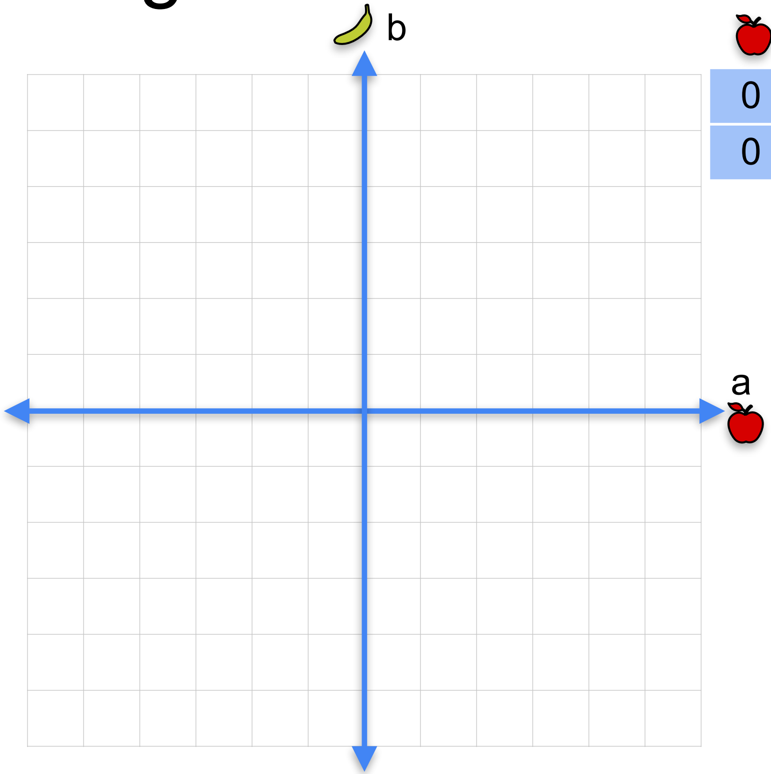
Singular transformation



$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Singular transformation



$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

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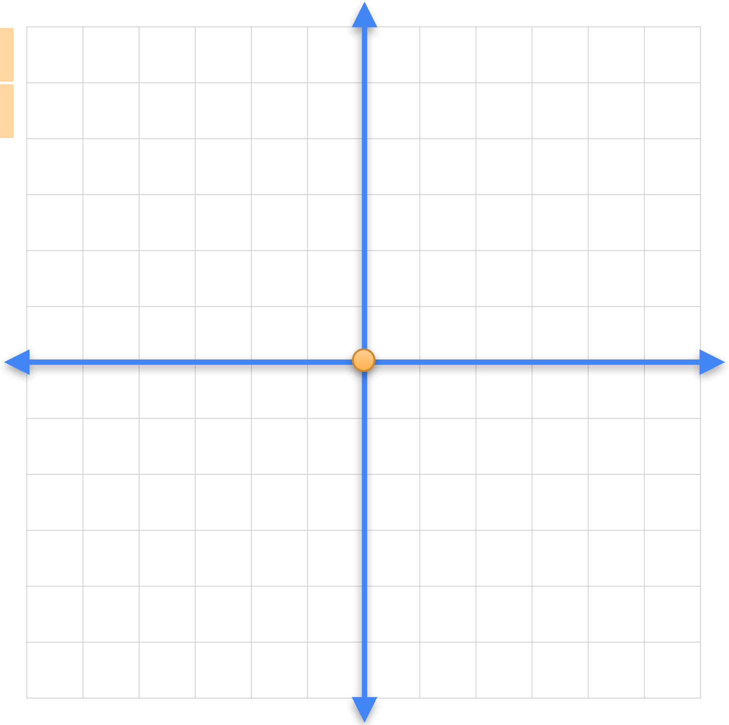
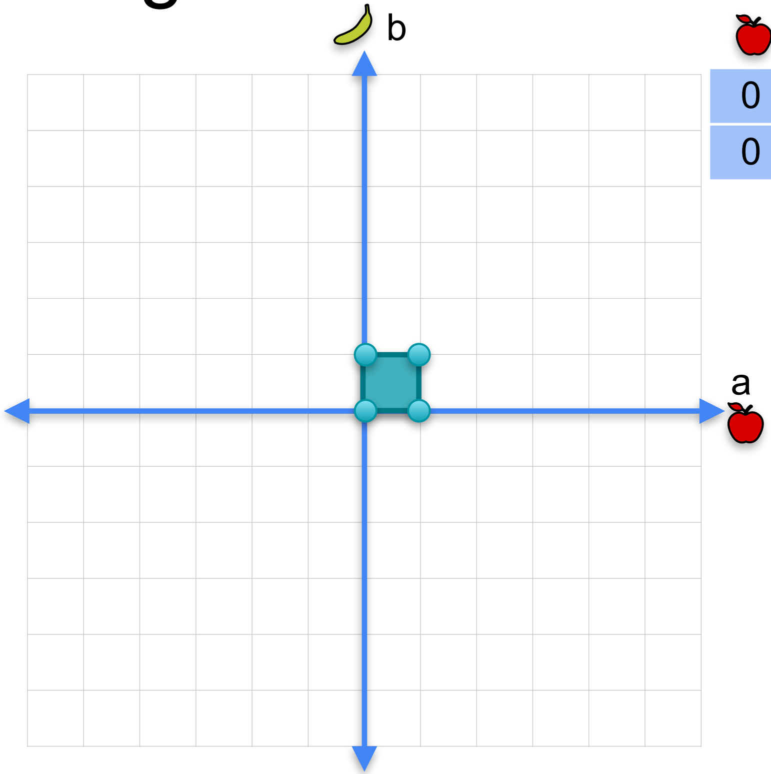


Singular transformation



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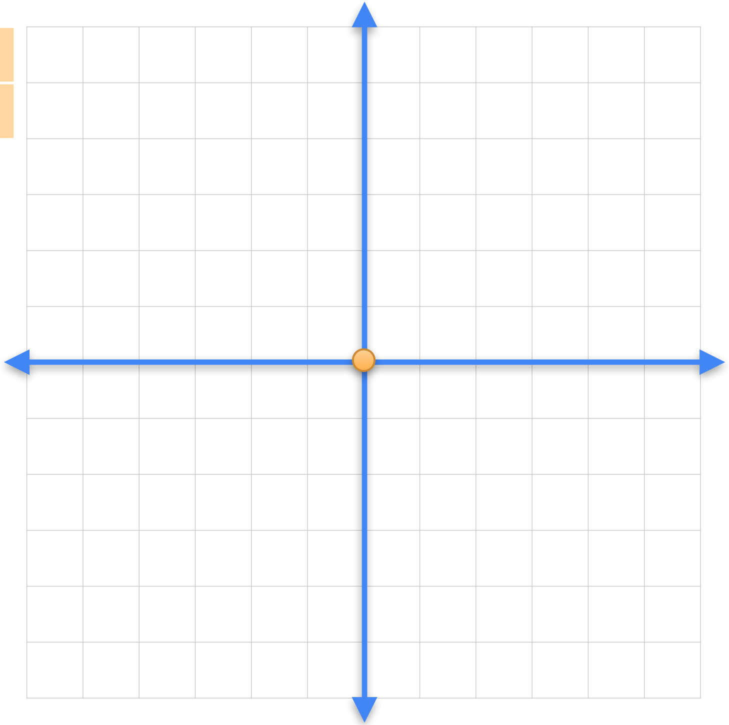
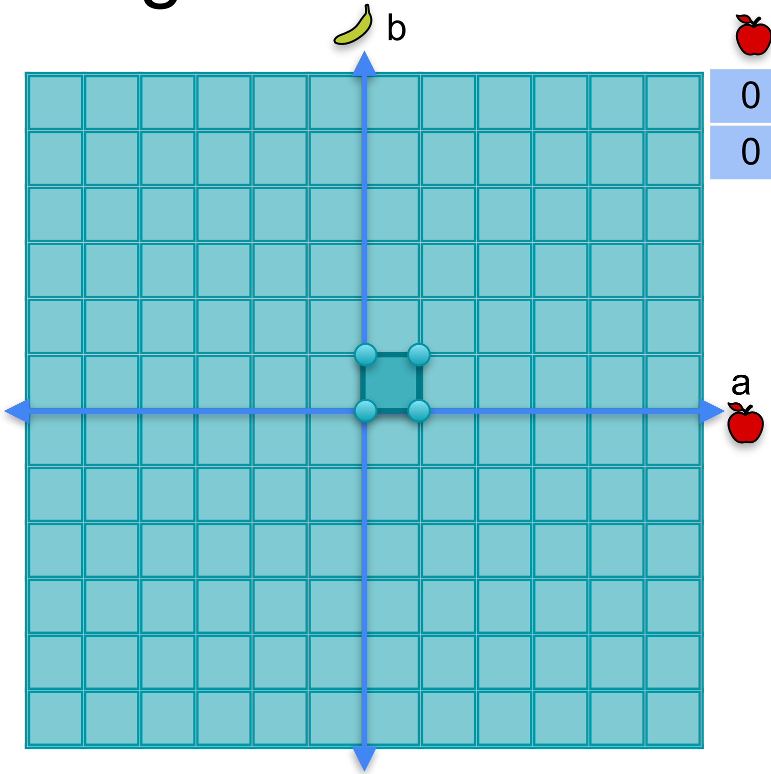


Singular transformation



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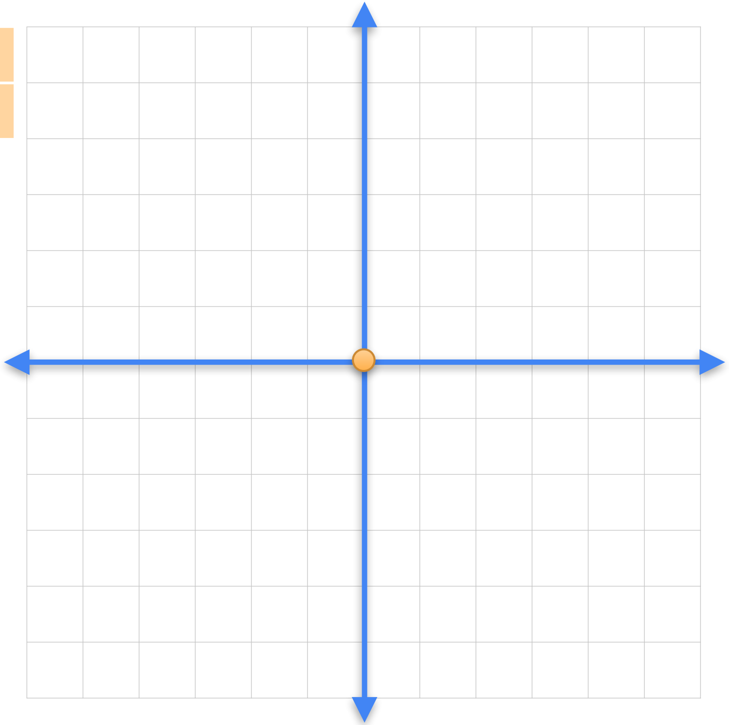
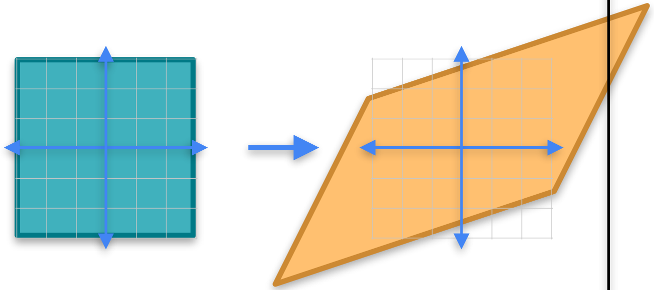


Singular and non-singular transformations

Singular and non-singular transformations

Non-singular

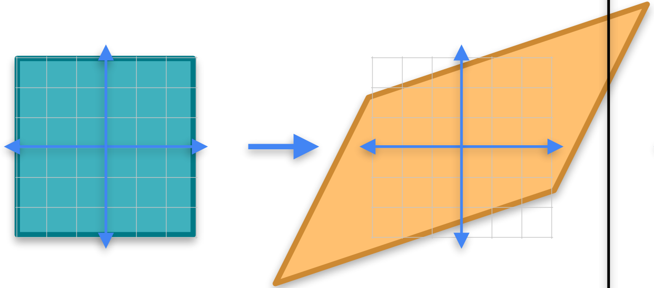
3	1
1	2



Singular and non-singular transformations

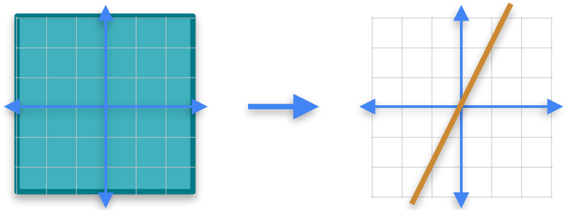
Non-singular

3	1
1	2



Singular

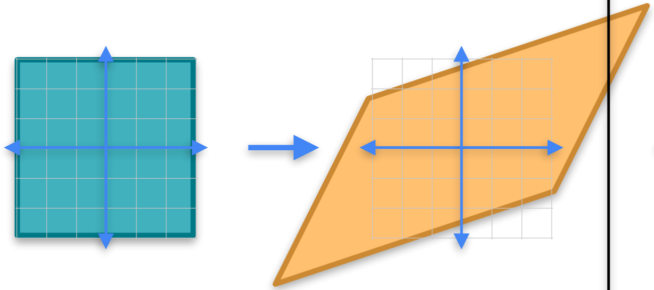
1	1
2	2



Singular and non-singular transformations

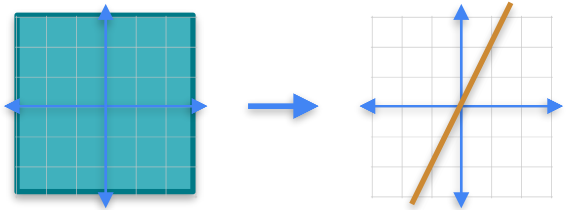
Non-singular

3	1
1	2



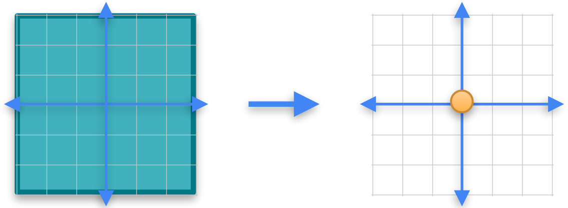
Singular

1	1
2	2



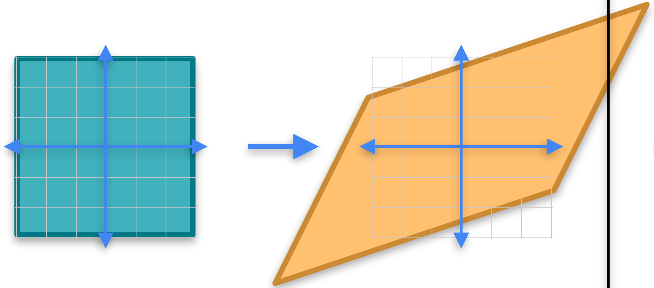
Singular

0	0
0	0

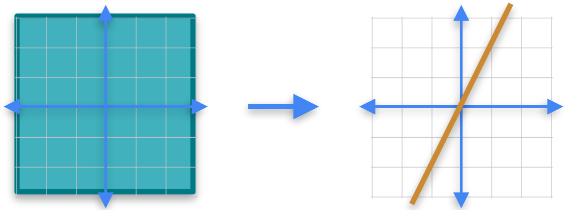


Rank of linear transformations

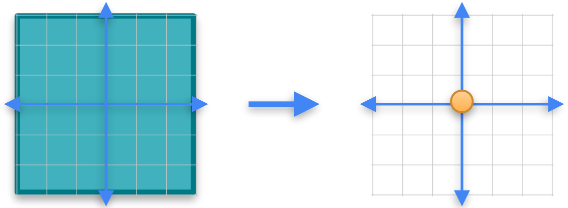
3	1
1	2



1	1
2	2

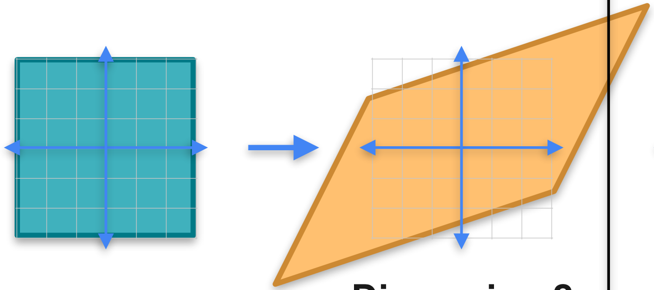


0	0
0	0



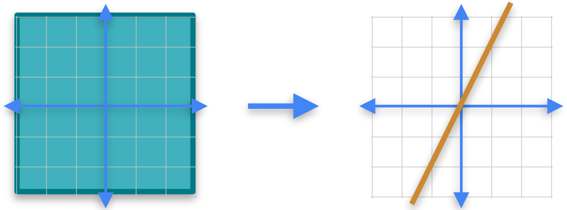
Rank of linear transformations

3	1
1	2

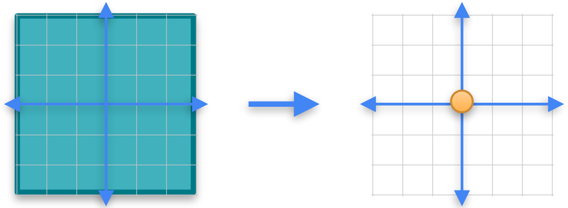


Dimension 2

1	1
2	2



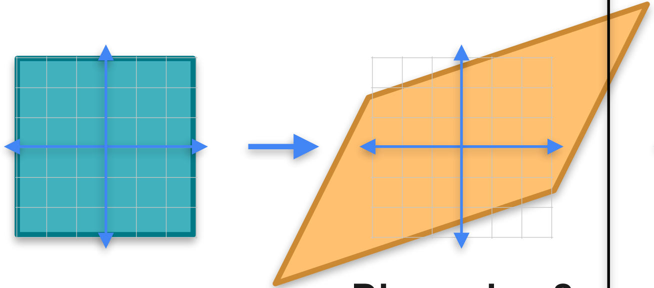
0	0
0	0



Rank of linear transformations

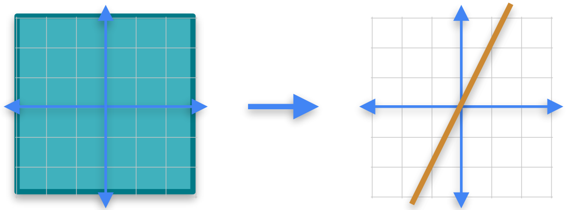
Rank 2

3	1
1	2

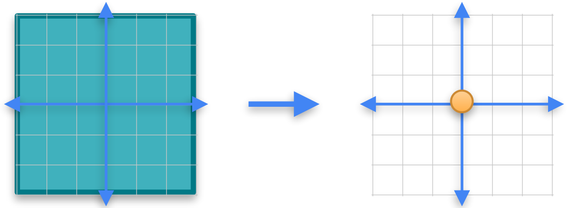


Dimension 2

1	1
2	2



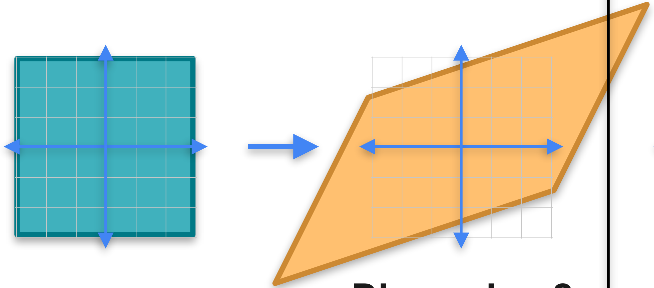
0	0
0	0



Rank of linear transformations

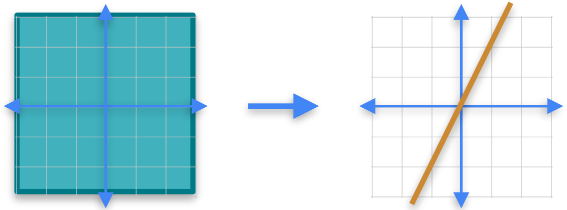
Rank 2

3	1
1	2



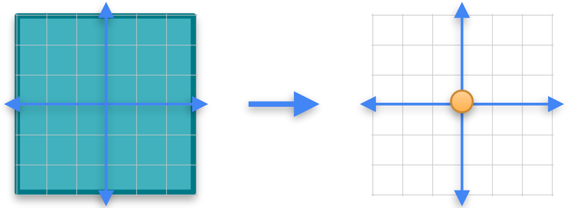
Dimension 2

1	1
2	2



Dimension 1

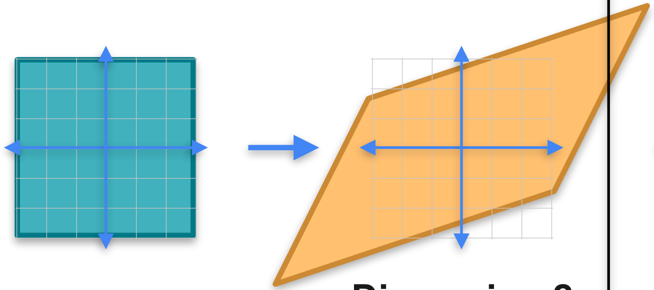
0	0
0	0



Rank of linear transformations

Rank 2

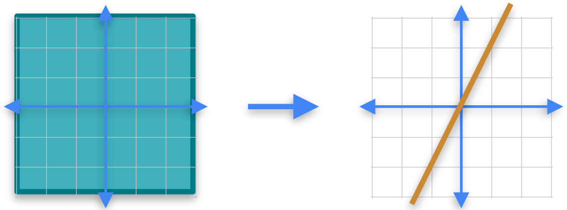
3	1
1	2



Dimension 2

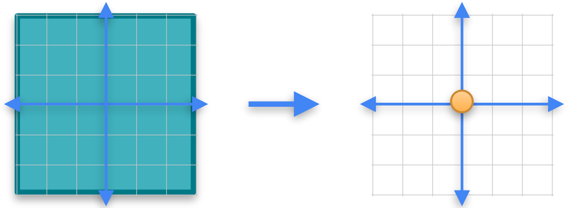
Rank 1

1	1
2	2



Dimension 1

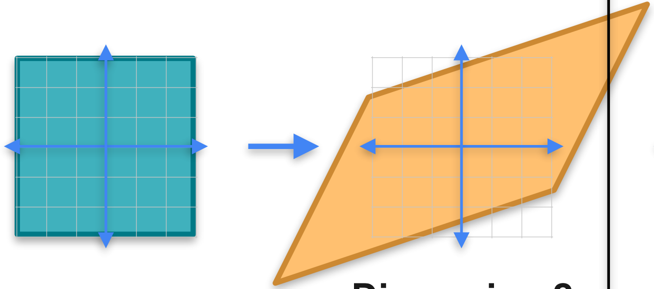
0	0
0	0



Rank of linear transformations

Rank 2

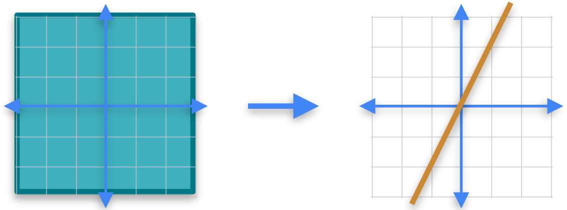
3	1
1	2



Dimension 2

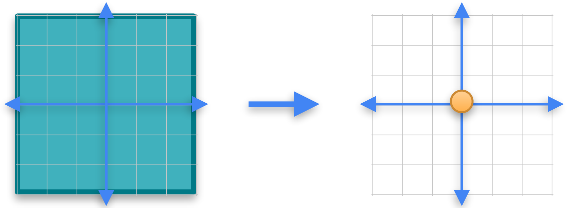
Rank 1

1	1
2	2



Dimension 1

0	0
0	0

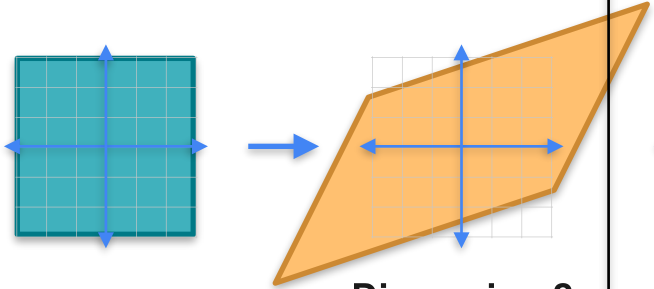


Dimension 0

Rank of linear transformations

Rank 2

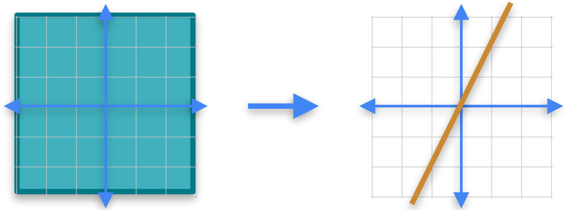
3	1
1	2



Dimension 2

Rank 1

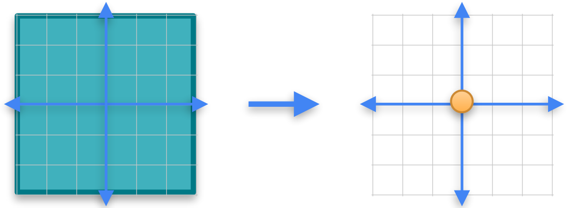
1	1
2	2



Dimension 1

Rank 0

0	0
0	0



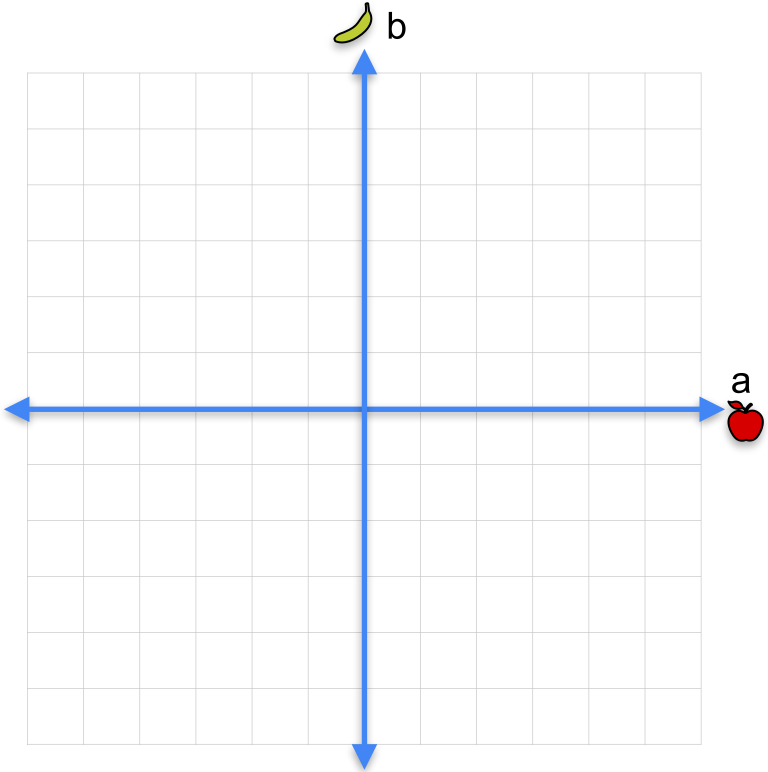
Dimension 0

DeepLearning.AI

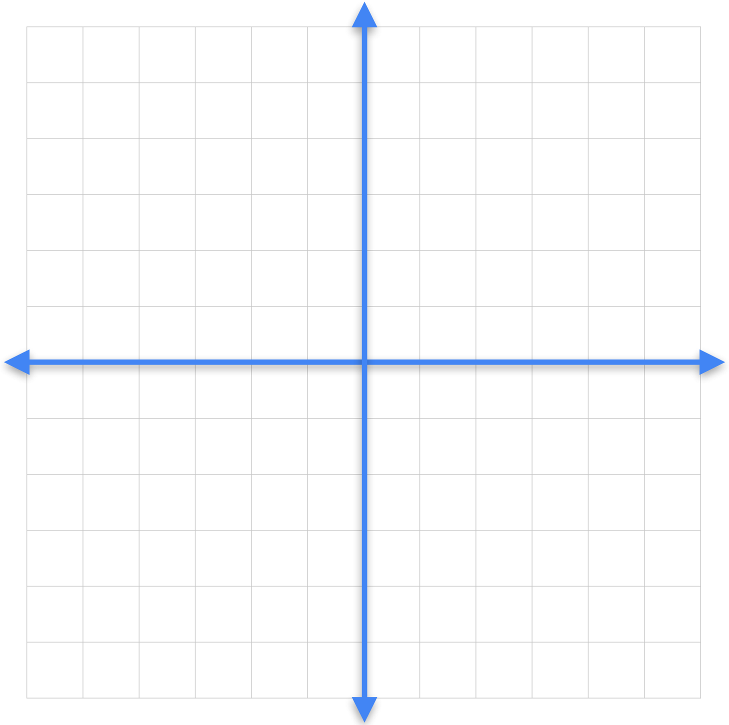
Determinants and Eigenvectors

Determinant as an area

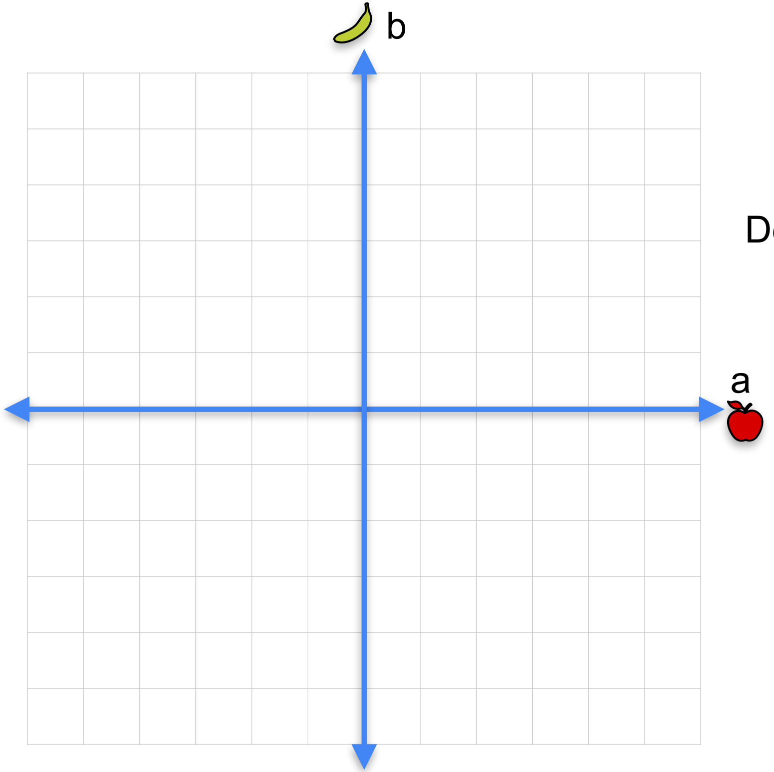
Determinant as an area



3	1
1	2



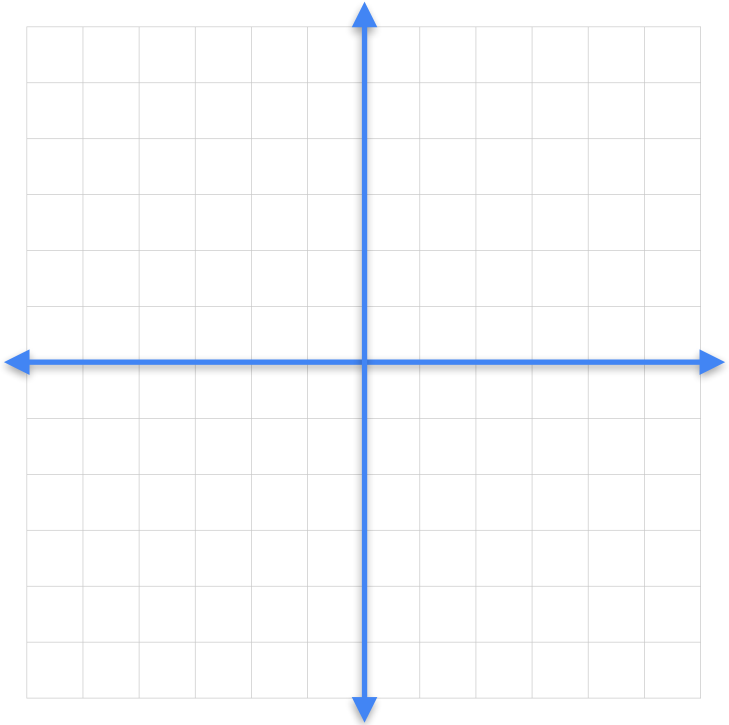
Determinant as an area



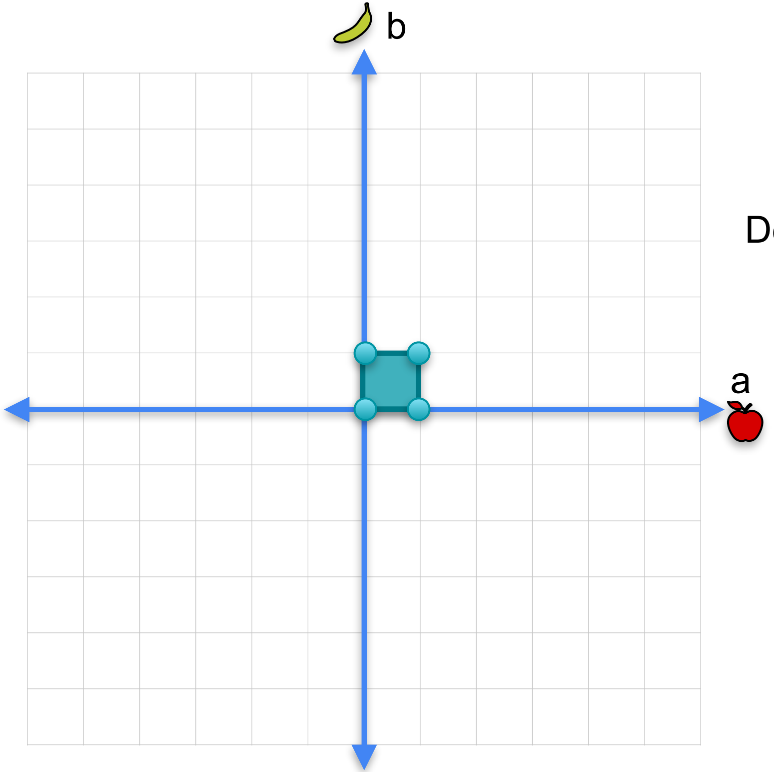
3	1
1	2

Det = $3 \cdot 2 - 1 \cdot 1$

Det = 5



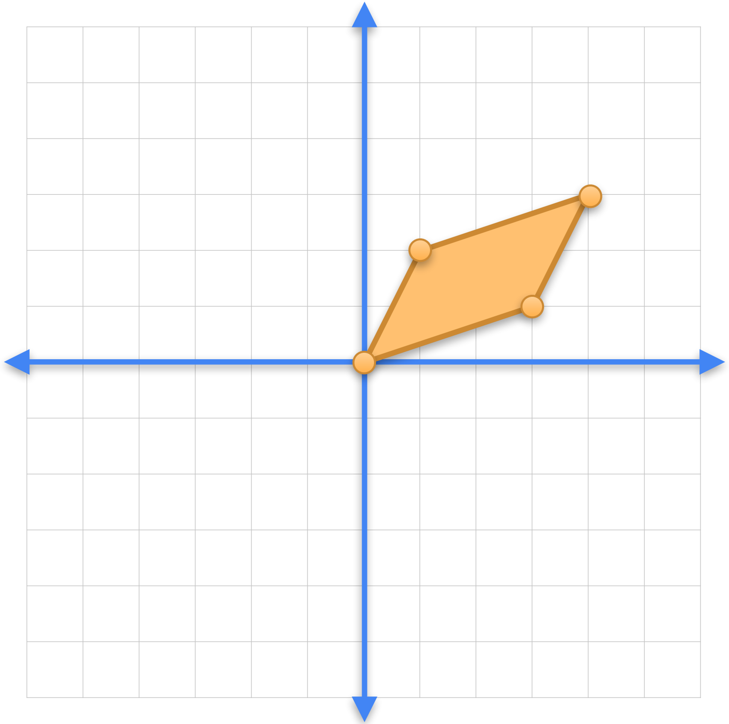
Determinant as an area



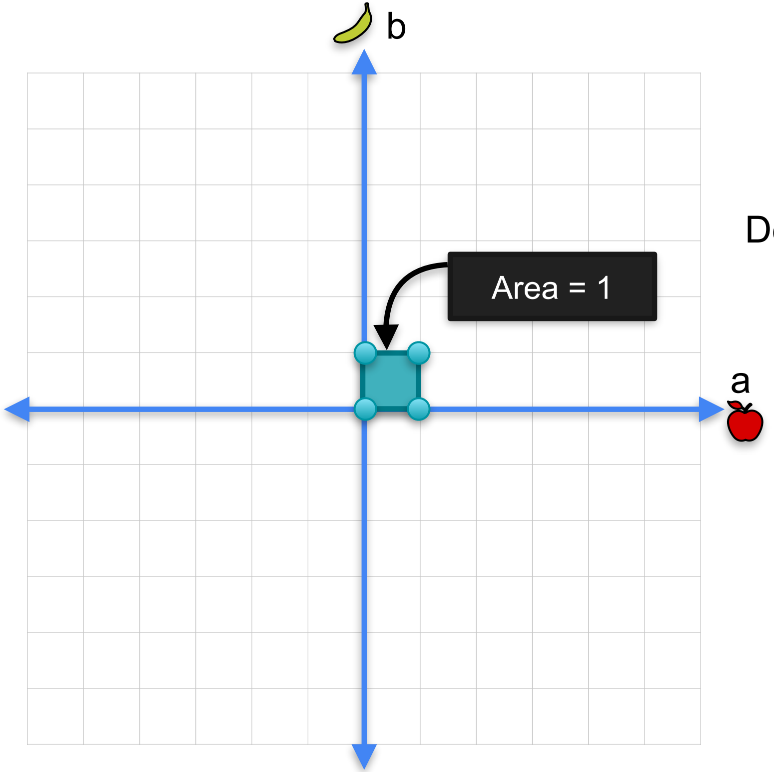
3	1
1	2

Det = $3 \cdot 2 - 1 \cdot 1$

Det = 5



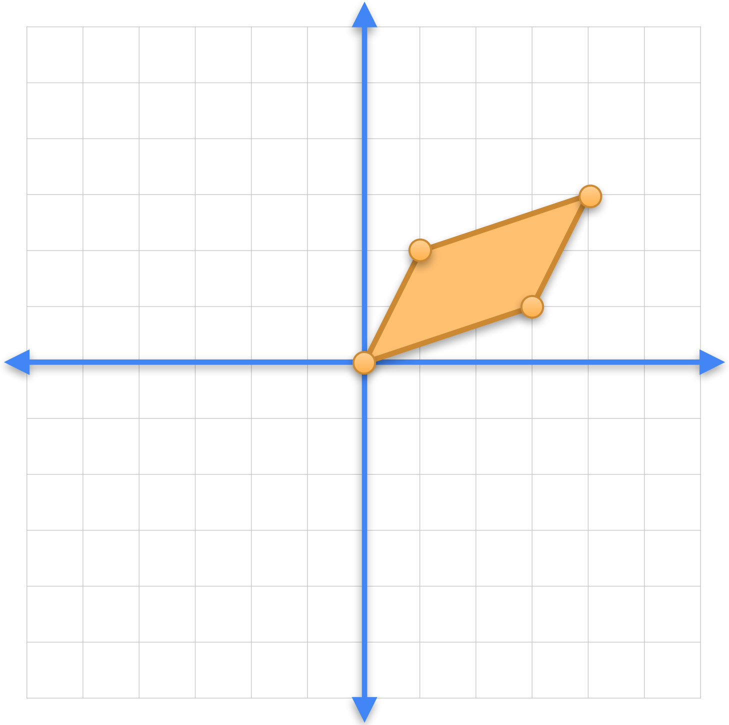
Determinant as an area



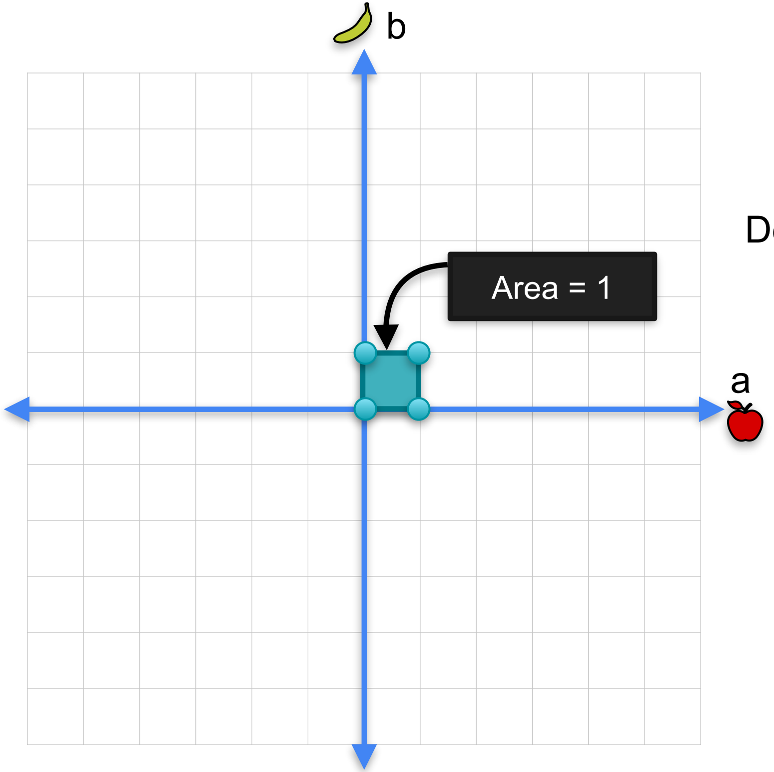
3	1
1	2

Det = $3 \cdot 2 - 1 \cdot 1$

Det = 5



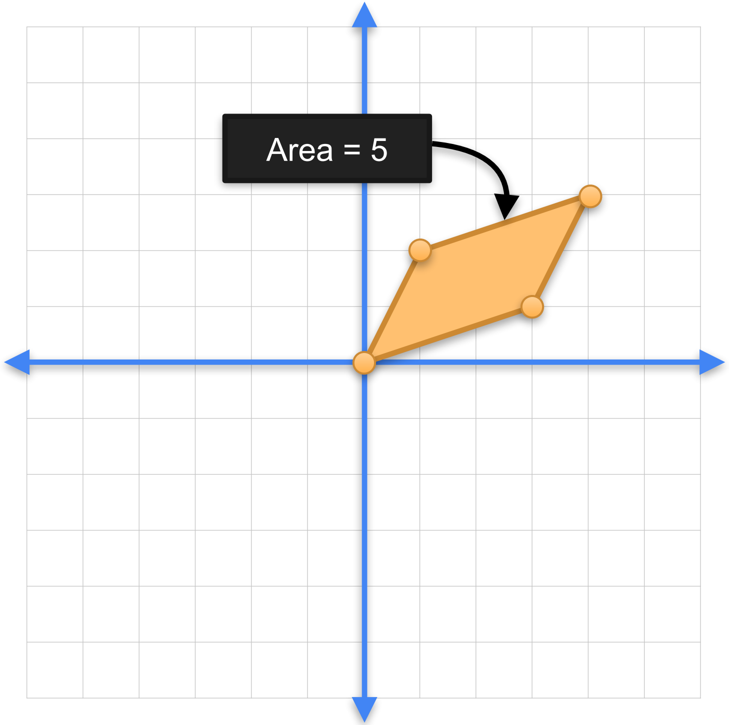
Determinant as an area



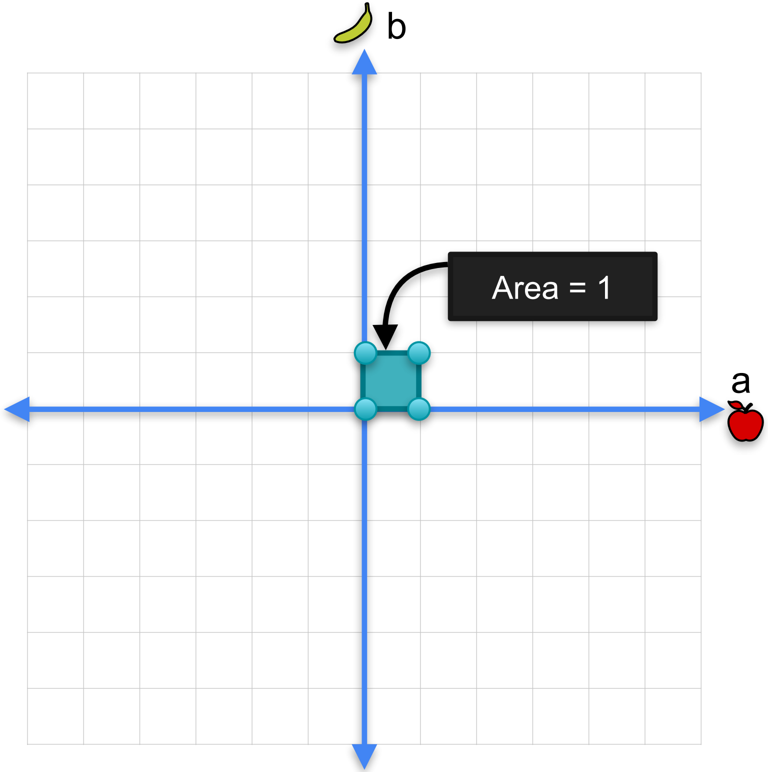
3	1
1	2

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

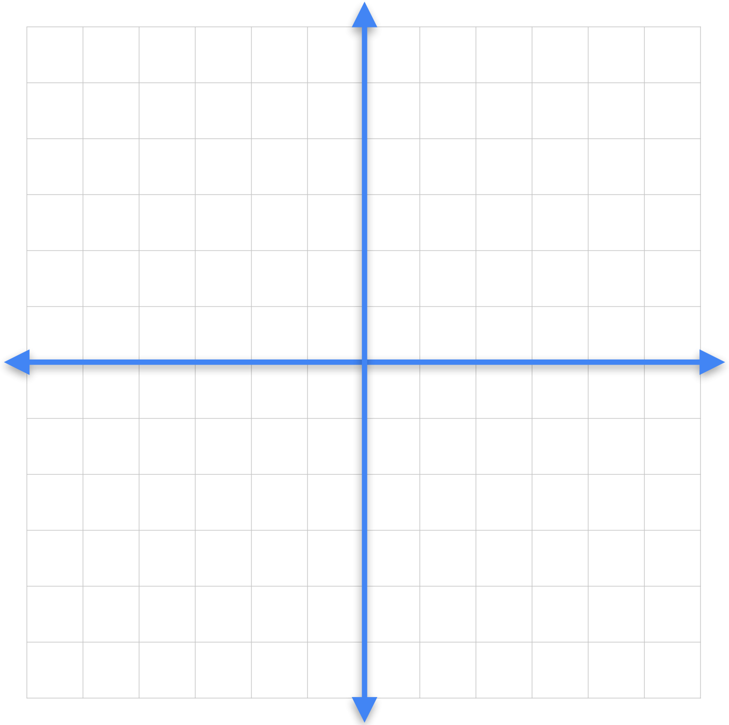
$$\text{Det} = 5$$



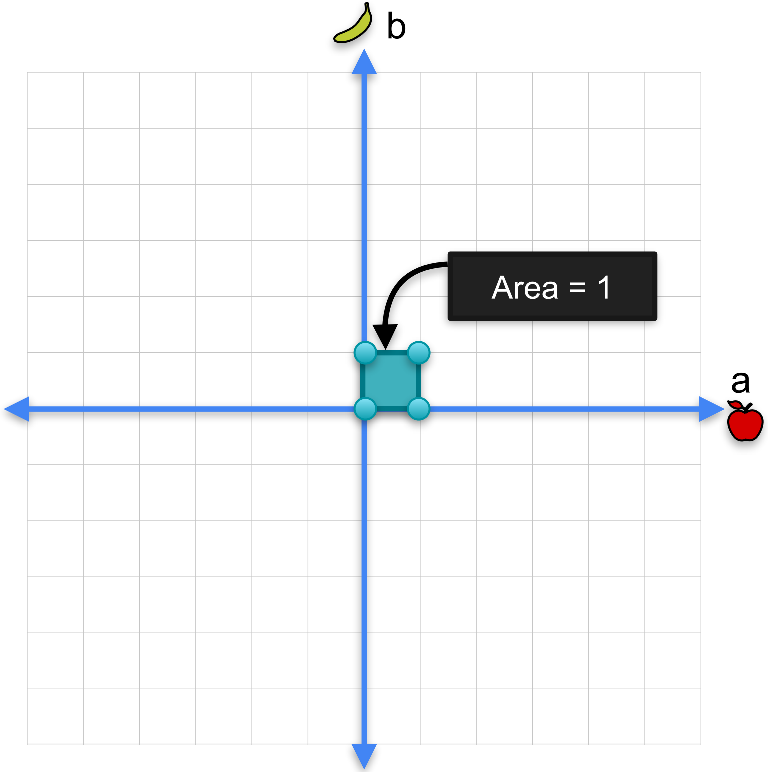
Determinant as an area



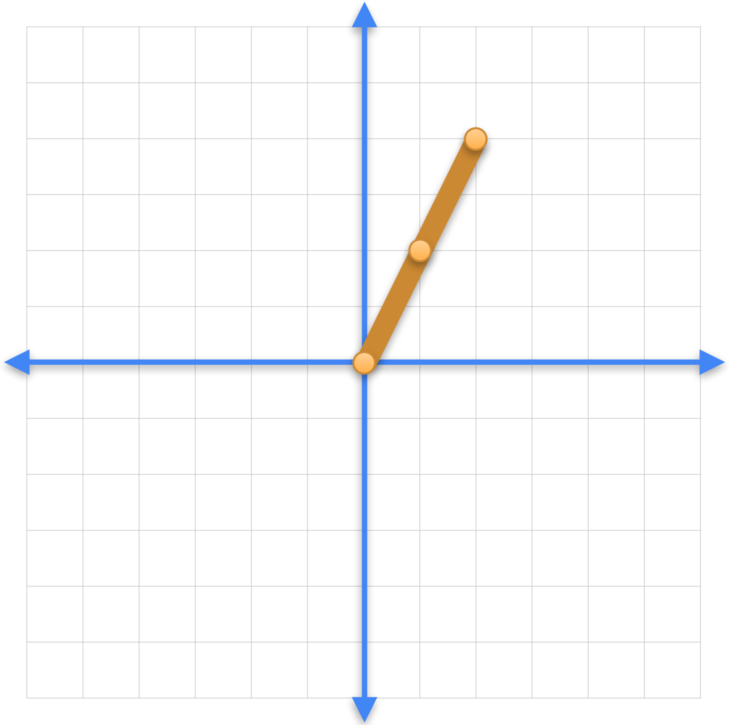
🍎	🍌
1	1
2	2



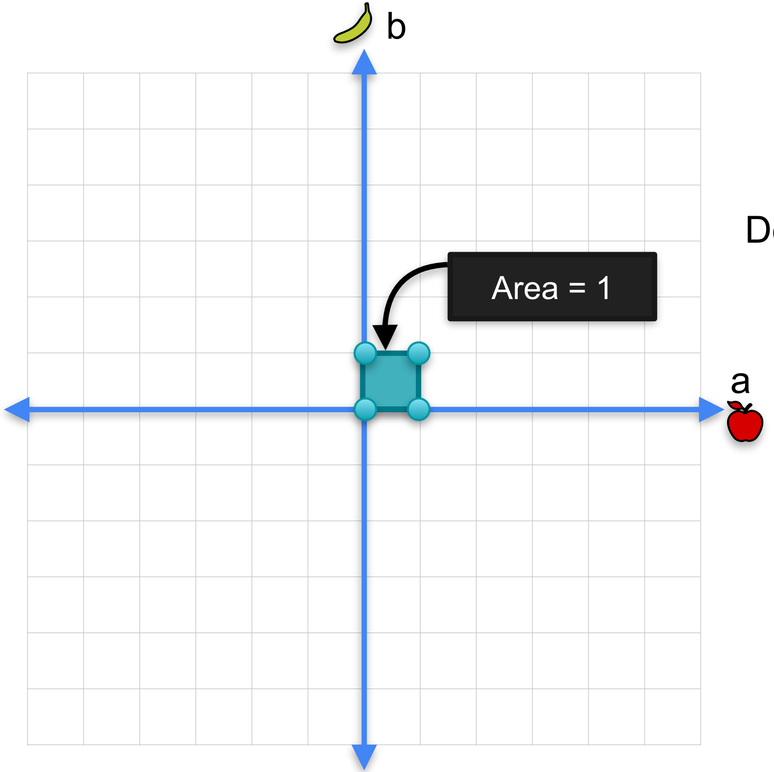
Determinant as an area



1	1
2	2



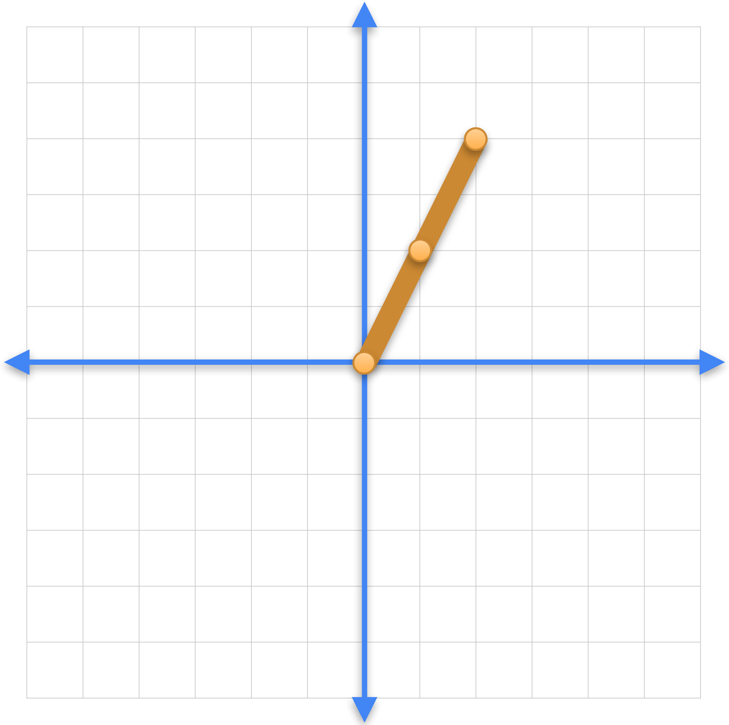
Determinant as an area



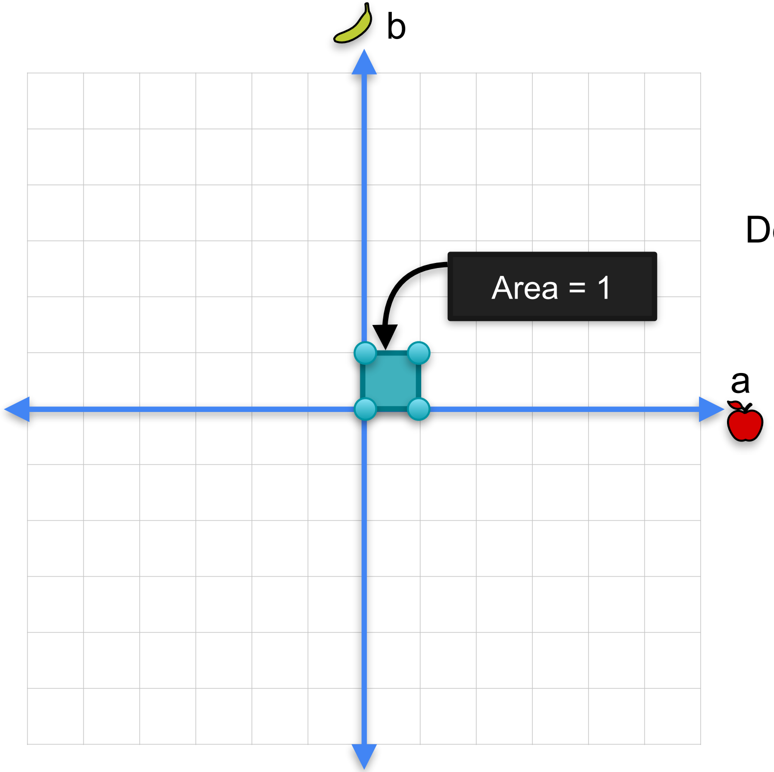
🍎	🍌
1	1
2	2

Det = $1 \cdot 2 - 1 \cdot 2$

Det = 0



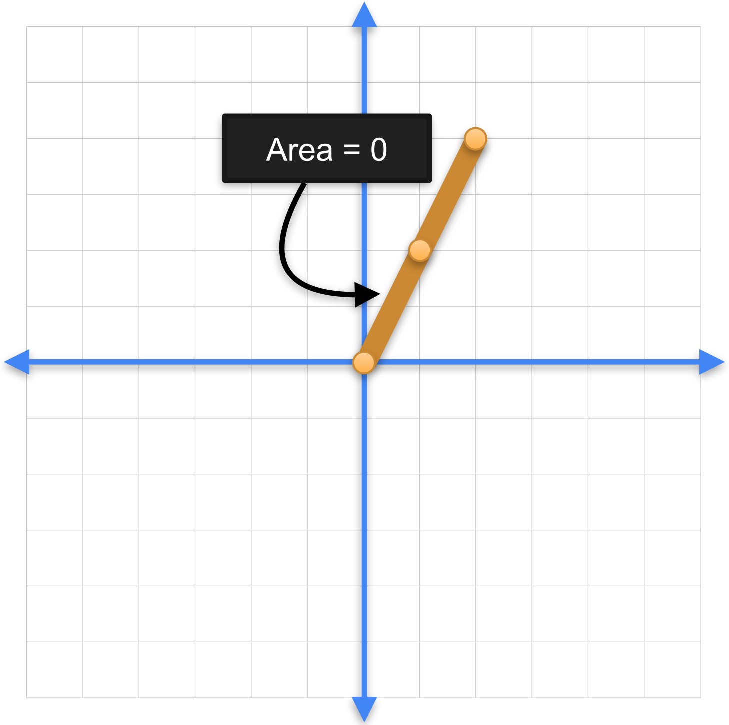
Determinant as an area



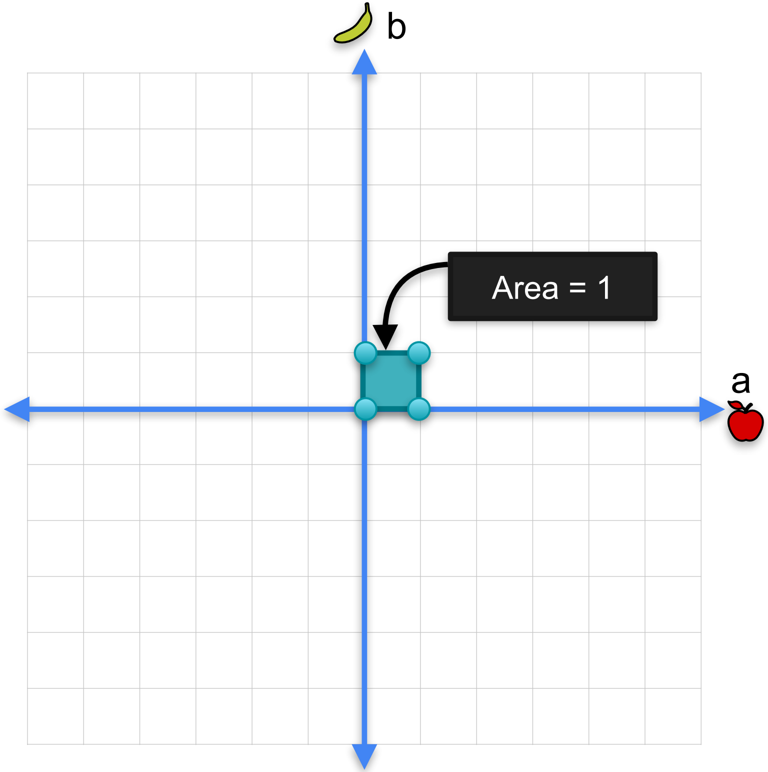
🍎	🍌
1	1
2	2

Det = $1 \cdot 2 - 1 \cdot 2$

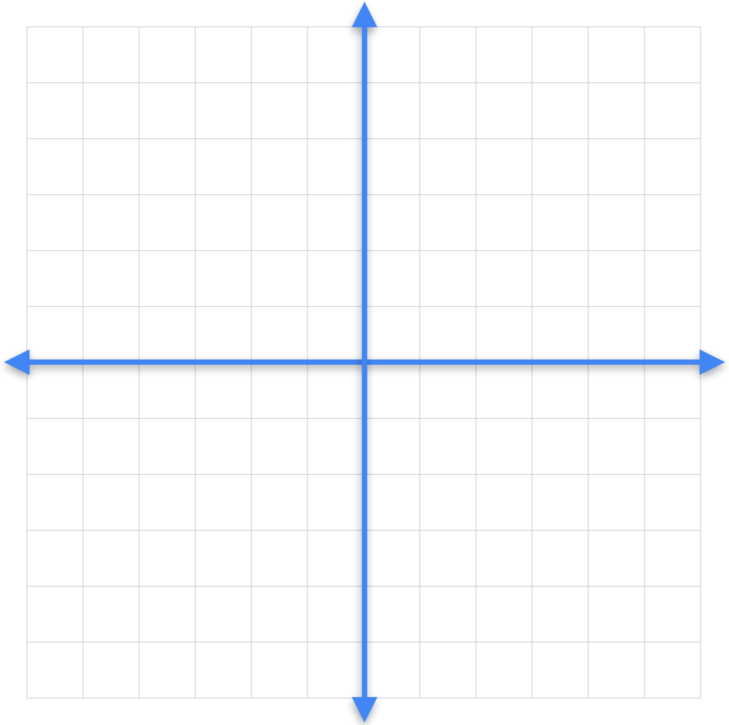
Det = 0



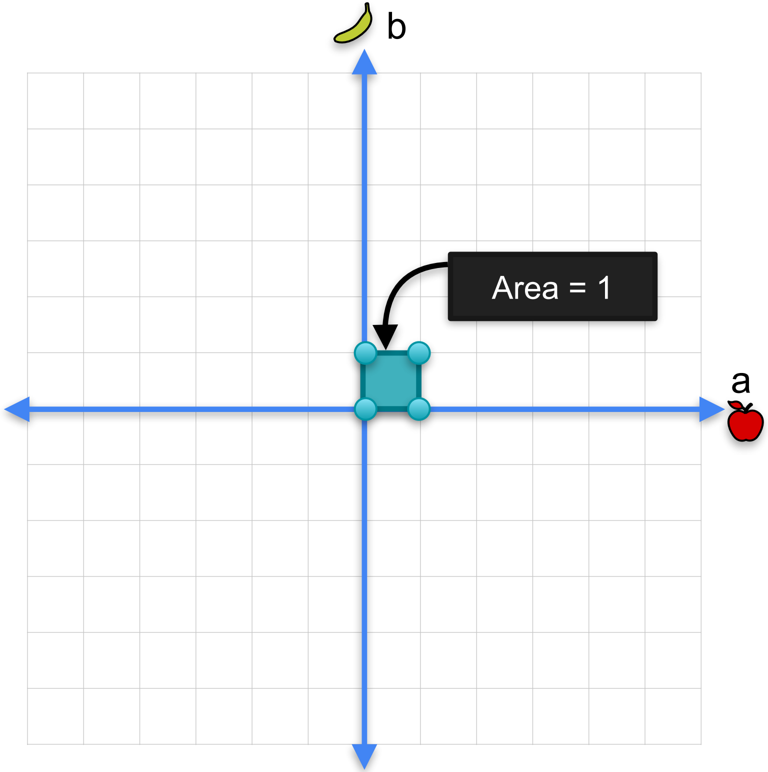
Determinant as an area



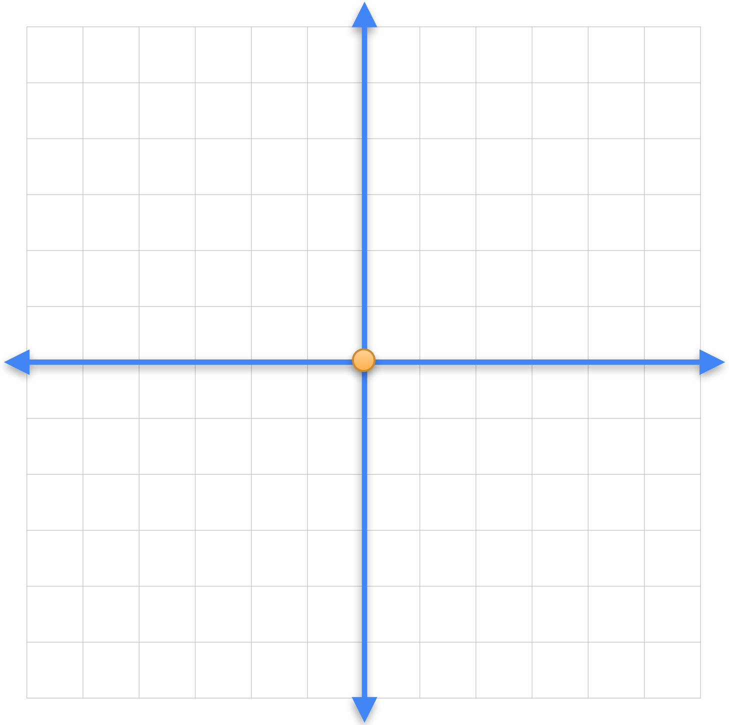
0	0
0	0



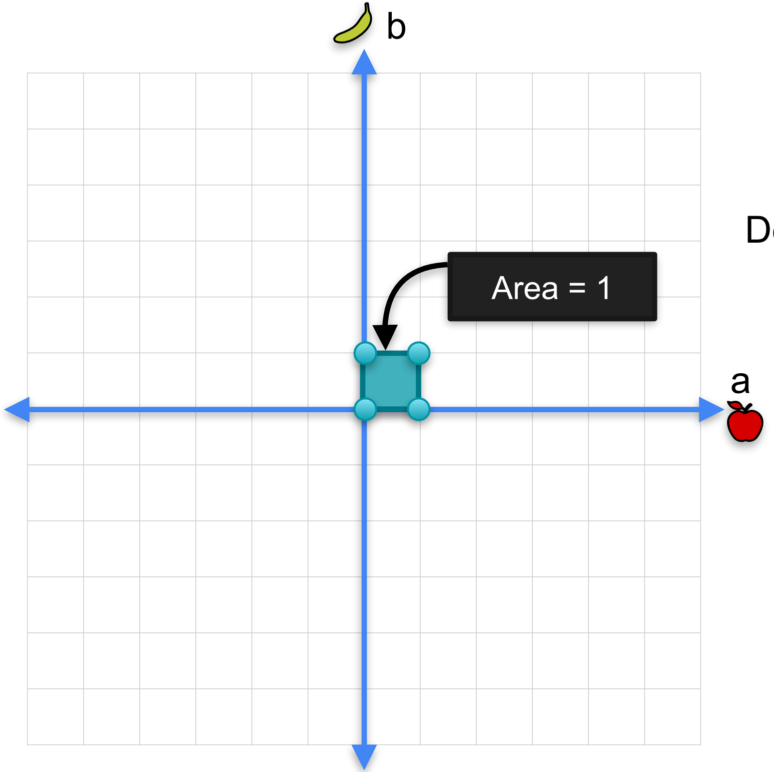
Determinant as an area



🍎	🍌
0	0
0	0



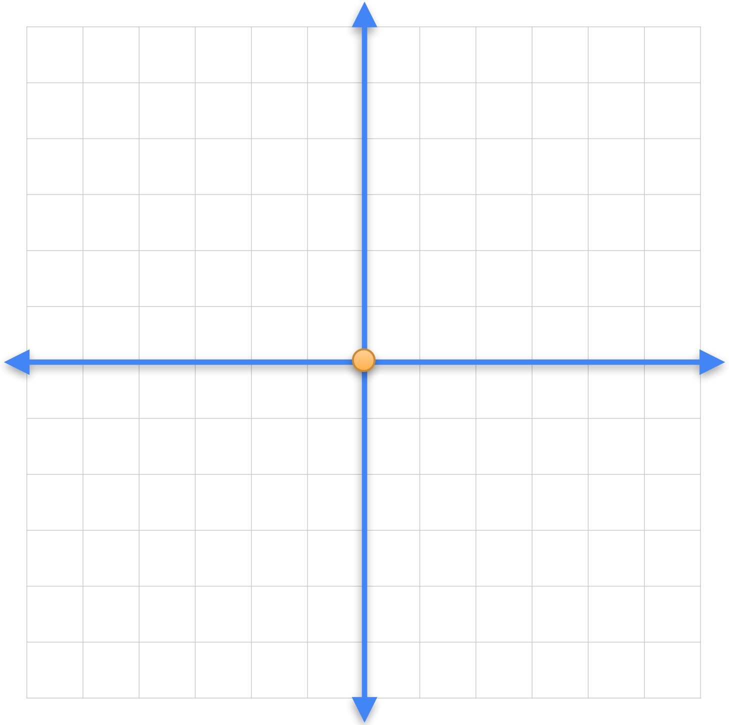
Determinant as an area



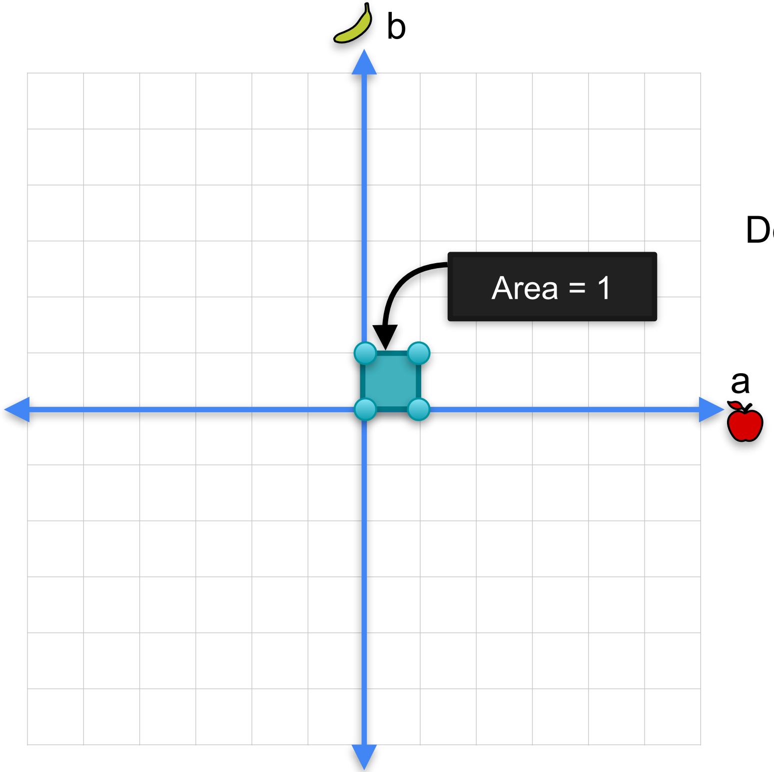
🍎	🍌
0	0
0	0

Det = $0 \cdot 0 - 0 \cdot 0$

Det = 0



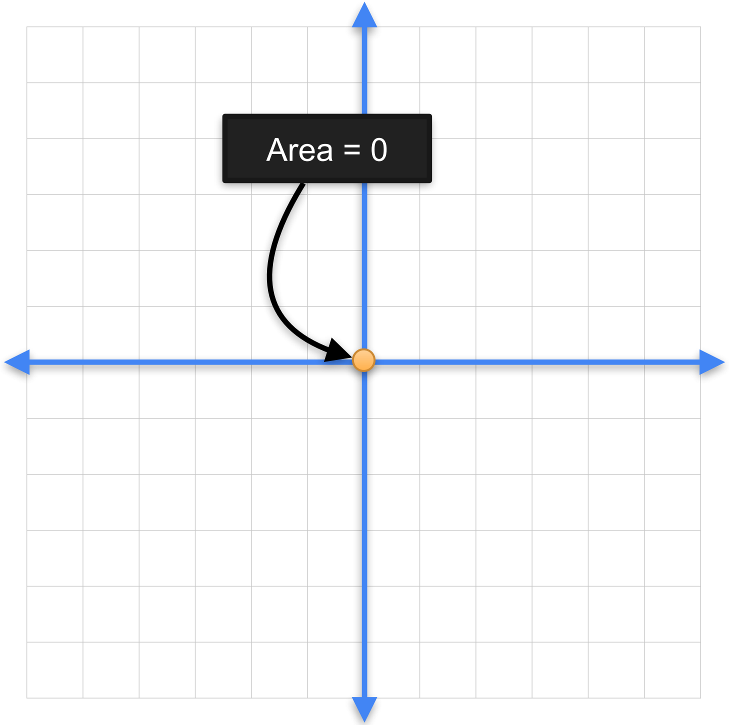
Determinant as an area



🍎	🍌
0	0
0	0

Det = $0 \cdot 0 - 0 \cdot 0$

Det = 0



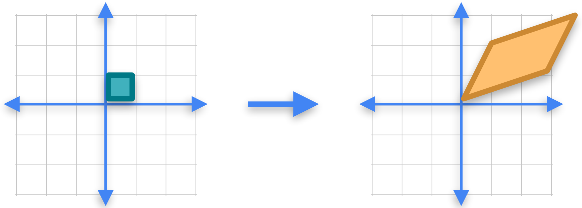
Determinant as an area

Determinant as an area

Non-singular

3	1
1	2

Determinant = 5



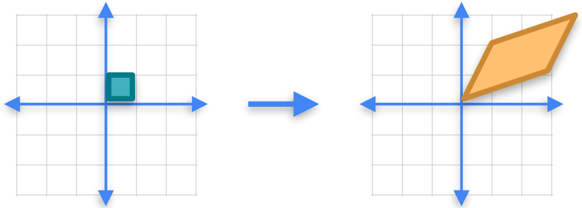
Area = 5

Determinant as an area

Non-singular

3	1
1	2

Determinant = 5

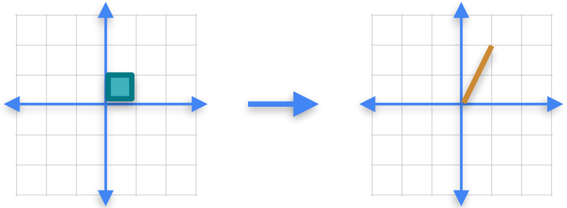


Area = 5

Singular

1	1
2	2

Determinant = 0



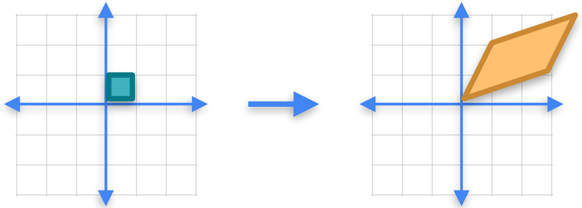
Area = 0

Determinant as an area

Non-singular

3	1
1	2

Determinant = 5

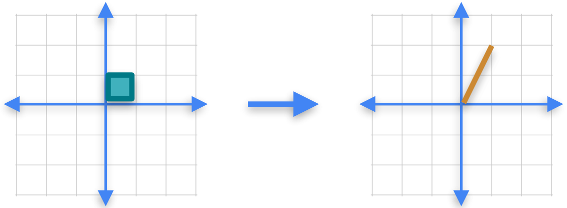


Area = 5

Singular

1	1
2	2

Determinant = 0

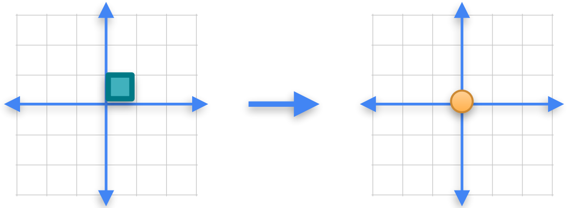


Area = 0

Singular

0	0
0	0

Determinant = 0



Area = 0

Negative determinants?

	
3	1
1	2

	
1	3
2	1

Negative determinants?

3	1
1	2

Det = $3 \cdot 2 - 1 \cdot 1$

Det = 5

1	3
2	1

Negative determinants?

3	1
1	2

Det = $3 \cdot 2 - 1 \cdot 1$

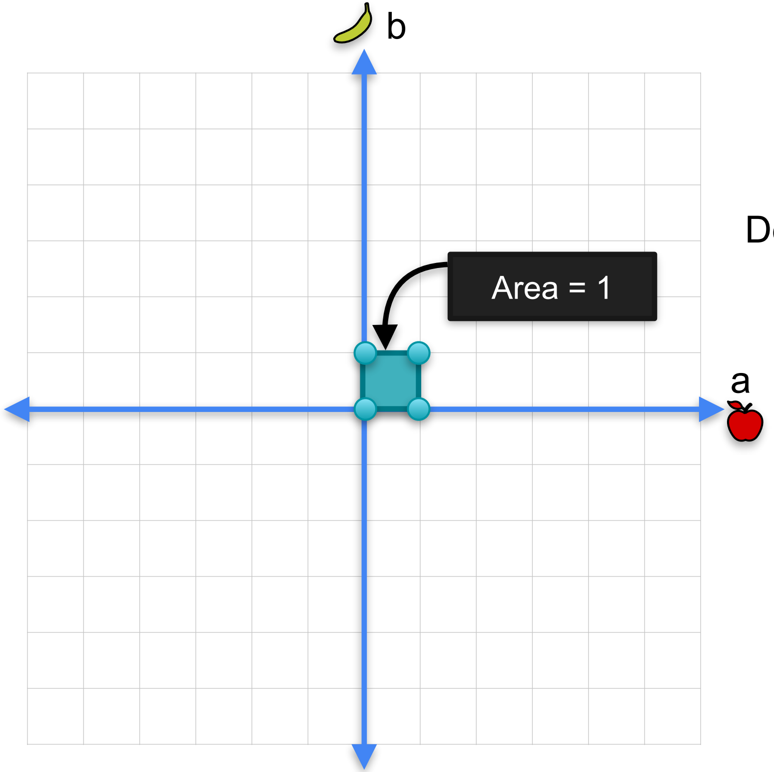
Det = 5

1	3
2	1

Det = $1 \cdot 1 - 3 \cdot 2$

Det = -5

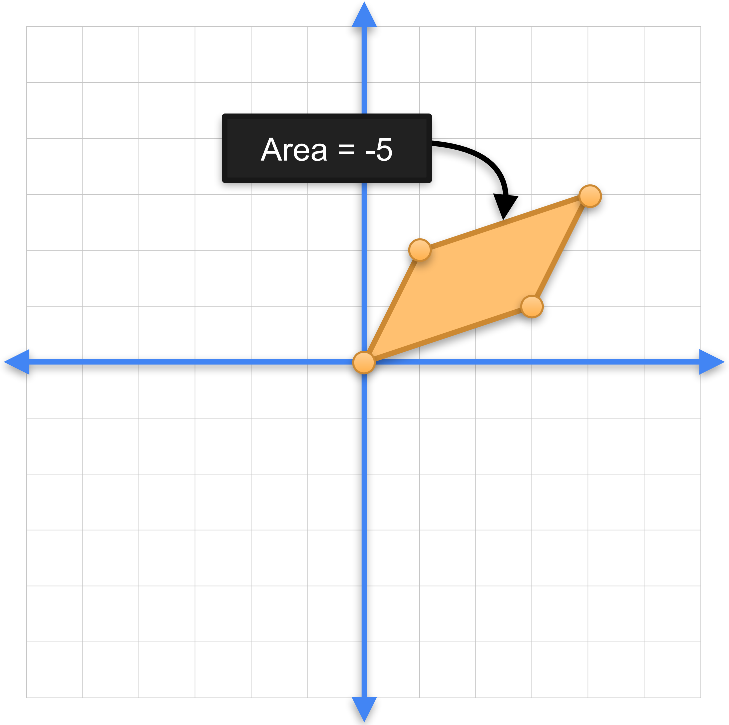
Determinant as an area



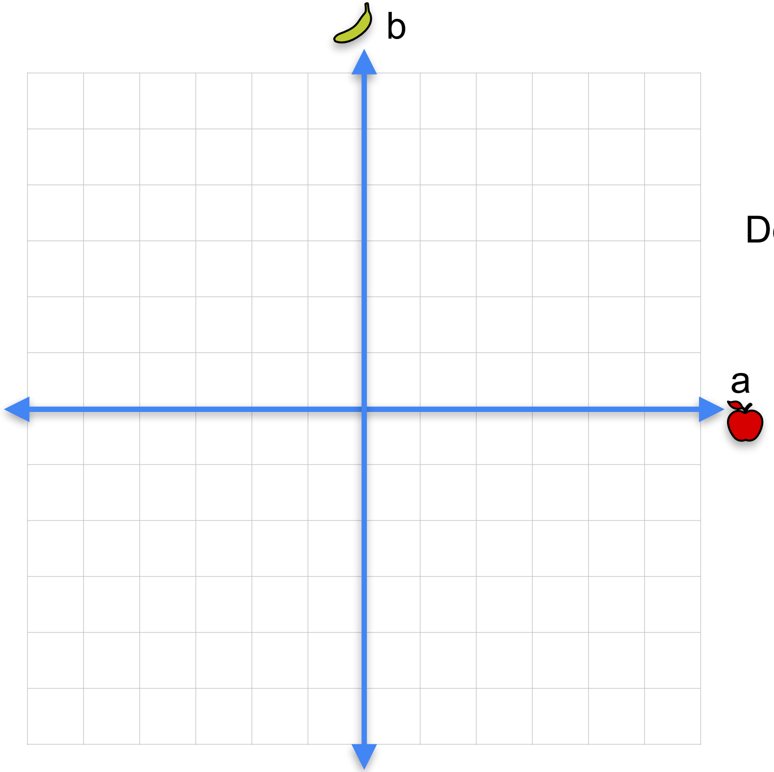
🍎	🍌
1	3
2	1

Det = $1 \cdot 1 - 3 \cdot 2$

Det = -5



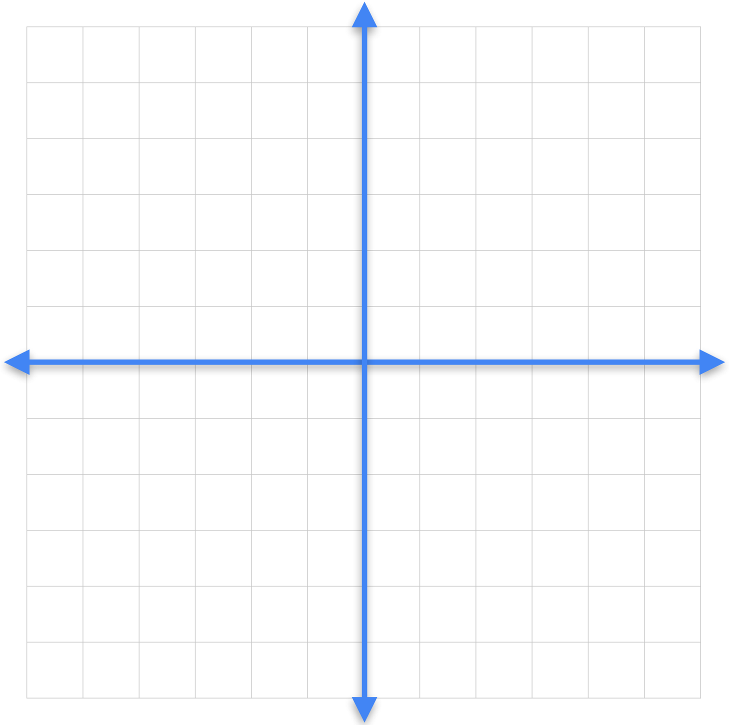
Determinant as an area



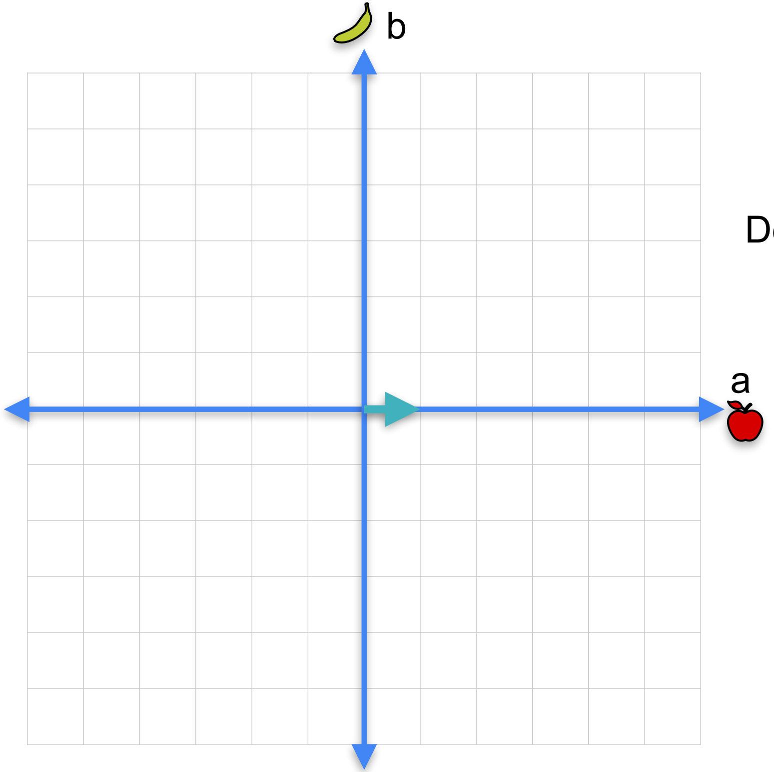
1	3
2	1

$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$



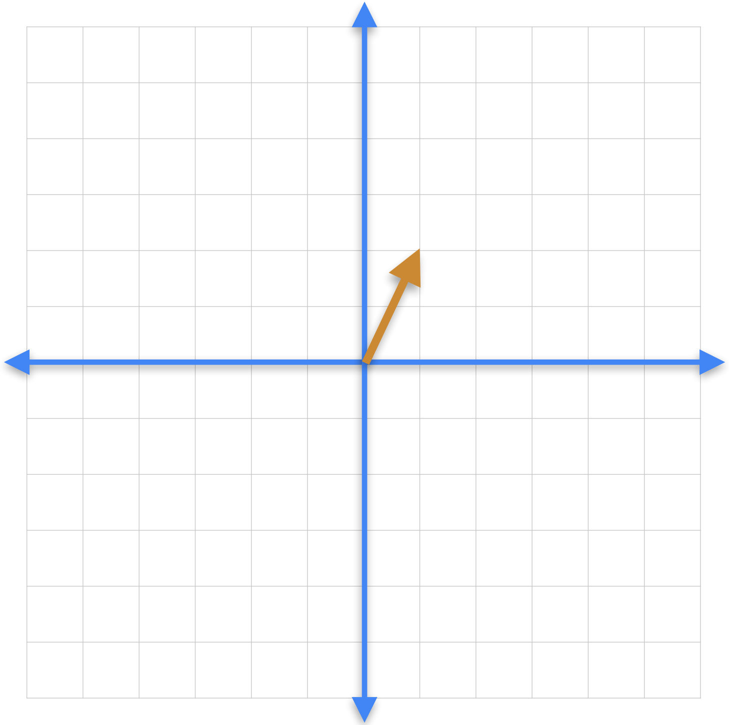
Determinant as an area



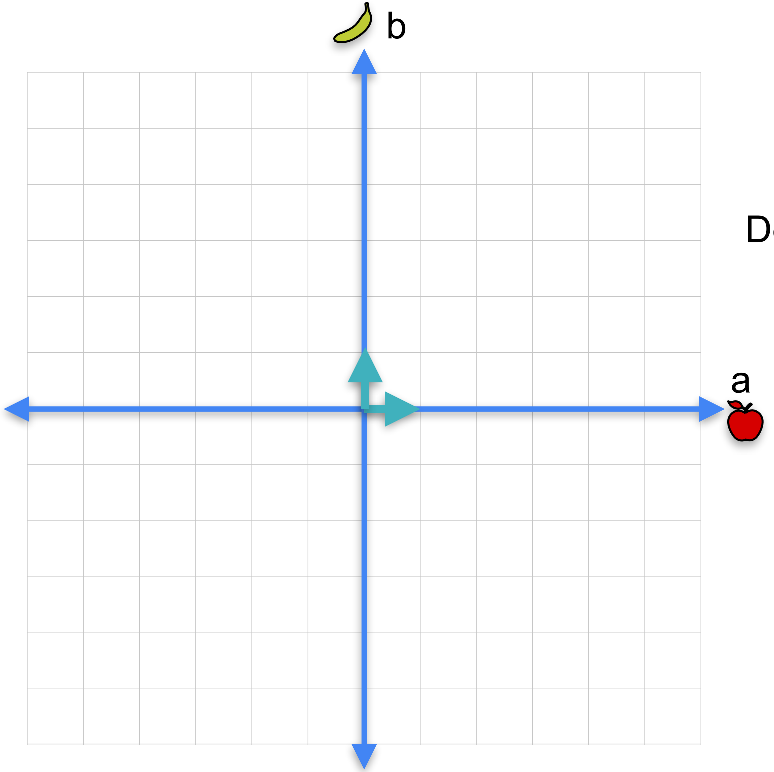
🍎	🍌
1	3
2	1

$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$



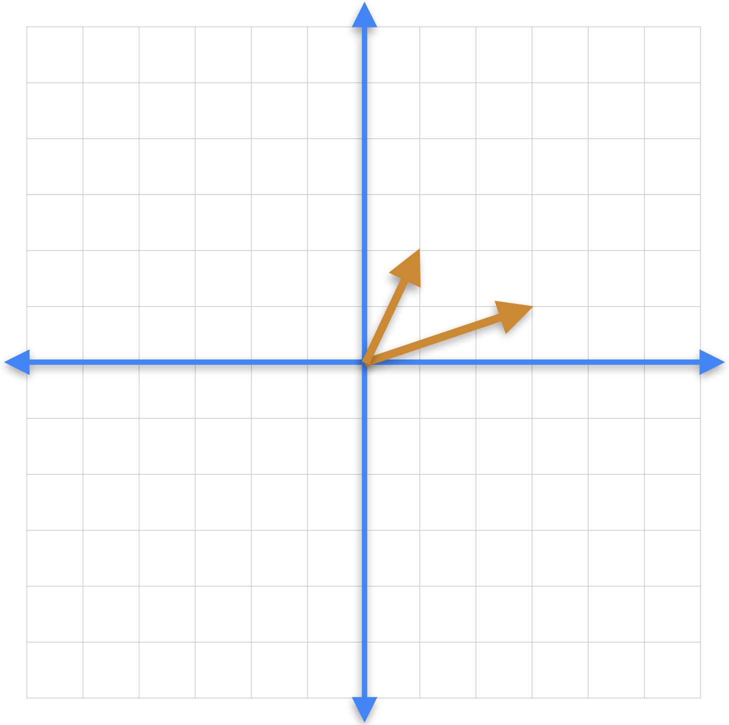
Determinant as an area



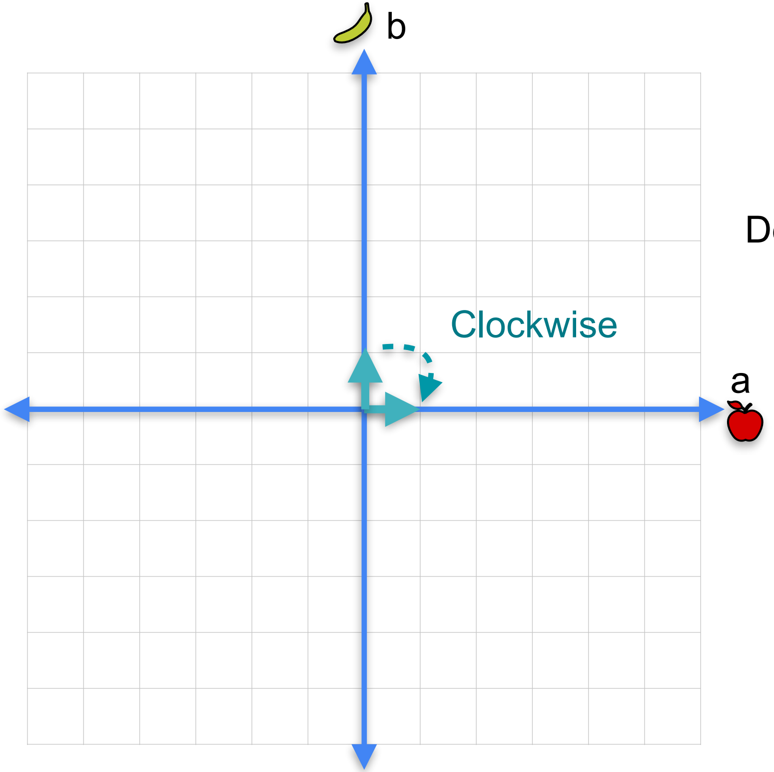
🍎	🍌
1	3
2	1

Det = $1 \cdot 1 - 3 \cdot 2$

Det = -5



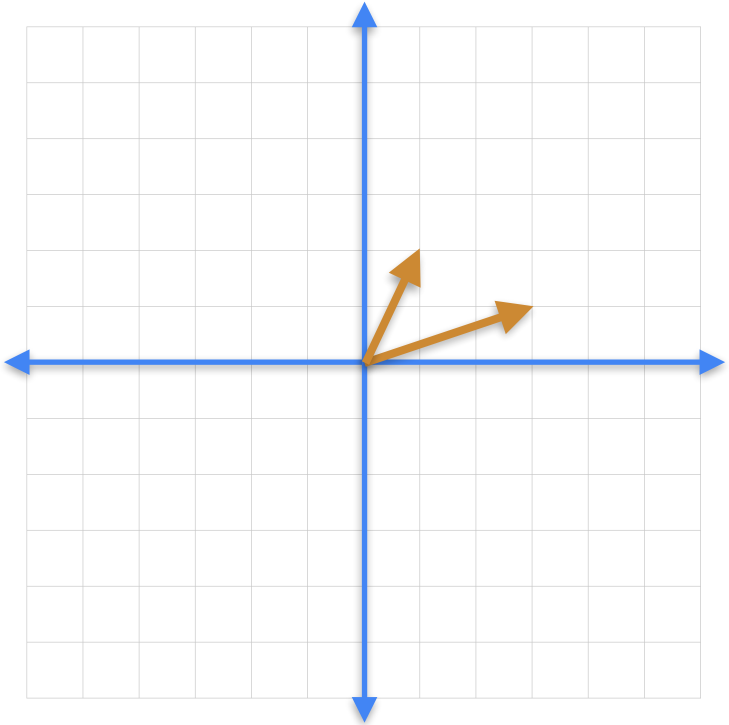
Determinant as an area



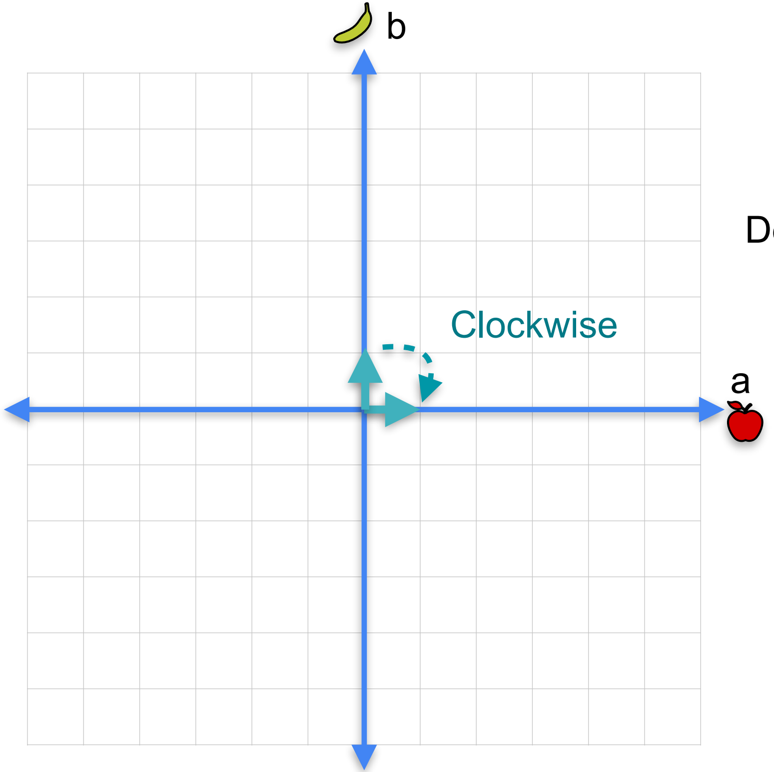
1	3
2	1

Det = $1 \cdot 1 - 3 \cdot 2$

Det = -5



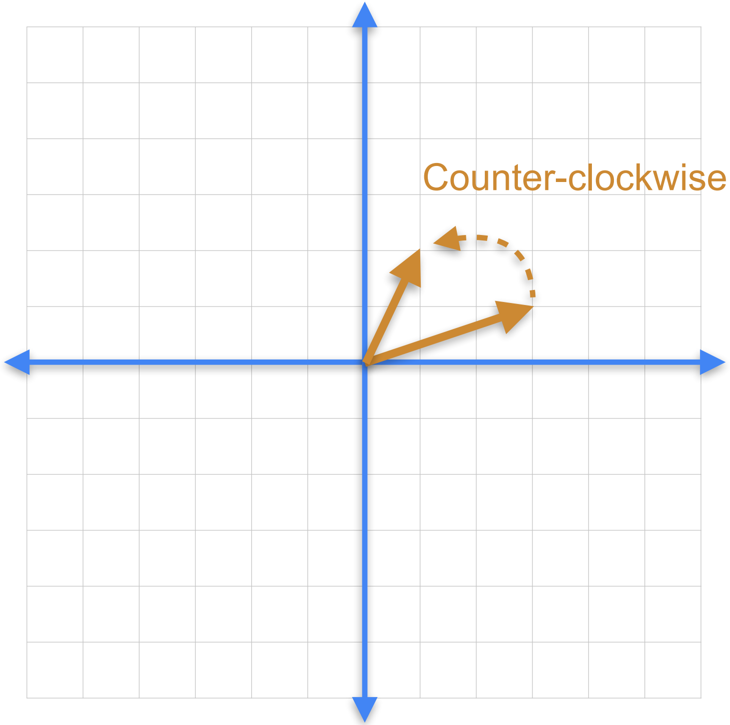
Determinant as an area



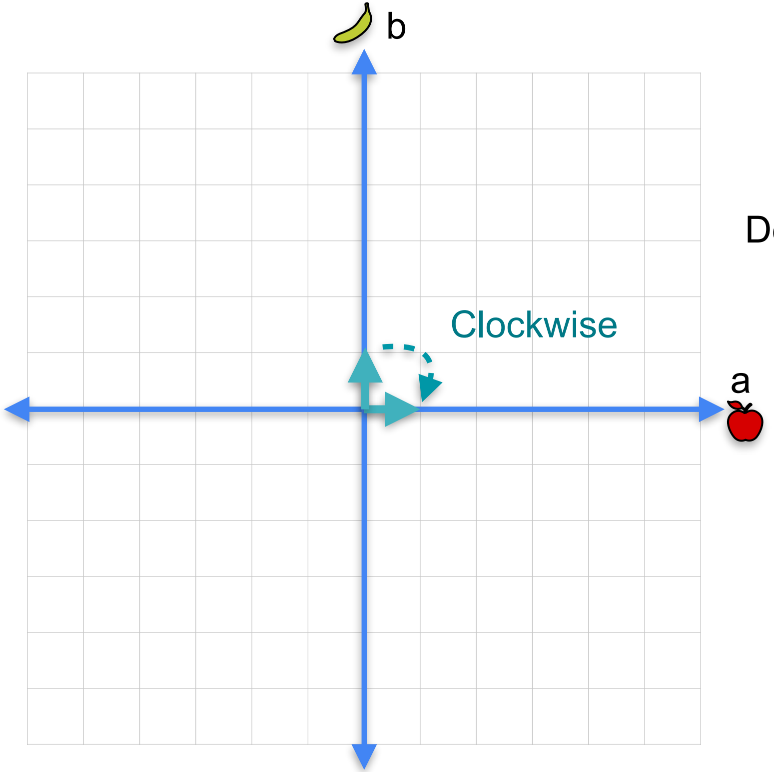
🍎	🍌
1	3
2	1

Det = $1 \cdot 1 - 3 \cdot 2$

Det = -5



Determinant as an area

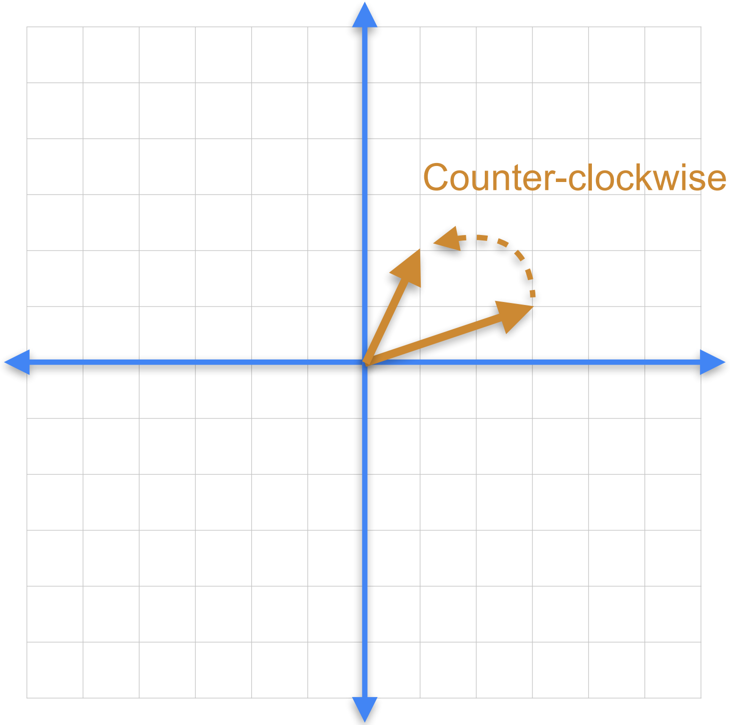


$$\begin{array}{cc} 🍎 & 🍌 \\ \hline 1 & 3 \\ 2 & 1 \end{array}$$

Det = $1 \cdot 1 - 3 \cdot 2$

Det = -5

Negative



DeepLearning.AI

Determinants and Eigenvectors

Determinant of a product

Determinant of a product

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 7 & 6 \end{bmatrix}$$

Determinant of a product

3	1		5	2	=	16	8
1	2		1	2		7	6

det = 5

$3 \cdot 2 - 1 \cdot 1$

Determinant of a product

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 7 & 6 \end{bmatrix}$$

det = 5

$3 \cdot 2 - 1 \cdot 1$

det = 8

$5 \cdot 2 - 2 \cdot 1$

Determinant of a product

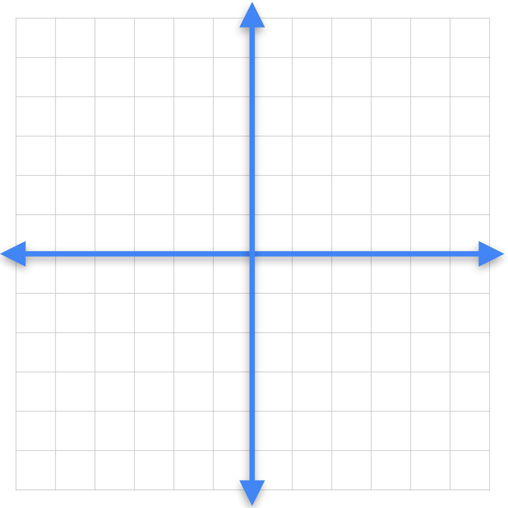
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 7 & 6 \end{bmatrix}$$

det = 5	det = 8	det = 40
$3 \cdot 2 - 1 \cdot 1$	$5 \cdot 2 - 2 \cdot 1$	$16 \cdot 6 - 8 \cdot 7$

Determinant of a product

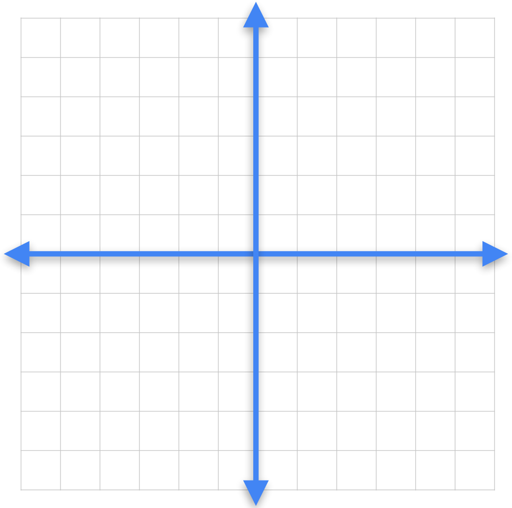
$$\det(AB) = \det(A)\det(B)$$

Determinant of a product

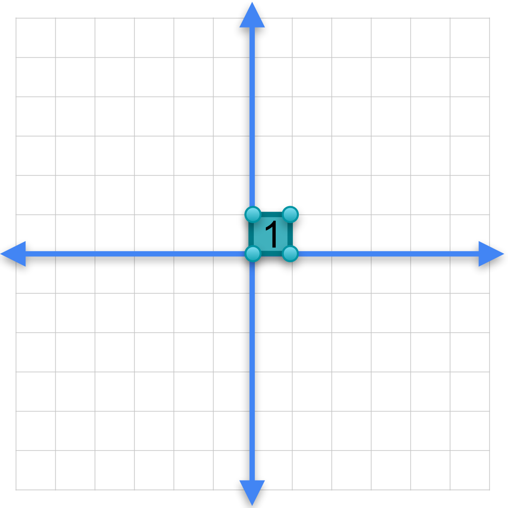


3	1
1	2

Det = 5

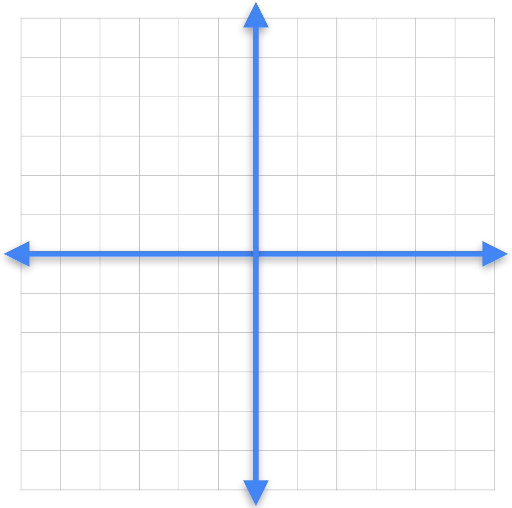


Determinant of a product

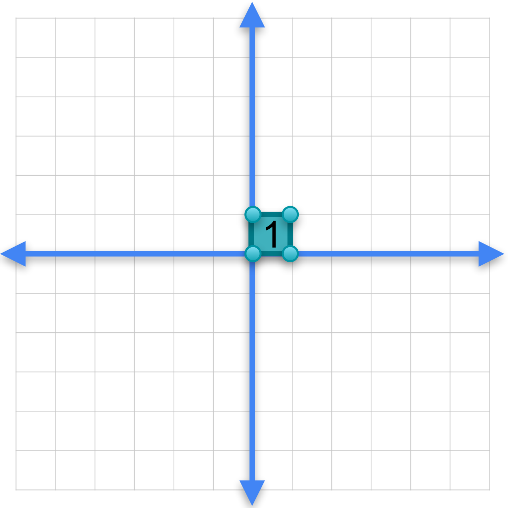


3	1
1	2

Det = 5

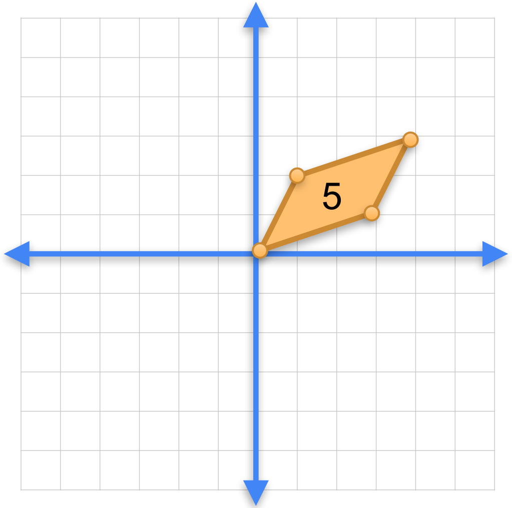


Determinant of a product

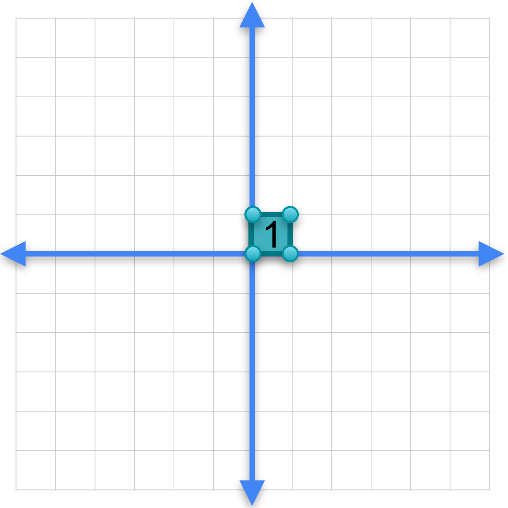


3	1
1	2

Det = 5

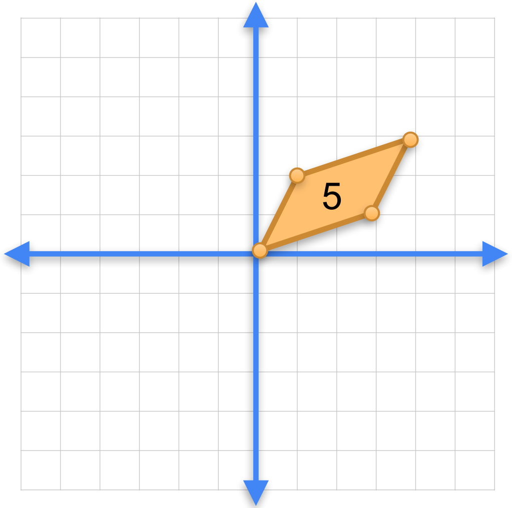


Determinant of a product



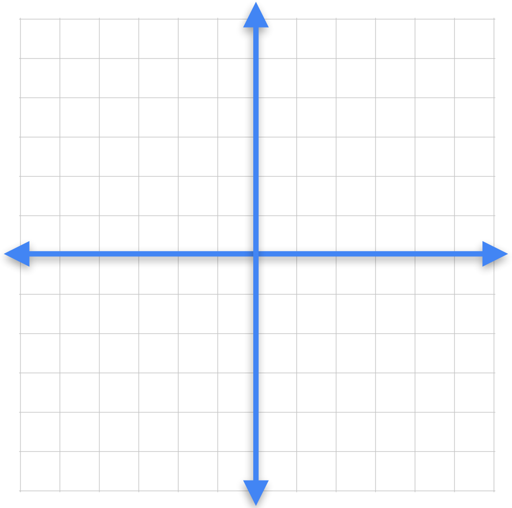
3	1
1	2

Det = 5



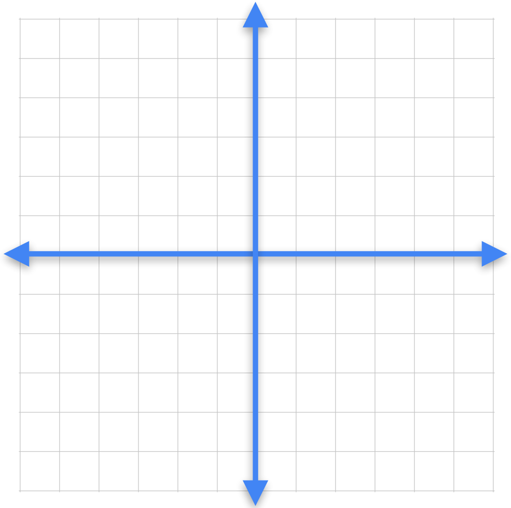
Area blows up by 5

Determinant of a product

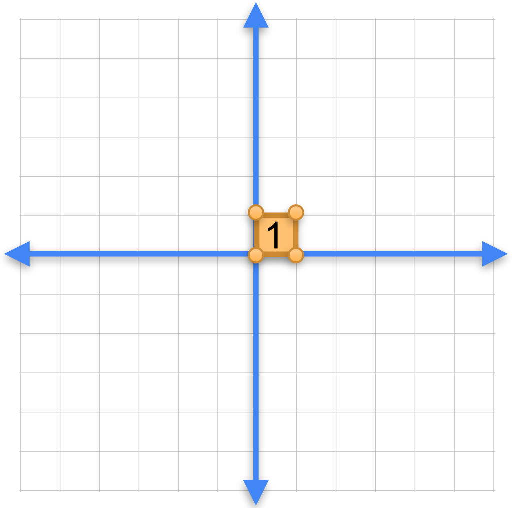


1	1
-2	1

Det = 3

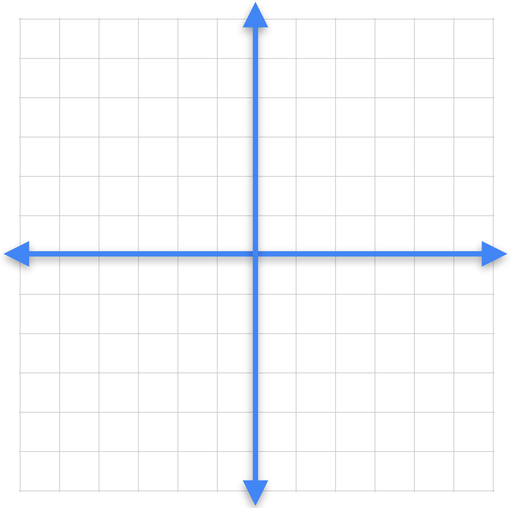


Determinant of a product

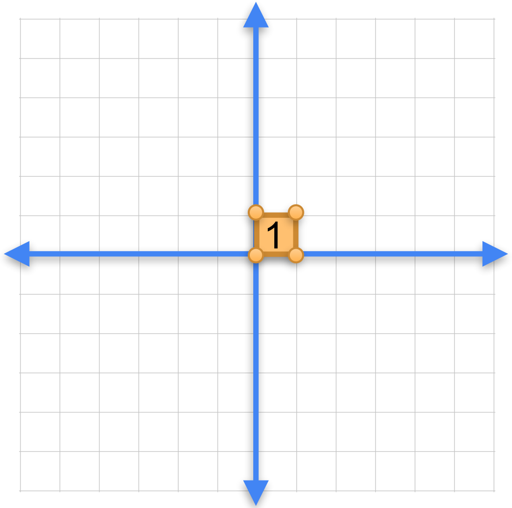


1	1
-2	1

Det = 3

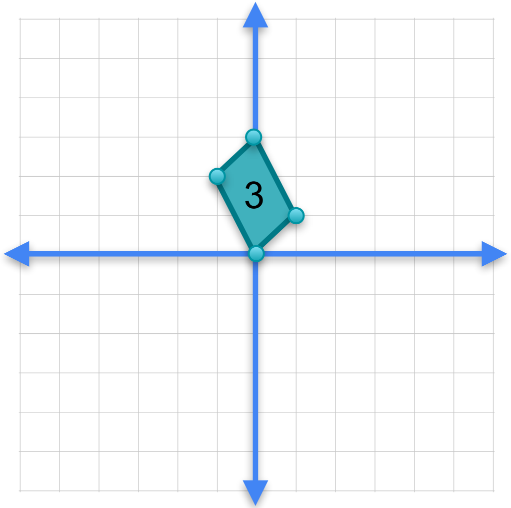


Determinant of a product

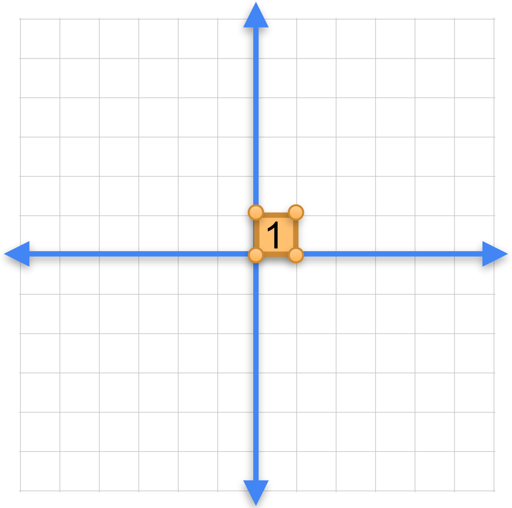


1	1
-2	1

Det = 3

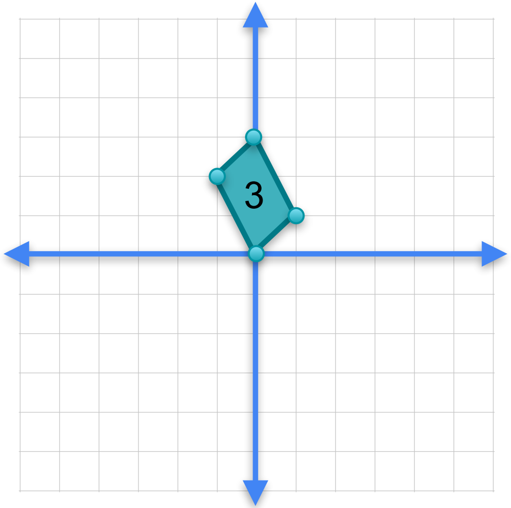


Determinant of a product



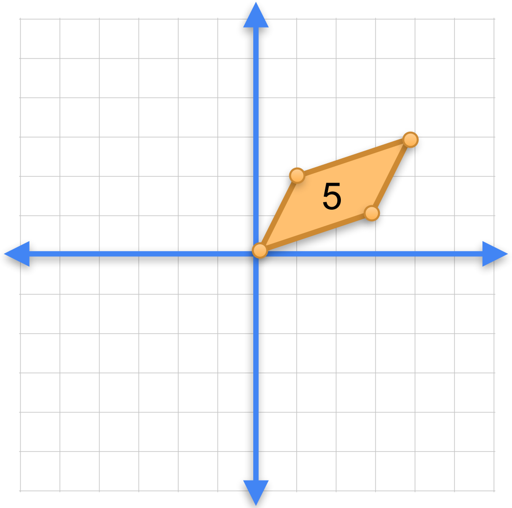
1	1
-2	1

Det = 3



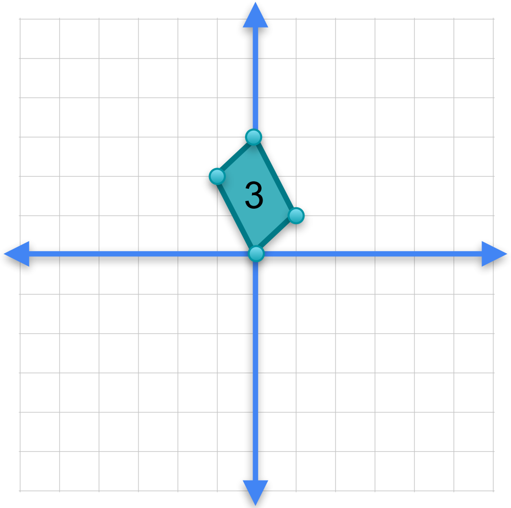
Area blows up by 3

Determinant of a product



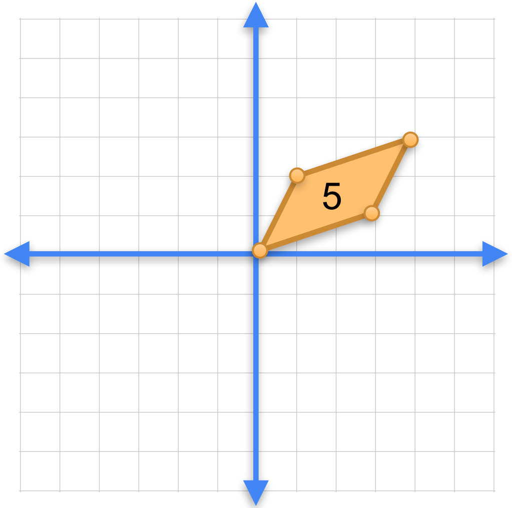
1	1
-2	1

Det = 3



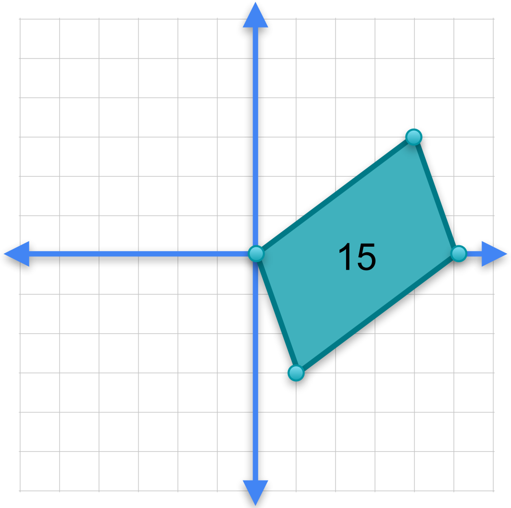
Area blows up by 3

Determinant of a product



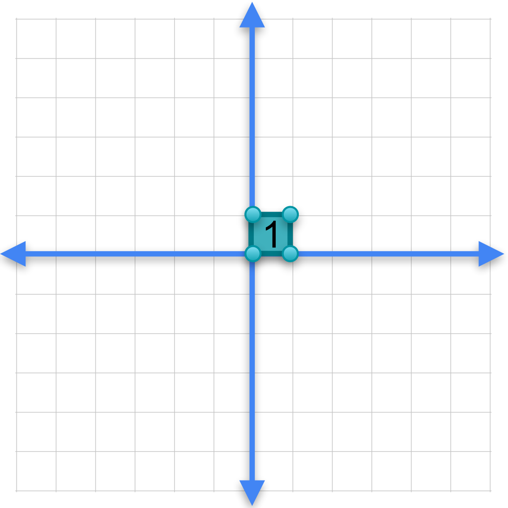
1	1
-2	1

Det = 3



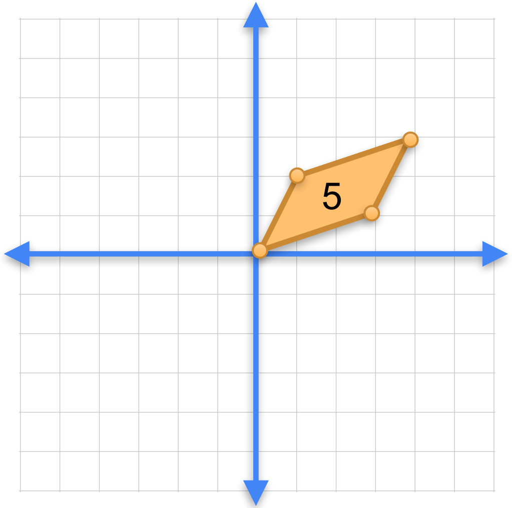
Area blows up by 3

Determinant of a product



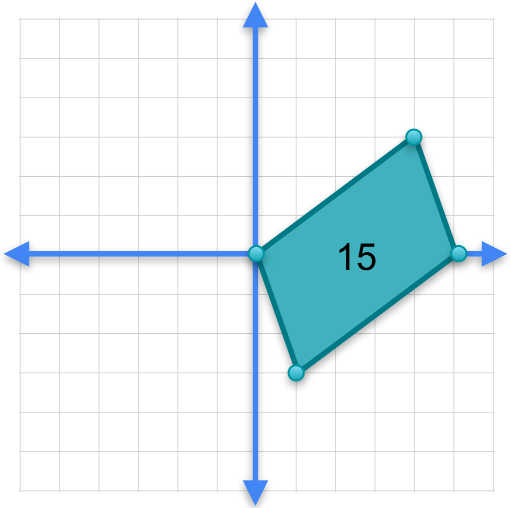
3	1
1	2

Det = 5

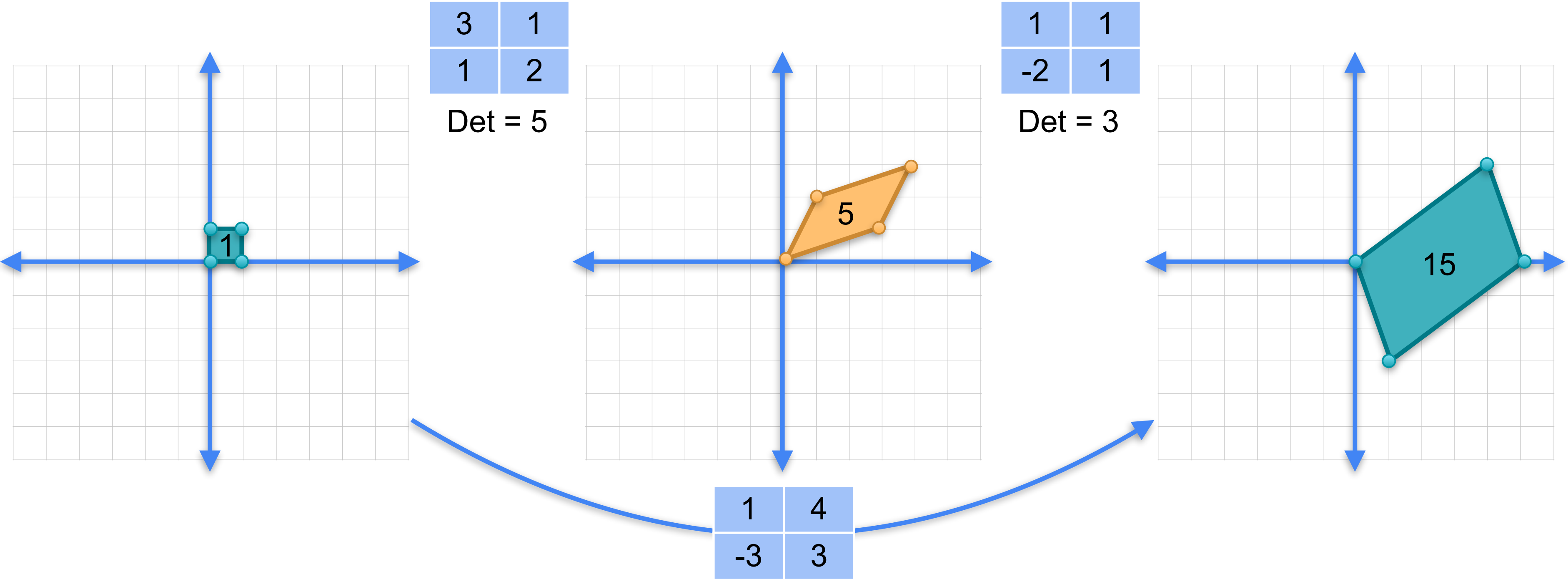


1	1
-2	1

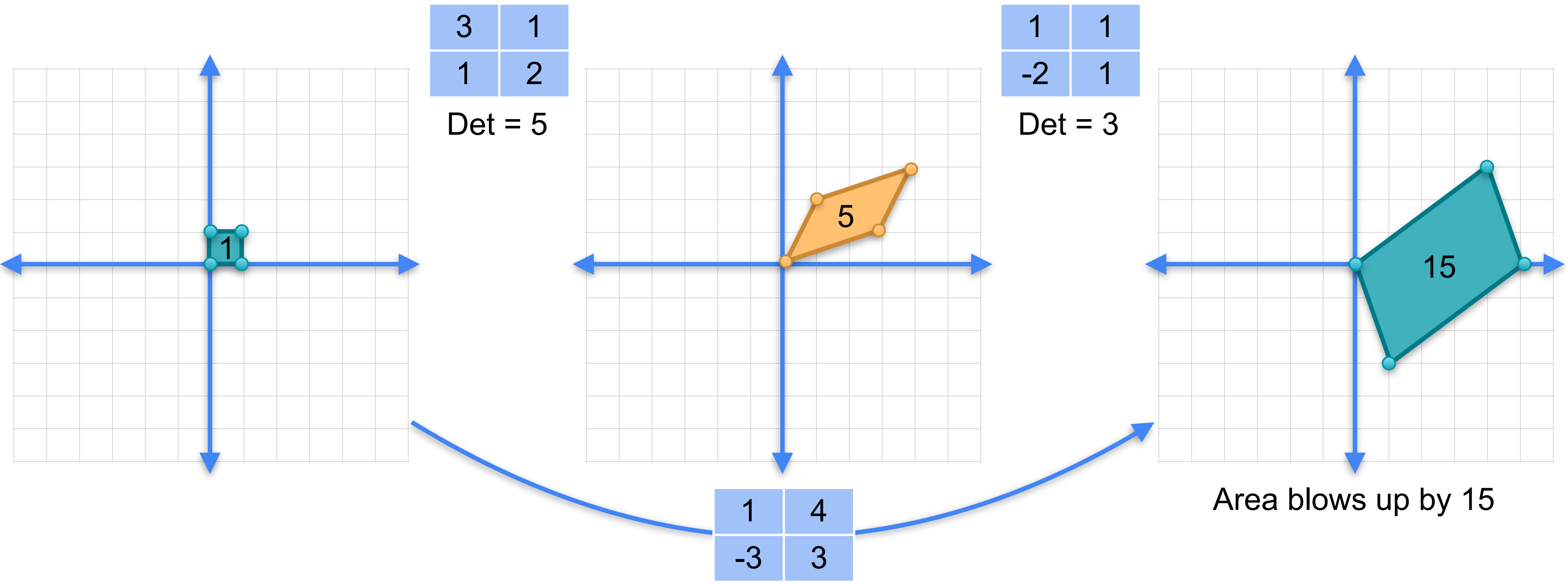
Det = 3



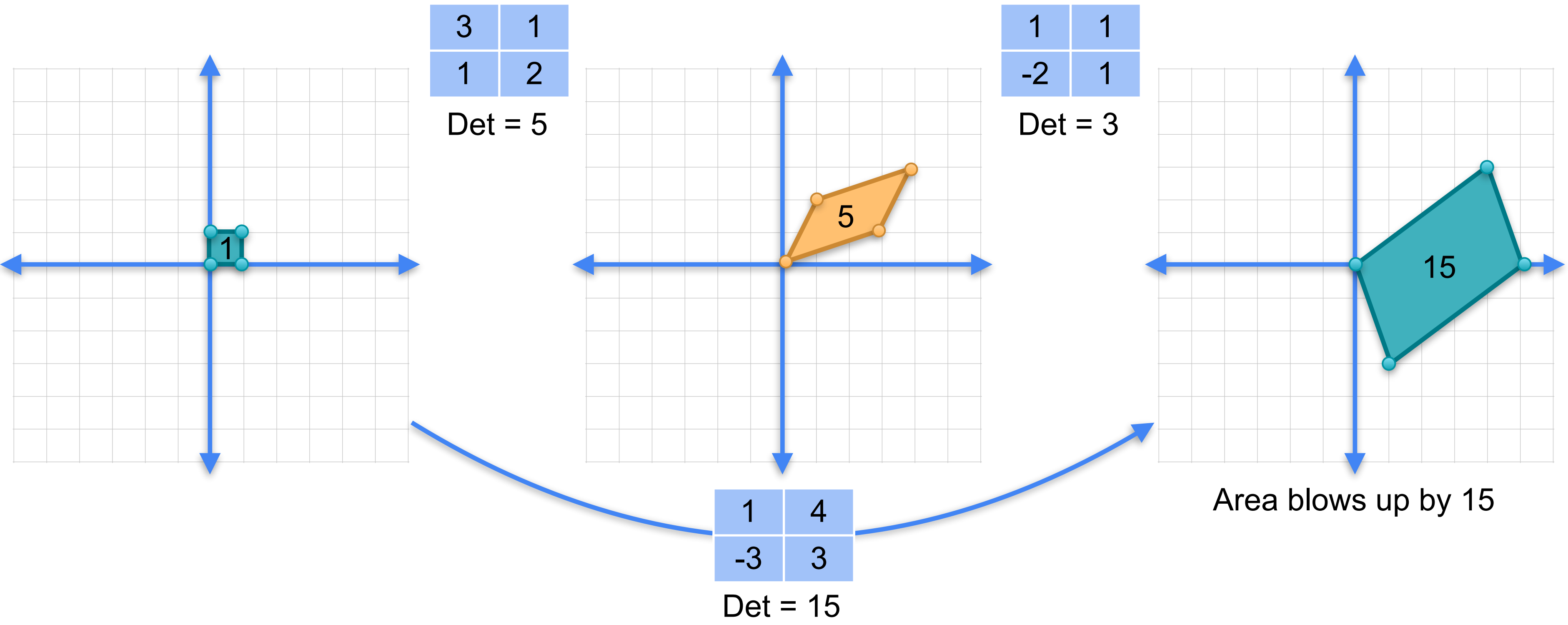
Determinant of a product



Determinant of a product



Determinant of a product



Quiz

- The product of a singular and a non-singular matrix (in any order) is:
  - Singular
  - Non-singular
  - Could be either one

Solution

- If A is non-singular and B is singular, then det(AB) = det(A) x det(B) = 0, since det(B) = 0. Therefore det(AB) = 0, so AB is **singular**.

When one factor is zero

When one factor is zero

$$5$$

When one factor is zero

$$5 \cdot 0$$

When one factor is zero

$$5 \cdot 0 = 0$$

When one factor is singular…

$$\underset{\text{Det} = 5}{\overset{\text{Non-singular}}{\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}}} \quad \underset{\text{Det} = 0}{\overset{\text{Singular}}{\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}}} = \underset{\text{Det} = 0}{\overset{\text{Singular}}{\begin{bmatrix} 4 & 8 \\ 3 & 6 \end{bmatrix}}}$$

If one factor is singular…

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Det = 5

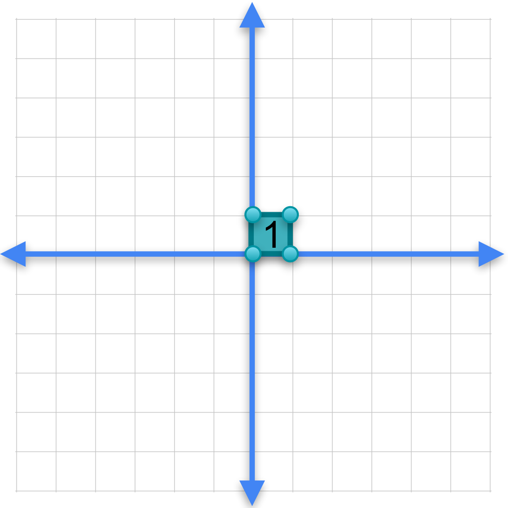
$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

Det = 0

$$\begin{bmatrix} 4 & 8 \\ 3 & 6 \end{bmatrix}$$

Det = 0

If one factor is singular…



3	1
1	2

Det = 5

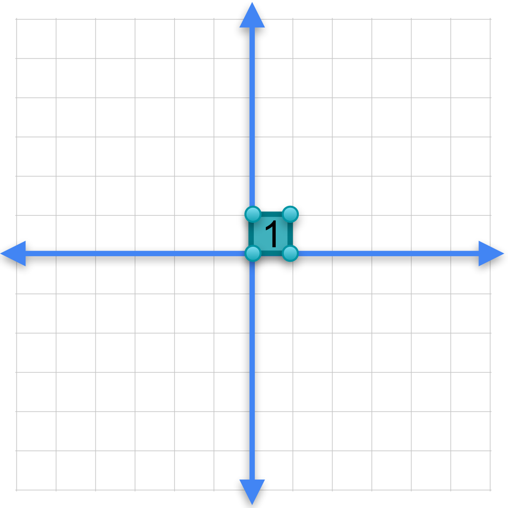
1	2
1	2

Det = 0

4	8
3	6

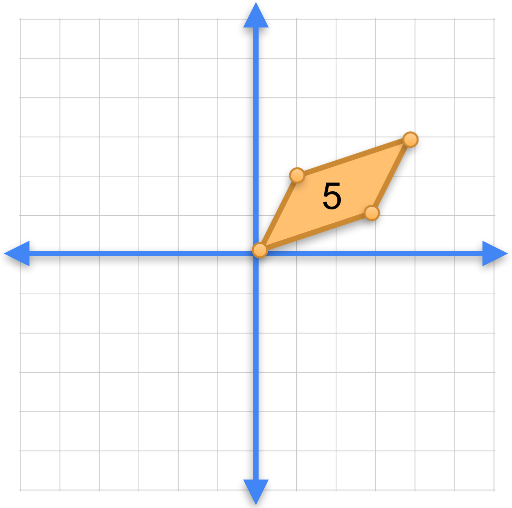
Det = 0

If one factor is singular…



3	1
1	2

Det = 5



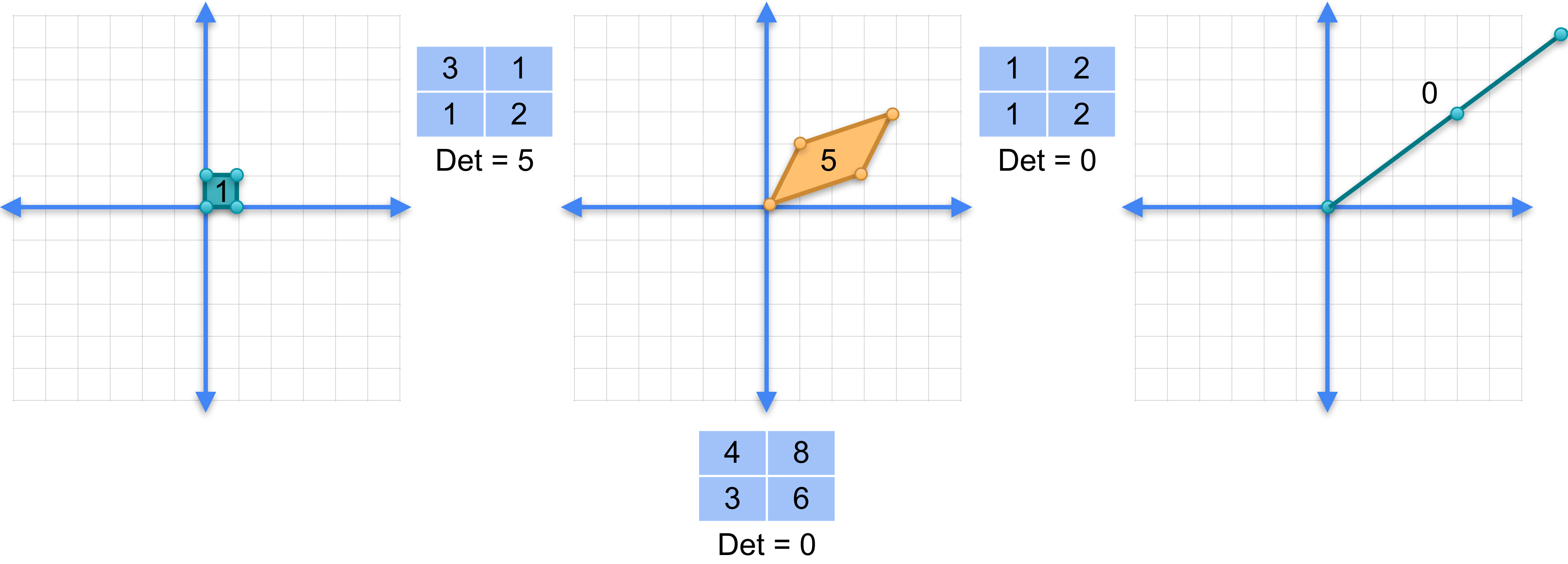
1	2
1	2

Det = 0

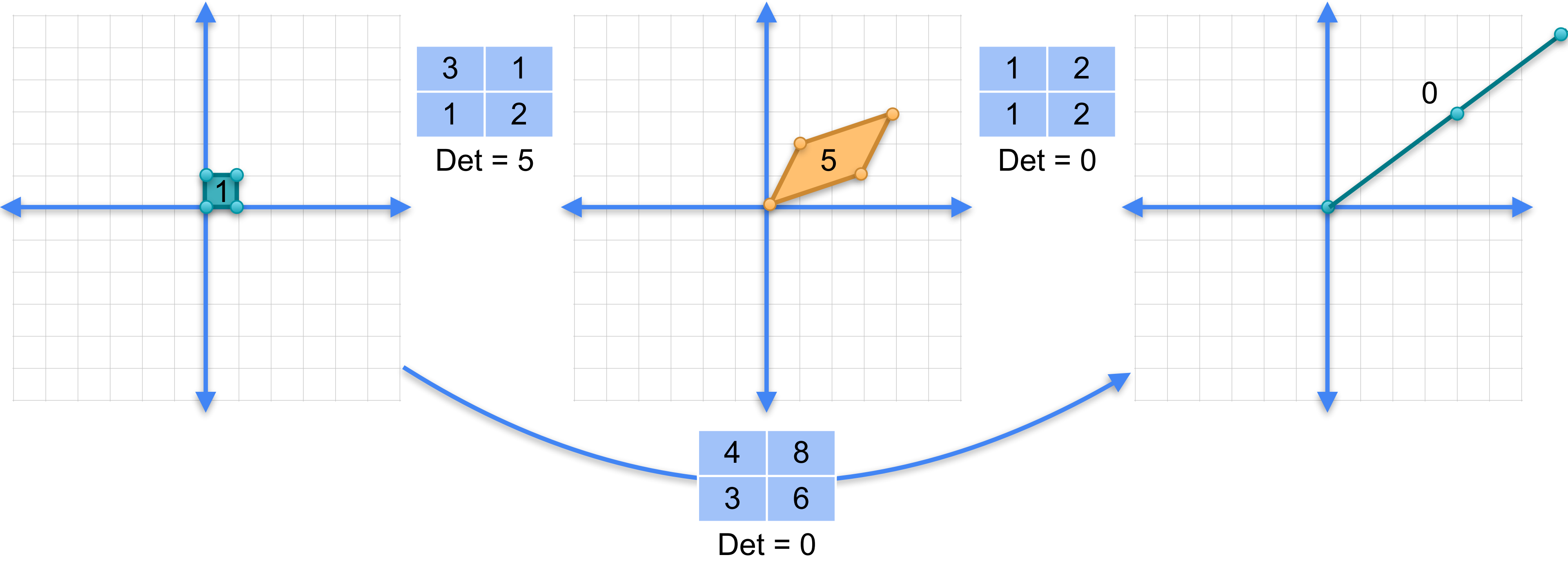
4	8
3	6

Det = 0

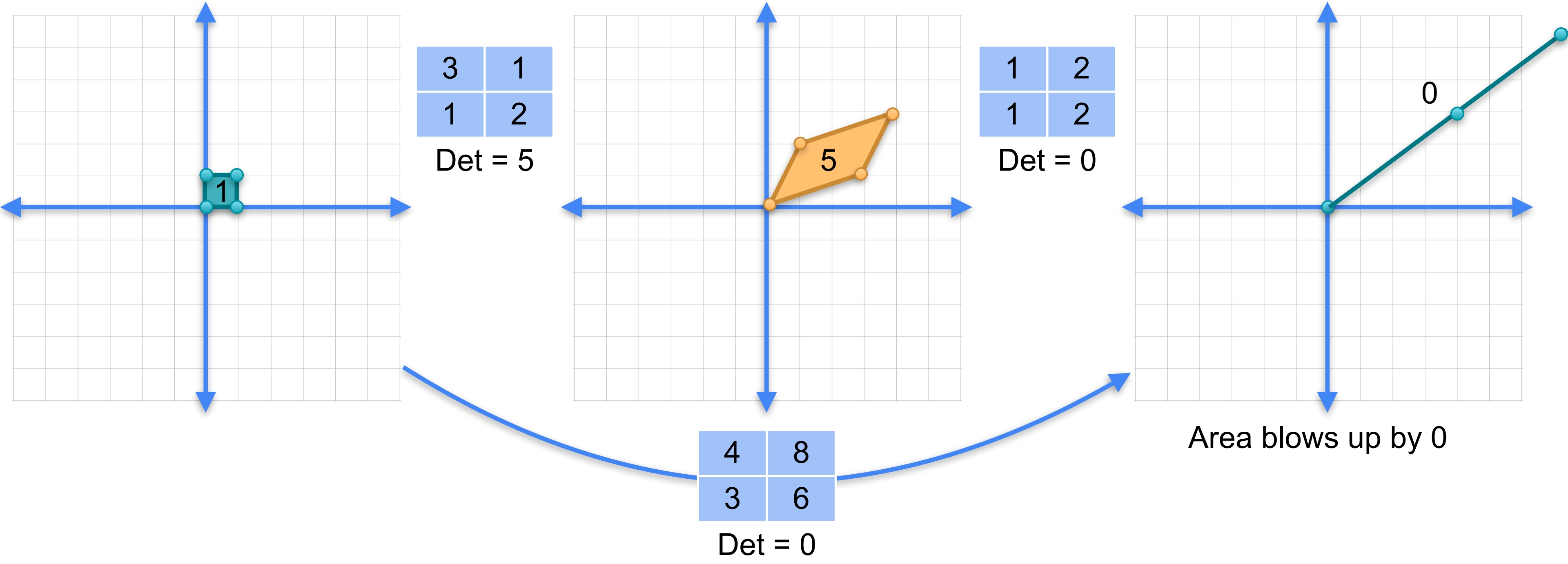
If one factor is singular…



If one factor is singular…



If one factor is singular…



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Determinants and Eigenvectors

Determinant of inverse

Quiz

- Find the determinants of the following matrices

0.4	-0.2
-0.2	0.6

0.25	-0.25
-0.125	0.625

Solution

$$\text{Det}\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} = (0.4)(0.6) - (-0.2)(-0.2) = 0.2$$

$$\text{Det}\begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} = (0.25)(0.625) - (-0.125)(-0.25) = 0.125$$

Determinant of an inverse

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5

det = 0.2

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5

det = 0.2

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 5 det = 0.2

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 5   det = 0.2   det = 8

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5 det = 0.2

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 8 det = 0.125

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 5 det = 0.2 det = 8 det = 0.125

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

det = 5 det = 0.2 det = 8 det = 0.125

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5   det = 0.2

$$5^{-1} = 0.2$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 8   det = 0.125

$$8^{-1} = 0.125$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

det = 0

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

det = 5   det = 0.2   det = 8   det = 0.125   det = 0   det = ???

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125$$

Determinant of an inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5 det = 0.2

$$5^{-1} = 0.2$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 8 det = 0.125

$$8^{-1} = 0.125$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

det = 0 det = ???

$$0^{-1} = ???$$

Determinant of an inverse

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

Why?

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

$$\det(AB) = \det(A)\det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

$$\det(AB) = \det(A)\det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A)\det(A^{-1})$$

Why?

$$\det(AB) = \det(A)\det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A)\det(A^{-1})$$

$$\det(I) = \det(A)\det(A^{-1})$$

Why?

$$\det(AB) = \det(A)\det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A)\det(A^{-1})$$

$$\det(I) = \det(A)\det(A^{-1})$$

$$\frac{1}{\det(A)}$$

Why?

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AB) = \det(A)\det(B)$$

$$\det(AA^{-1}) = \det(A)\det(A^{-1})$$

$$\underbrace{\det(I)}_{1} = \det(A)\underbrace{\det(A^{-1})}_{\frac{1}{\det(A)}}$$

Determinant of the identity matrix

$$\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \cdot 1 - 0 \cdot 0 = 1$$

Determinant of the identity matrix

$$\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \cdot 1 - 0 \cdot 0 = 1$$

$$\det(I) = 1$$

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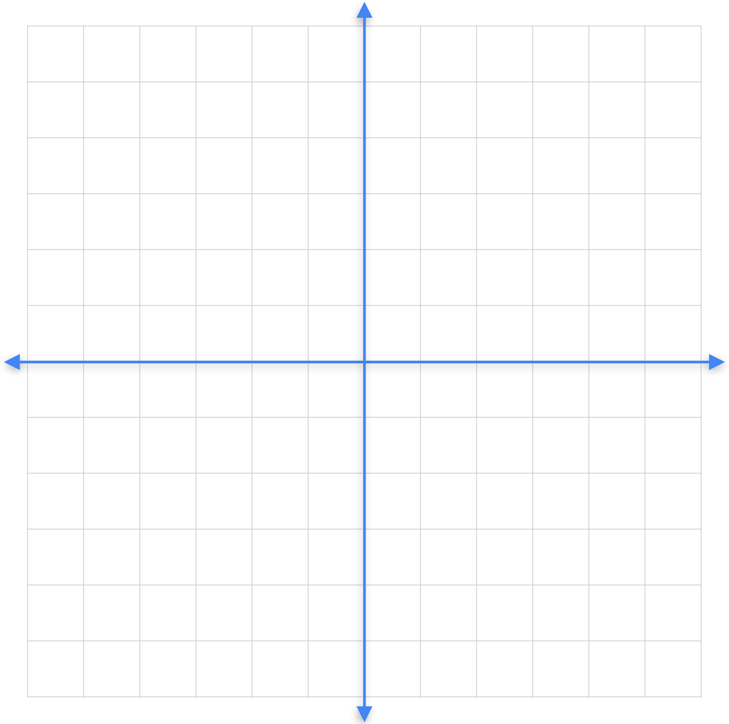
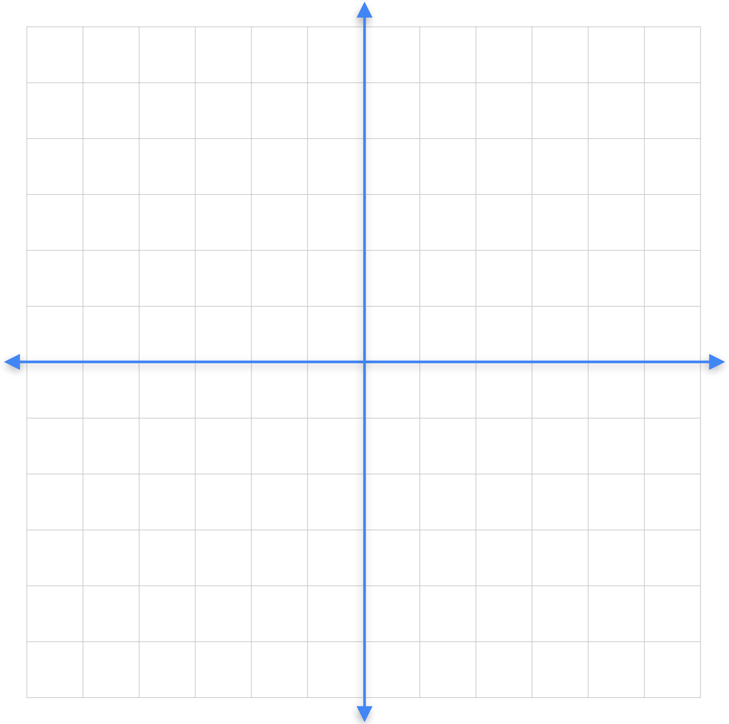
Determinants and Eigenvectors

Bases

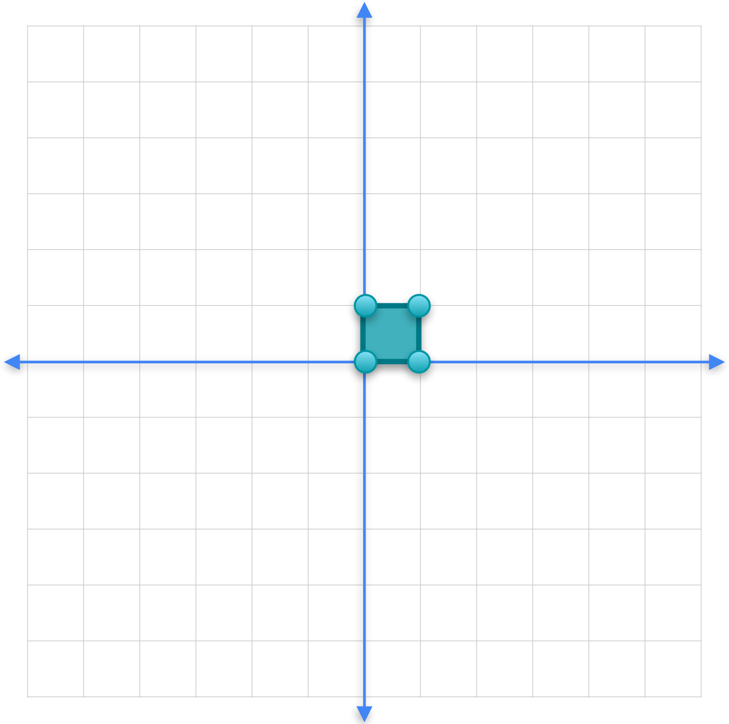
Bases



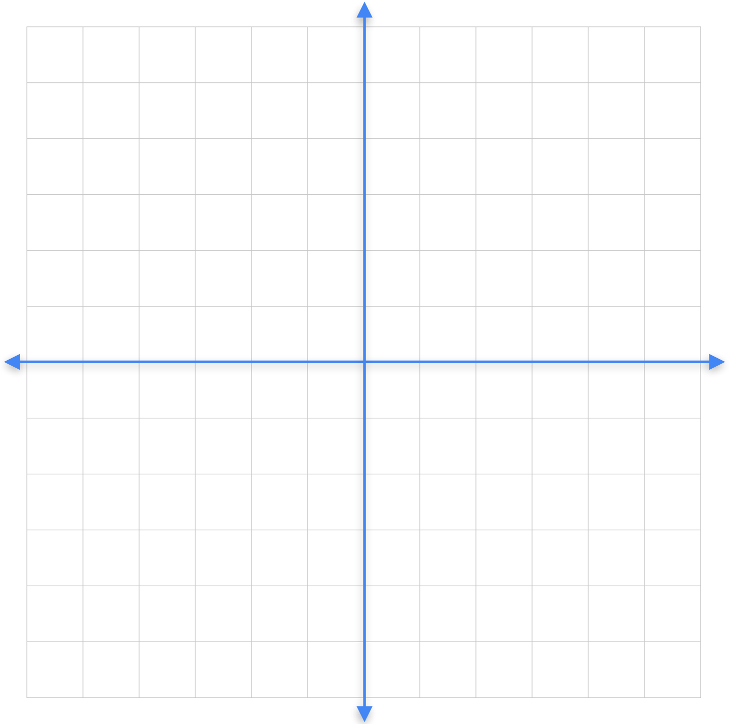
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$



Bases



3	1
1	2



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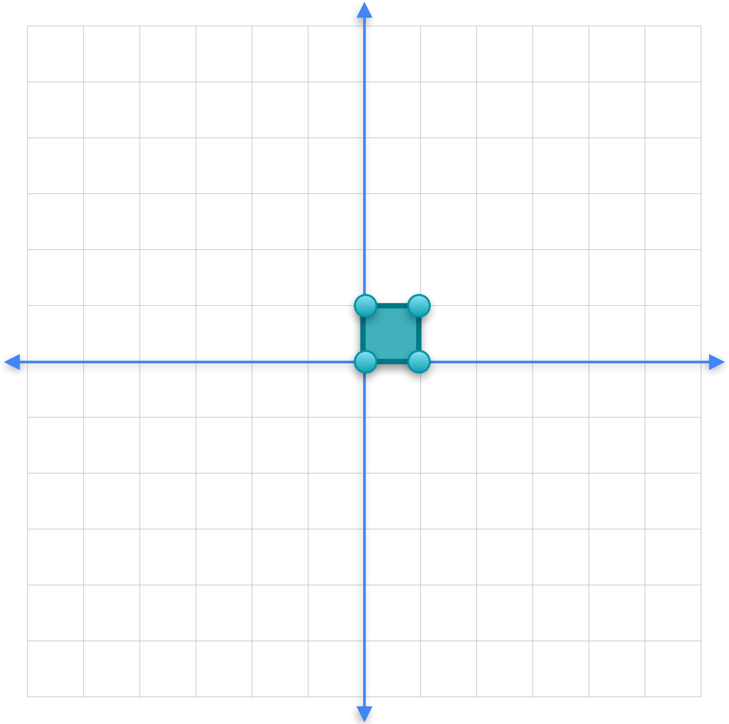
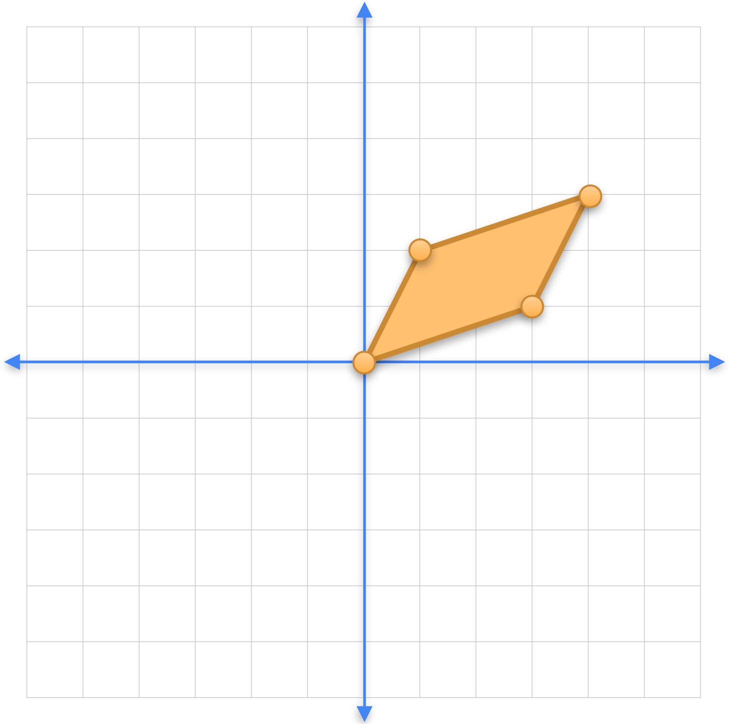
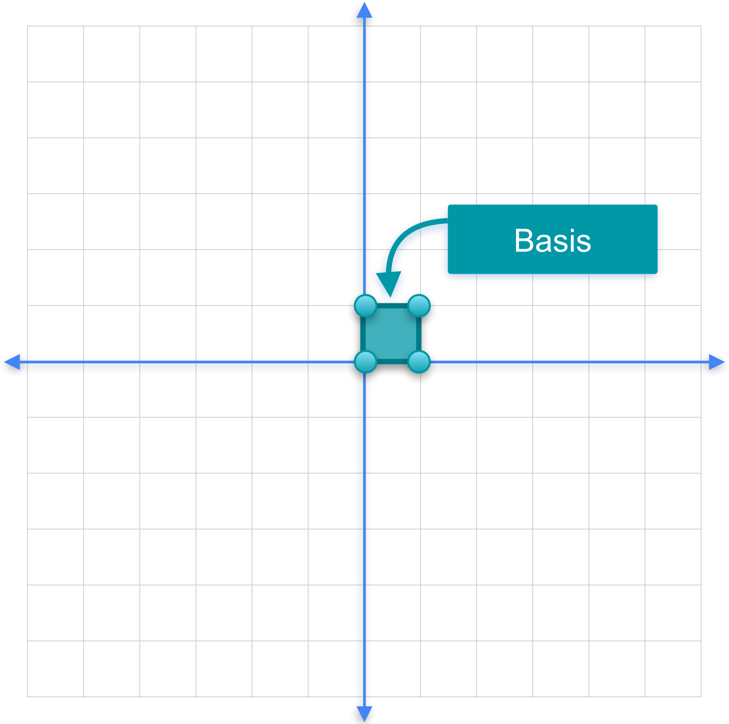


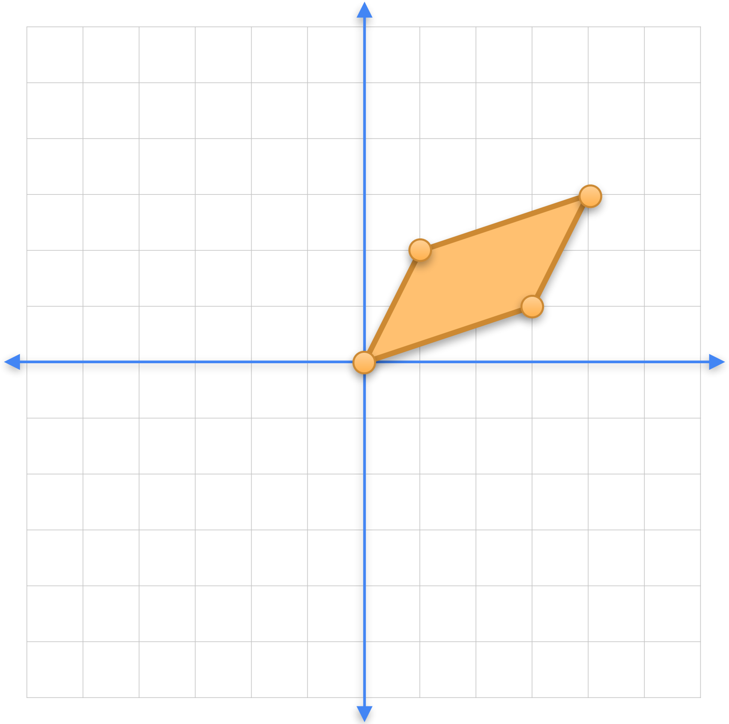
3	1
1	2



Bases



3	1
1	2



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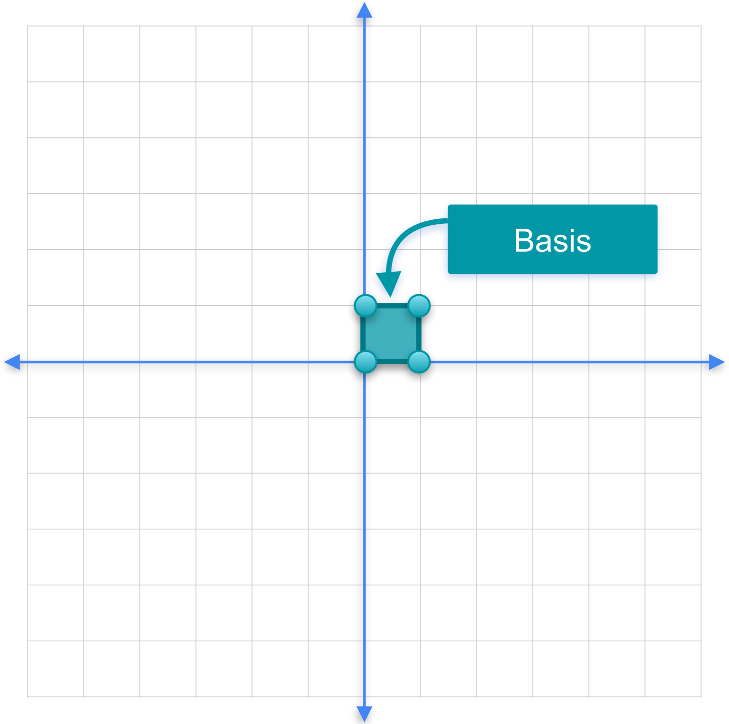

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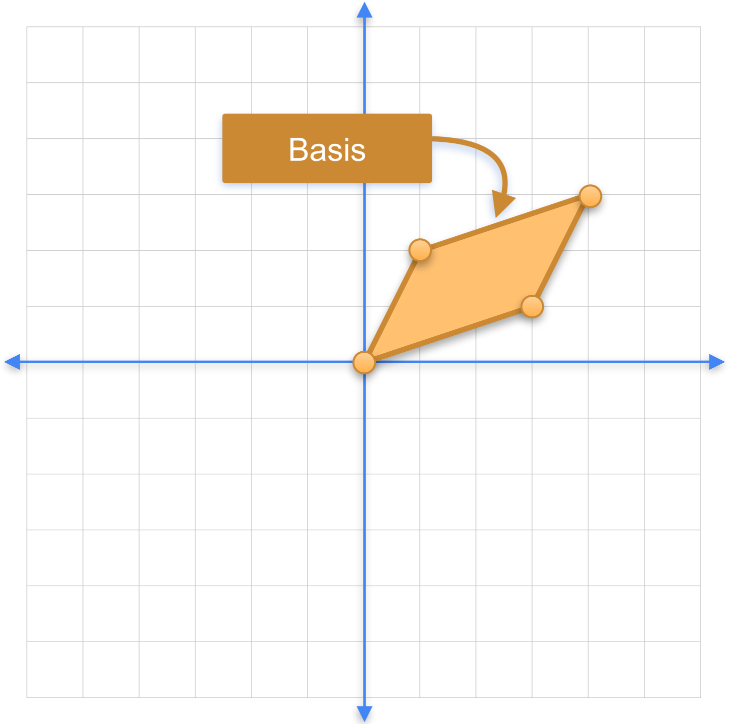

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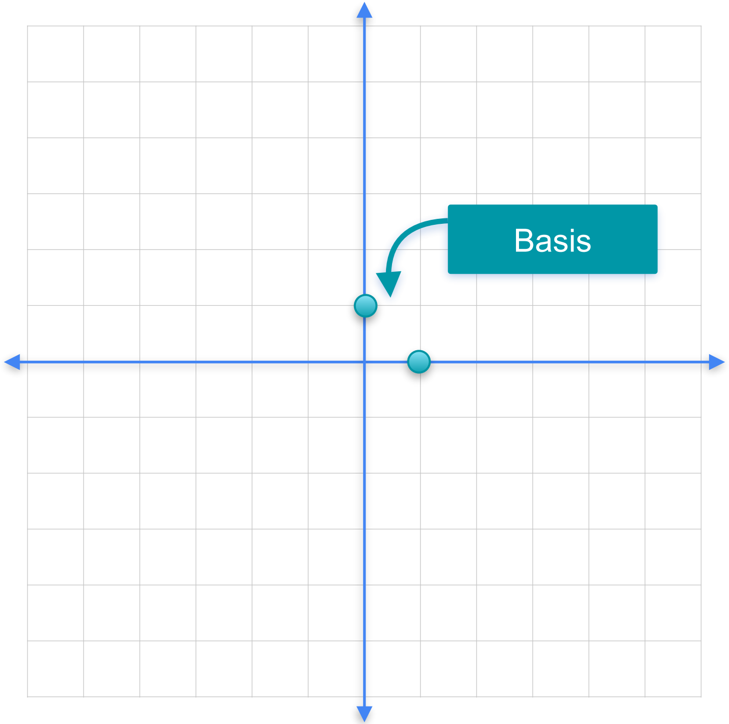
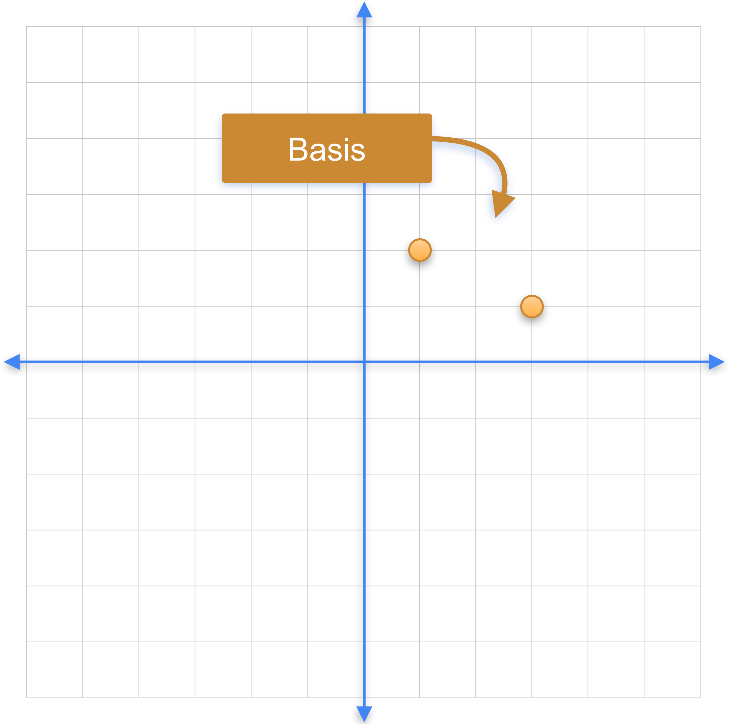


3	1
1	2



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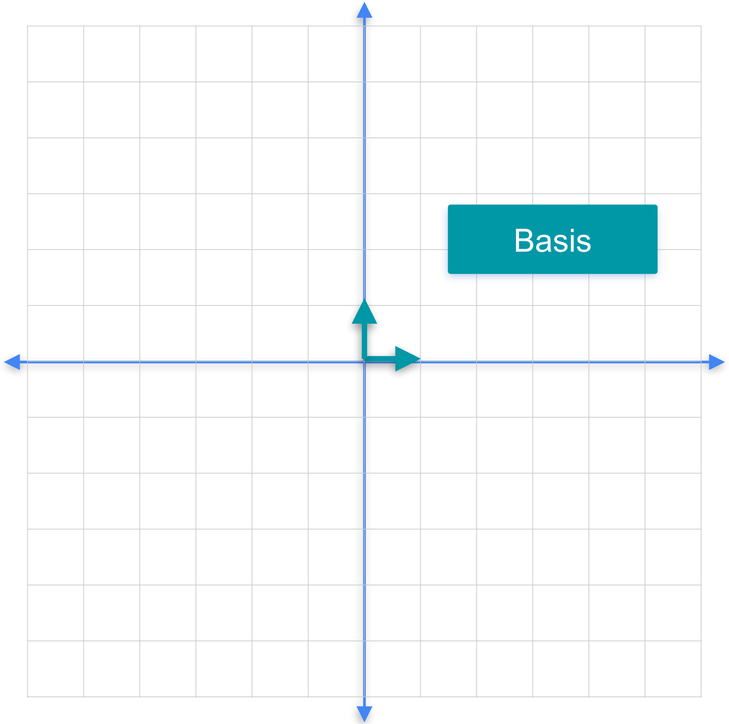
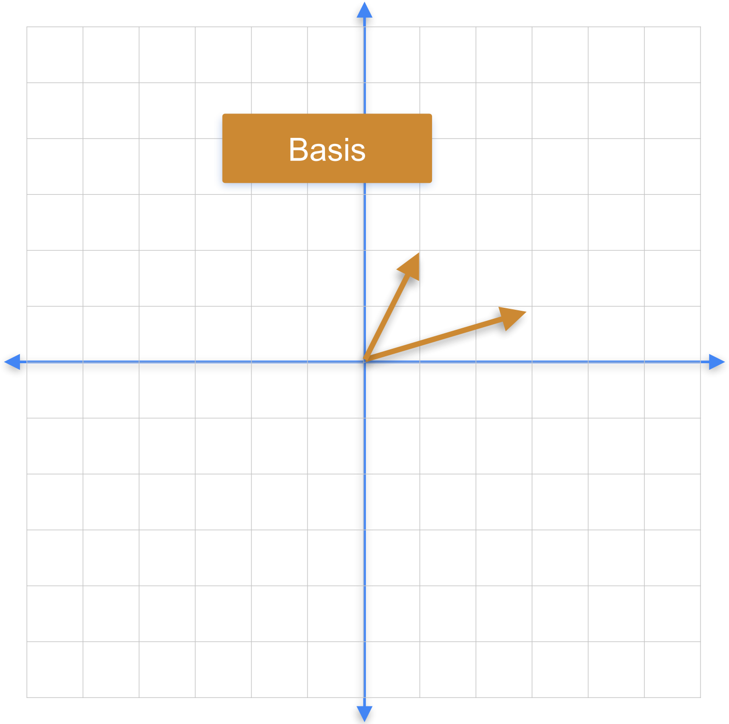


3	1
1	2



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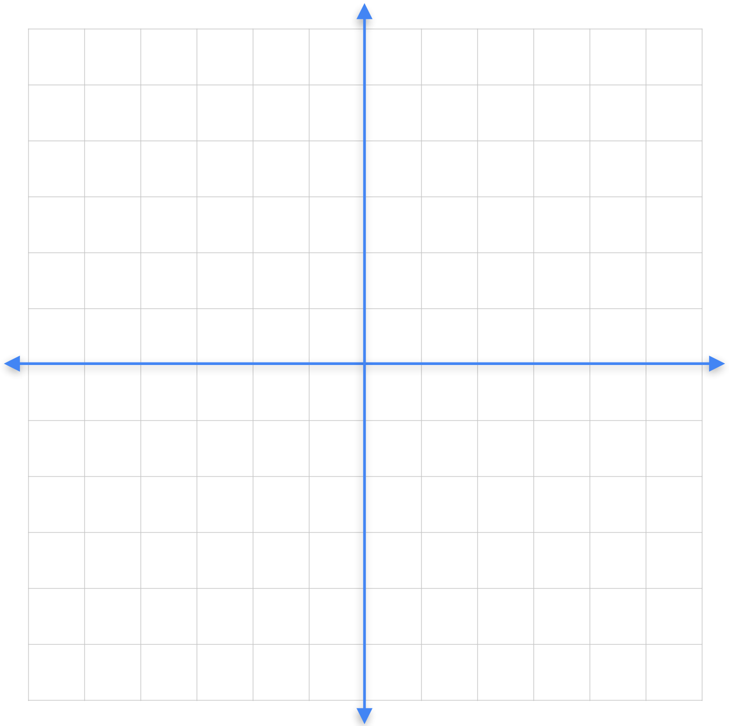
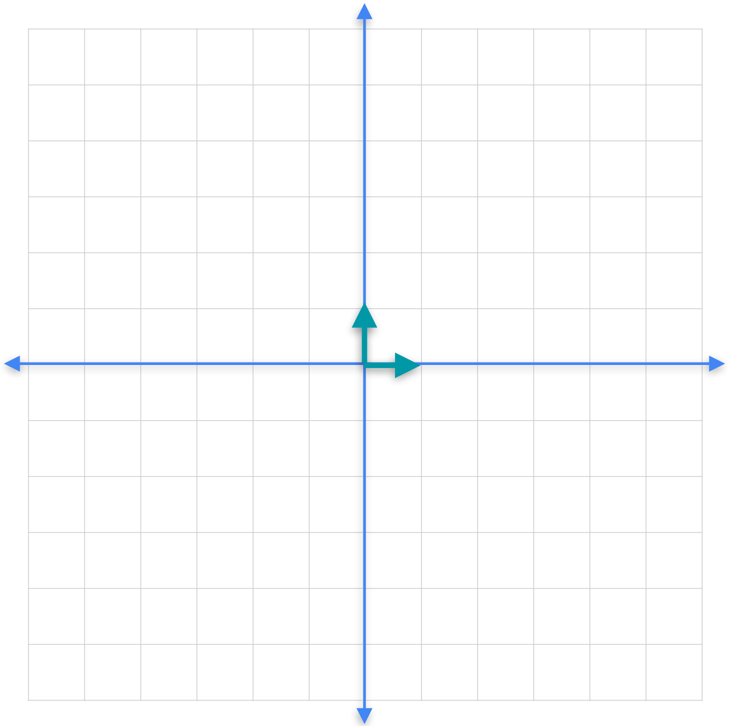


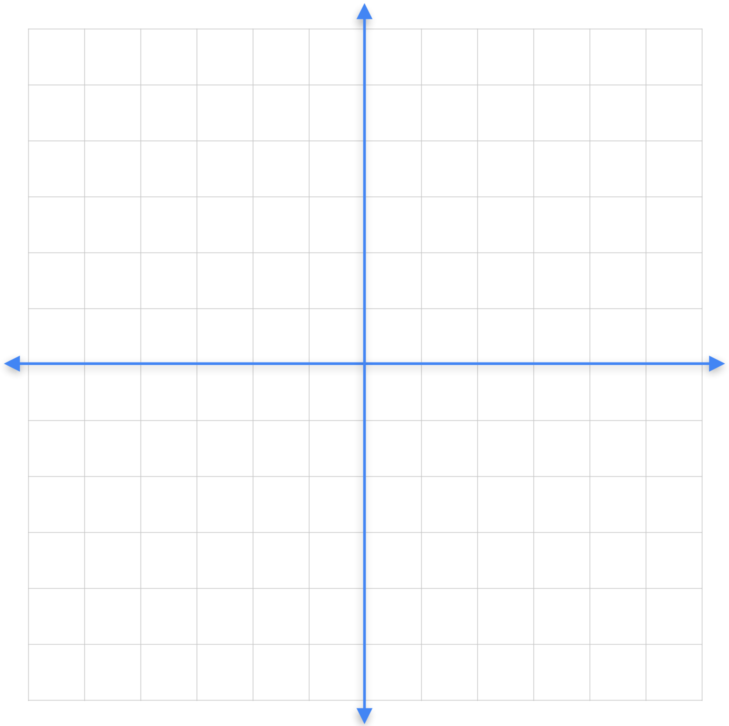
Bases

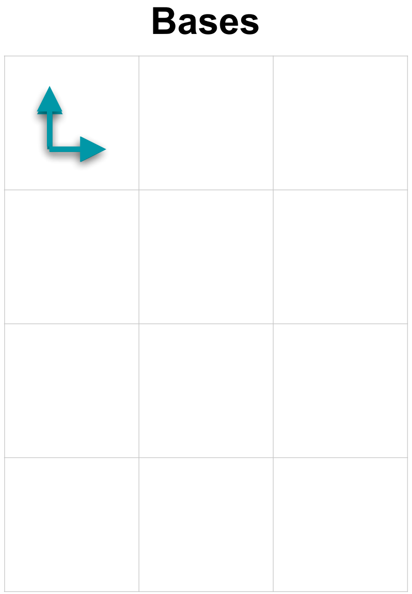
Bases



Bases

Bases





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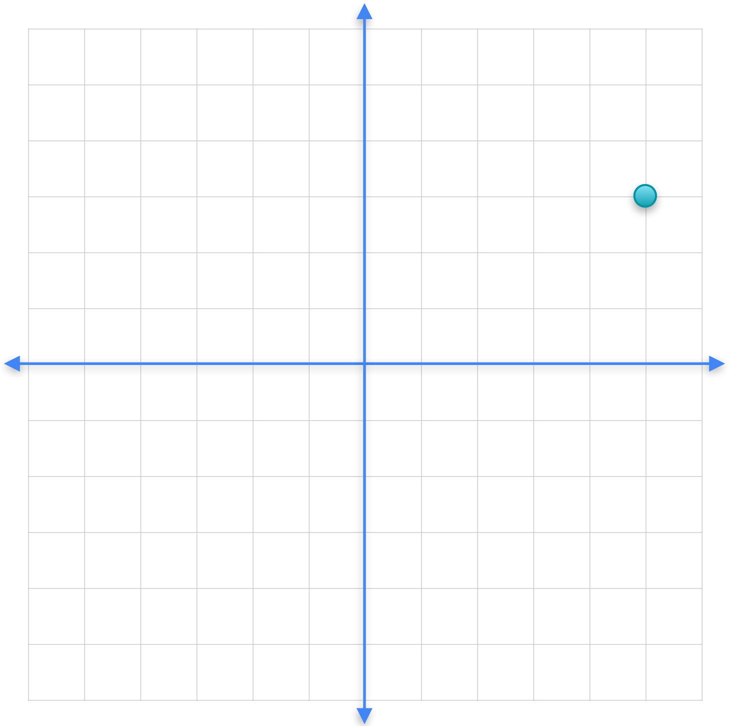

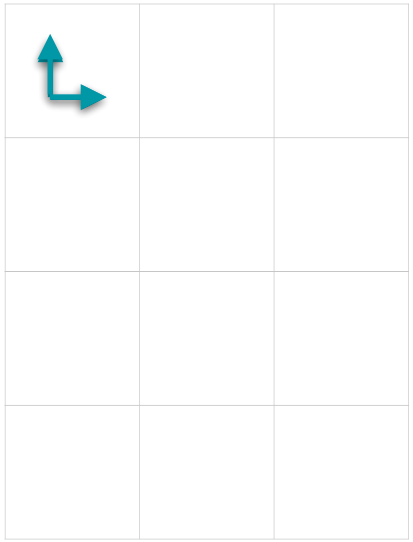

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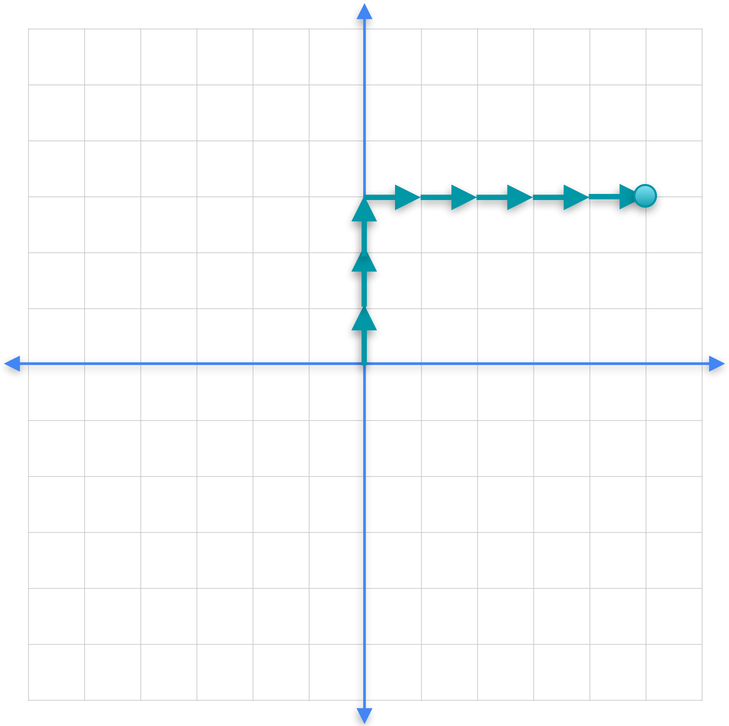

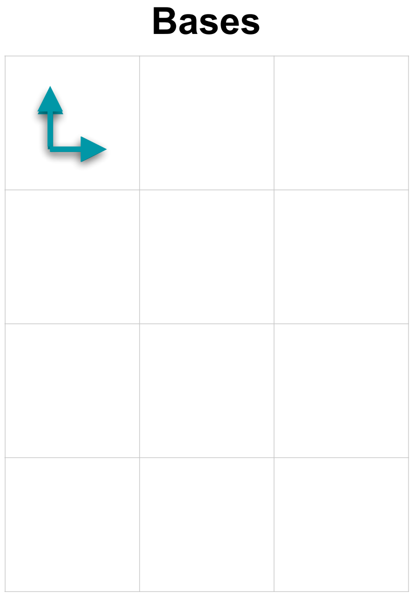

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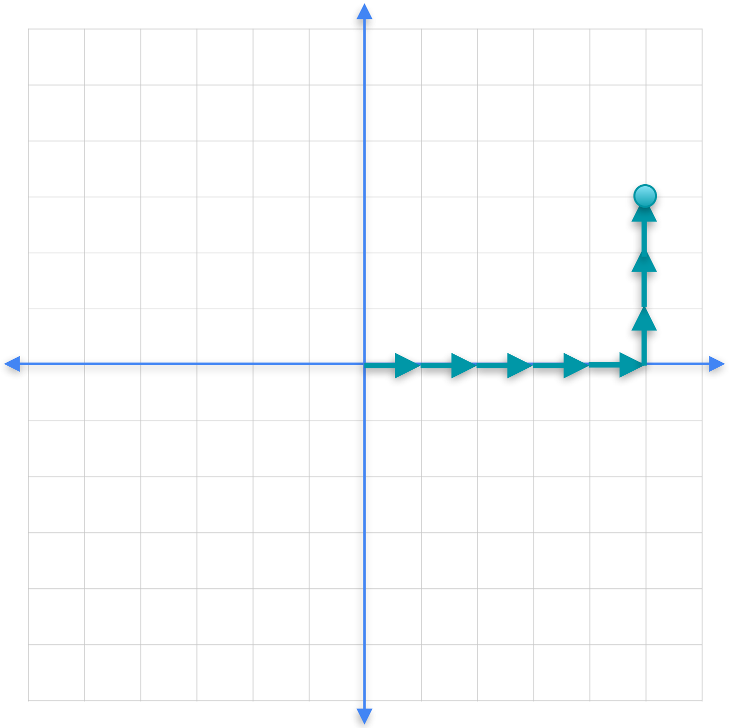

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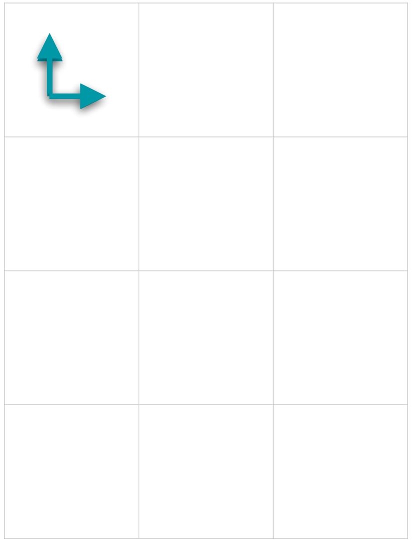

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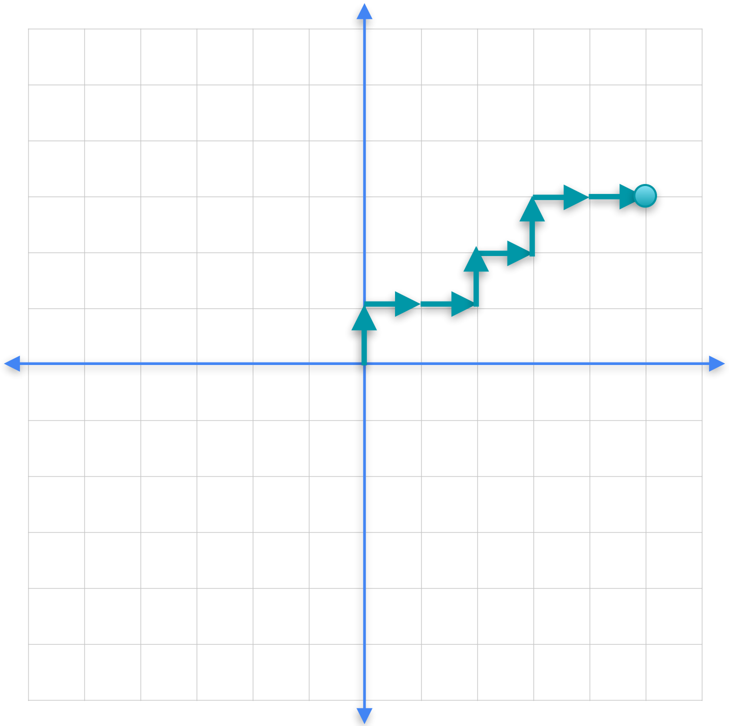

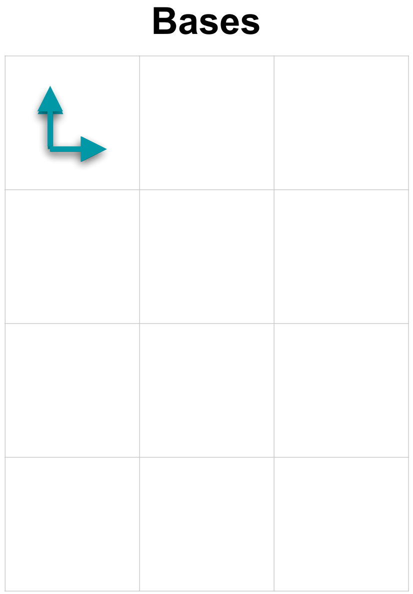

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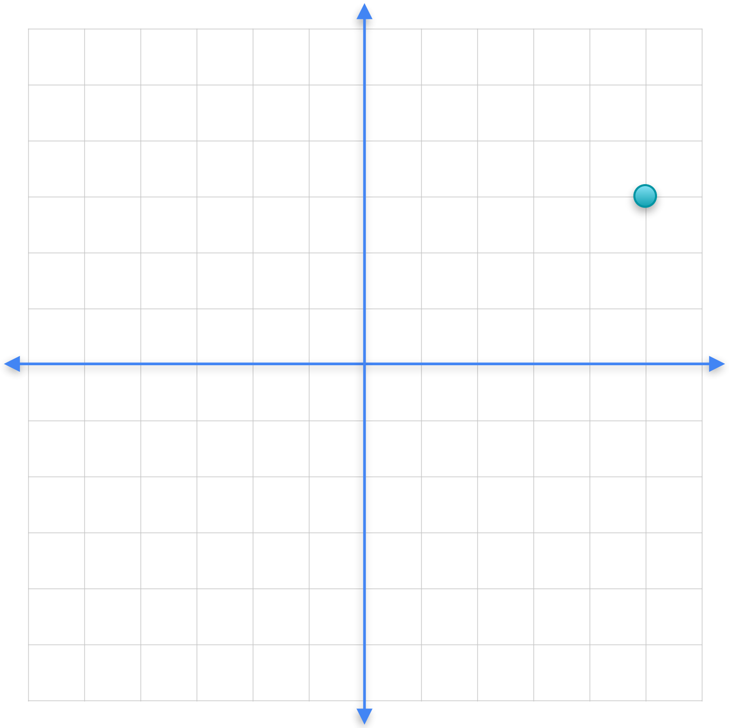

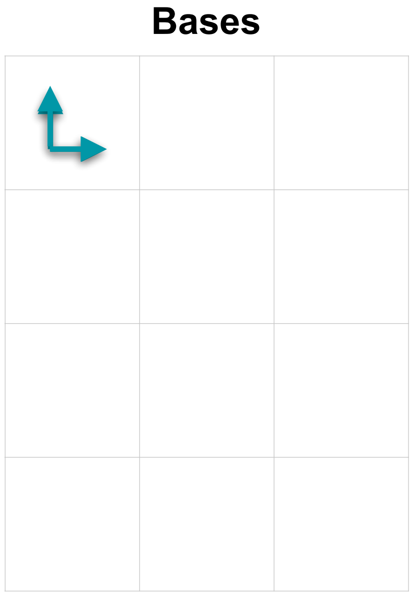

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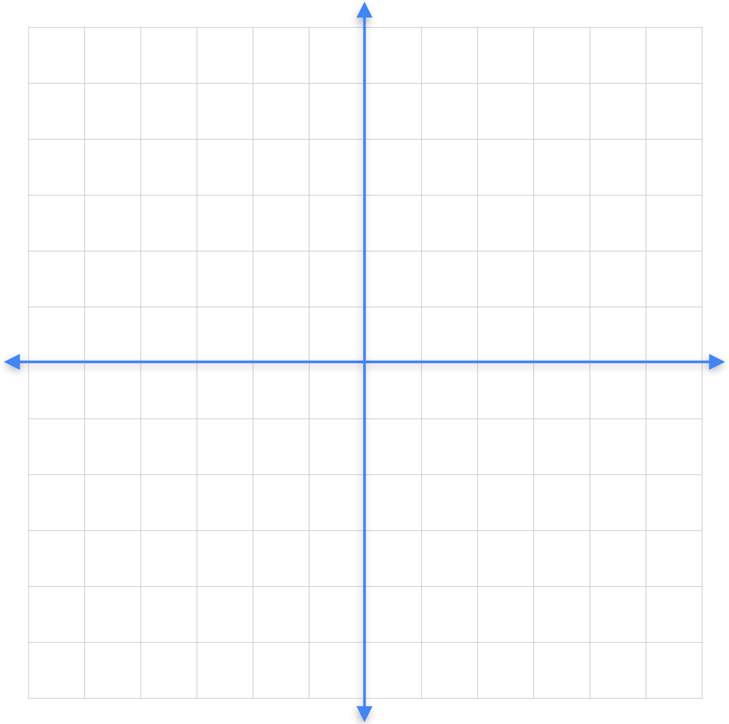

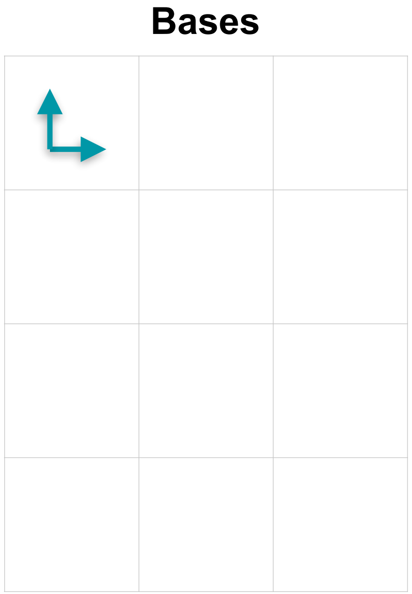

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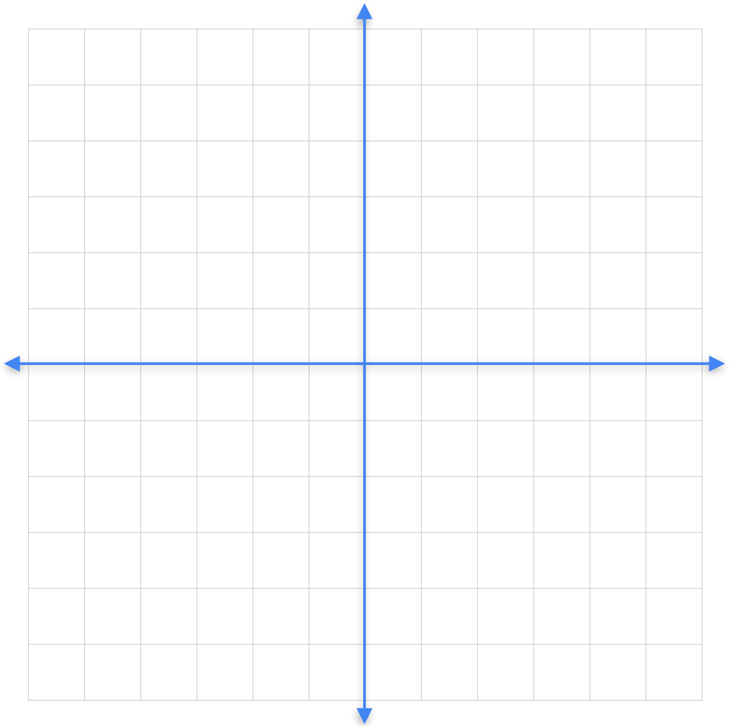

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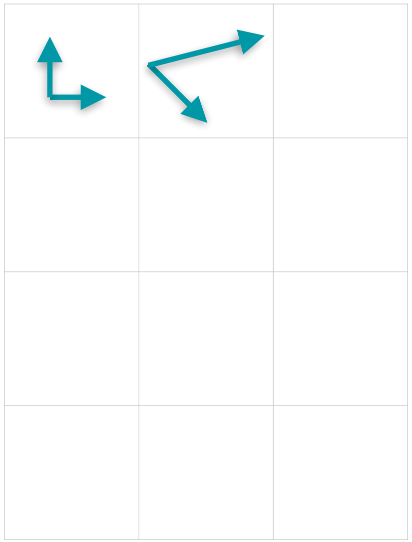

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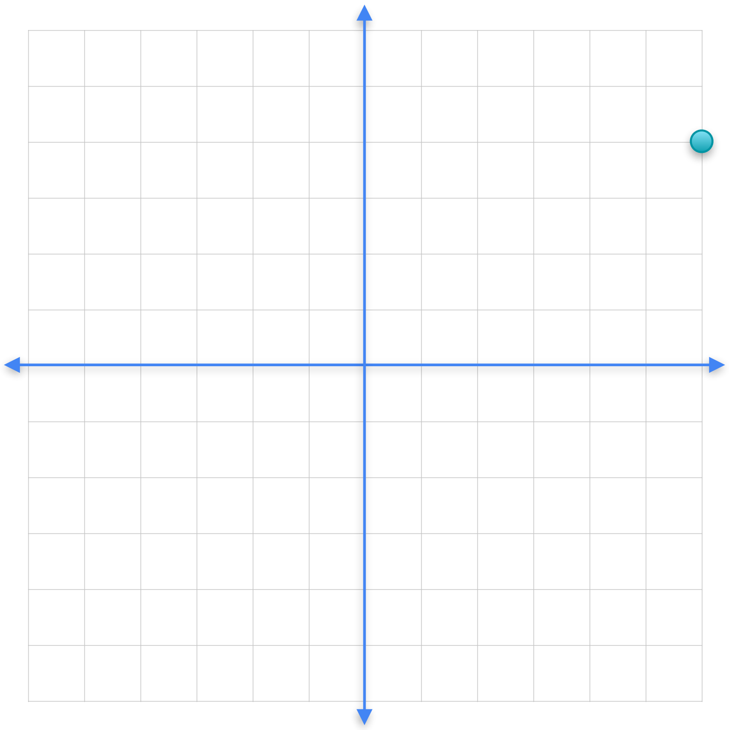

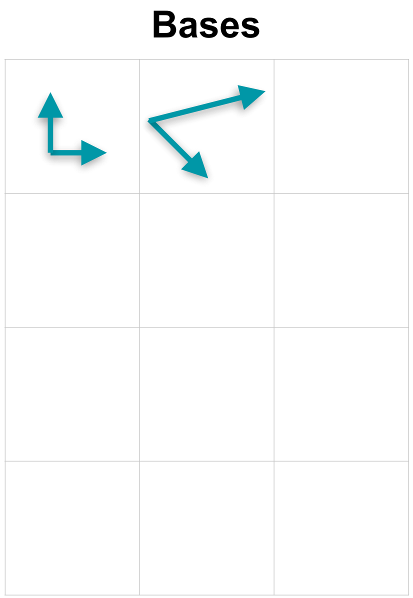

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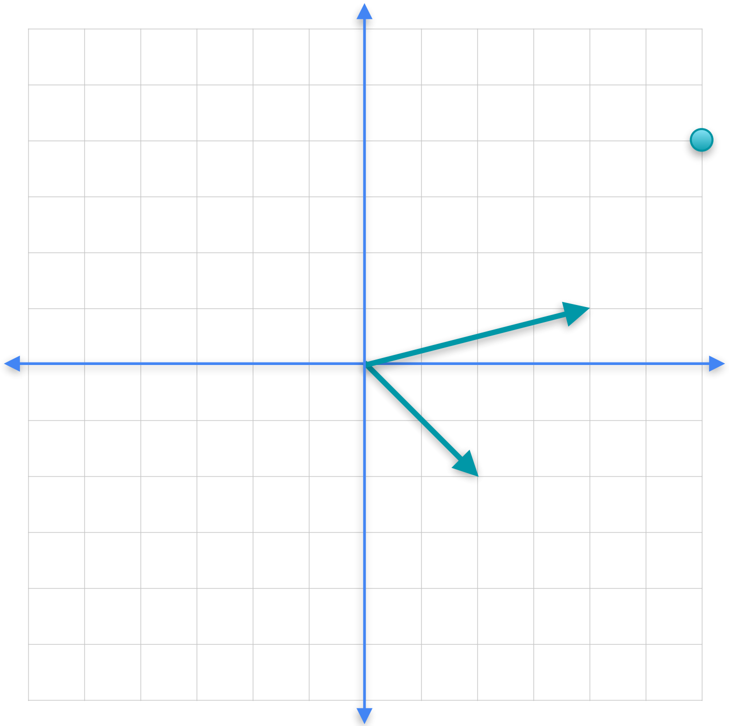

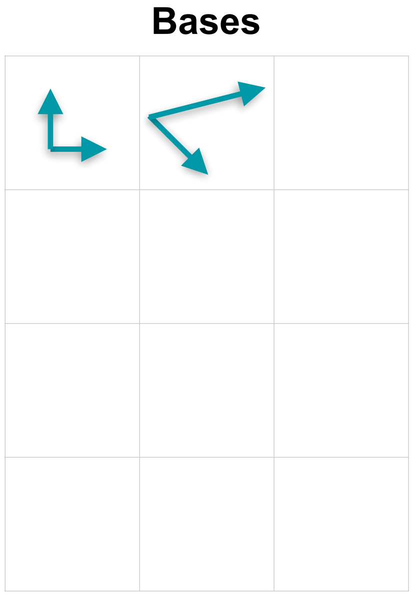

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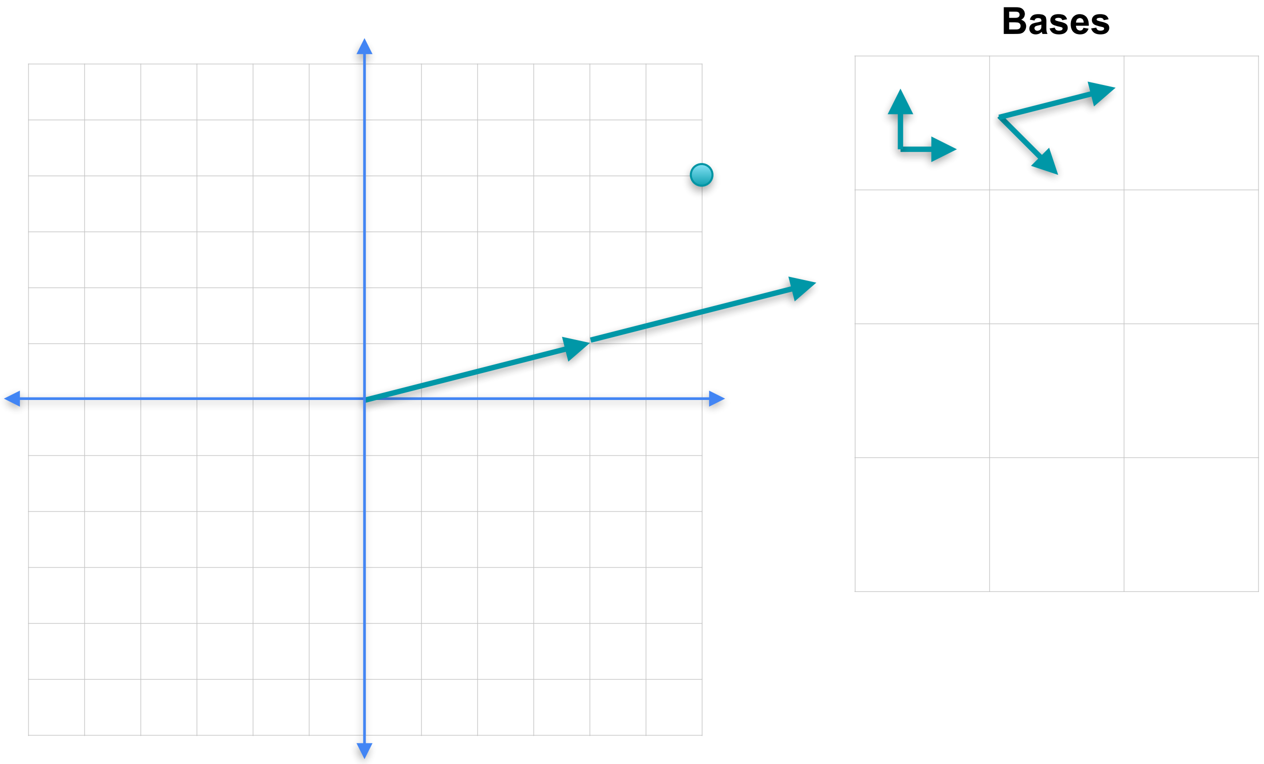

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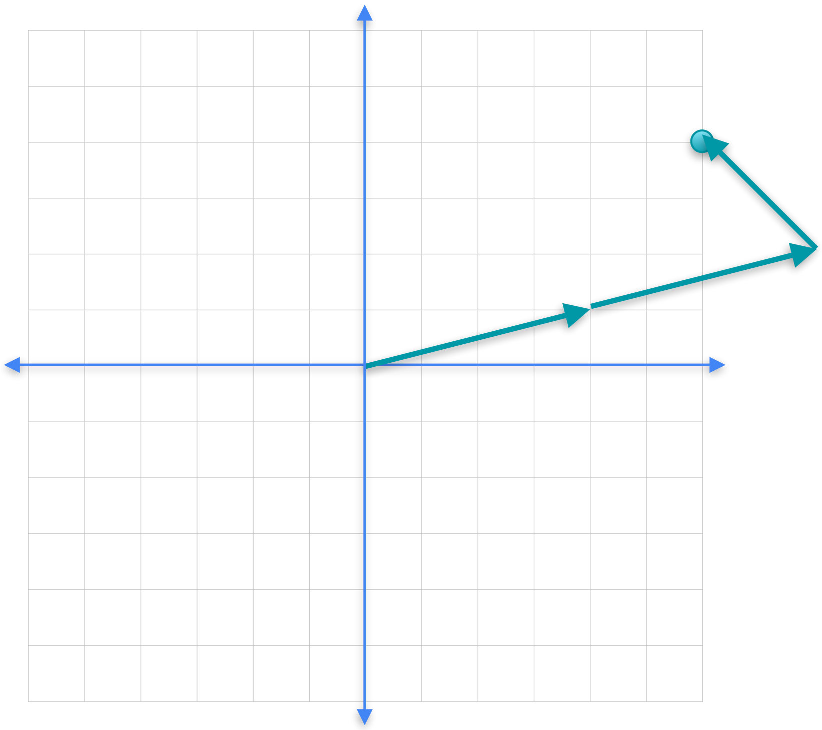

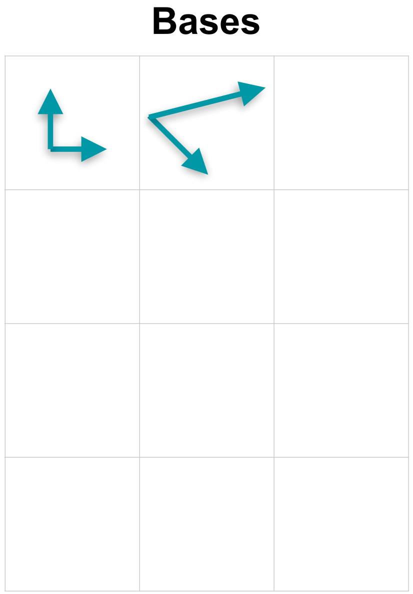

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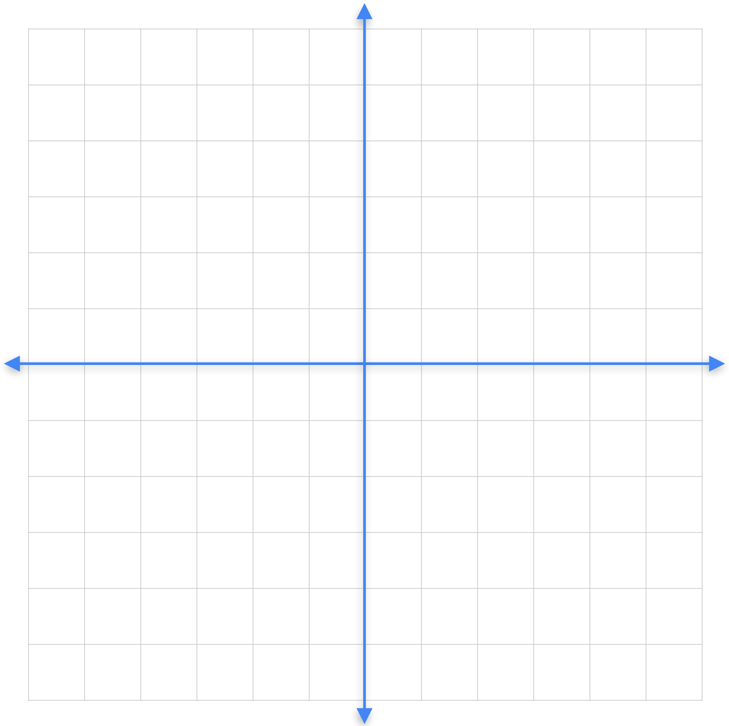

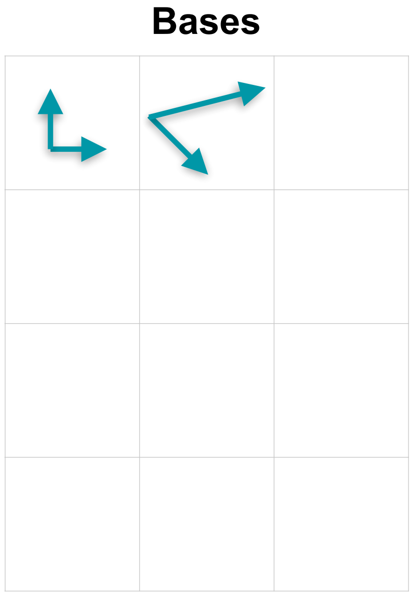

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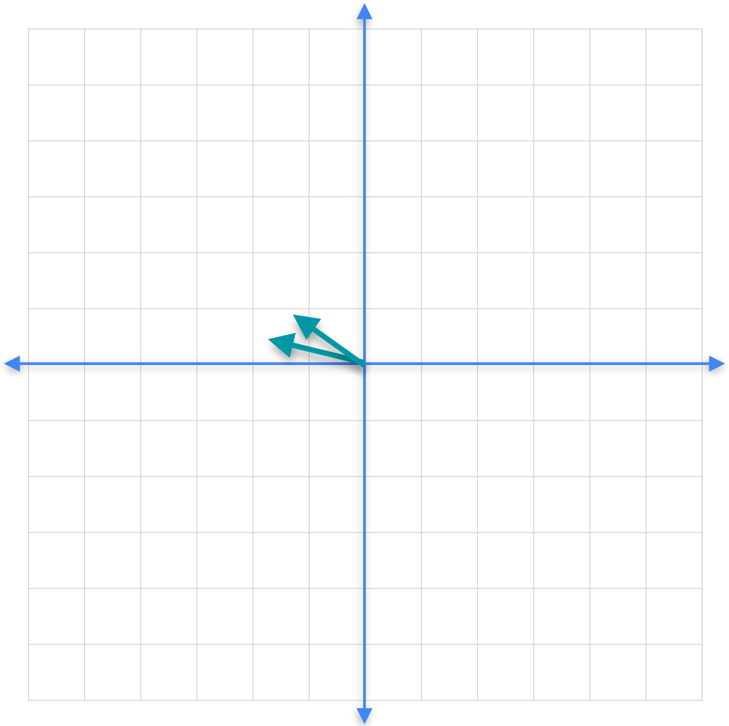

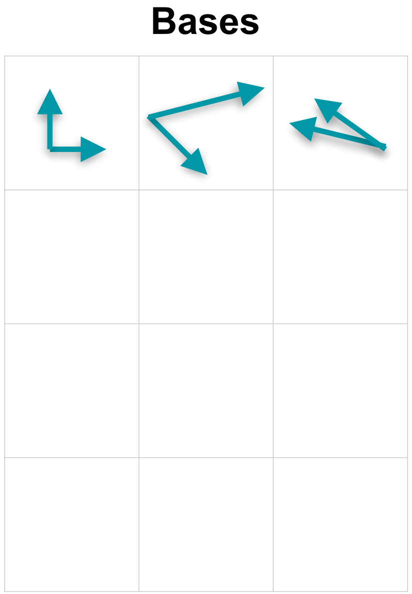

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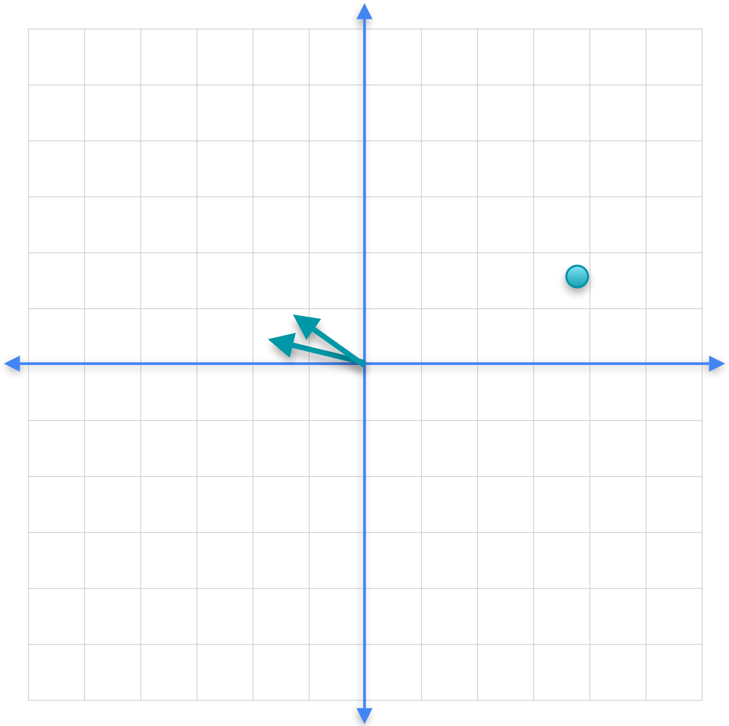

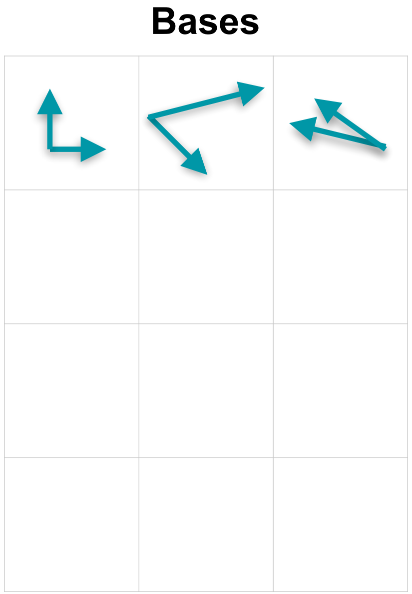

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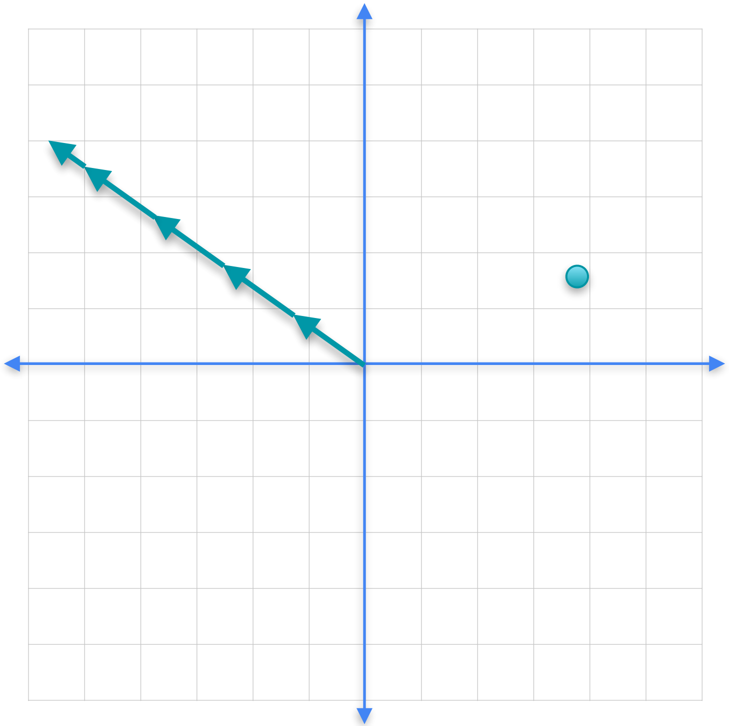

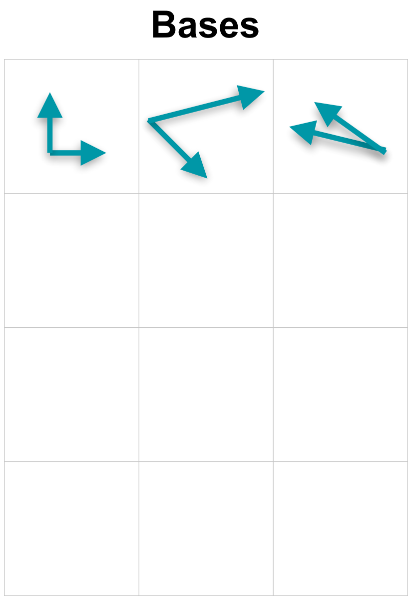

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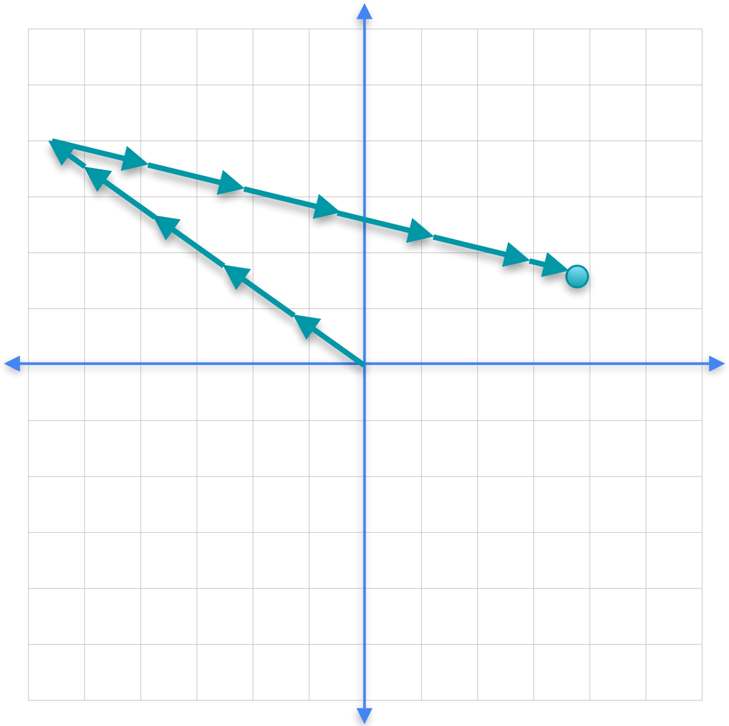

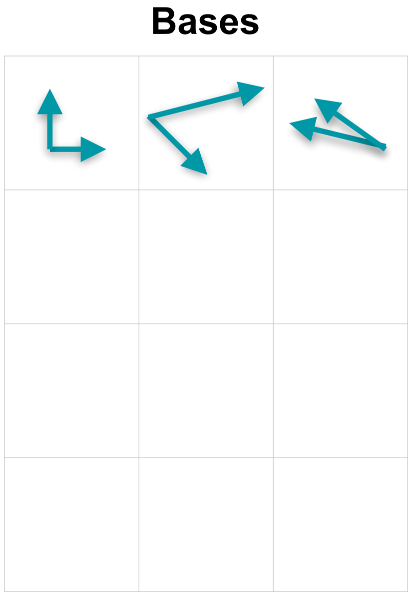

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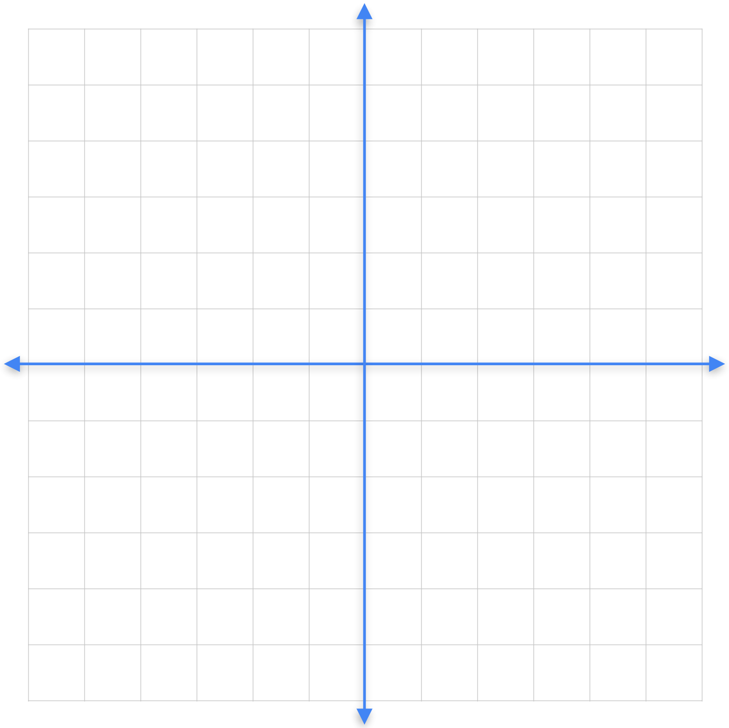

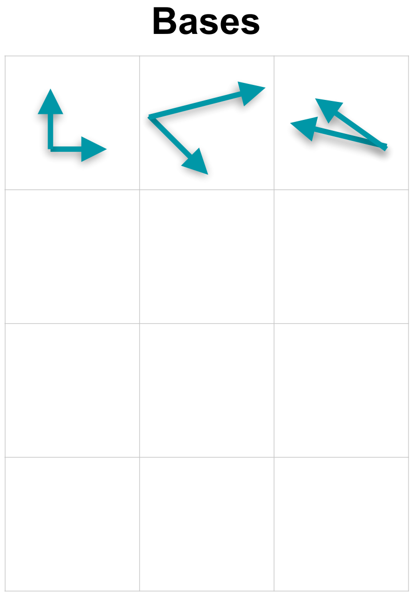

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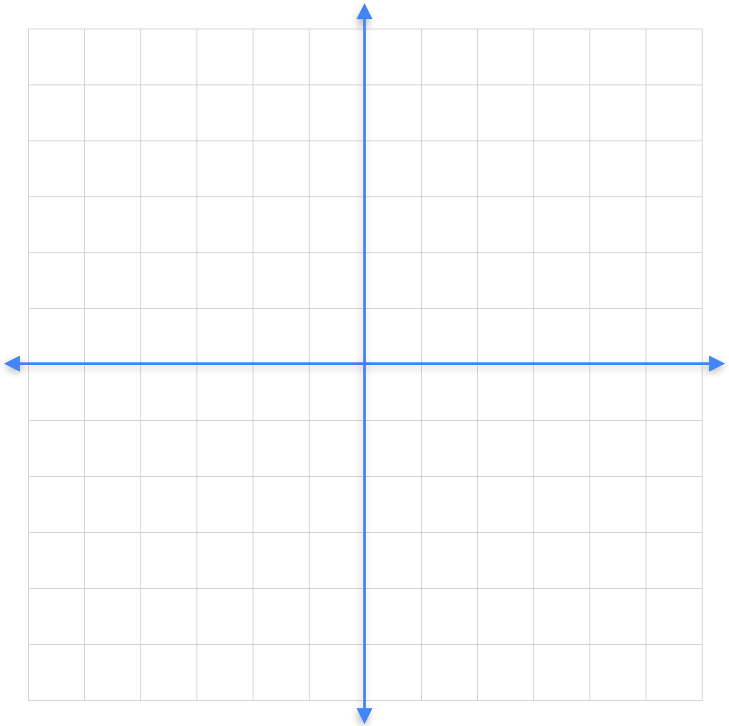

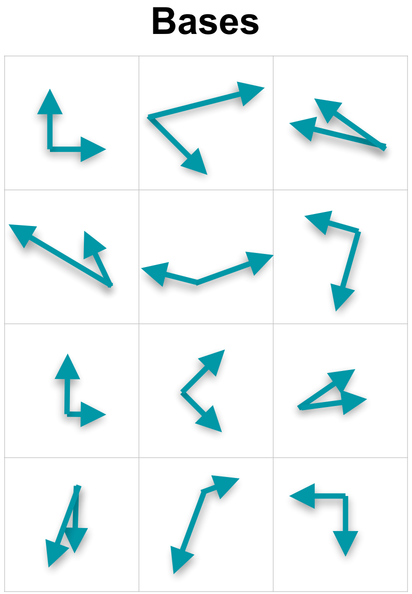

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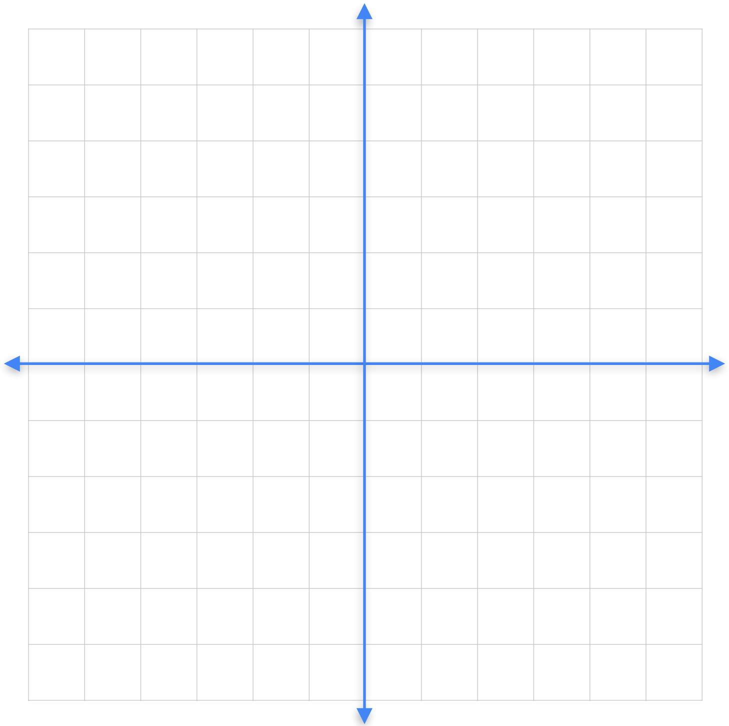

Not bases

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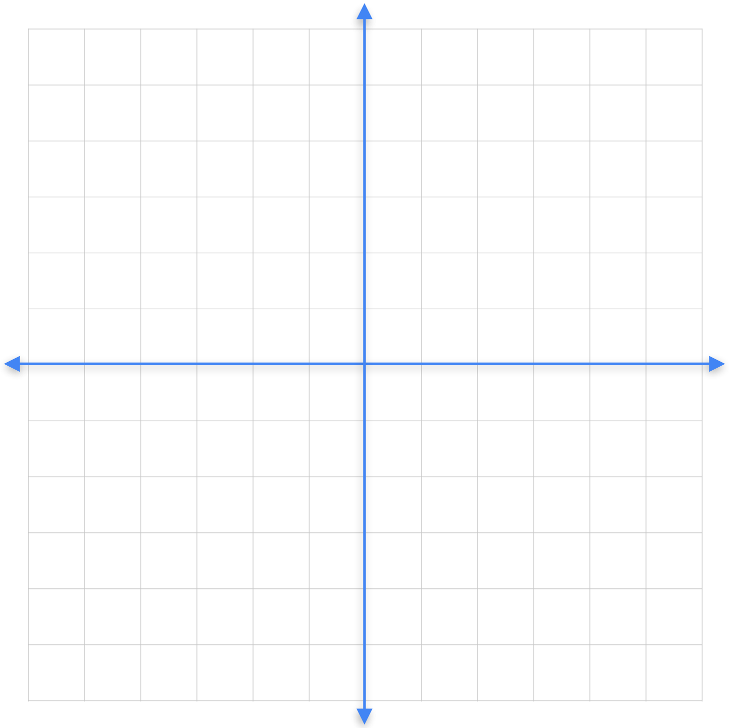

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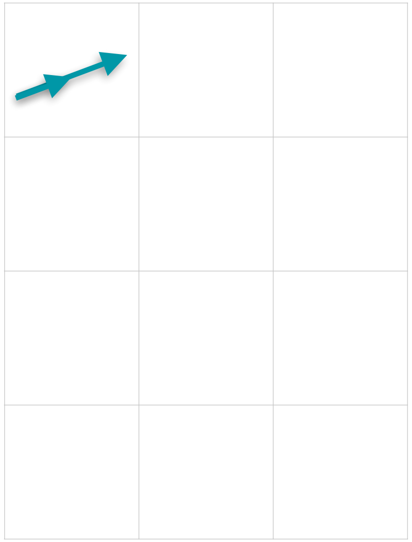

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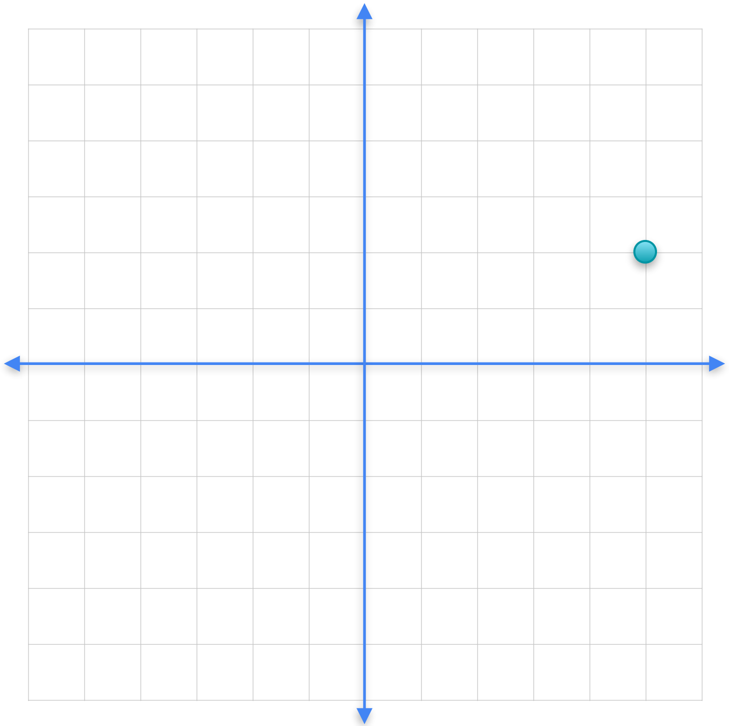

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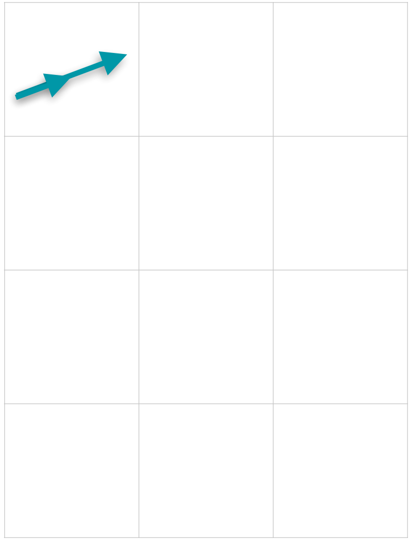

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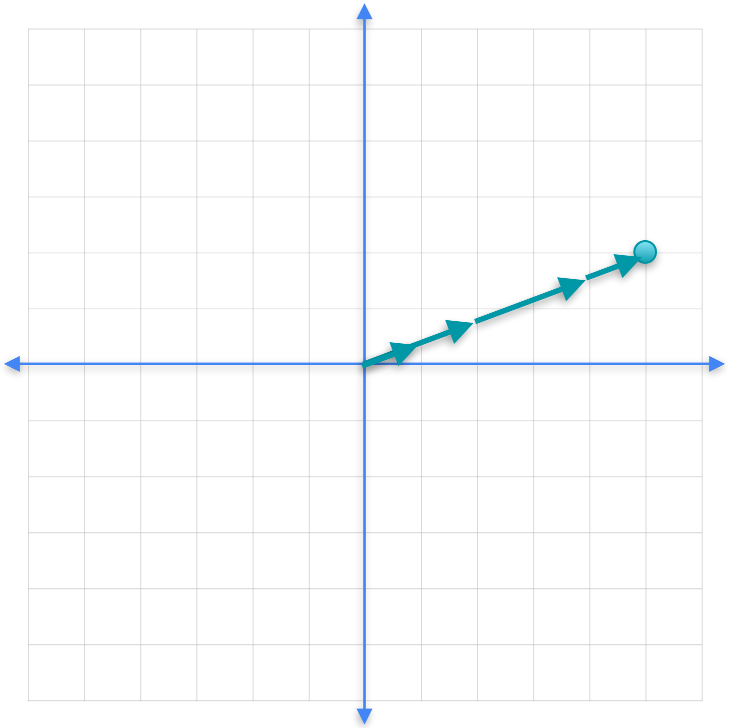

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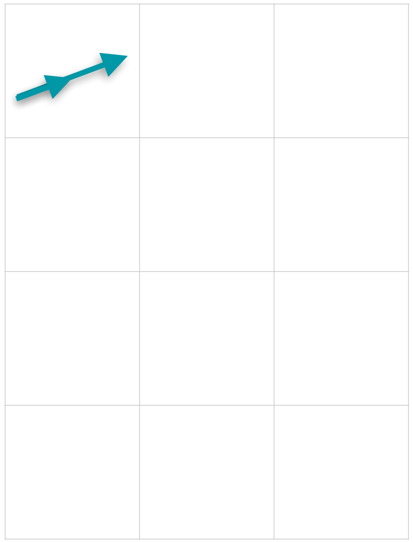

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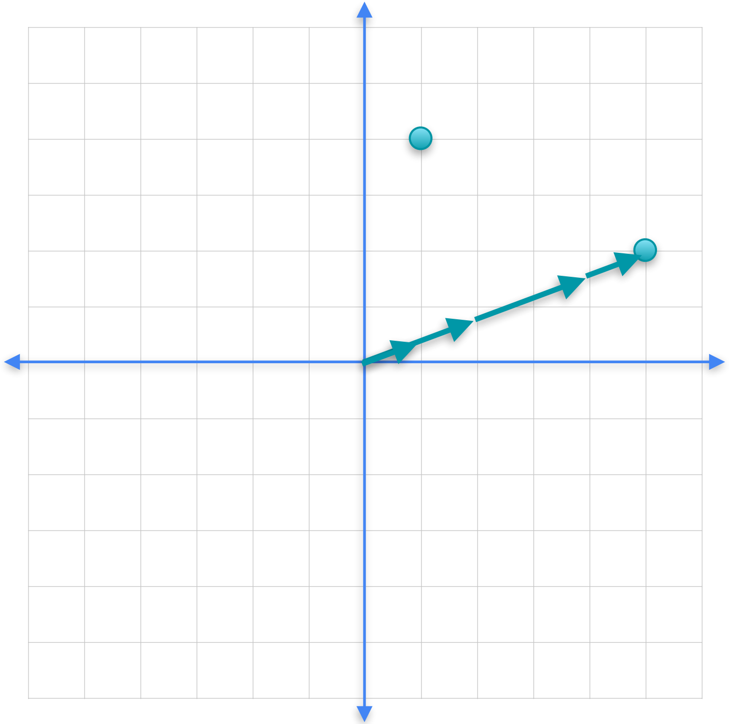

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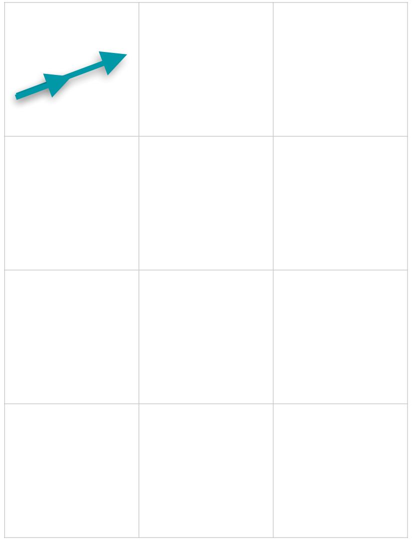

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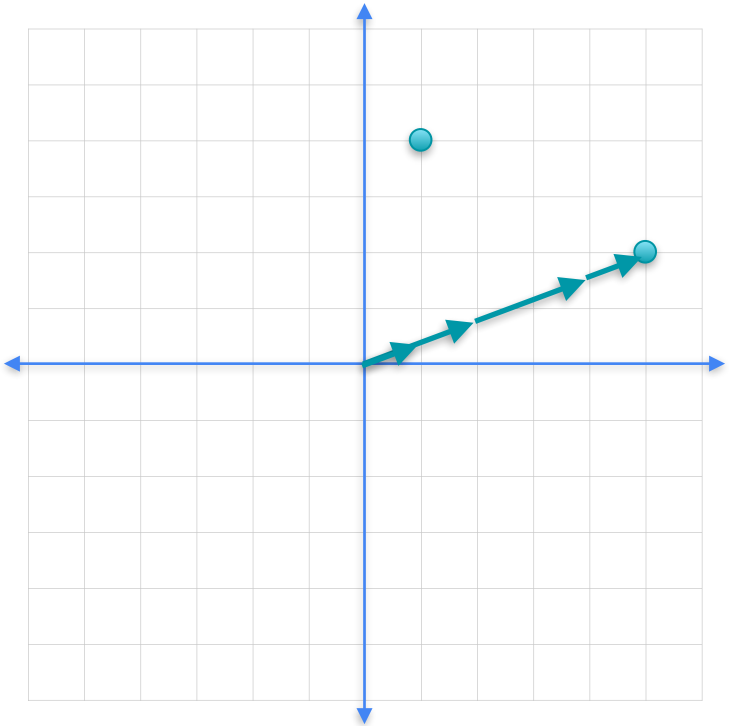

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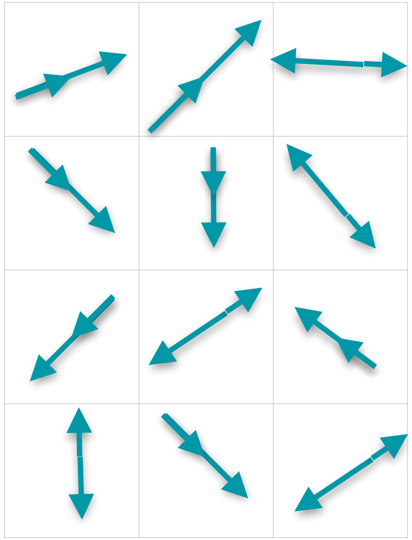
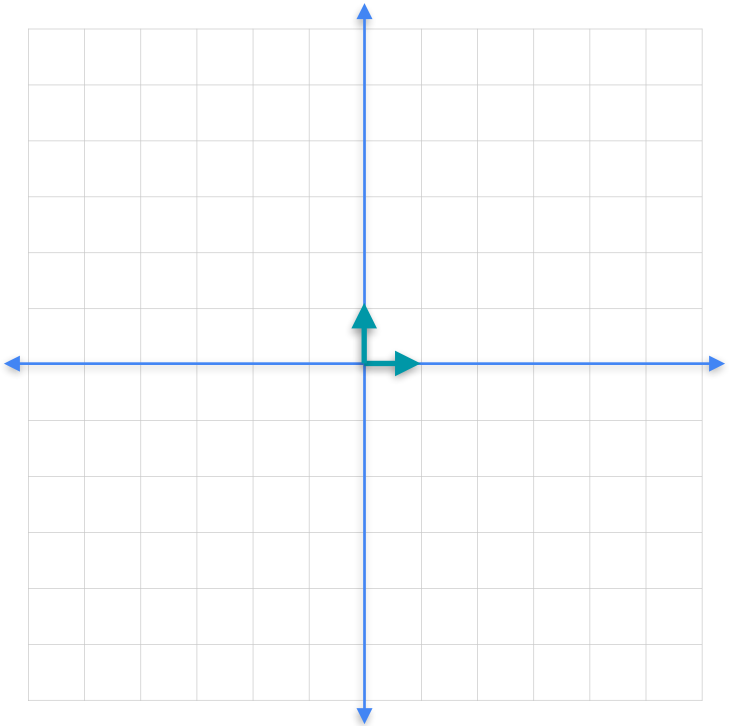


DeepLearning.AI

Determinants and Eigenvectors

Span

Span



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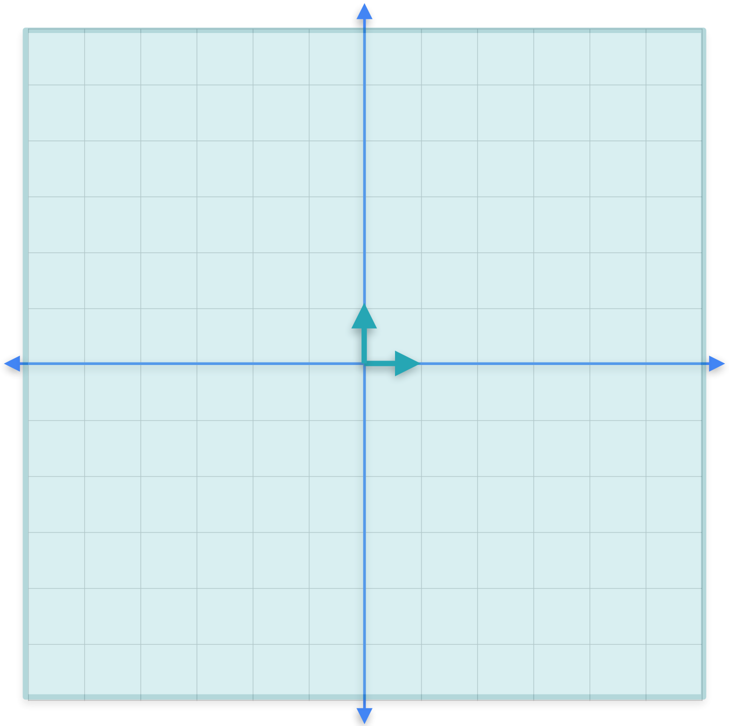

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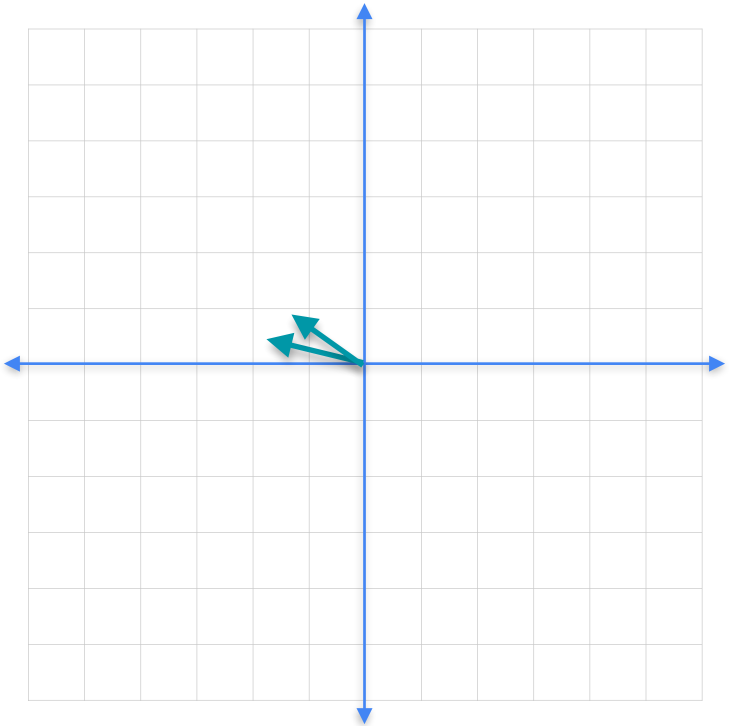

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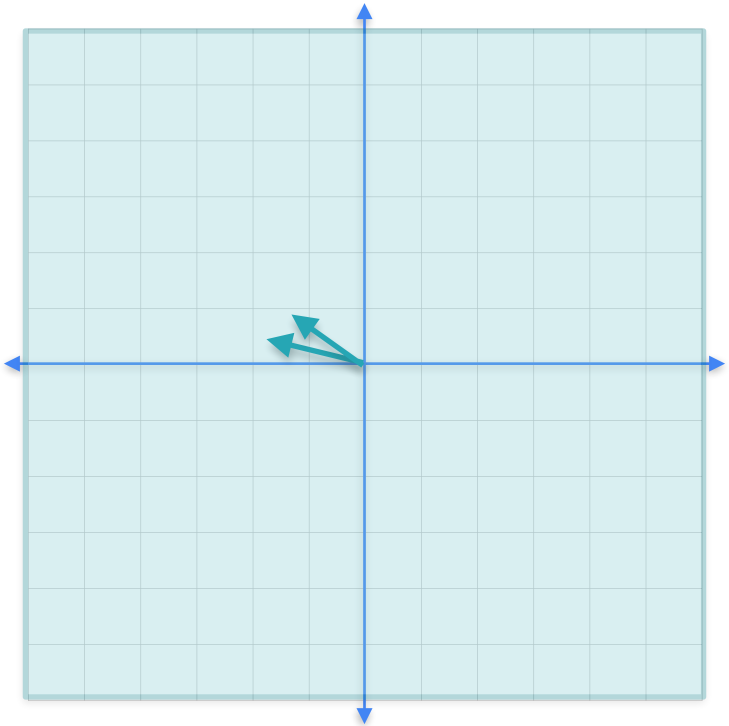

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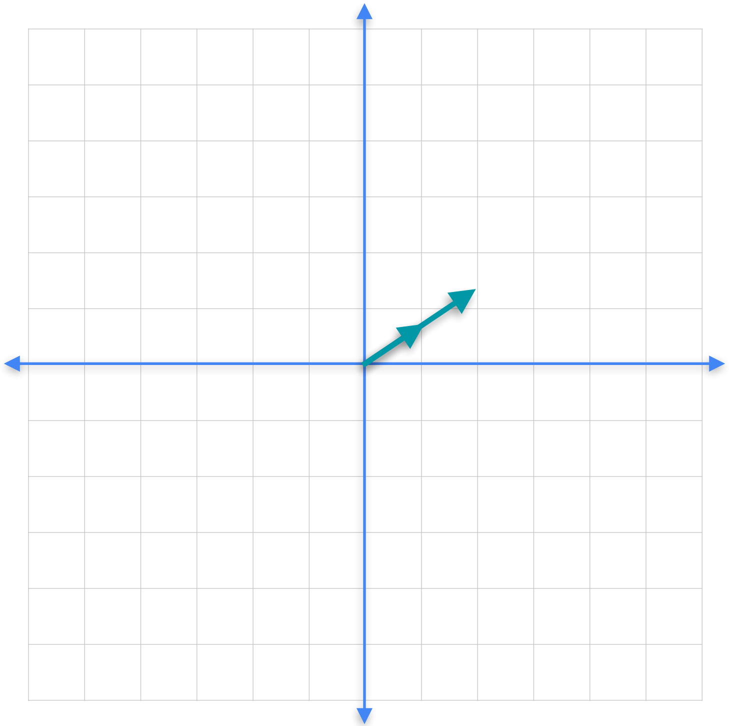

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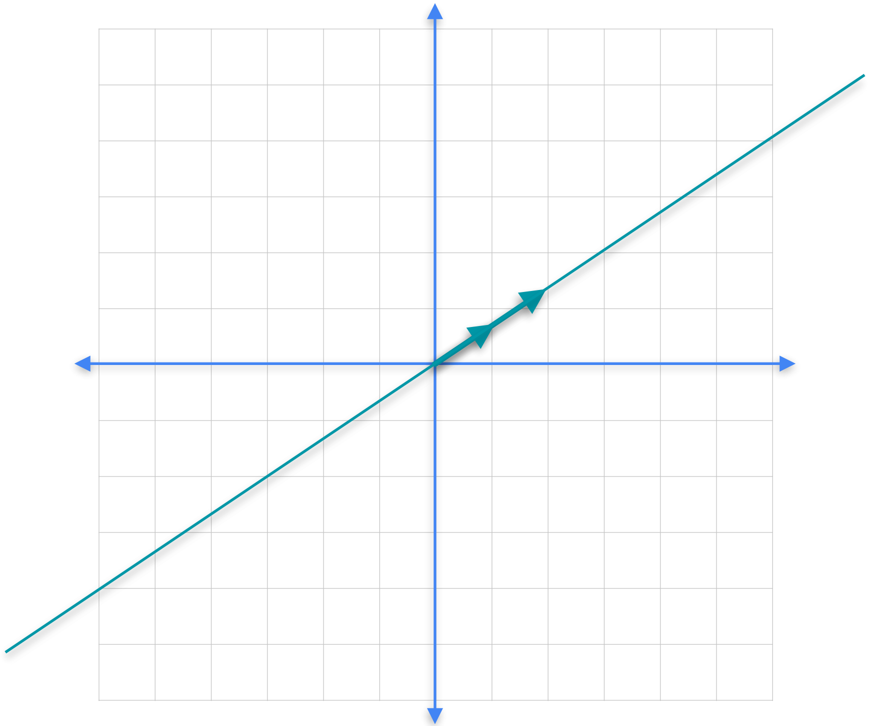

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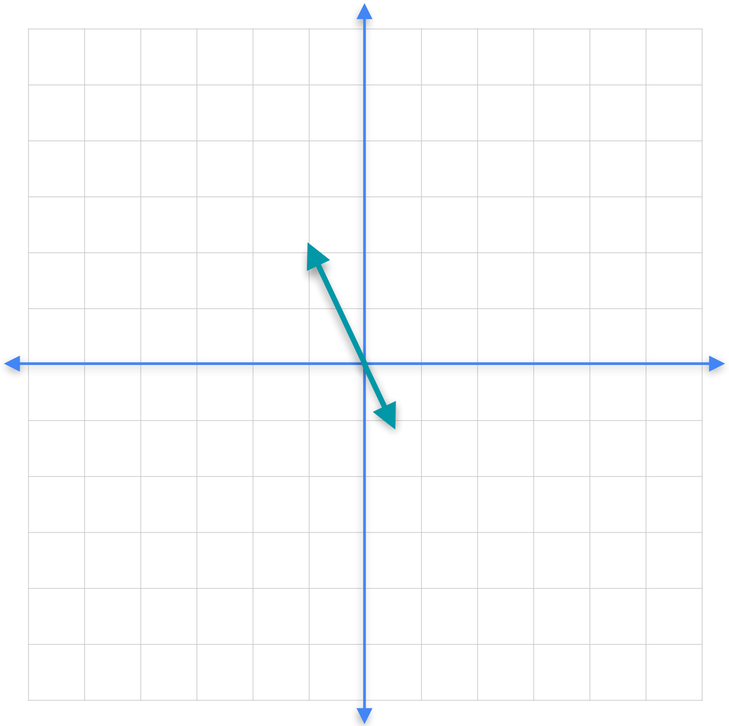

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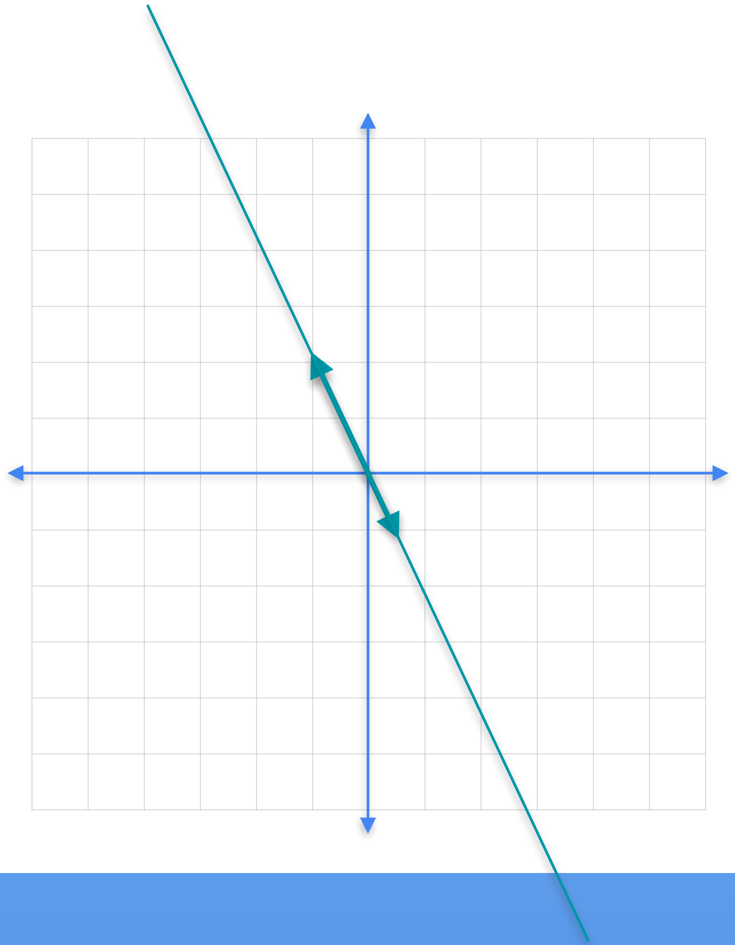

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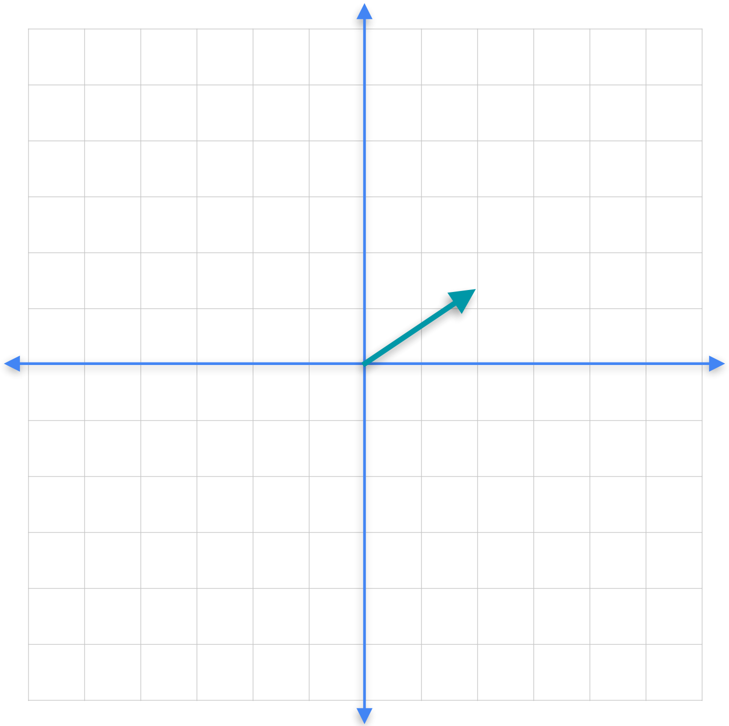

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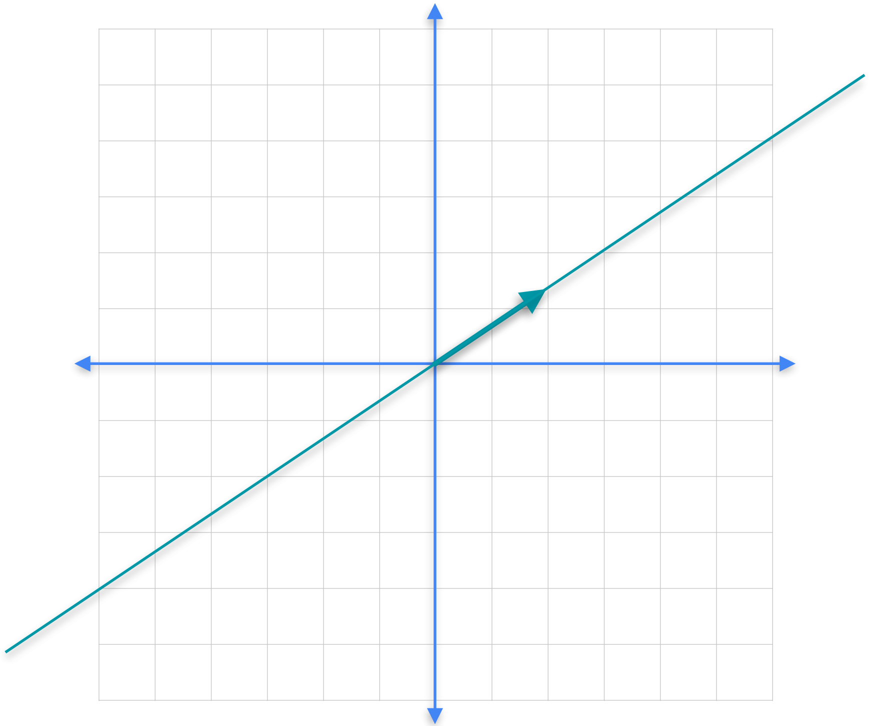

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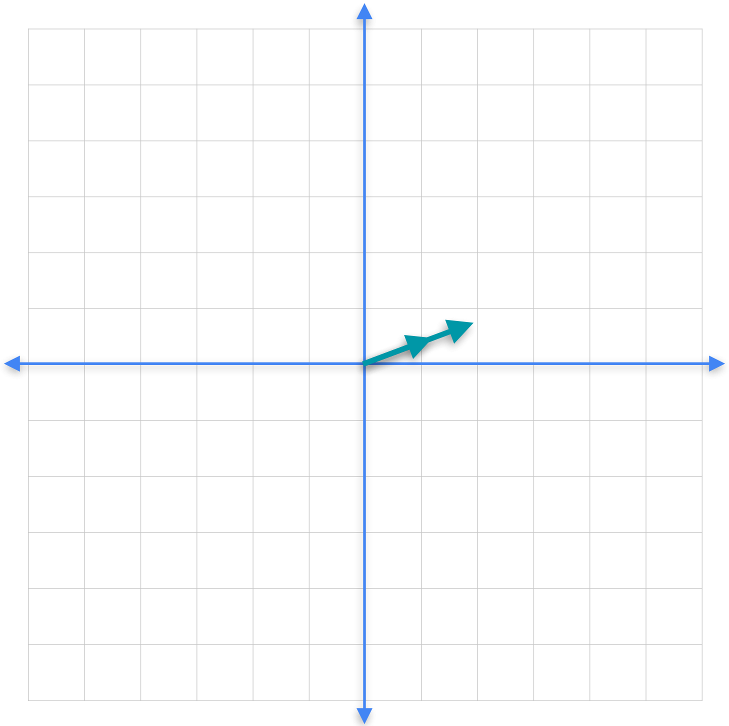

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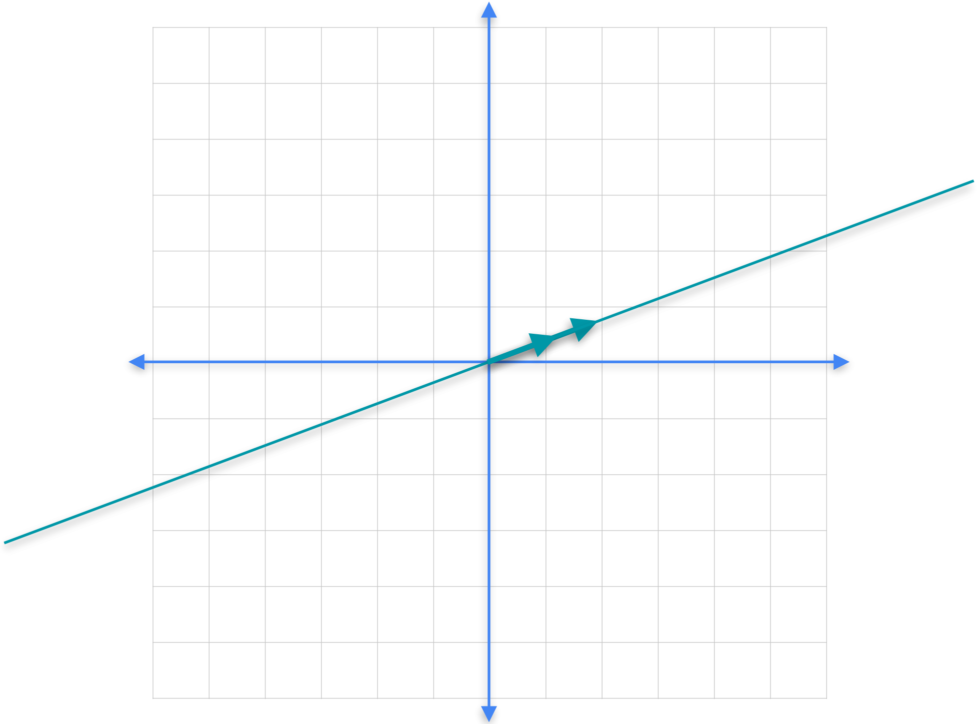

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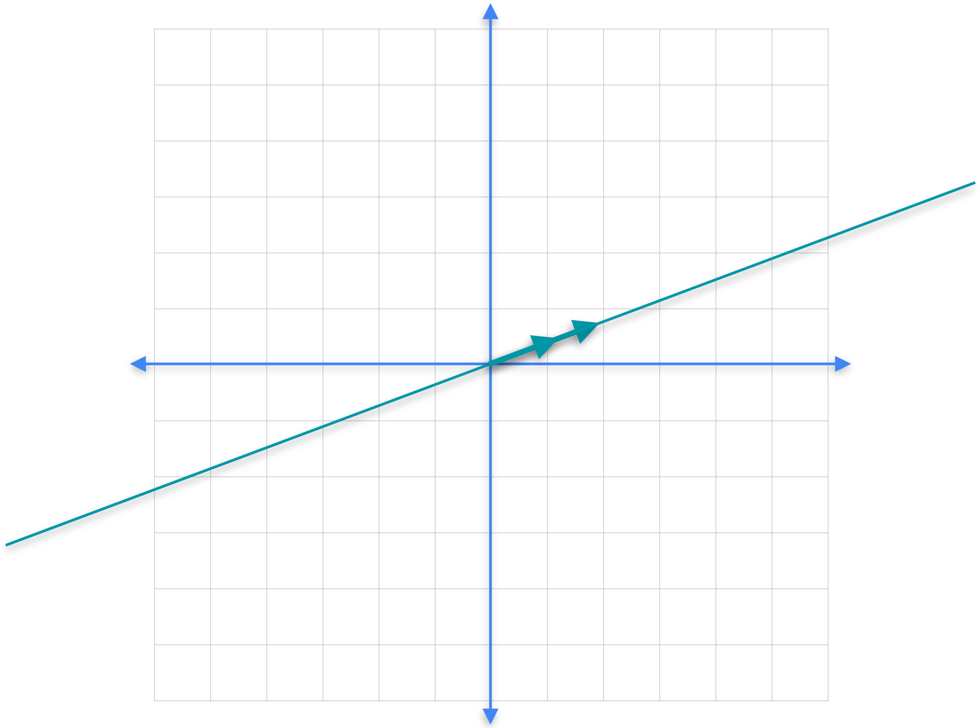

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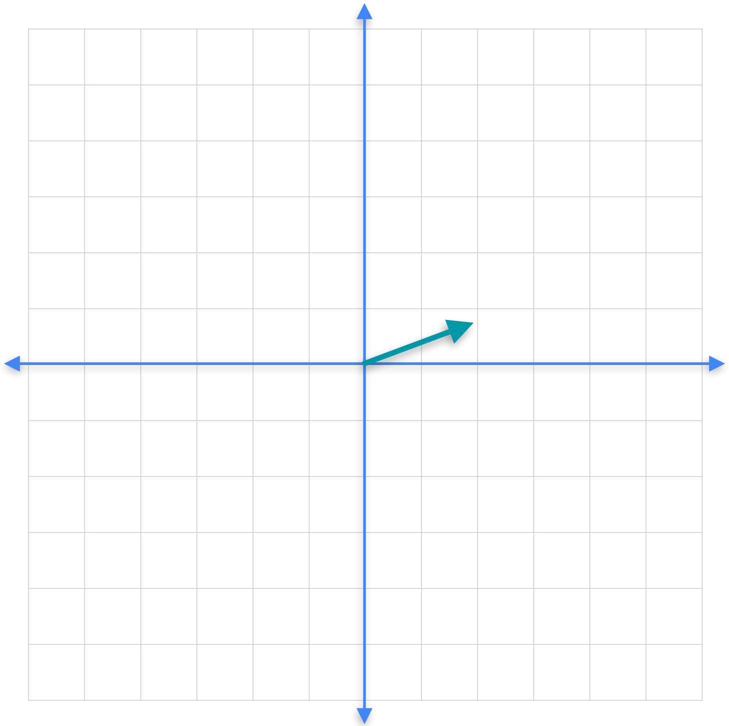
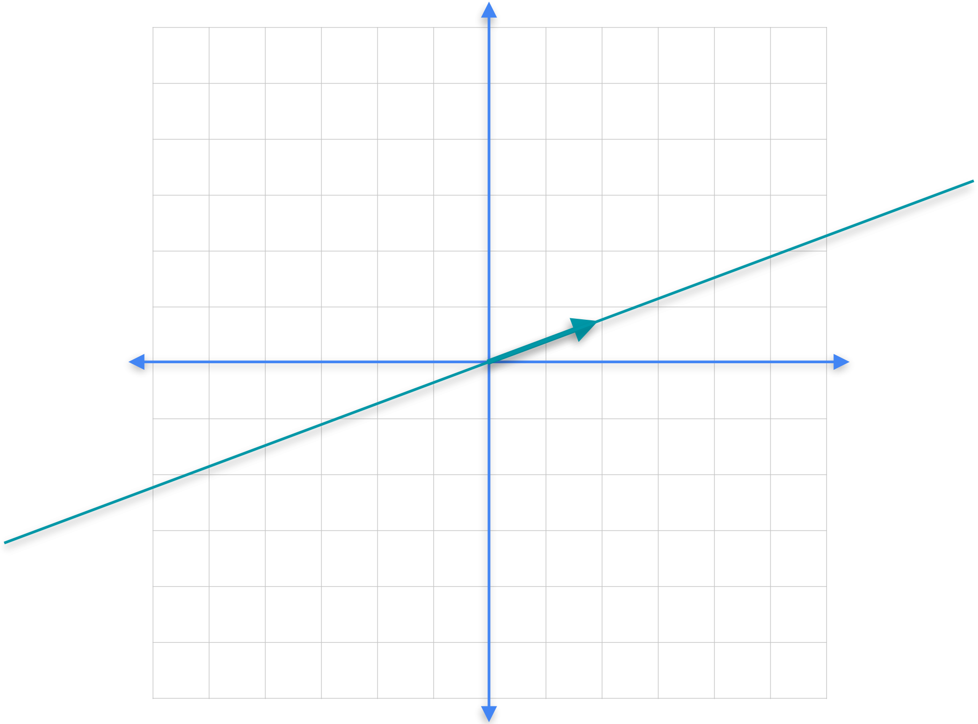
Is this a basis for something?



Is this a basis for something?

Bases



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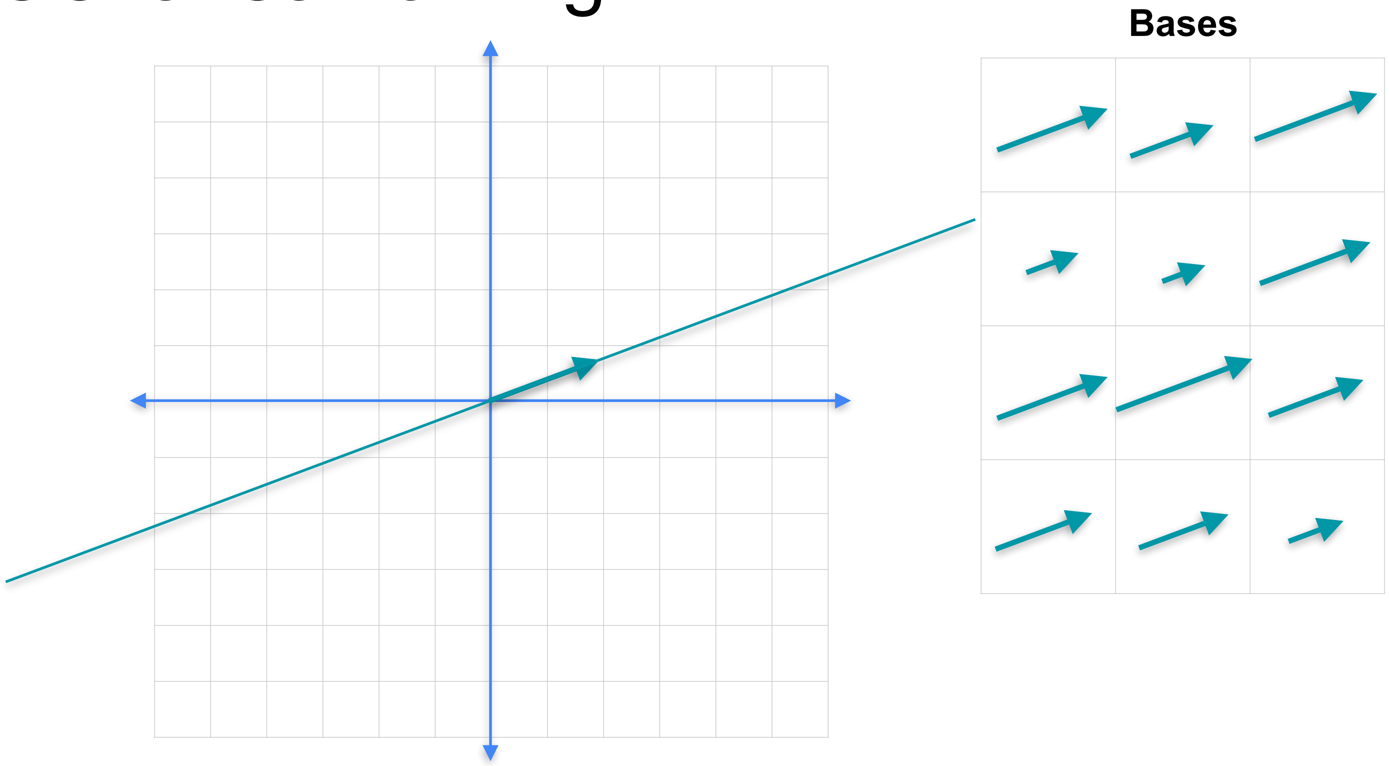
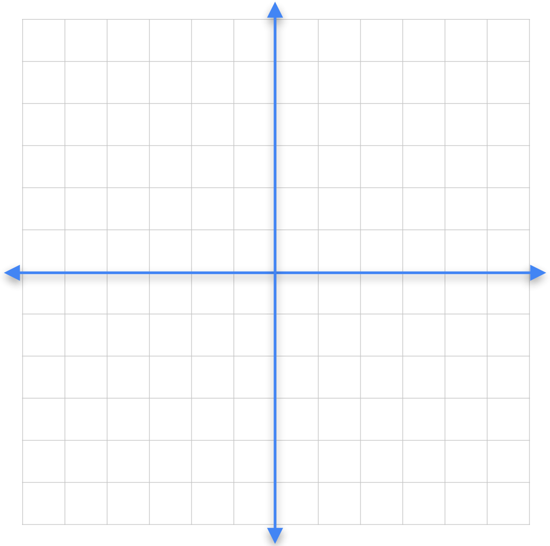
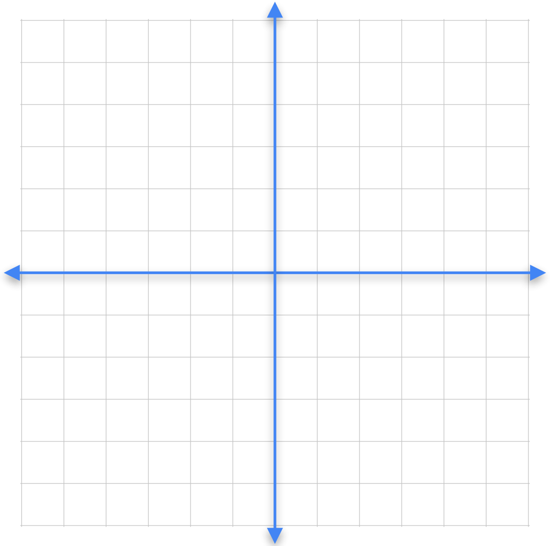


A basis is a minimal spanning set





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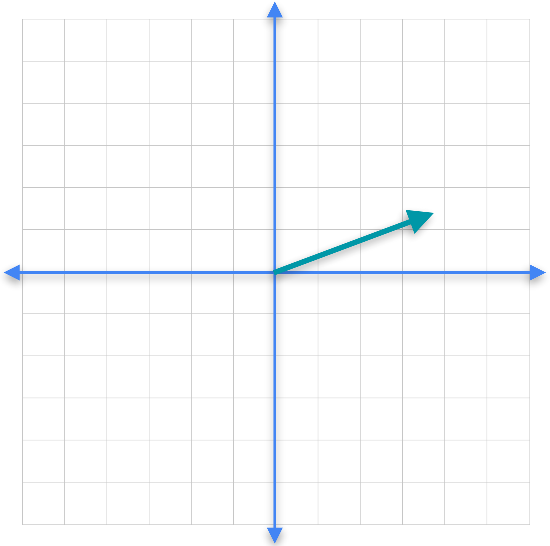

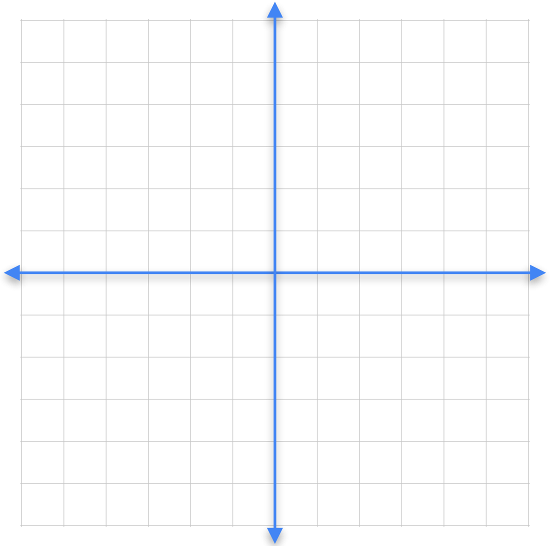

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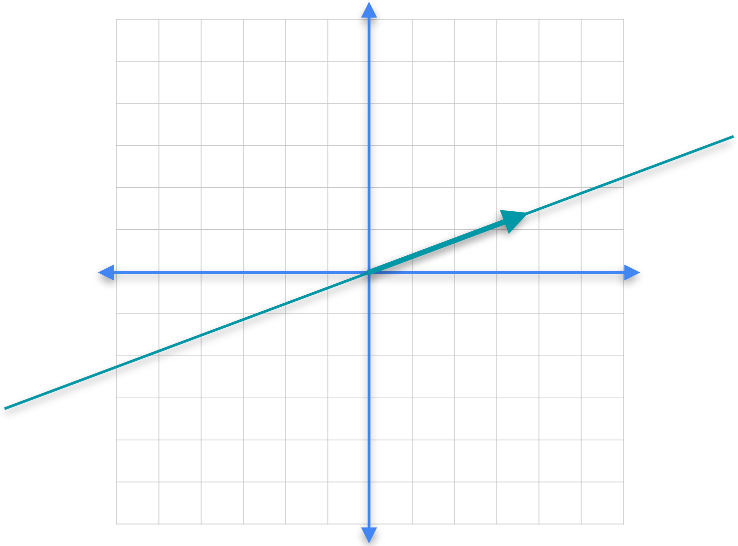

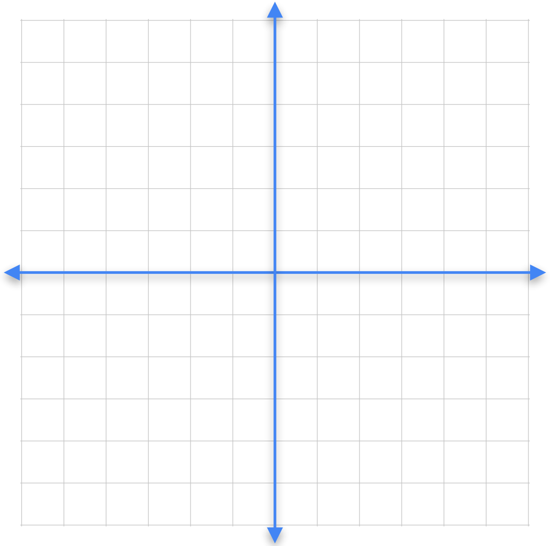

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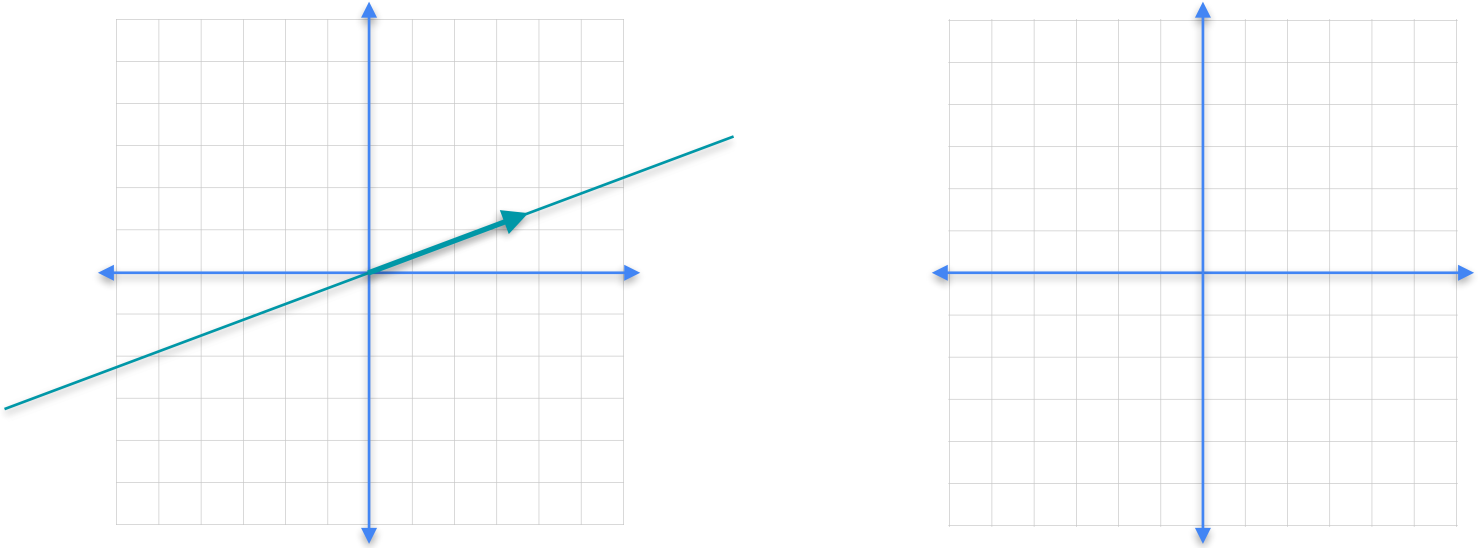

Basis

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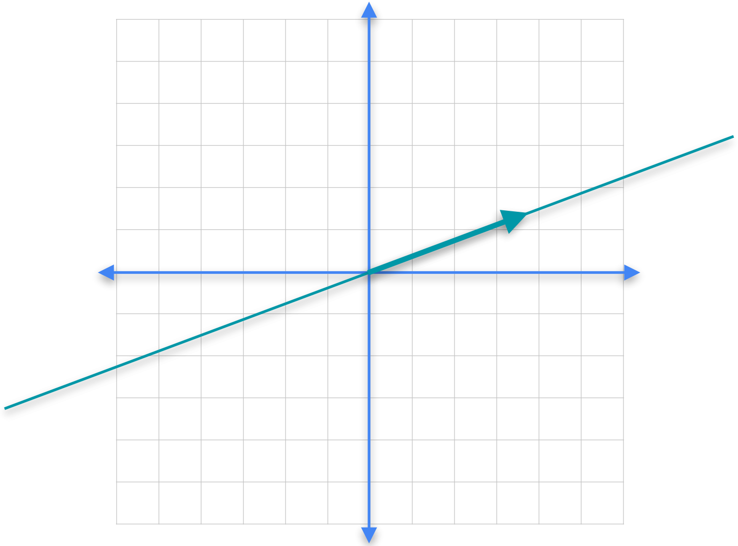

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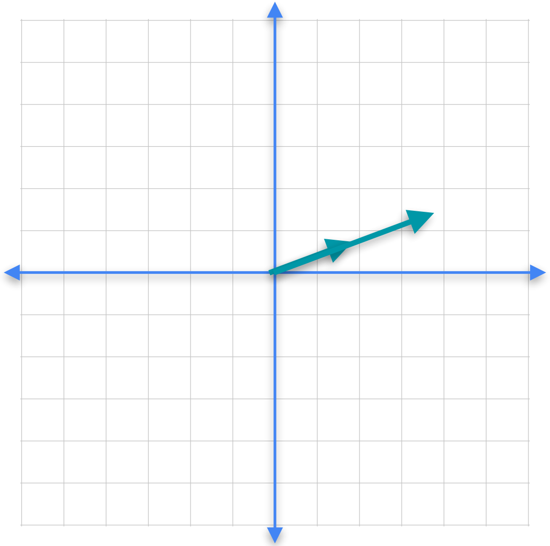

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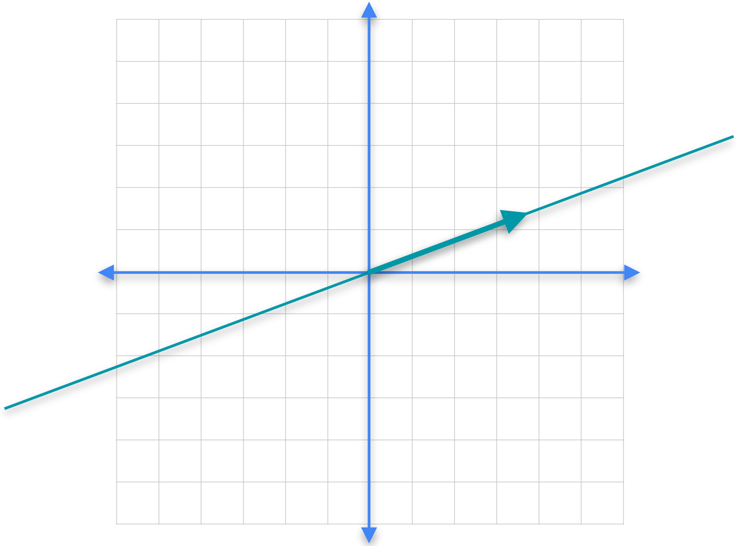

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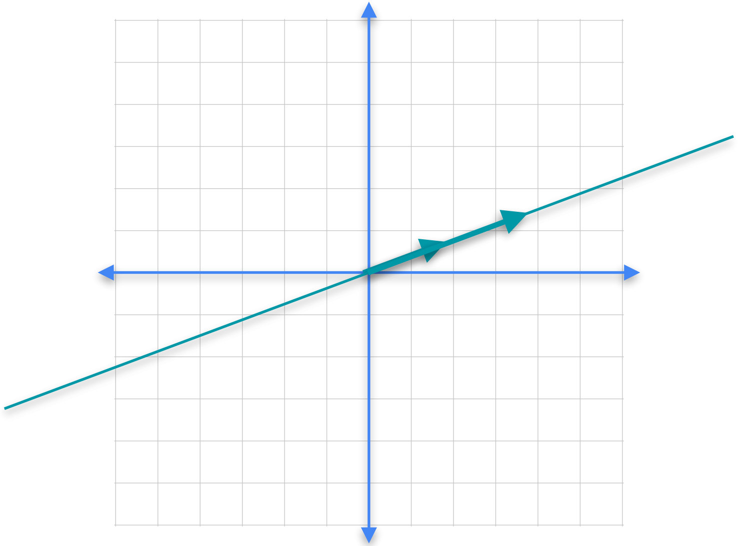
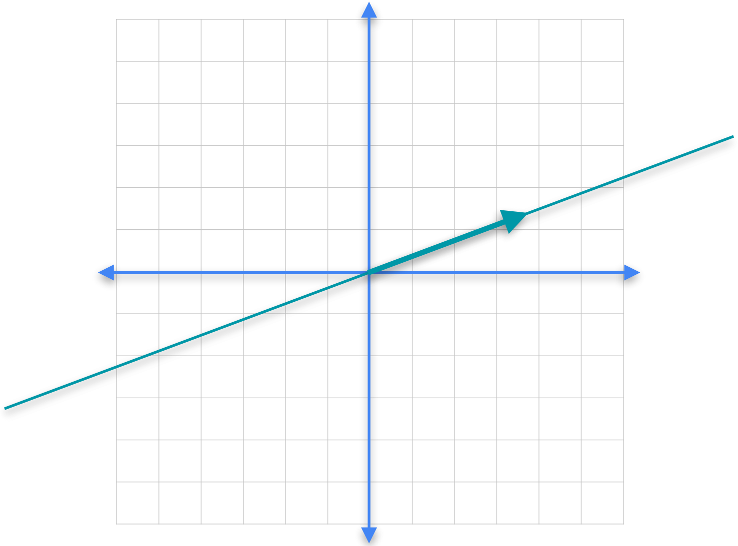
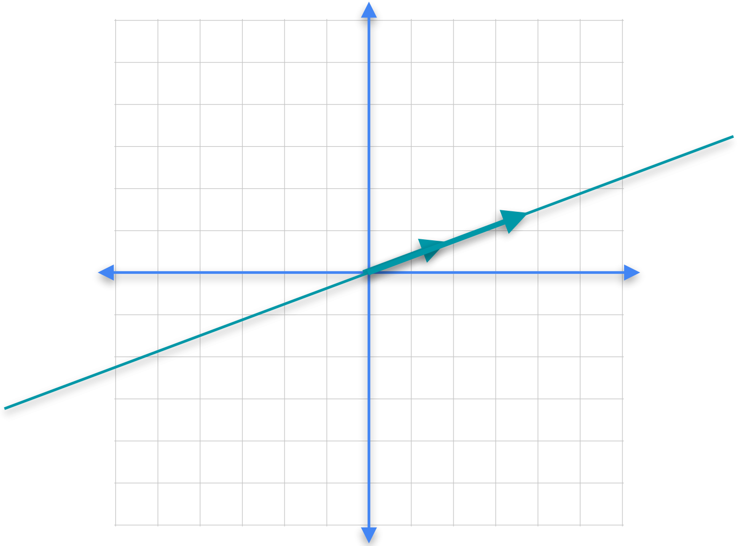


A basis is a minimal spanning set

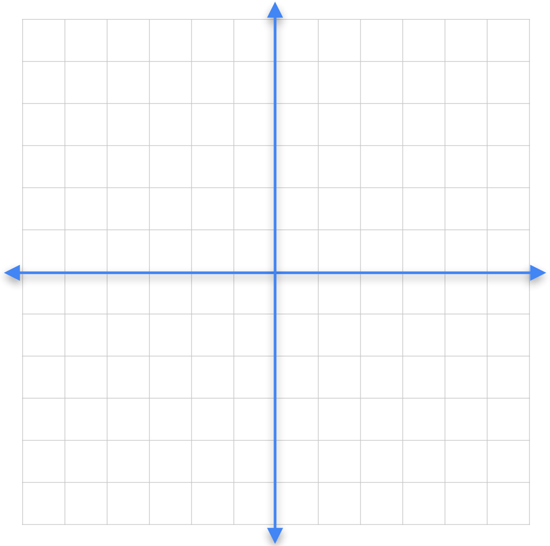


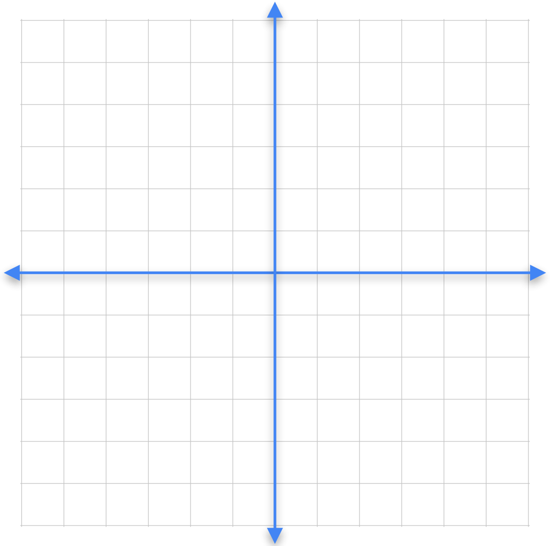
Basis



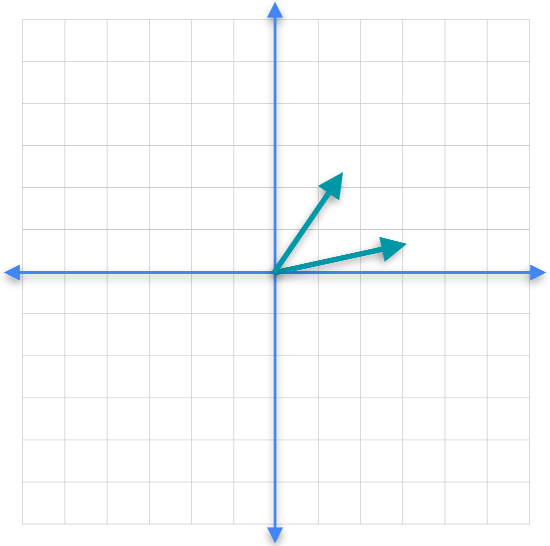
Not a basis

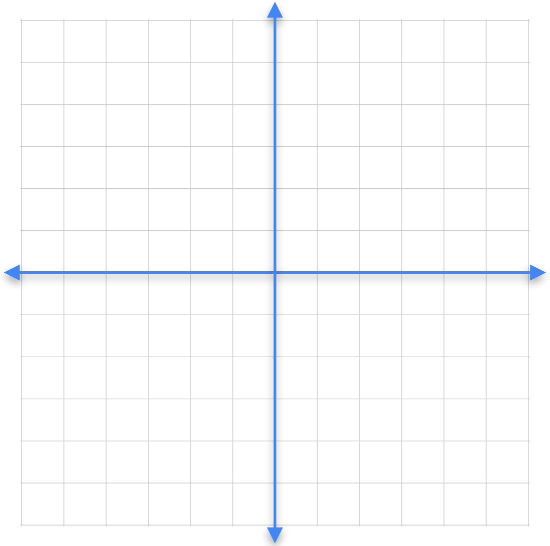
A basis is a minimal spanning set



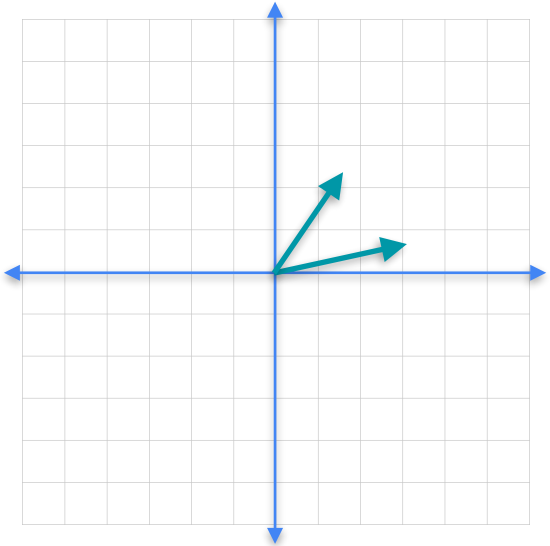


A basis is a minimal spanning set

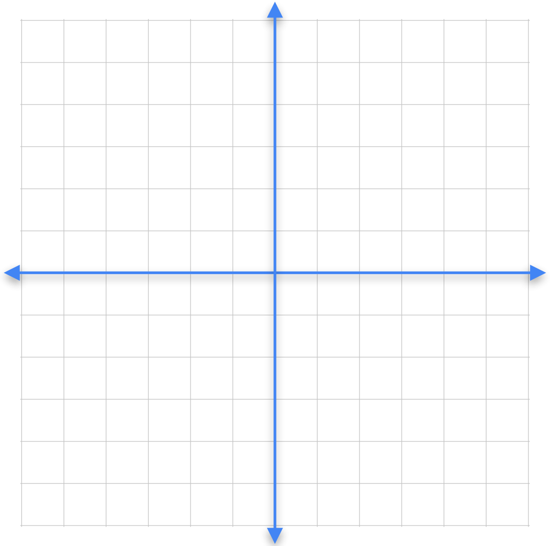




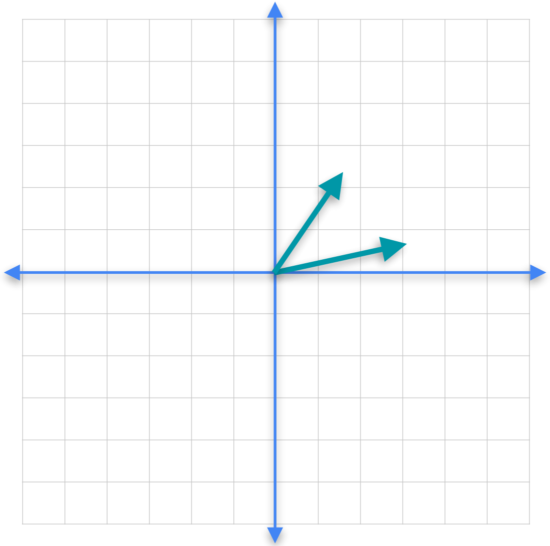
A basis is a minimal spanning set



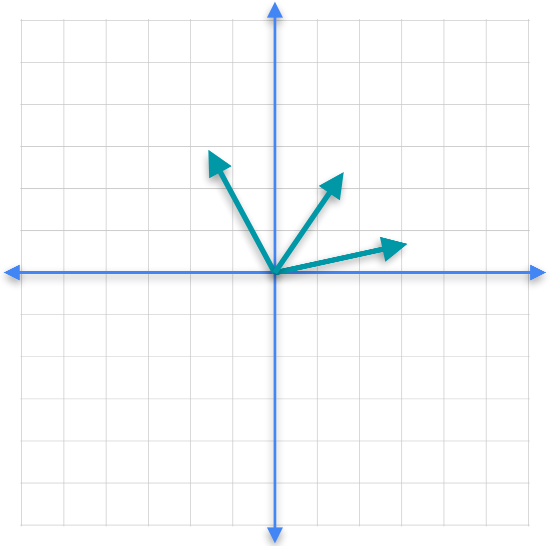
Basis



A basis is a minimal spanning set



Basis



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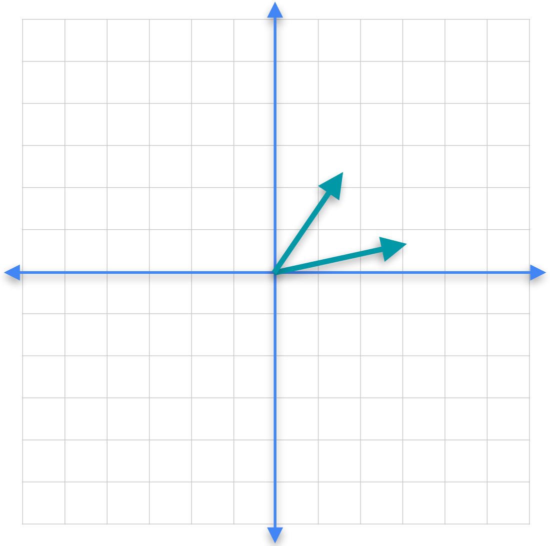
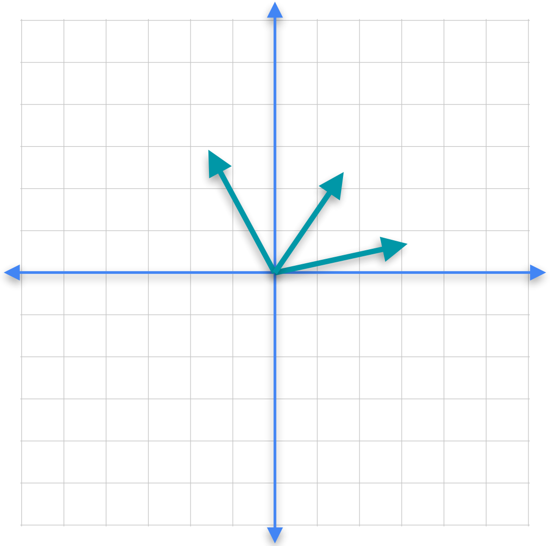
A basis is a minimal spanning set

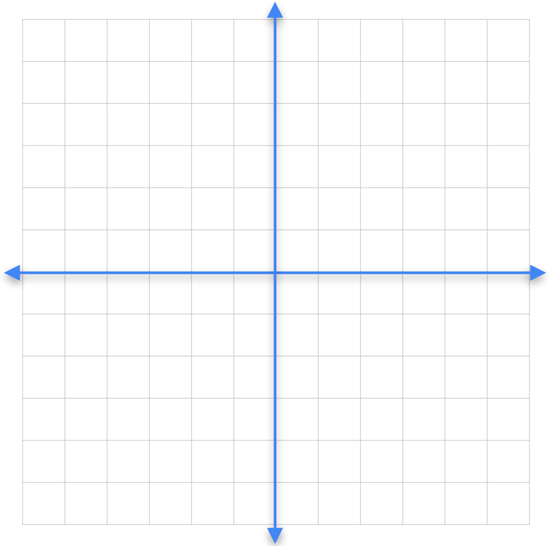


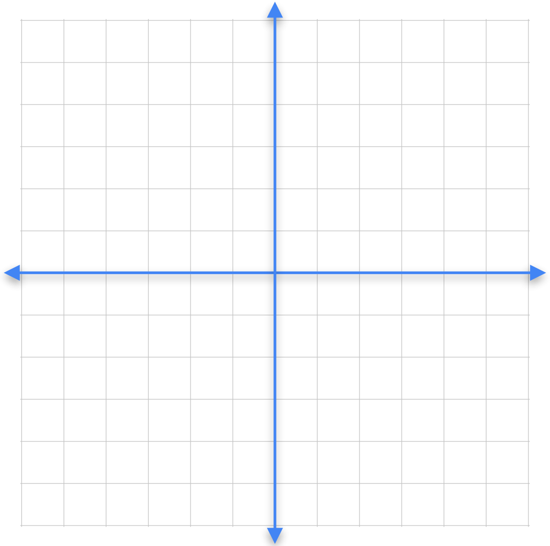
Basis



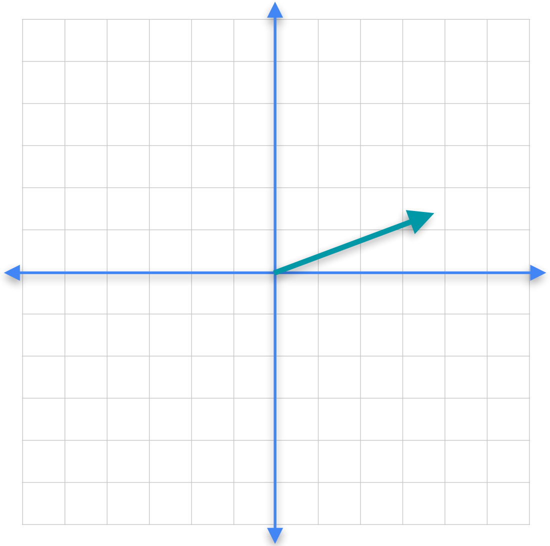
Not a basis

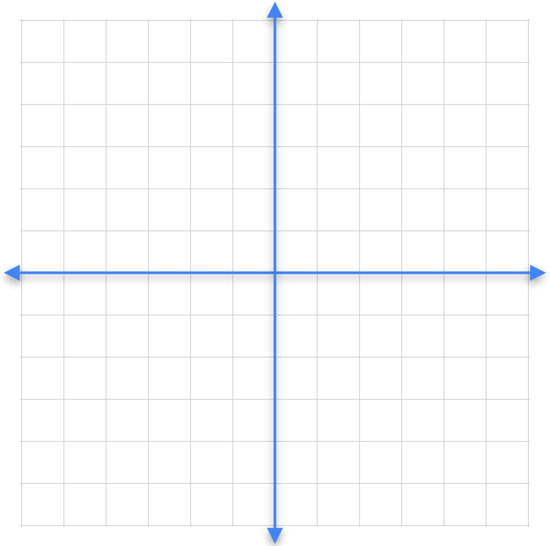
Number of elements in the basis is the dimension



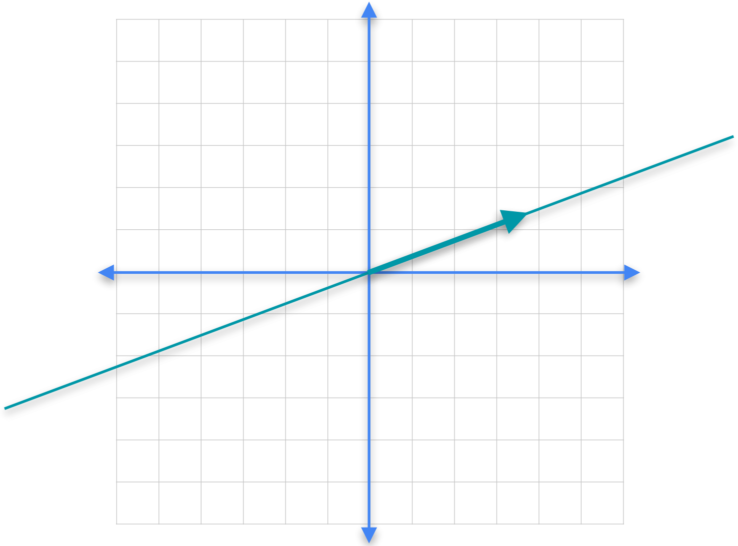


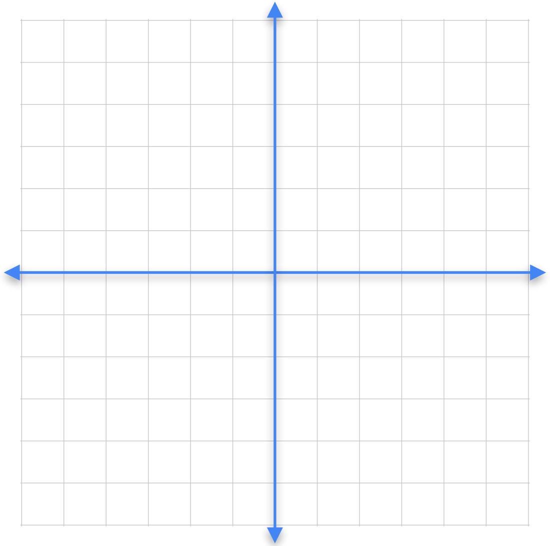
Number of elements in the basis is the dimension



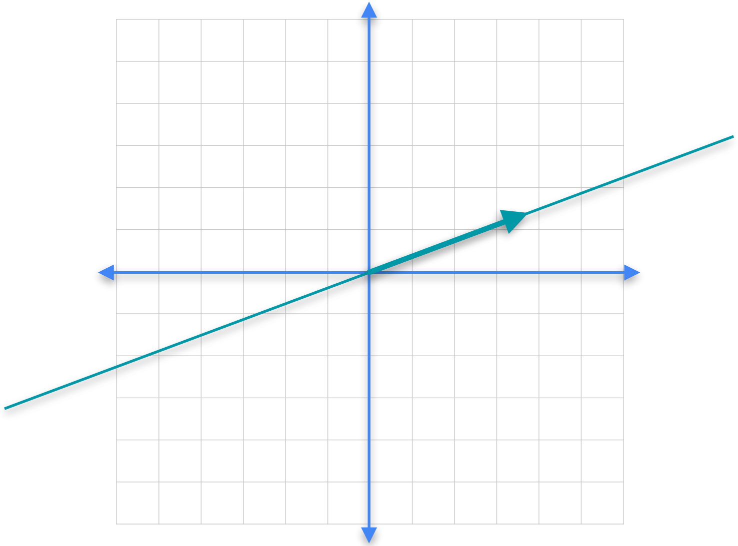


Number of elements in the basis is the dimension

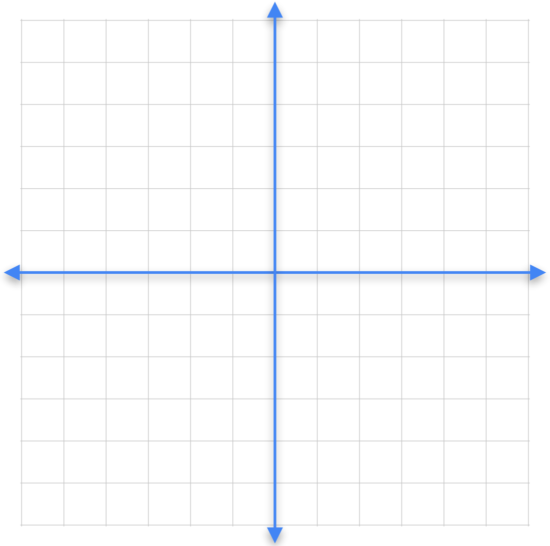




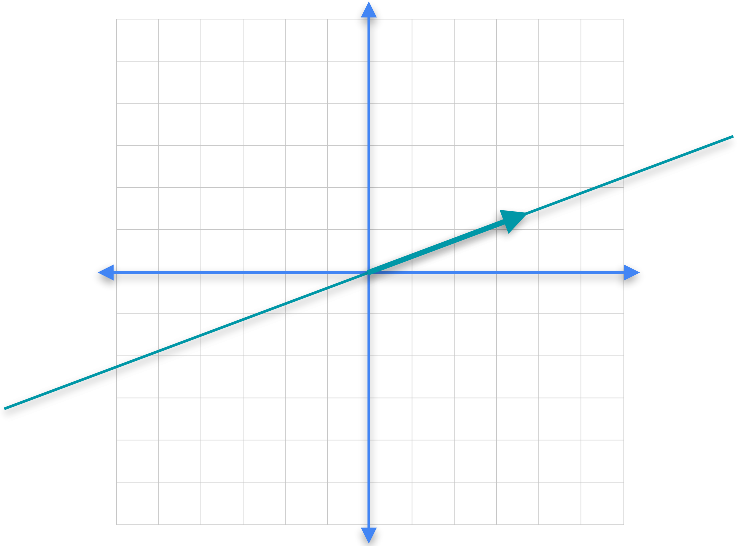
Number of elements in the basis is the dimension



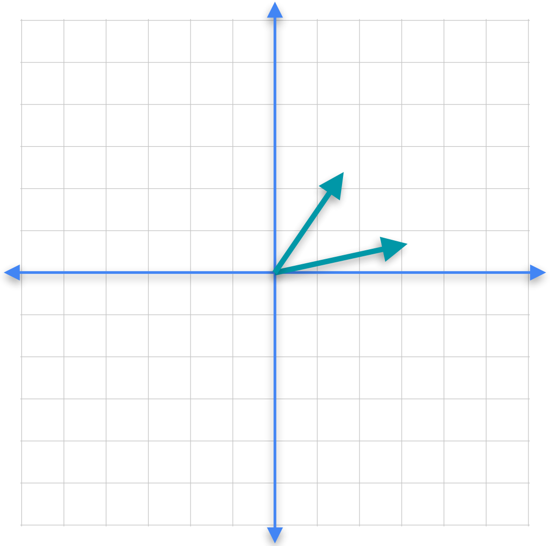
1 element
Dimension = 1



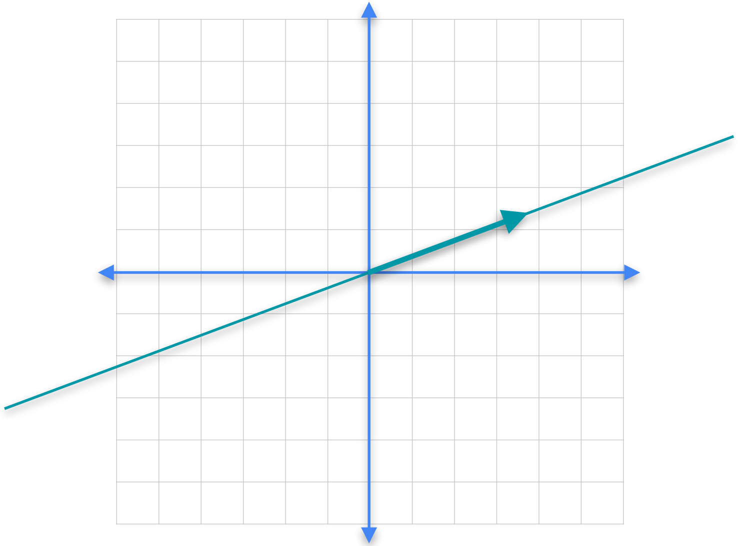
Number of elements in the basis is the dimension



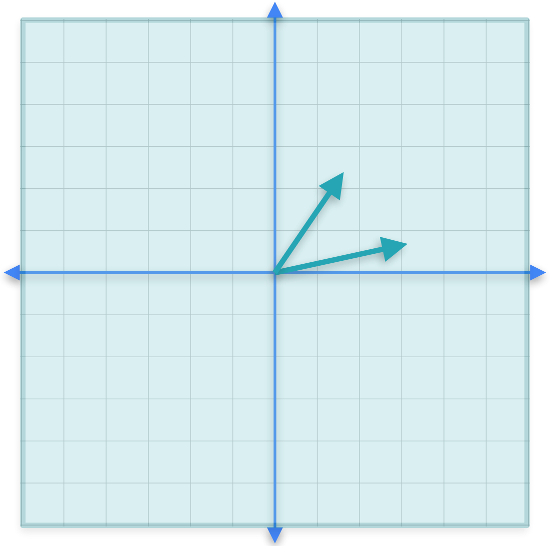
1 element
Dimension = 1



Number of elements in the basis is the dimension



1 element
Dimension = 1



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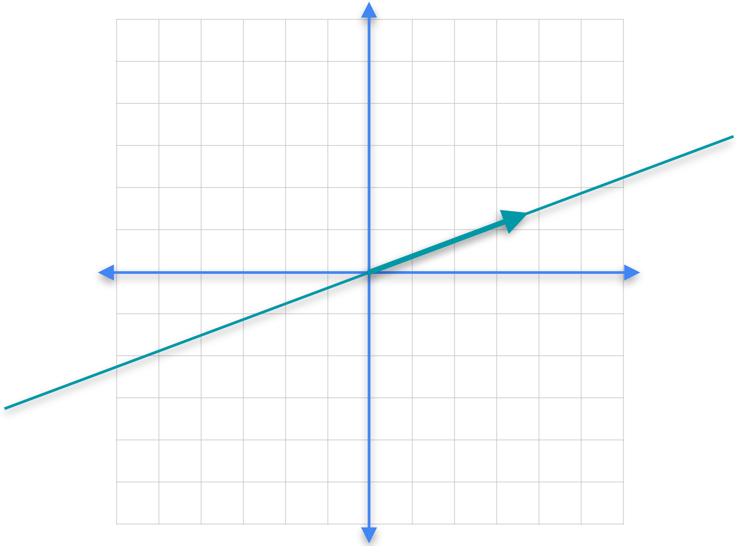
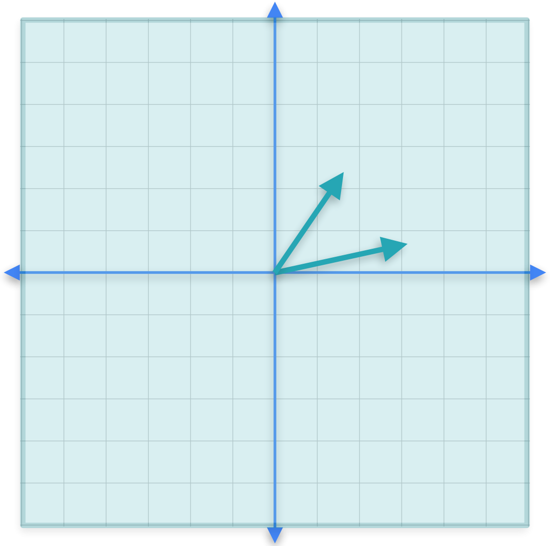
Number of elements in the basis is the dimension



1 element
Dimension = 1



2 elements in the basis
Dimension = 2

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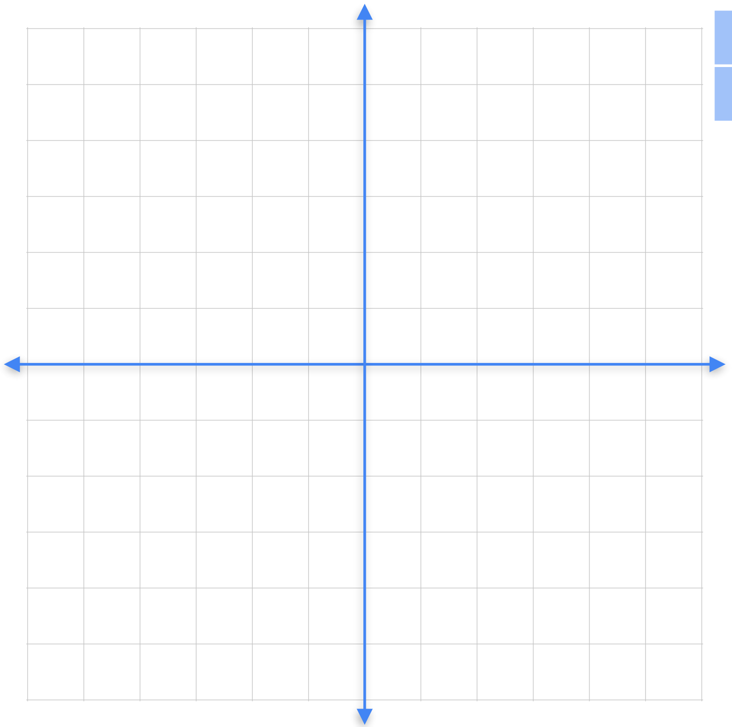
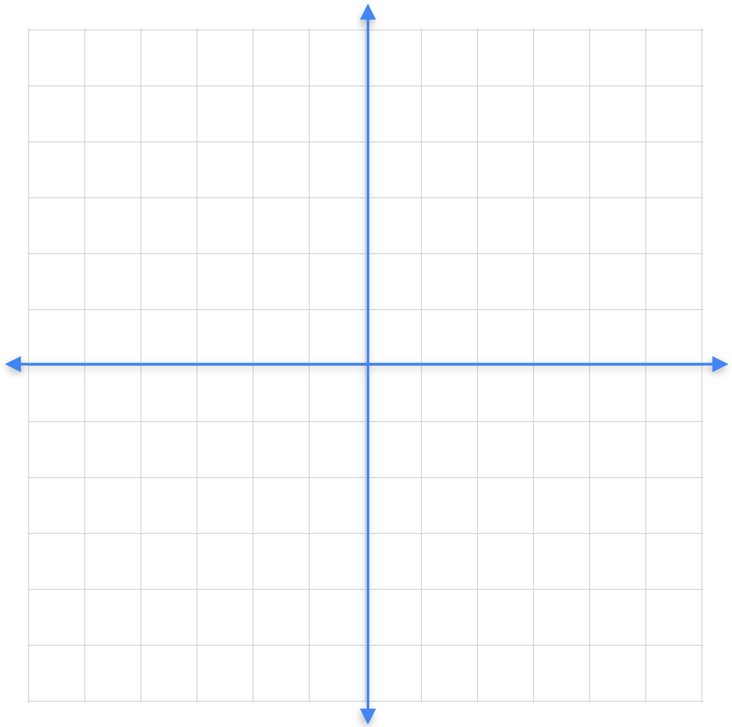
Determinants and Eigenvectors

Eigenbasis

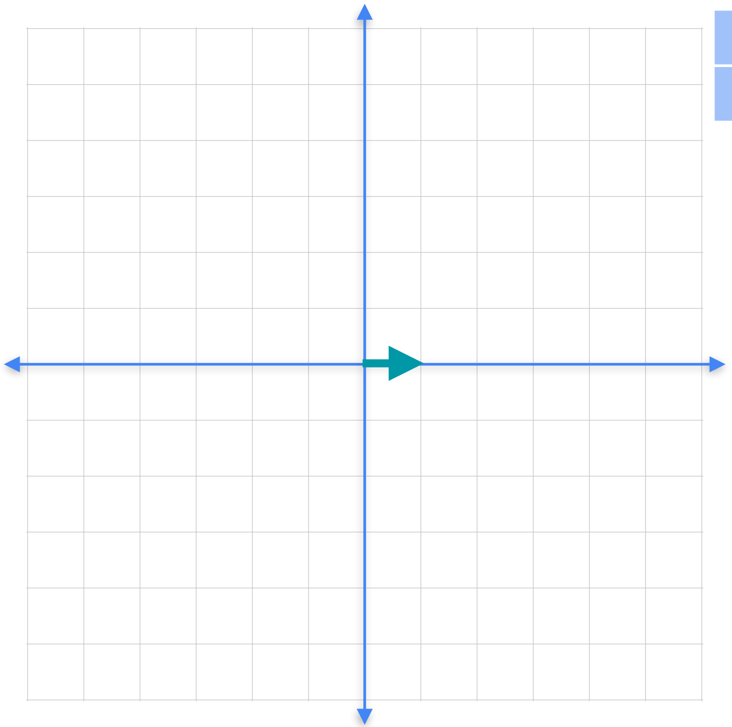
Basis



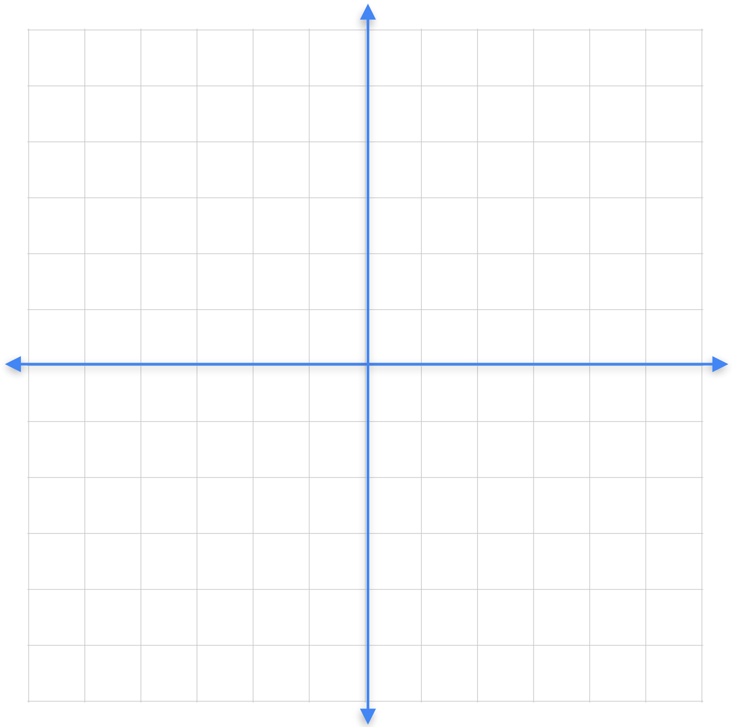
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$



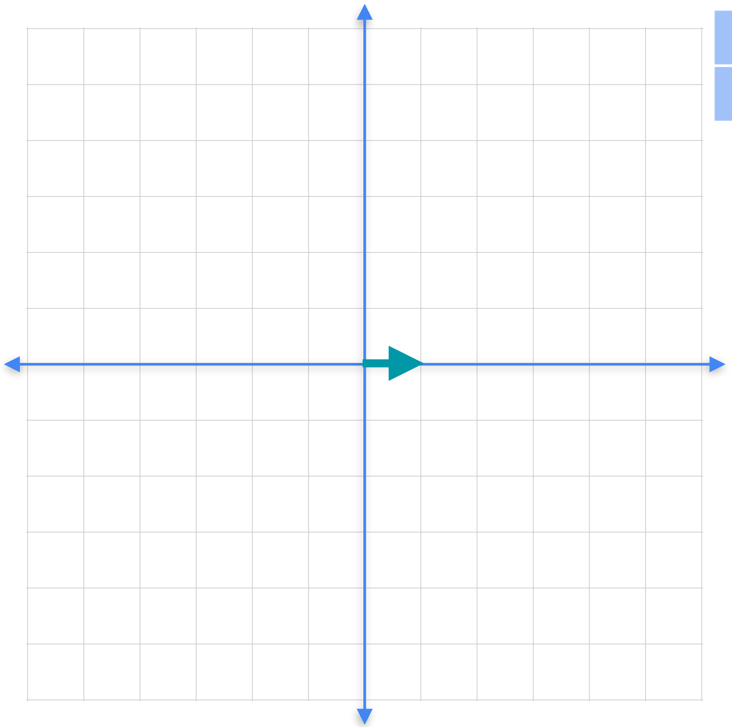
Basis



2	1
0	3

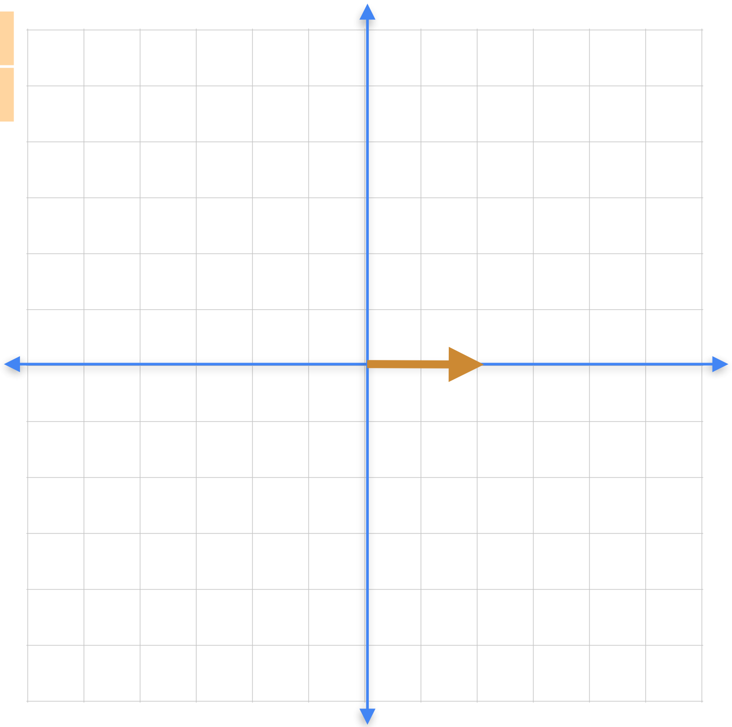


Basis

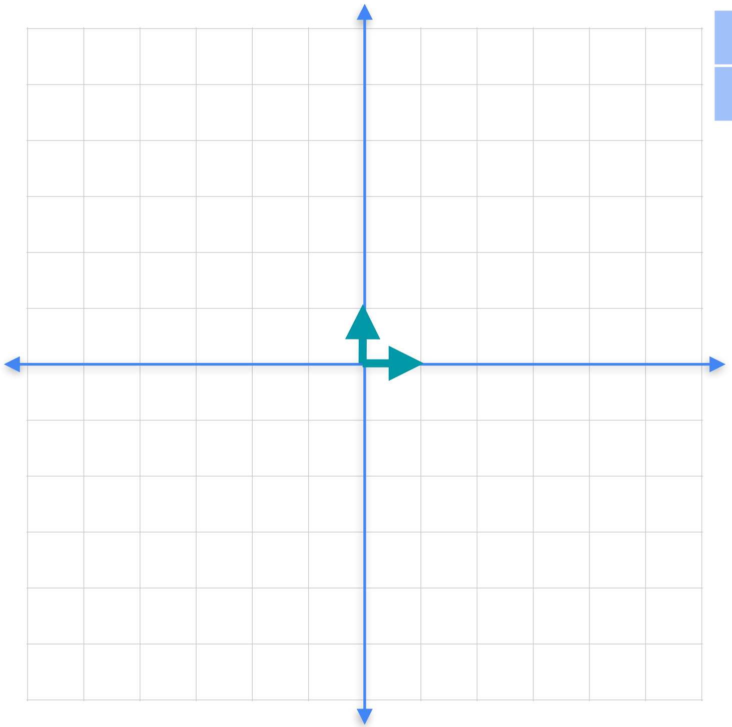


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(1,0) → (2,0)



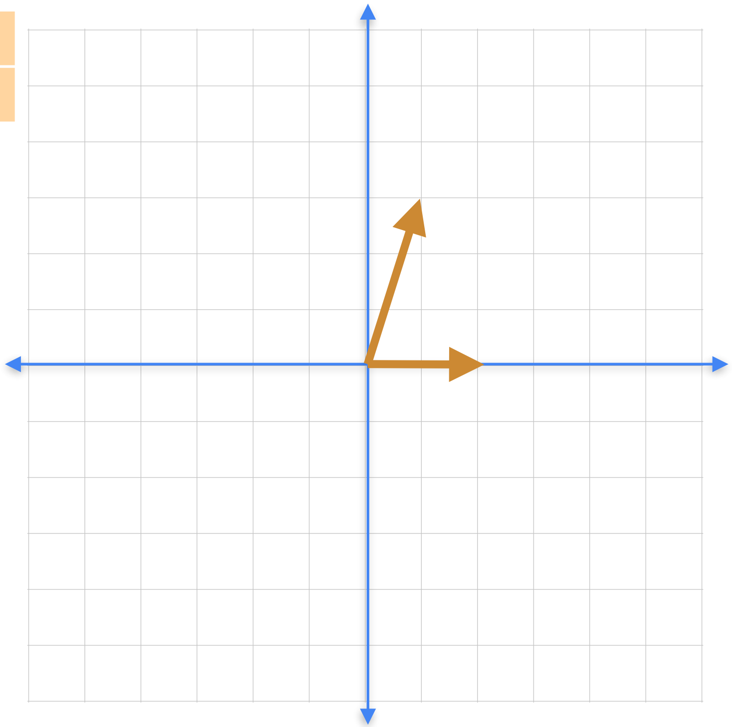
Basis



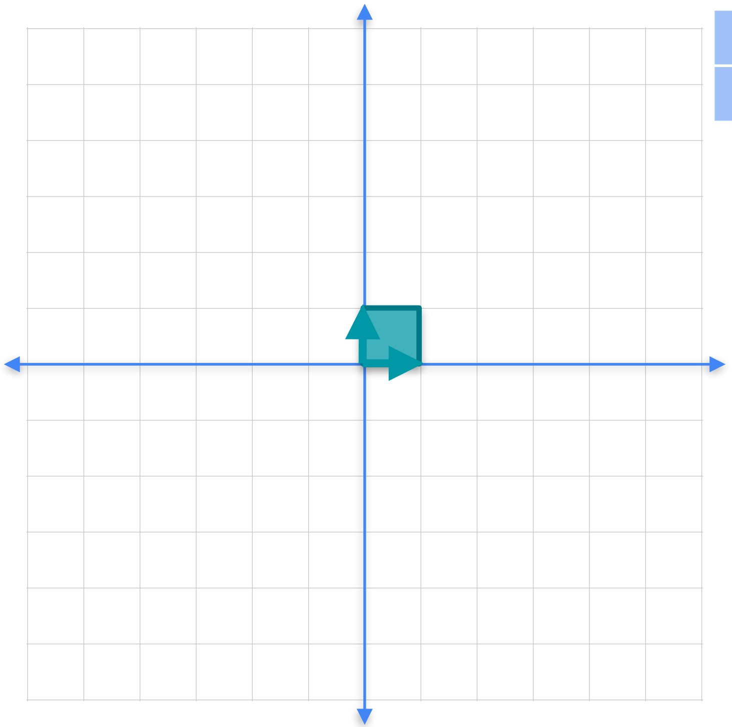
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$(1,0) \rightarrow (2,0)$

$(0,1) \rightarrow (1,3)$



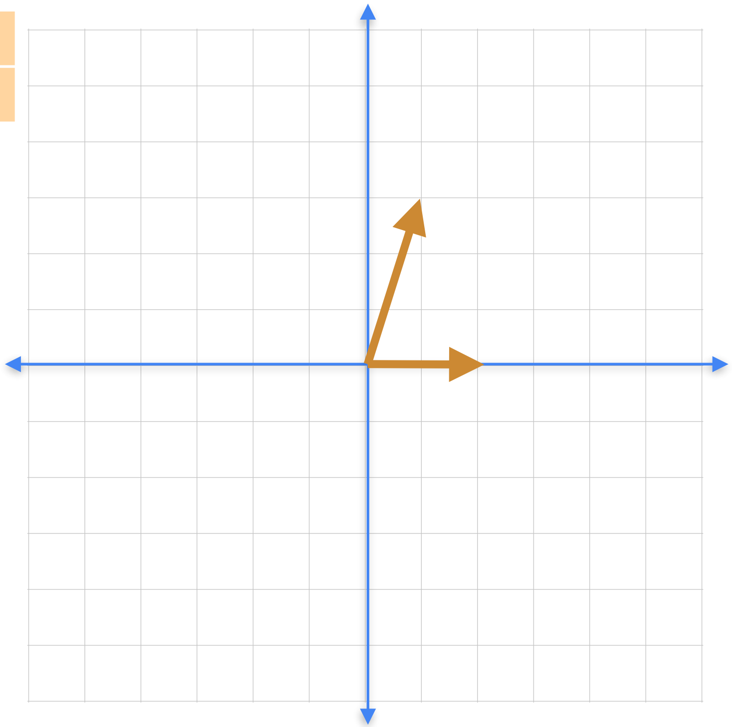
Basis



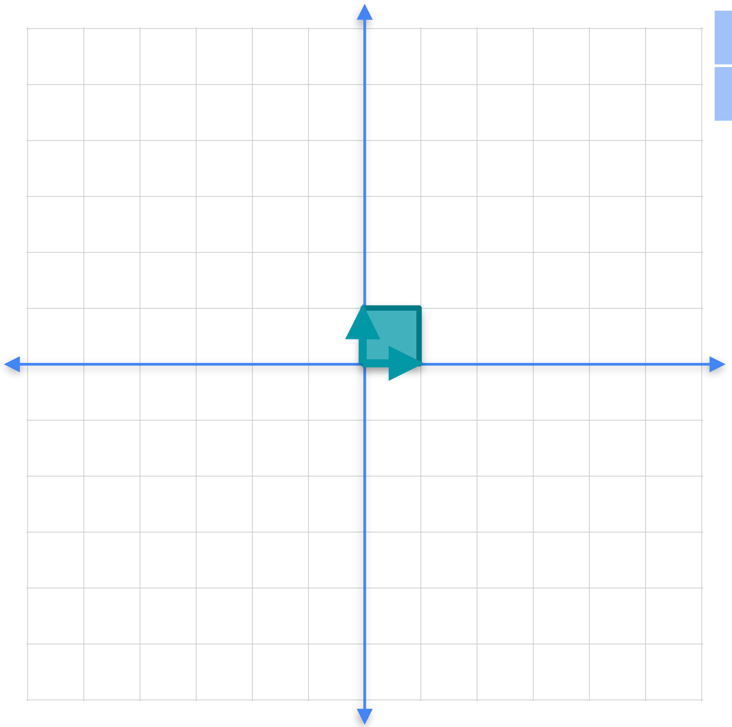
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$(1,0) \rightarrow (2,0)$

$(0,1) \rightarrow (1,3)$



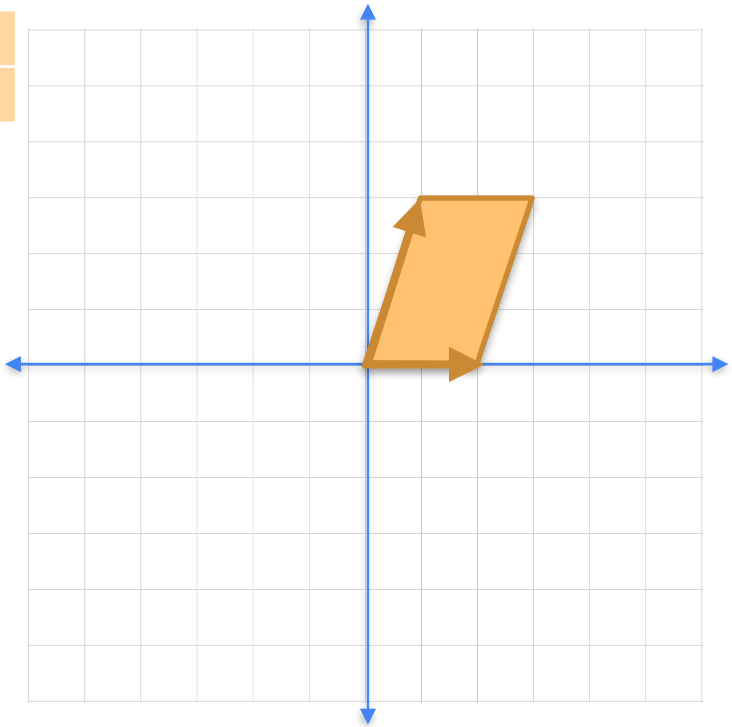
Basis



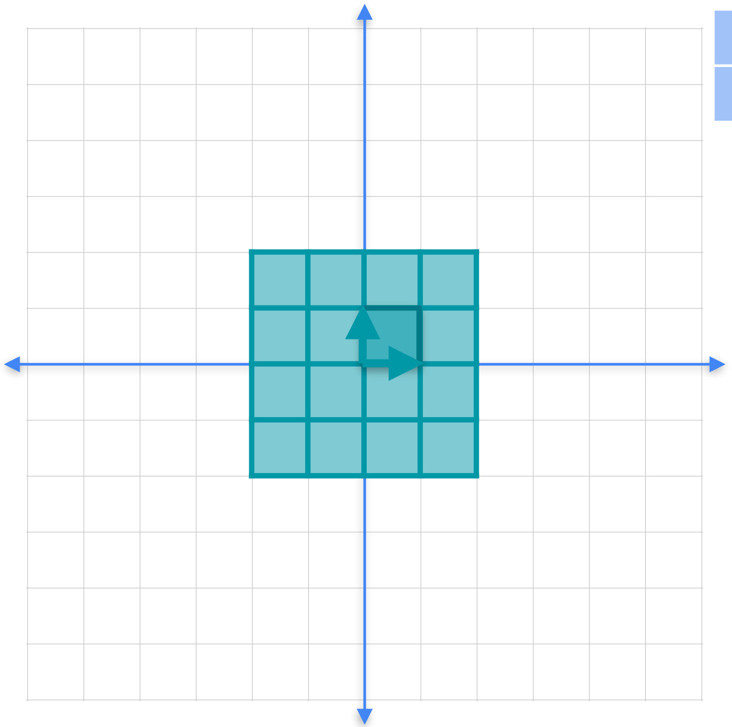
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$(1,0) \rightarrow (2,0)$

$(0,1) \rightarrow (1,3)$



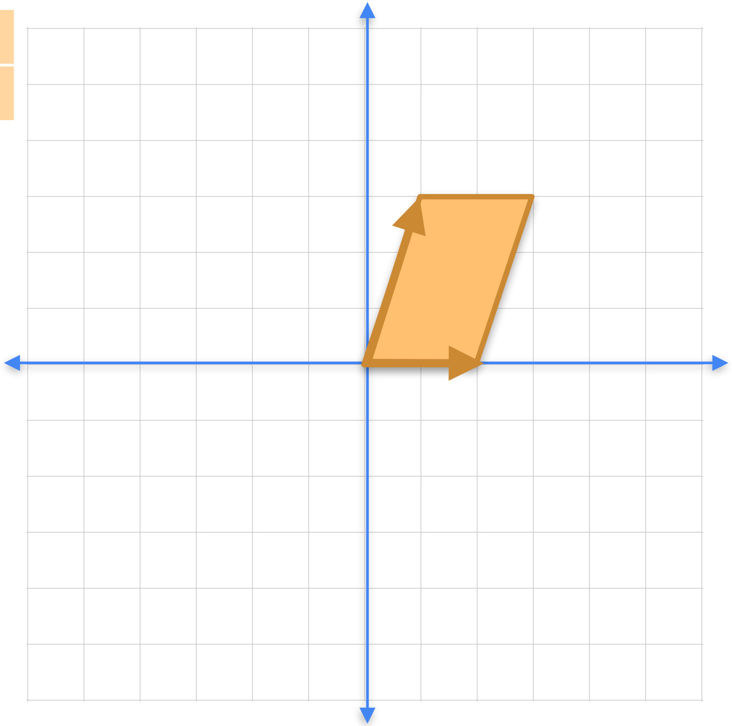
Basis



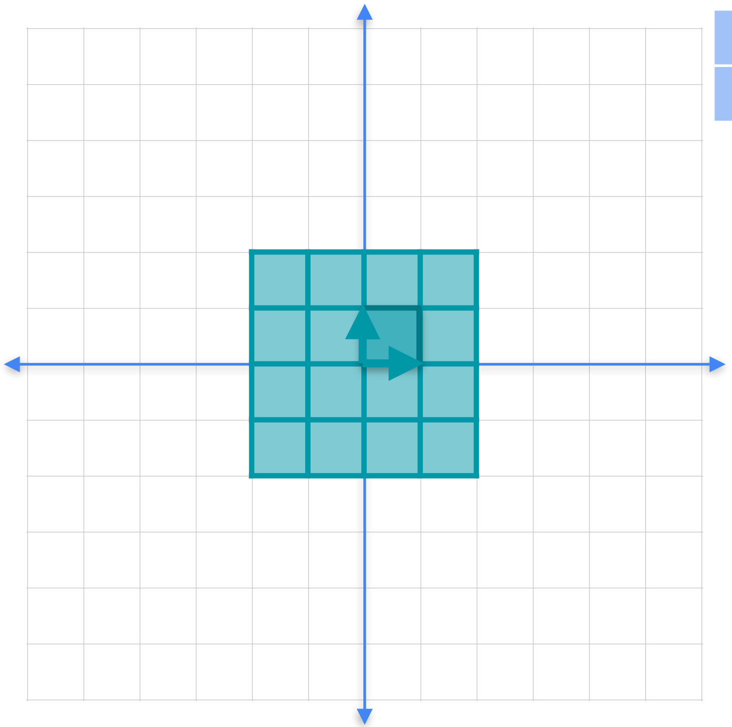
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(1,0) → (2,0)

(0,1) → (1,3)



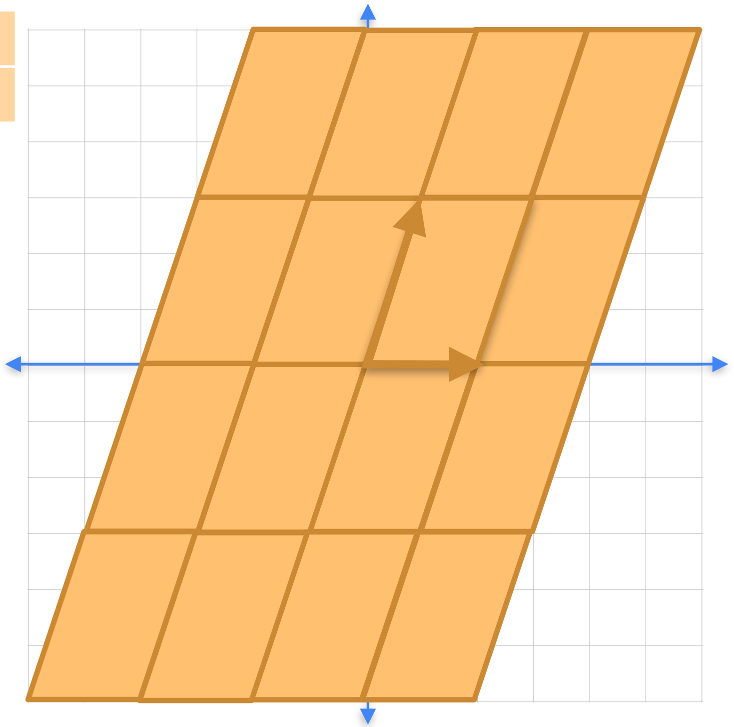
Basis



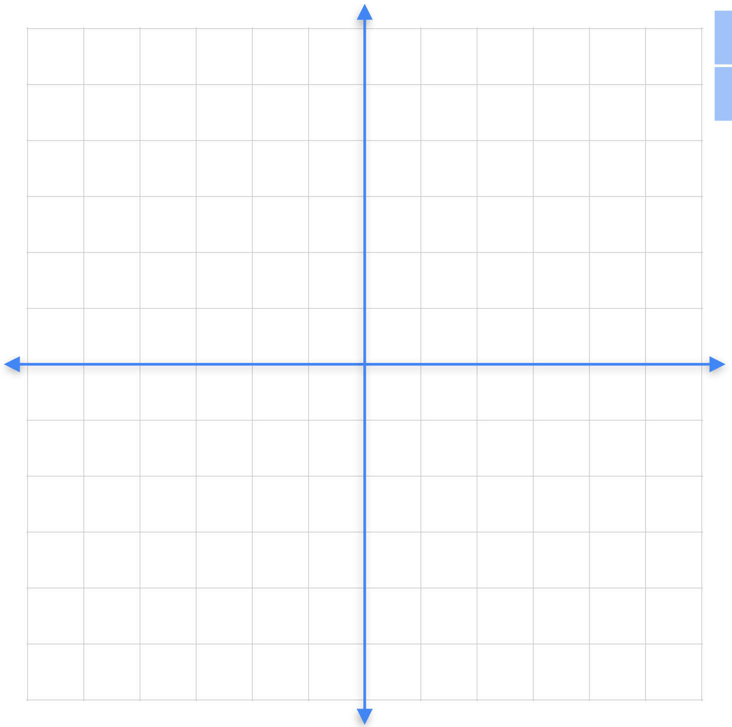
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$(1,0) \rightarrow (2,0)$

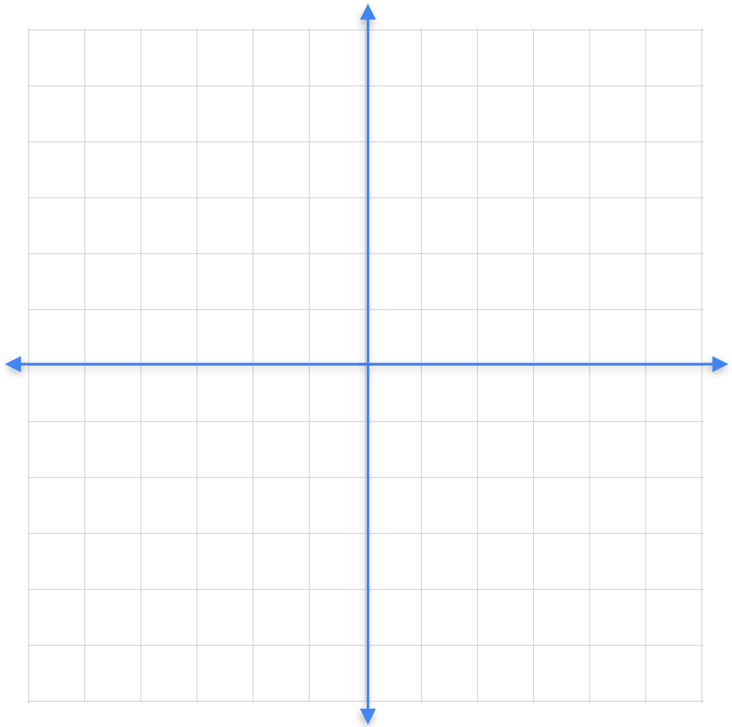
$(0,1) \rightarrow (1,3)$



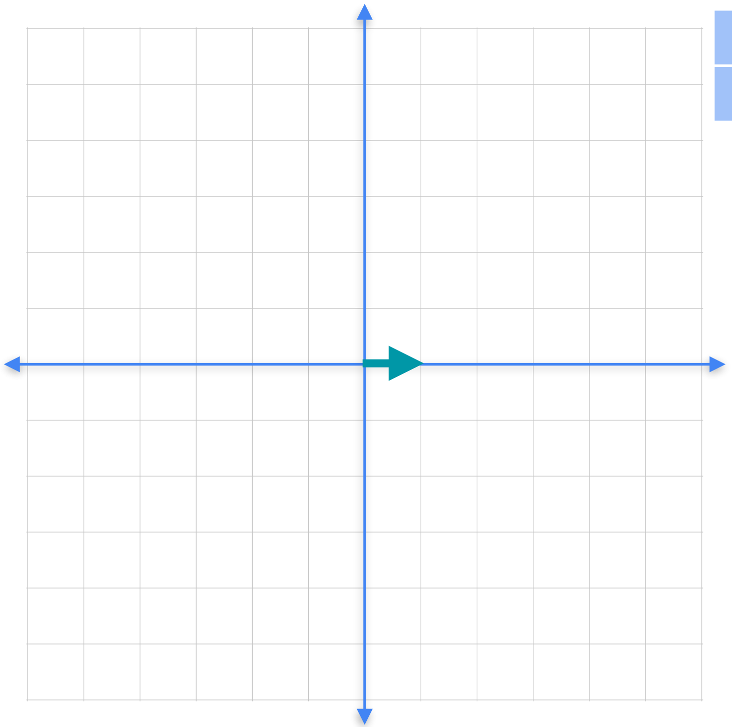
Eigenbasis



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

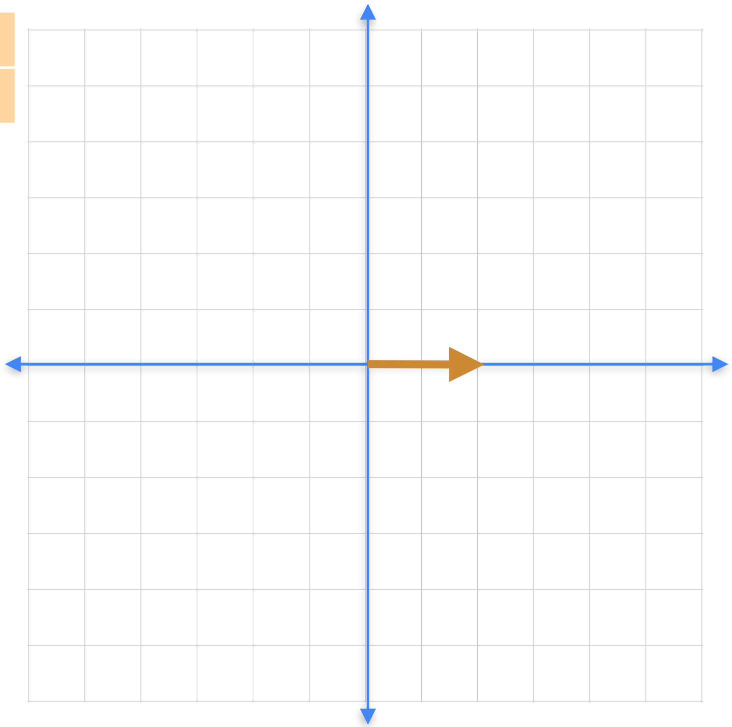


Eigenbasis

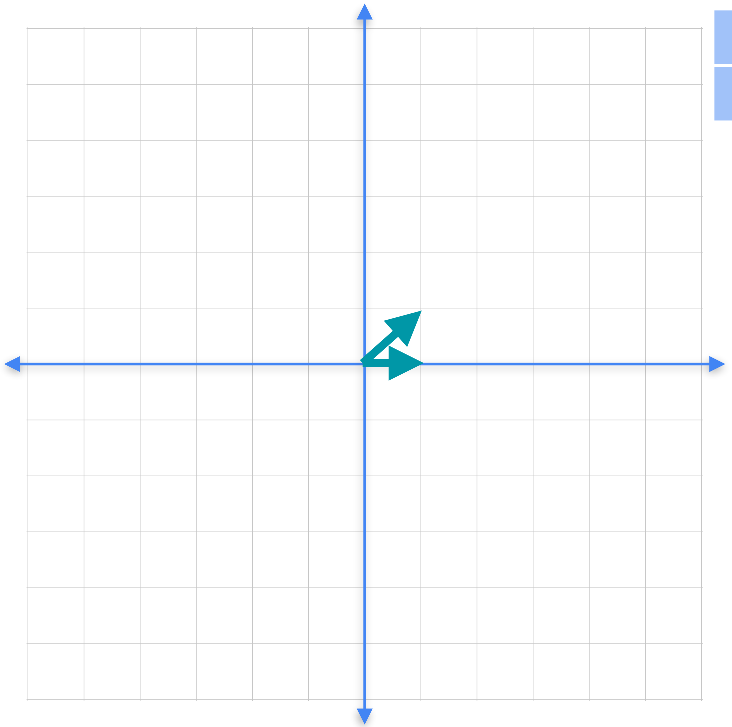


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$(1,0) \rightarrow (2,0)$



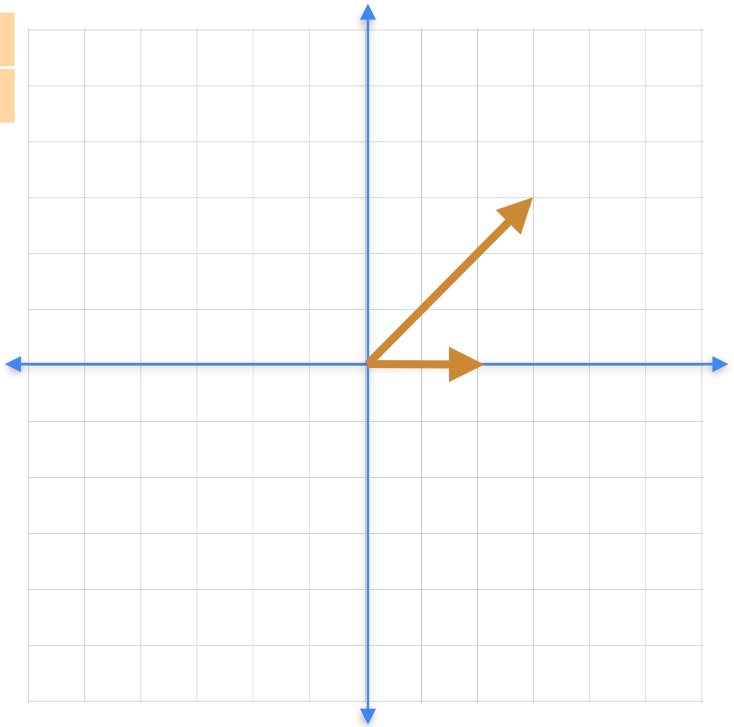
Eigenbasis



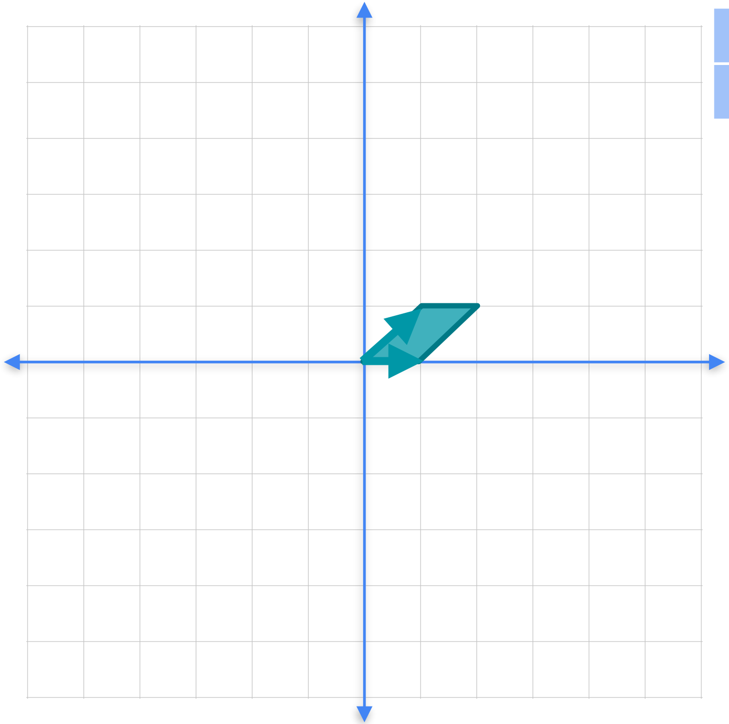
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$(1,0) \rightarrow (2,0)$

$(1,1) \rightarrow (3,3)$

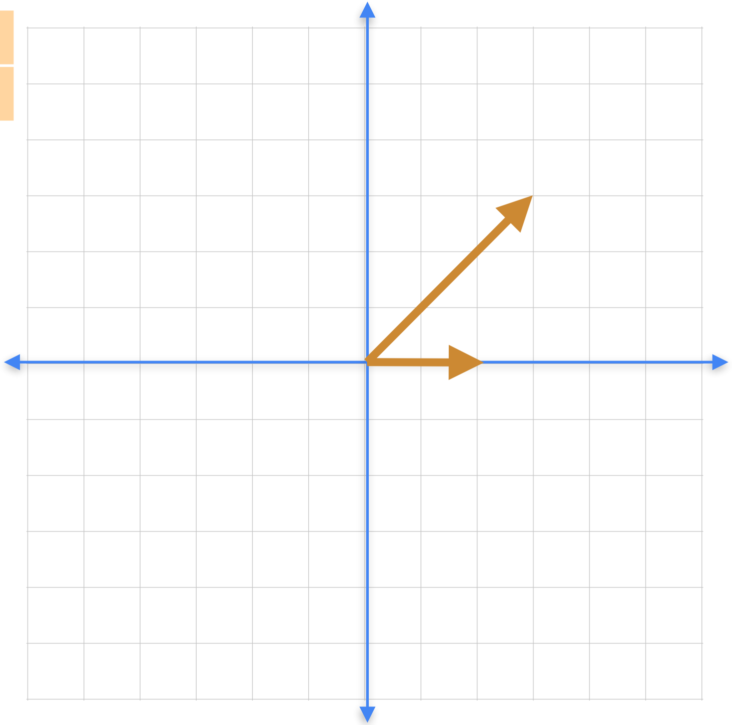


Eigenbasis

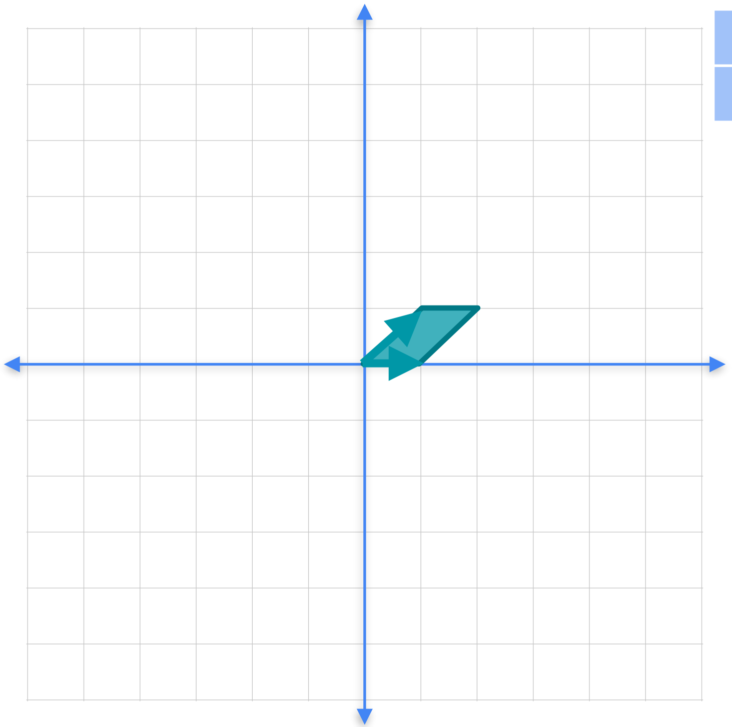


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

(1,0) → (2,0)
(1,1) → (3,3)



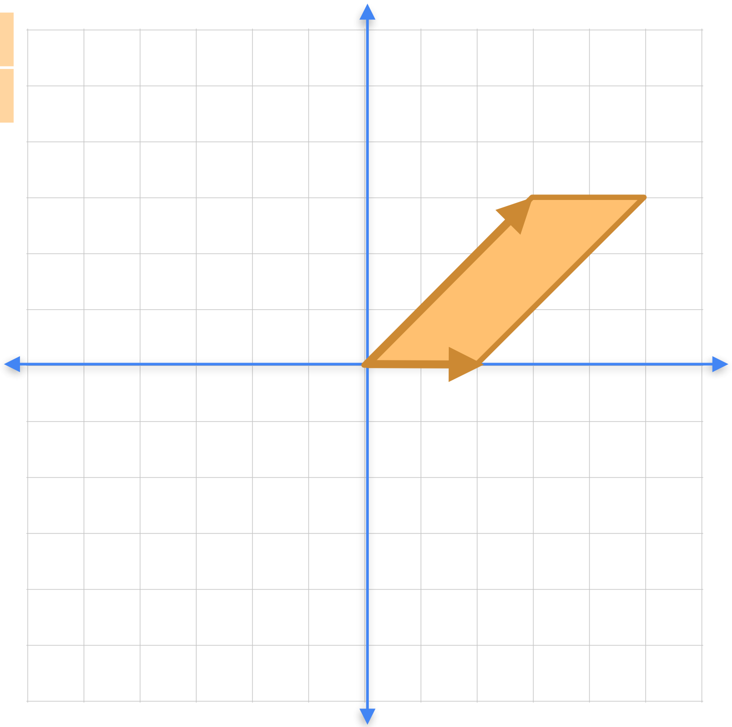
Eigenbasis



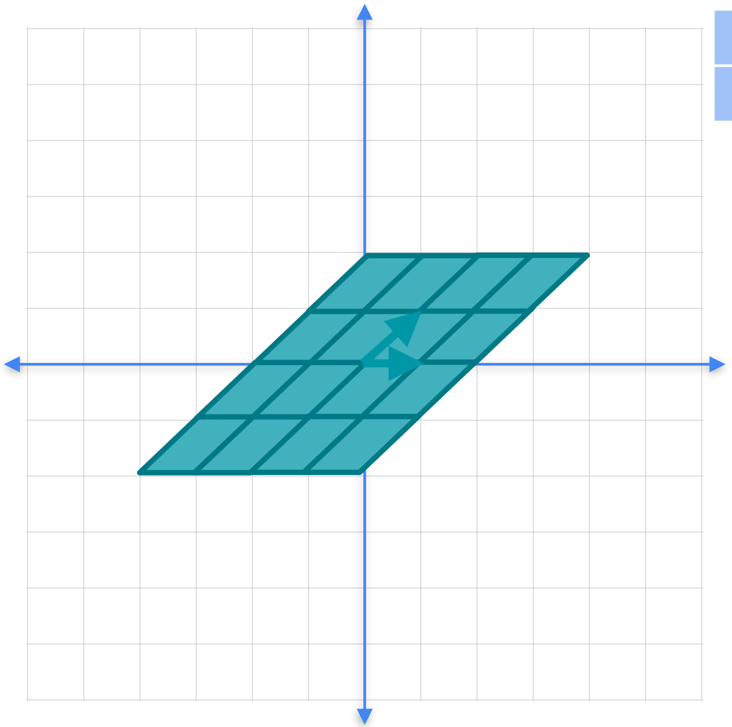
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

(1,0) → (2,0)

(1,1) → (3,3)



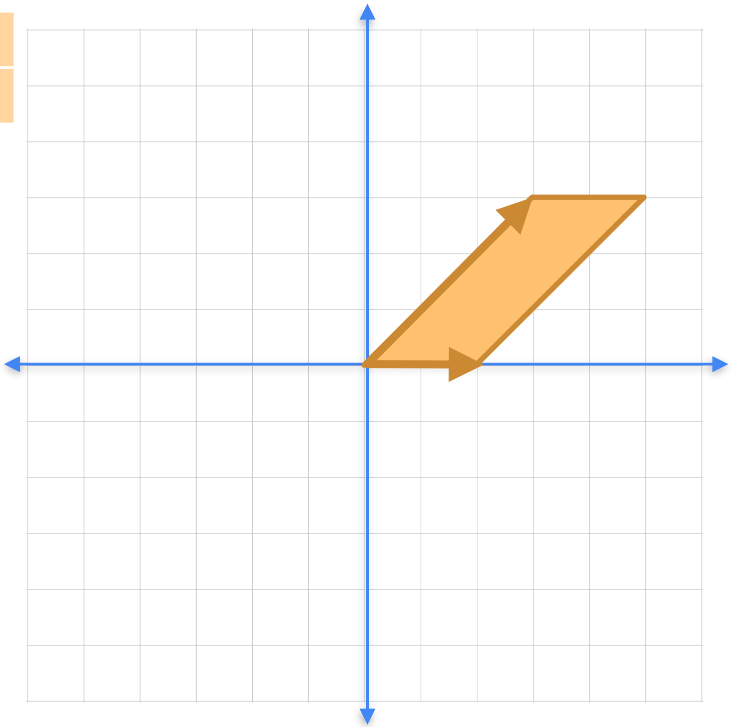
Eigenbasis



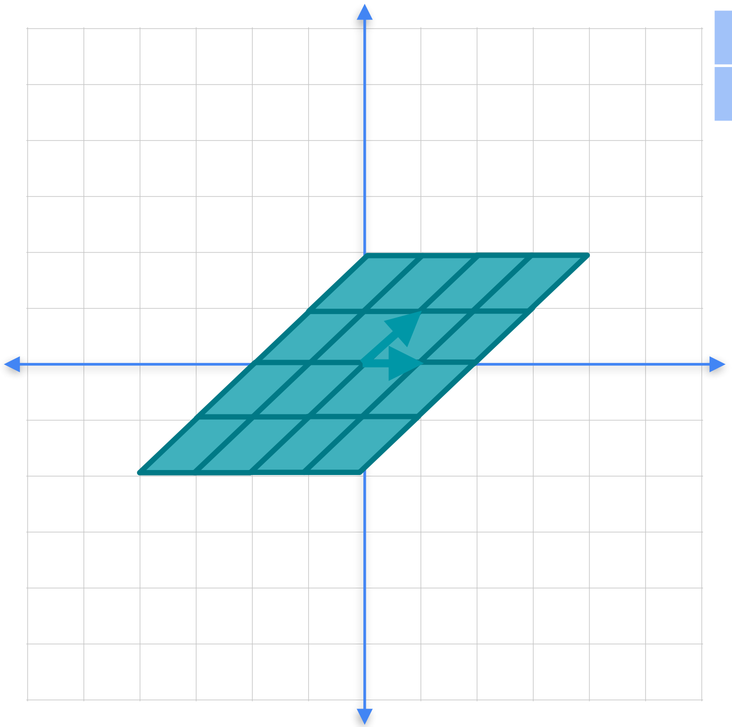
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

(1,0) → (2,0)

(1,1) → (3,3)



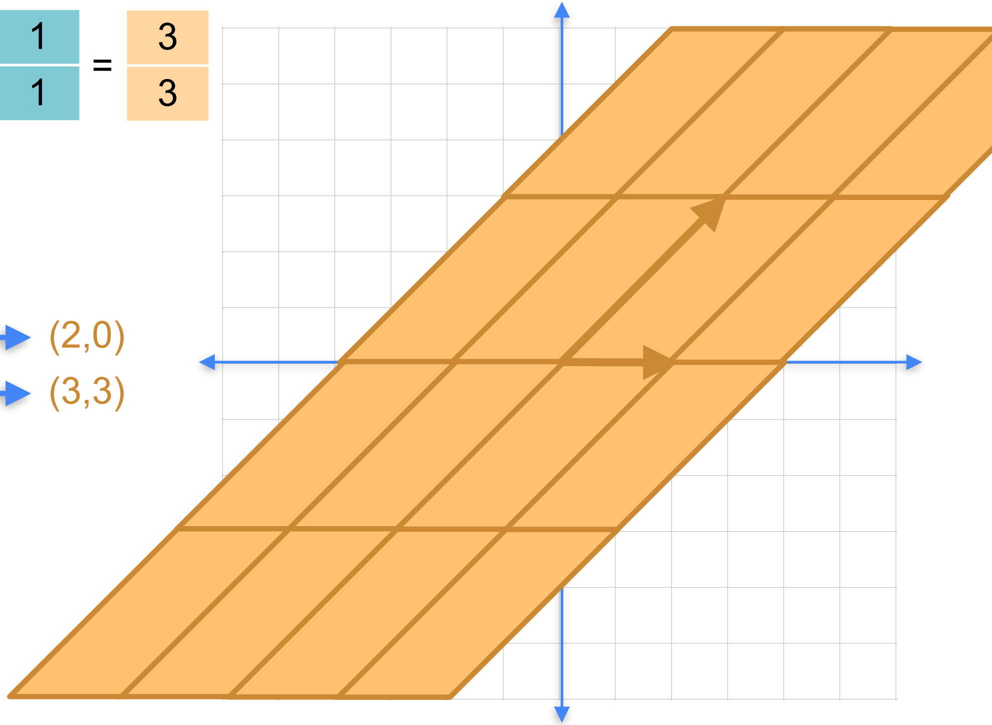
Eigenbasis



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

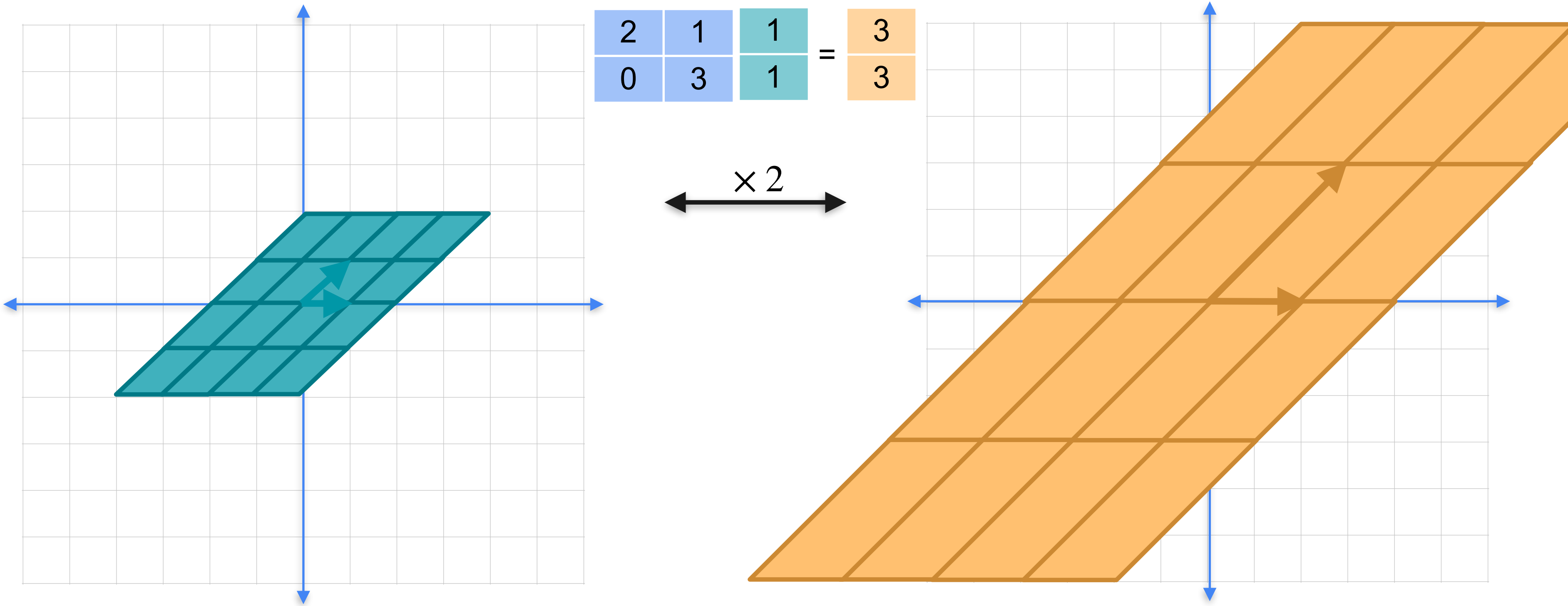
(1,0) → (2,0)

(1,1) → (3,3)



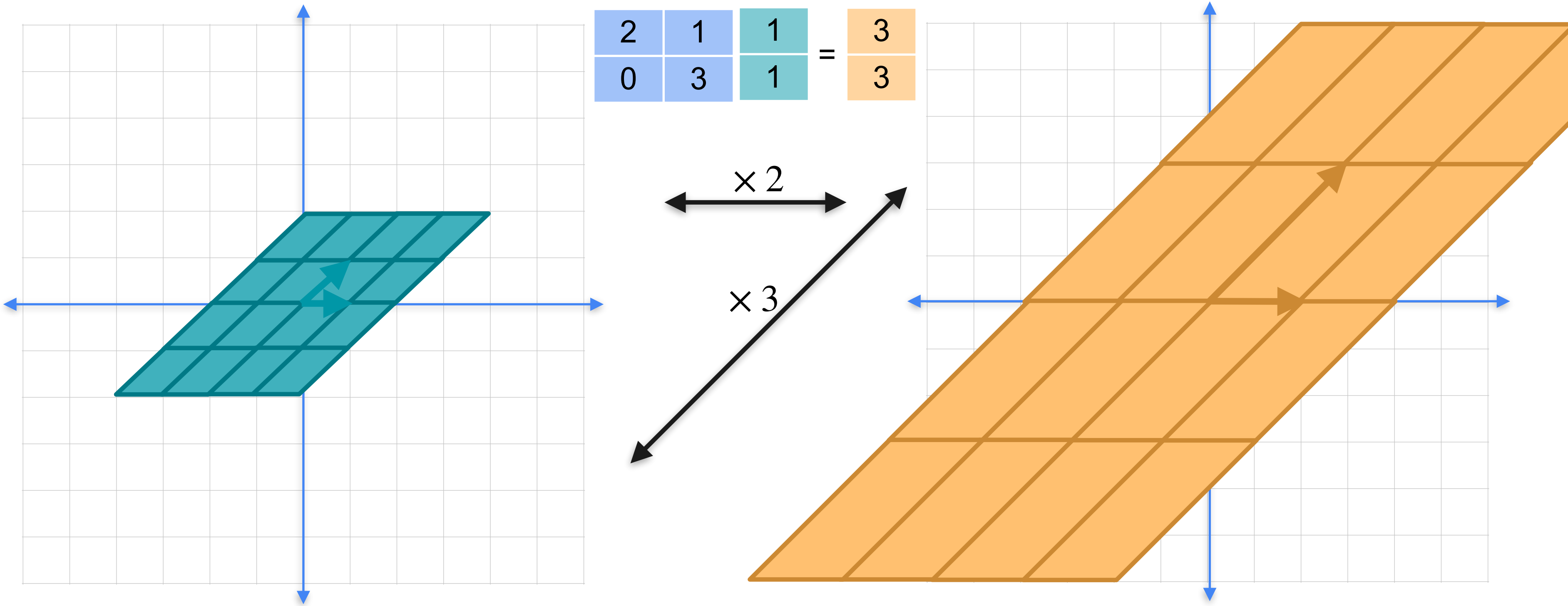
Eigenbasis

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

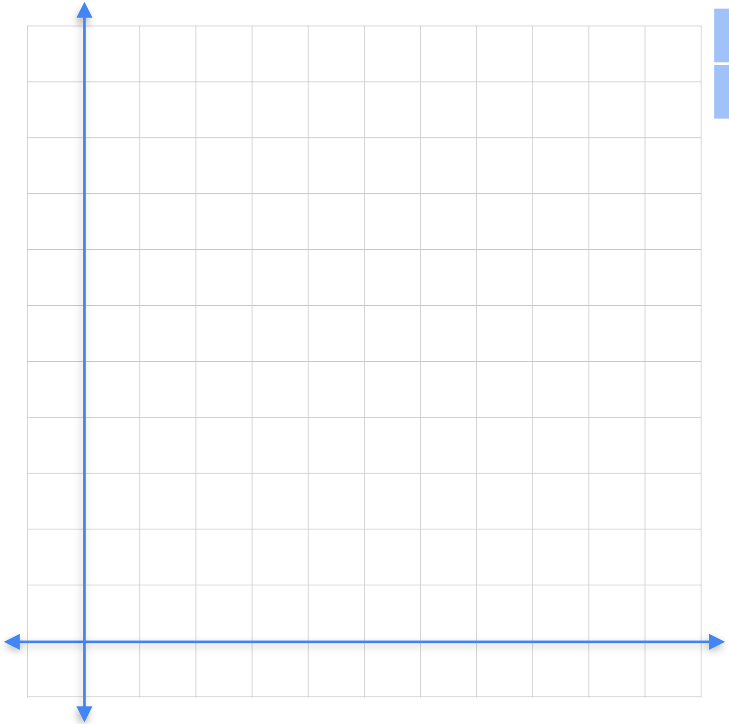


Eigenbasis

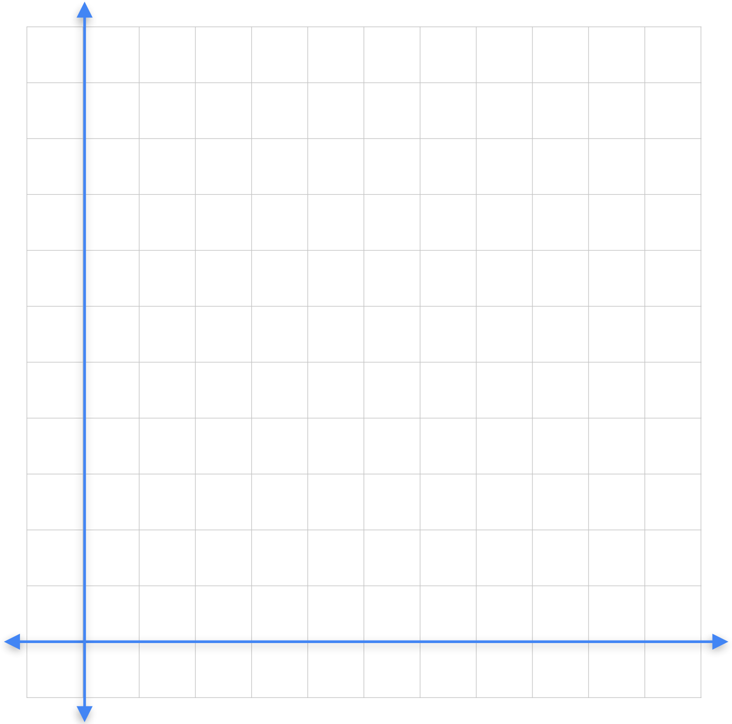
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



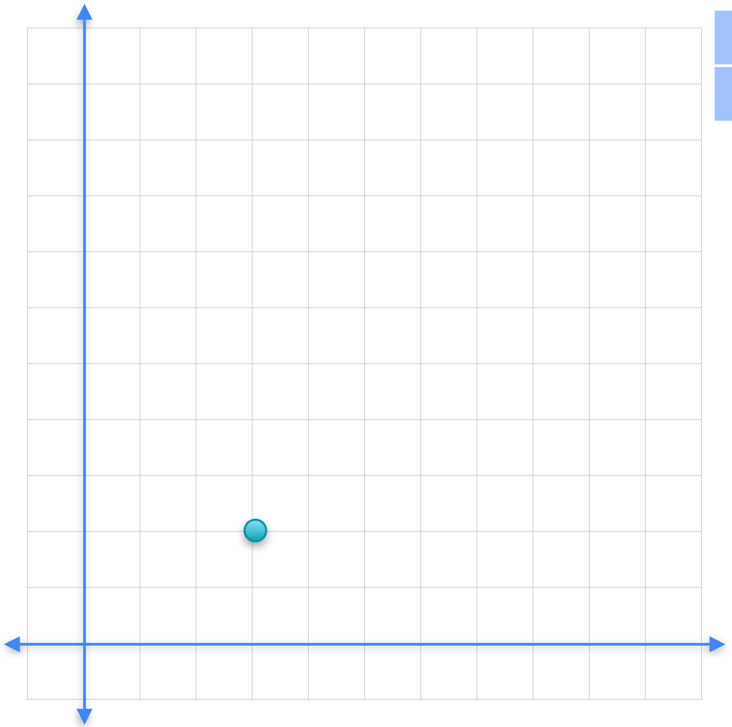
Eigenbasis



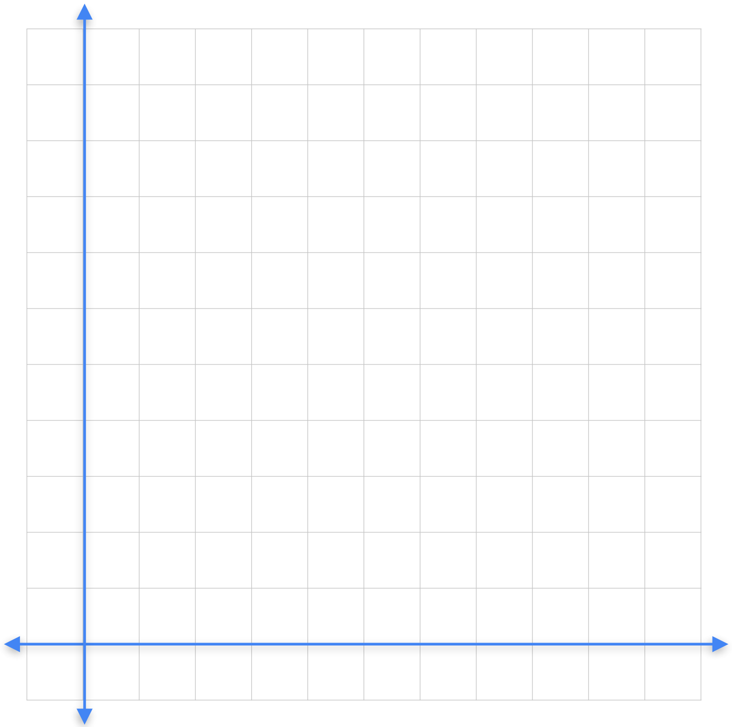
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$



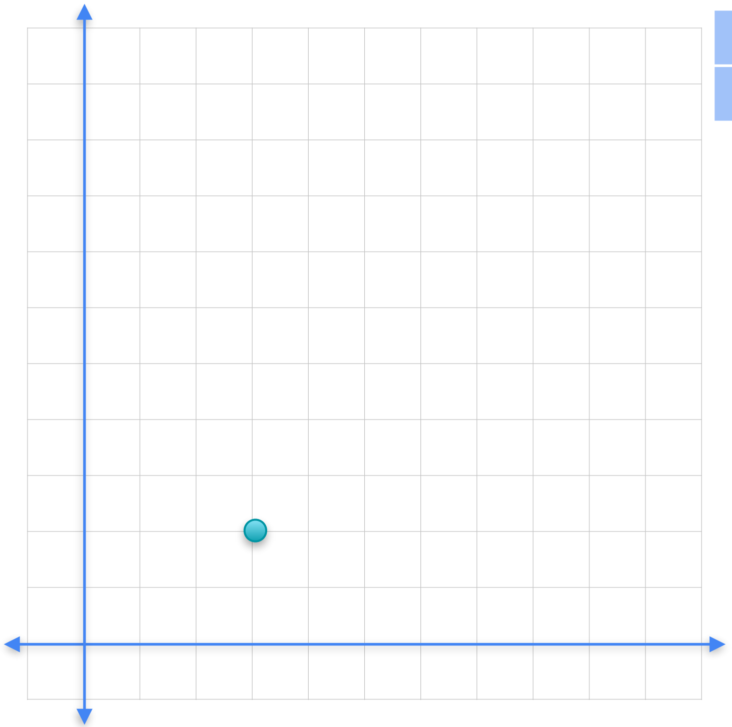
Eigenbasis



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

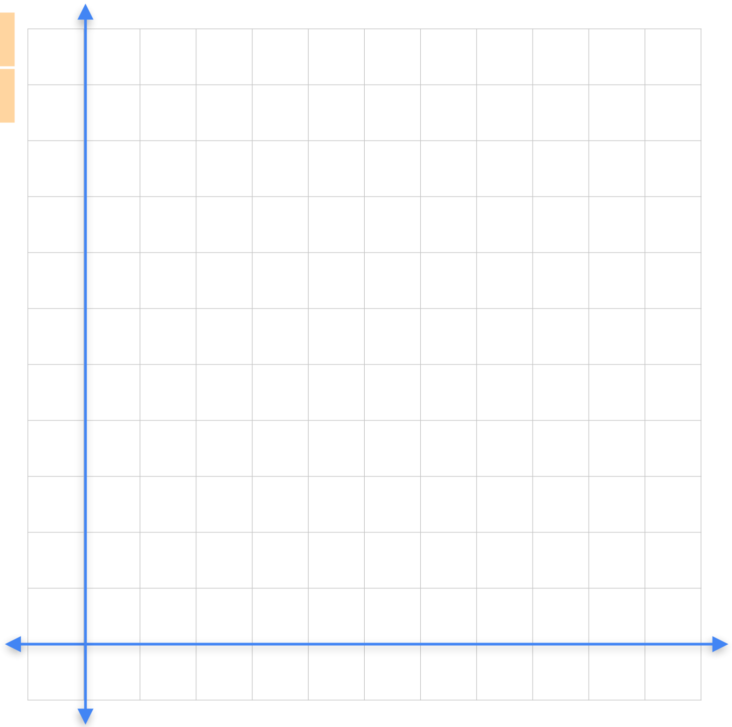


Eigenbasis

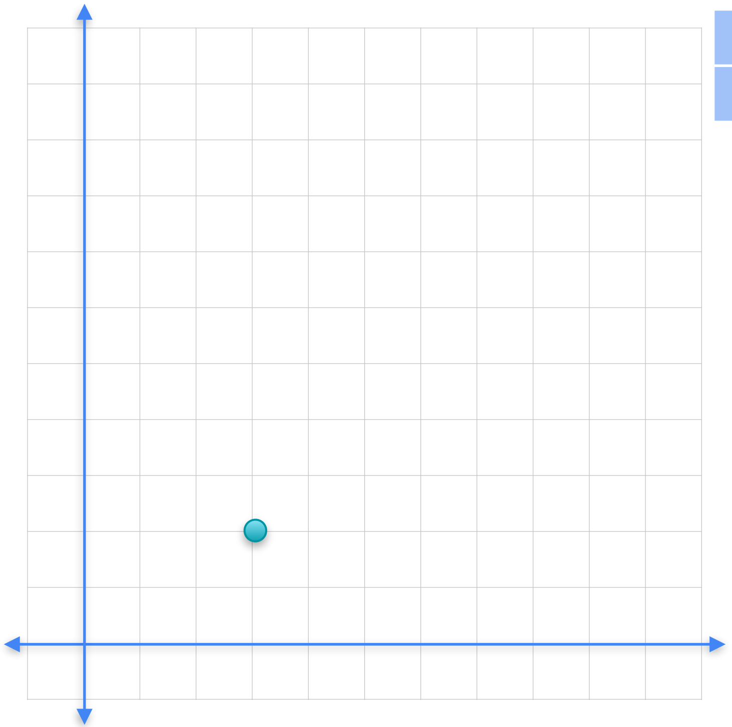


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$(3,2) \rightarrow (8,6)$

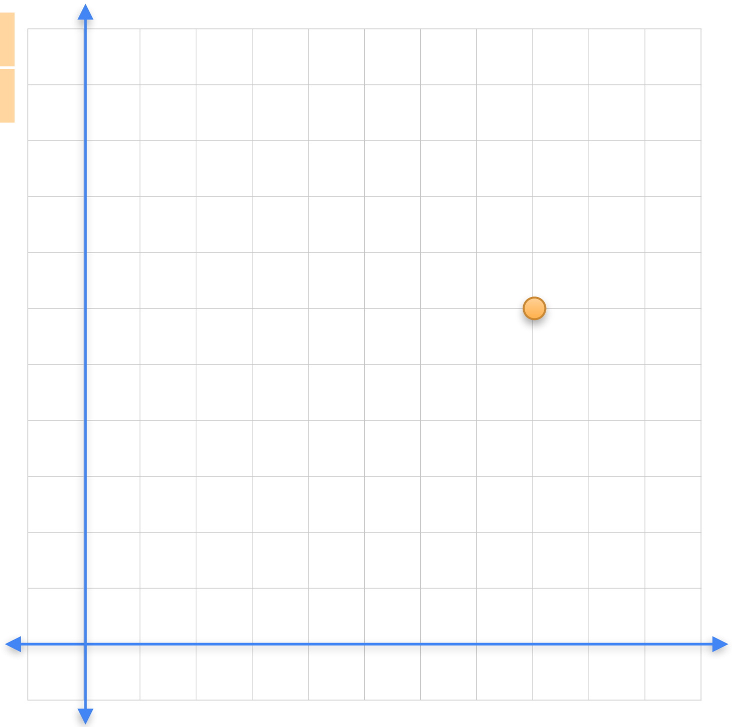


Eigenbasis

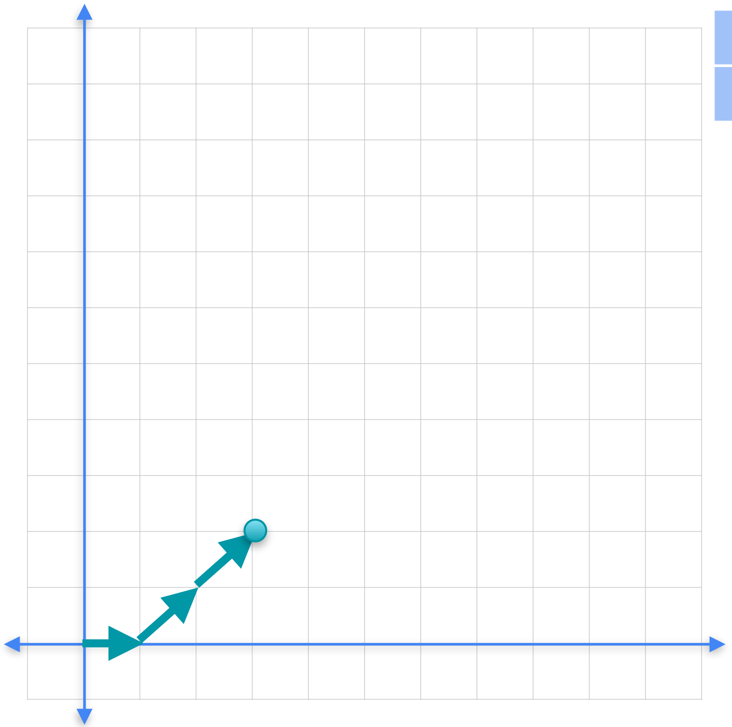


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$(3,2) \rightarrow (8,6)$

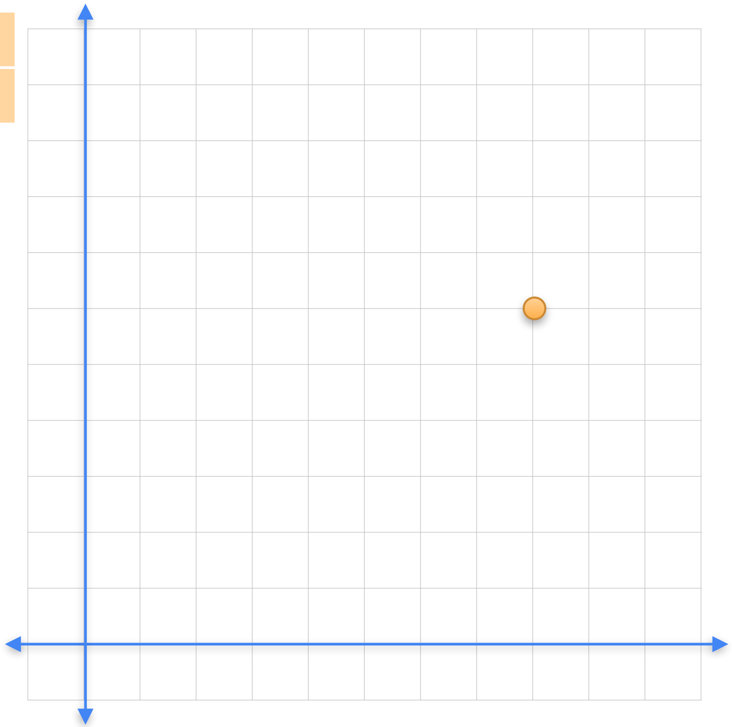


Eigenbasis

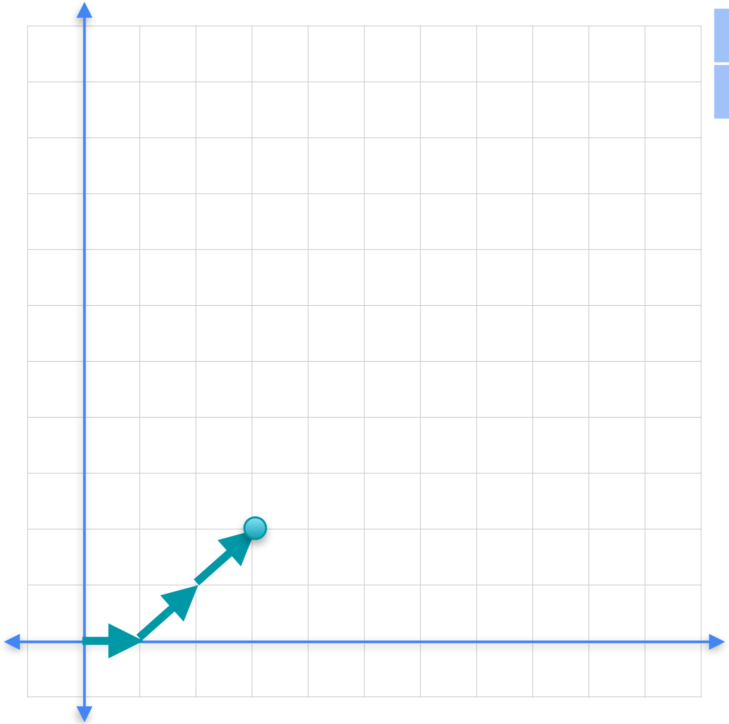


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$(3,2) \rightarrow (8,6)$

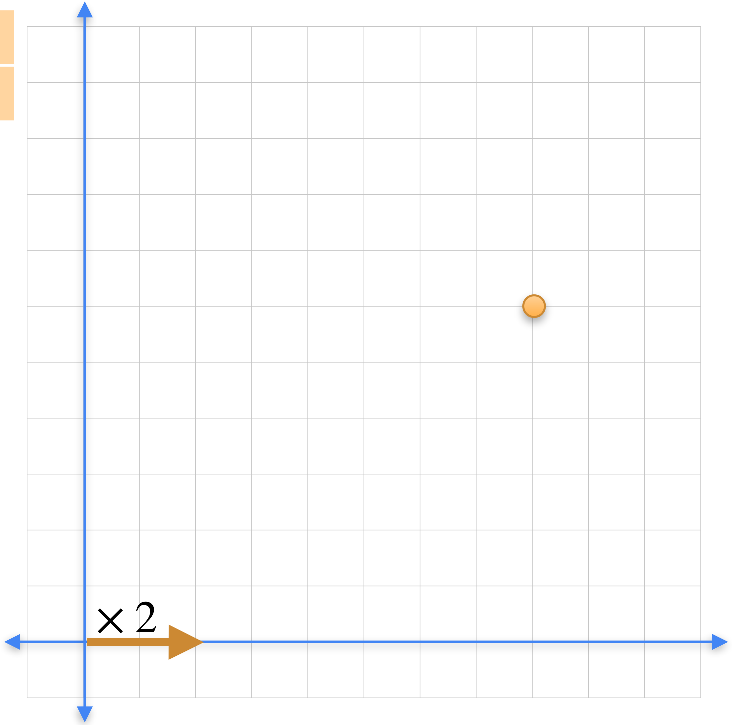


Eigenbasis

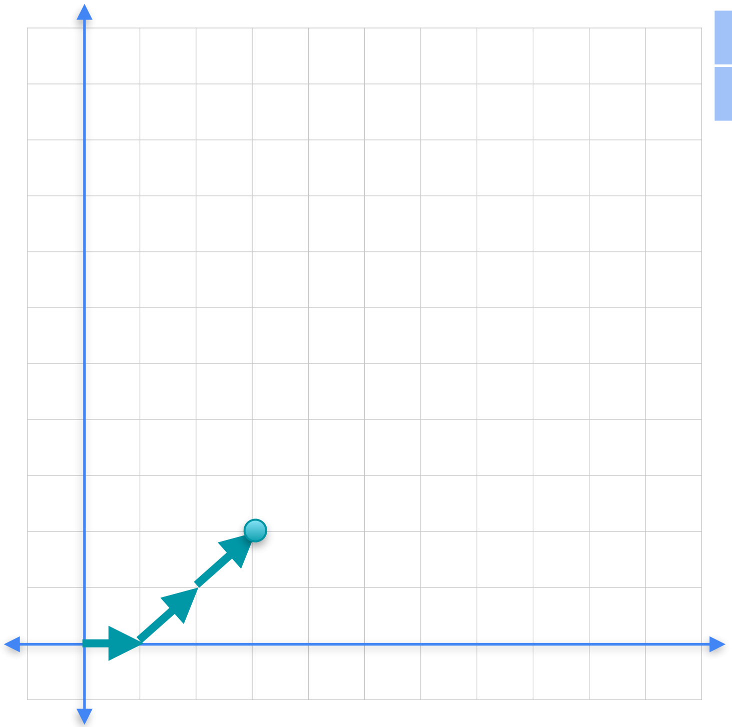


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$(3,2) \rightarrow (8,6)$

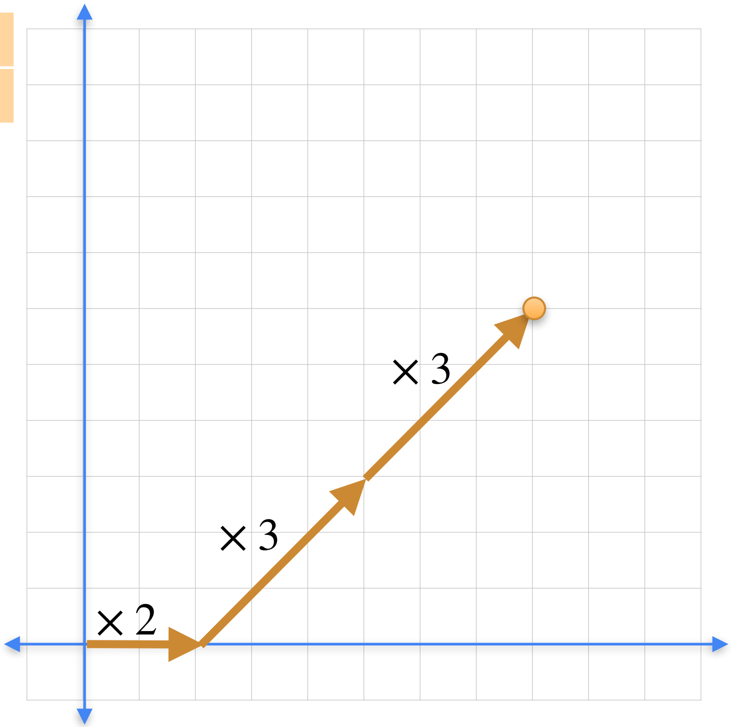


Eigenbasis



$$\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}8\\6\end{bmatrix}$$

$(3,2) \rightarrow (8,6)$



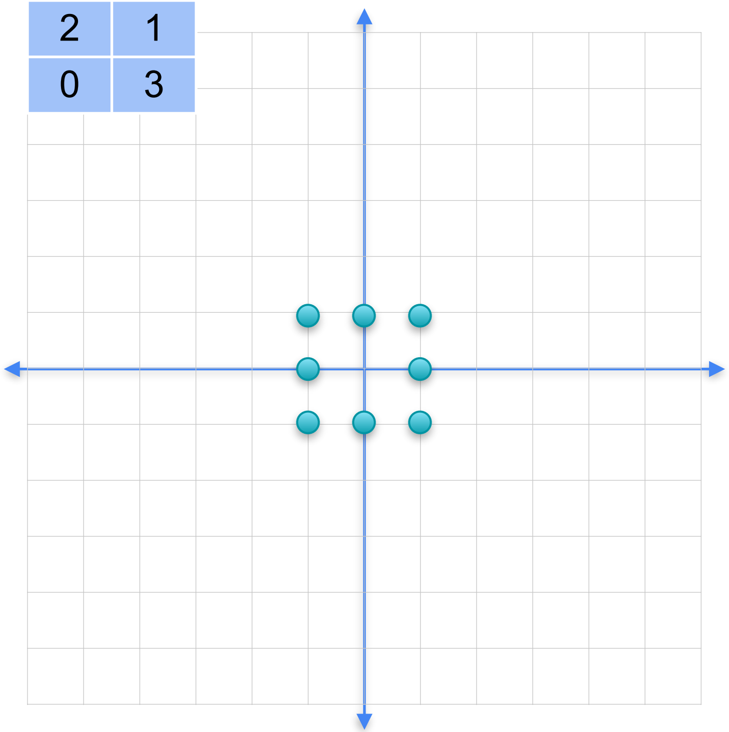
DeepLearning.AI

Determinants and Eigenvectors

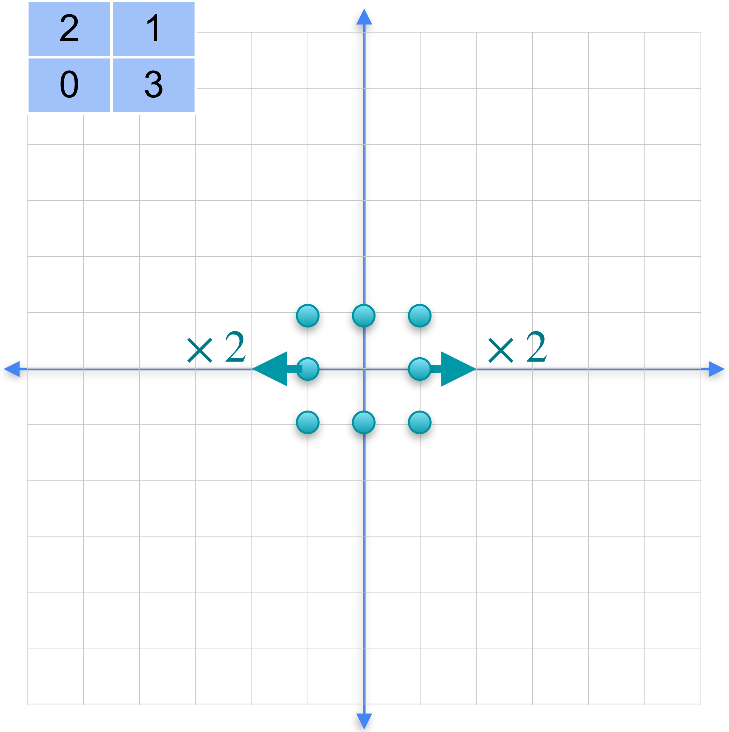
Eigenvalues and eigenvectors

Finding eigenvalues

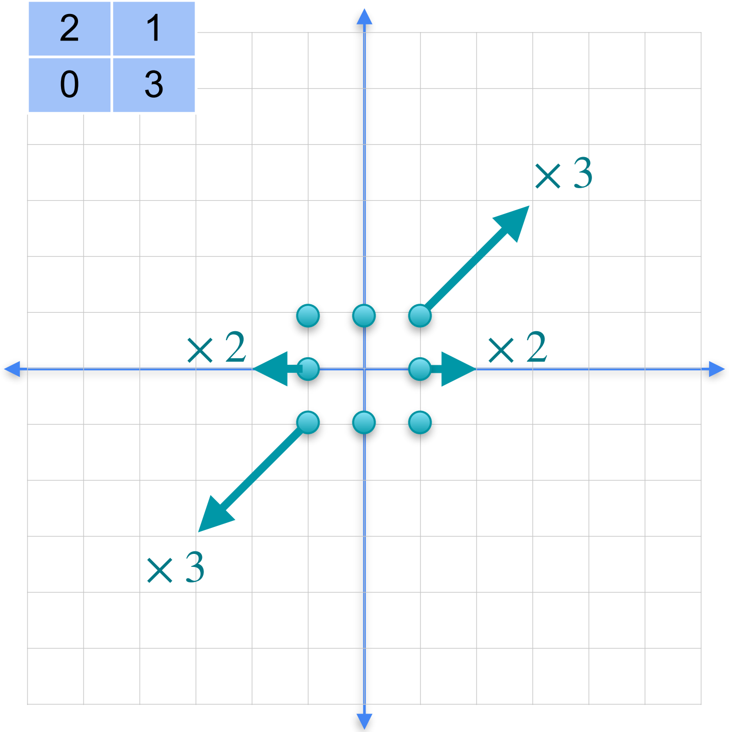
2	1
0	3



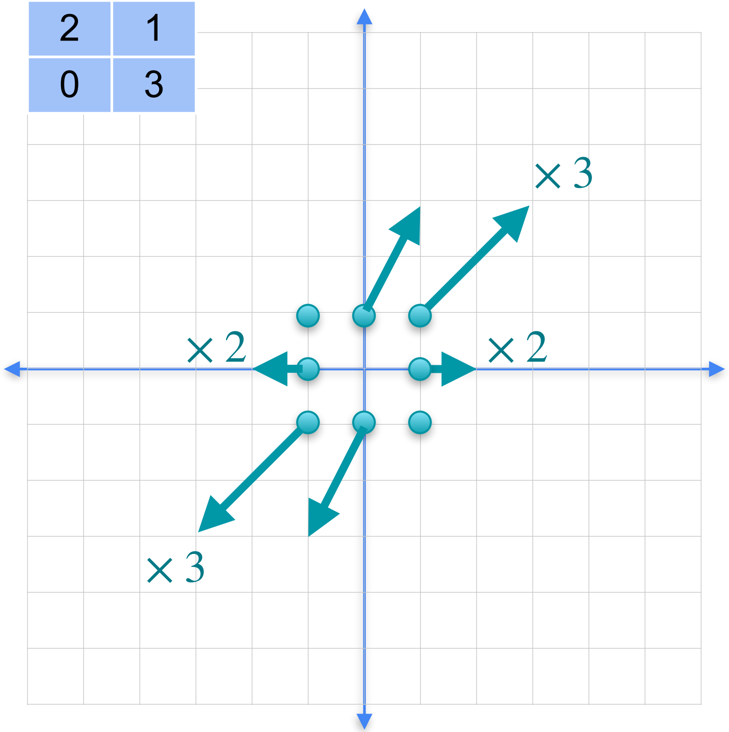
Finding eigenvalues



Finding eigenvalues

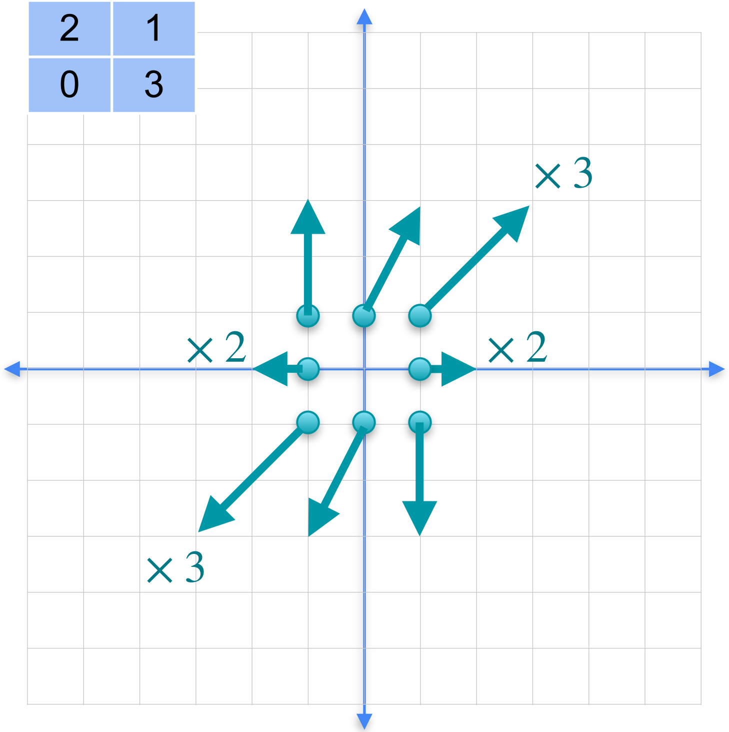


Finding eigenvalues

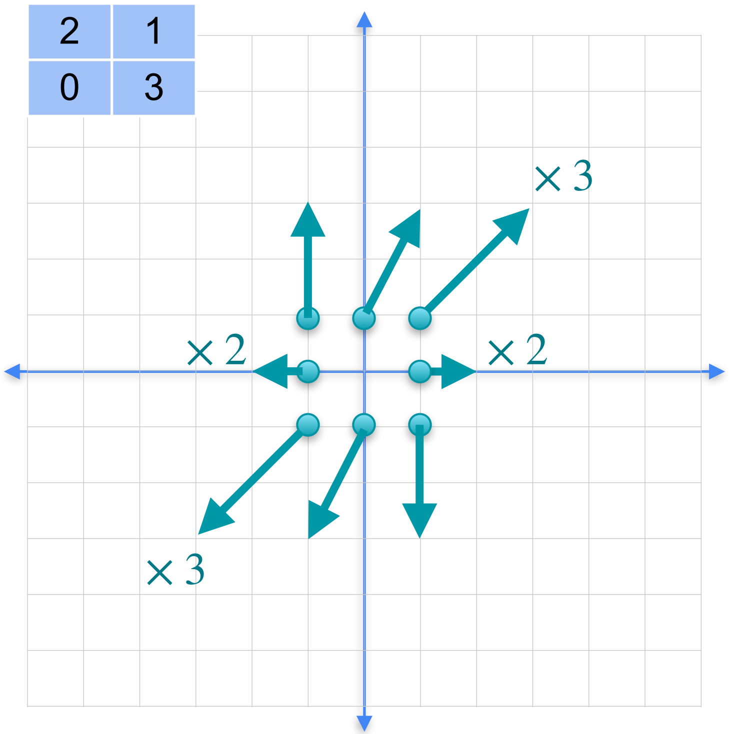


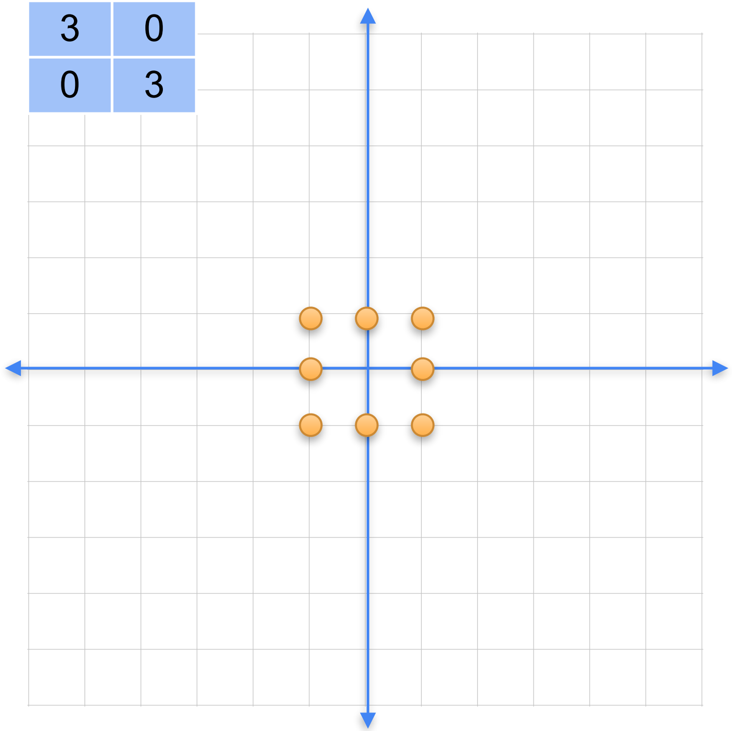
Finding eigenvalues

2	1
0	3

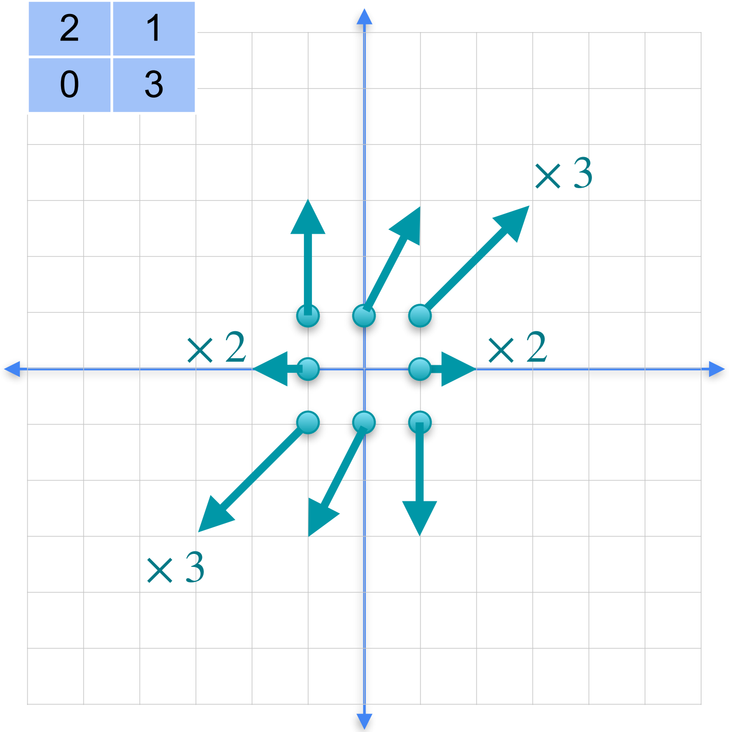


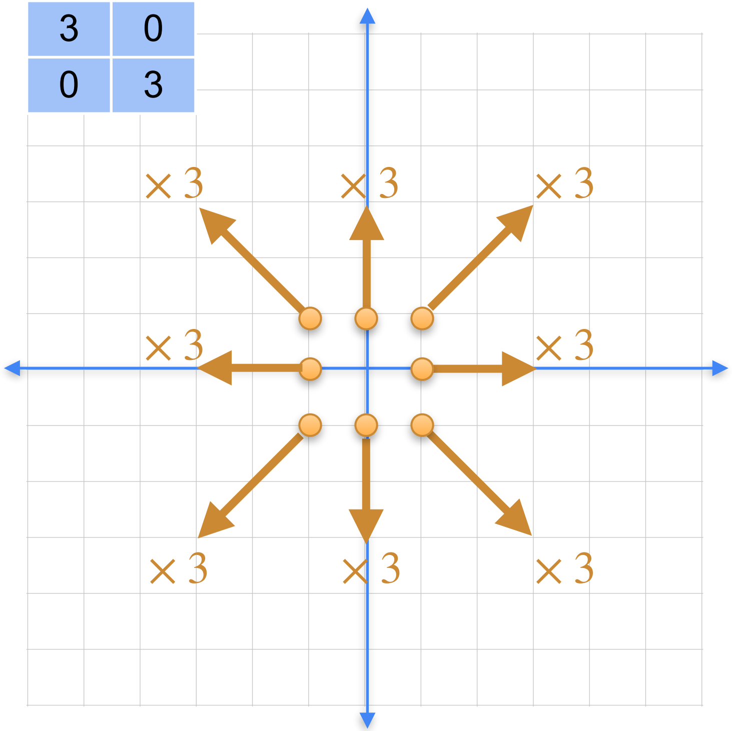
Finding eigenvalues





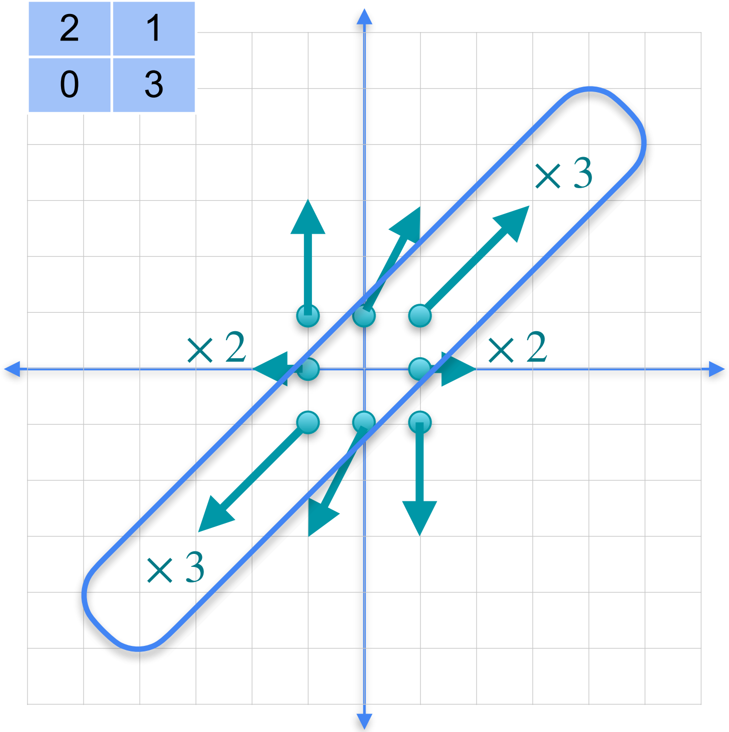
Finding eigenvalues

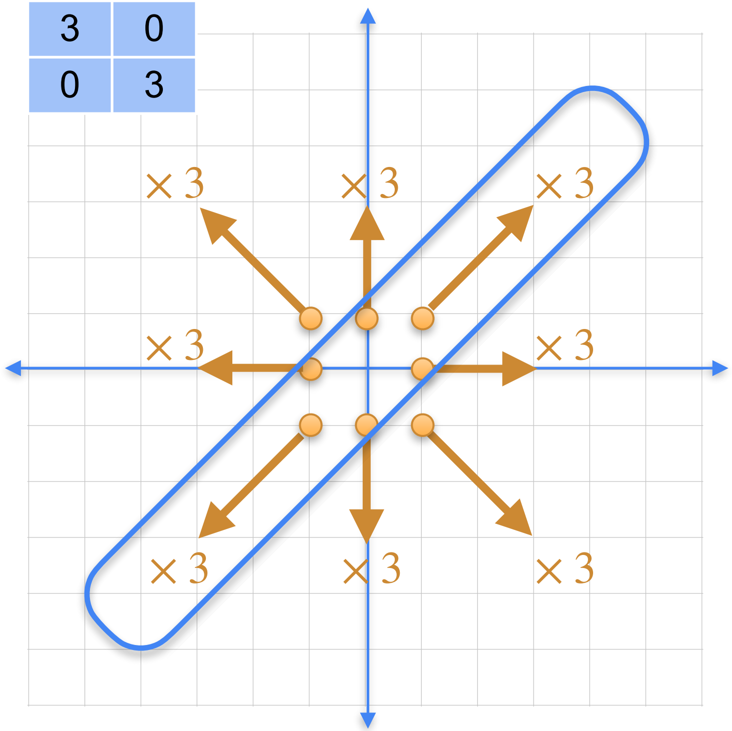




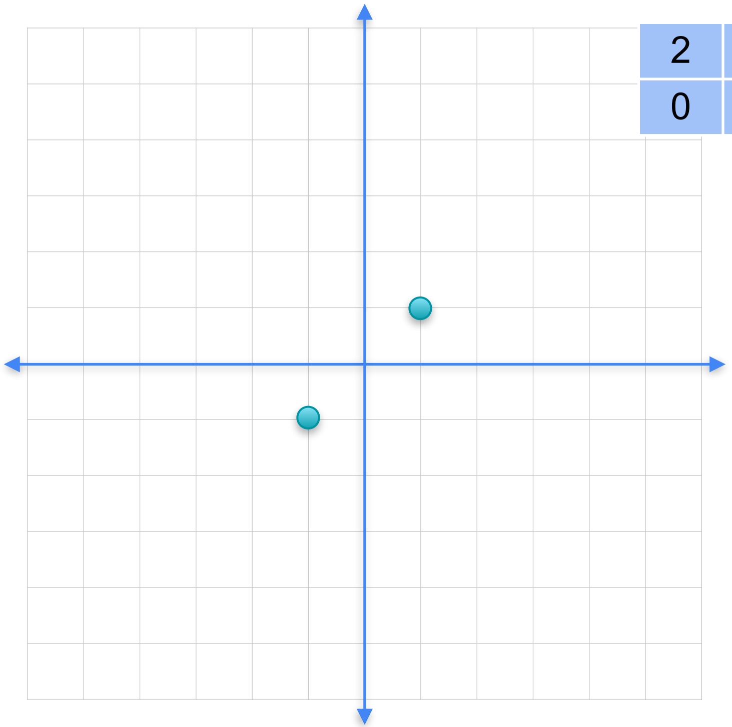
DeepLearning.AI

Finding eigenvalues

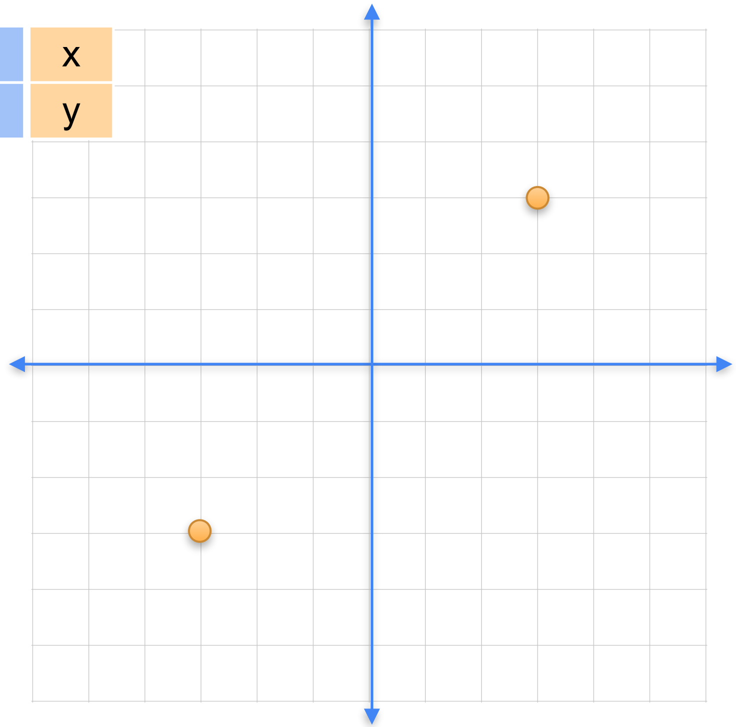




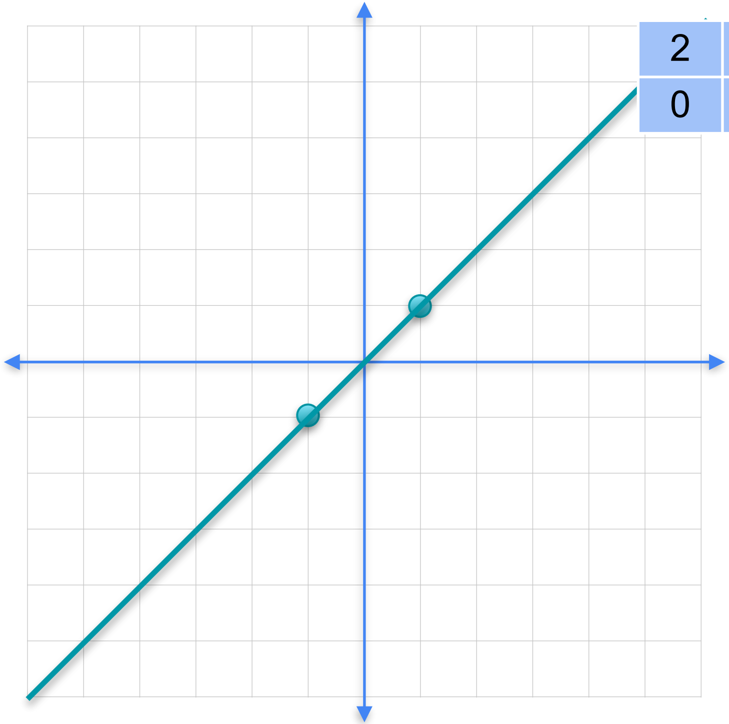
Finding eigenvalues



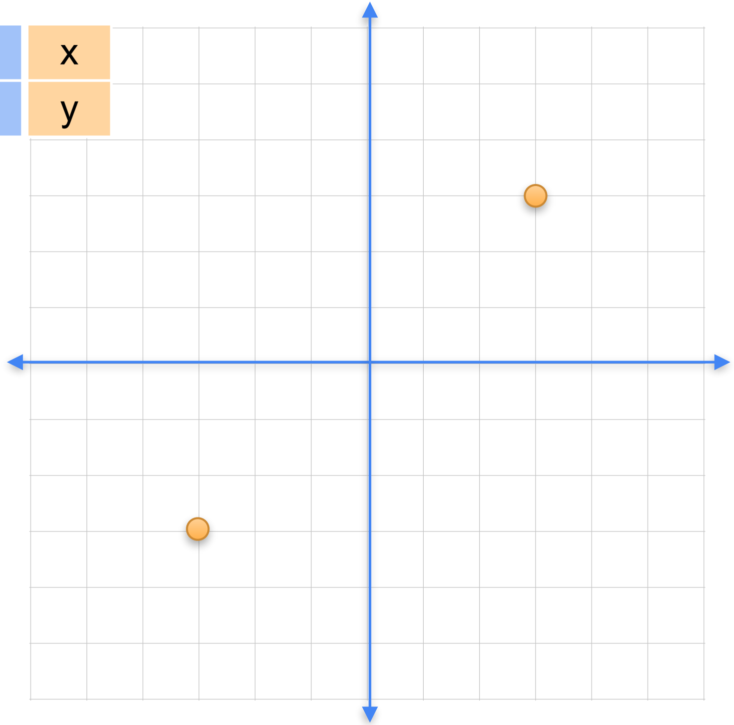
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



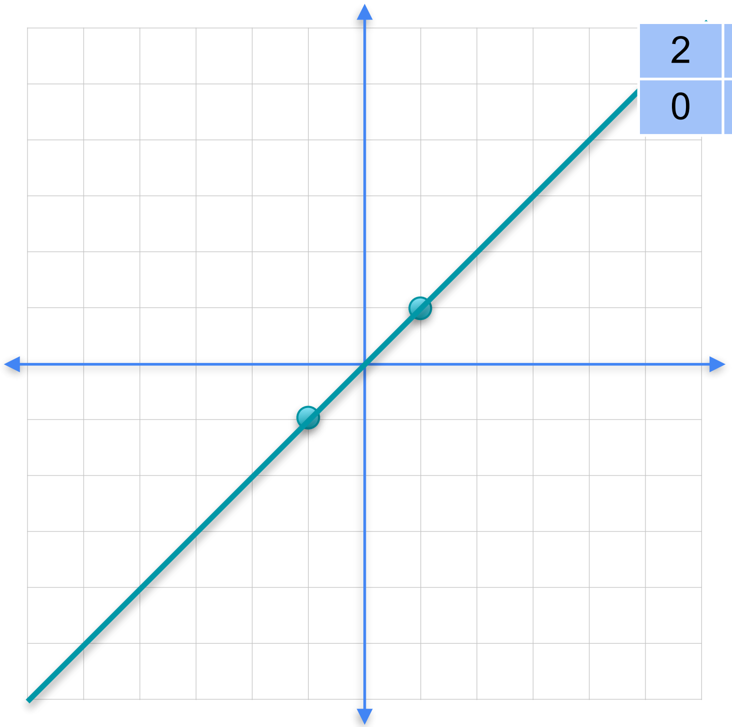
Finding eigenvalues



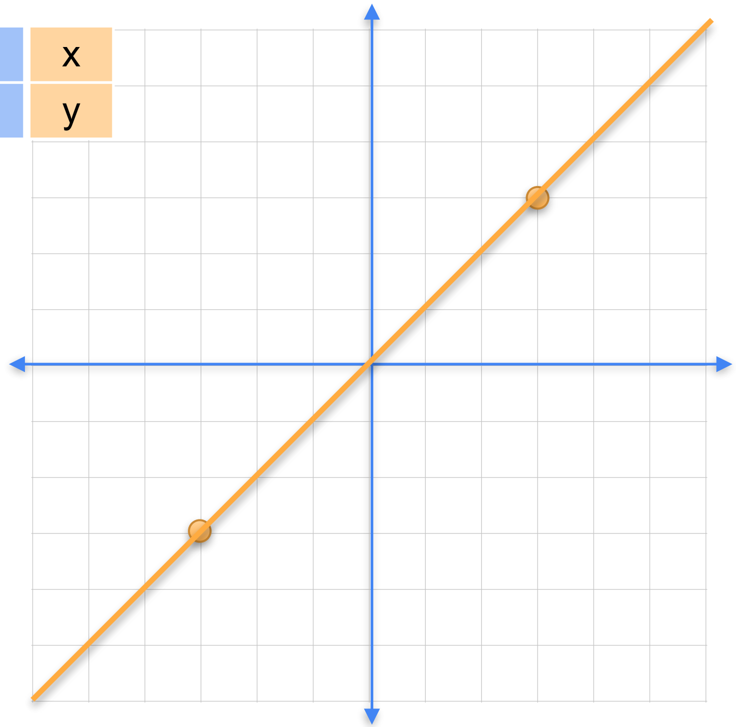
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



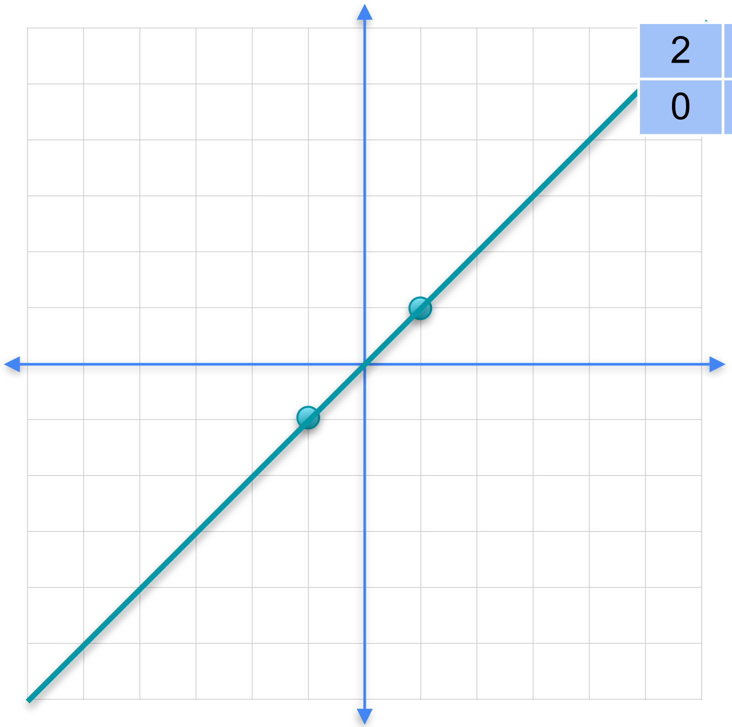
Finding eigenvalues



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

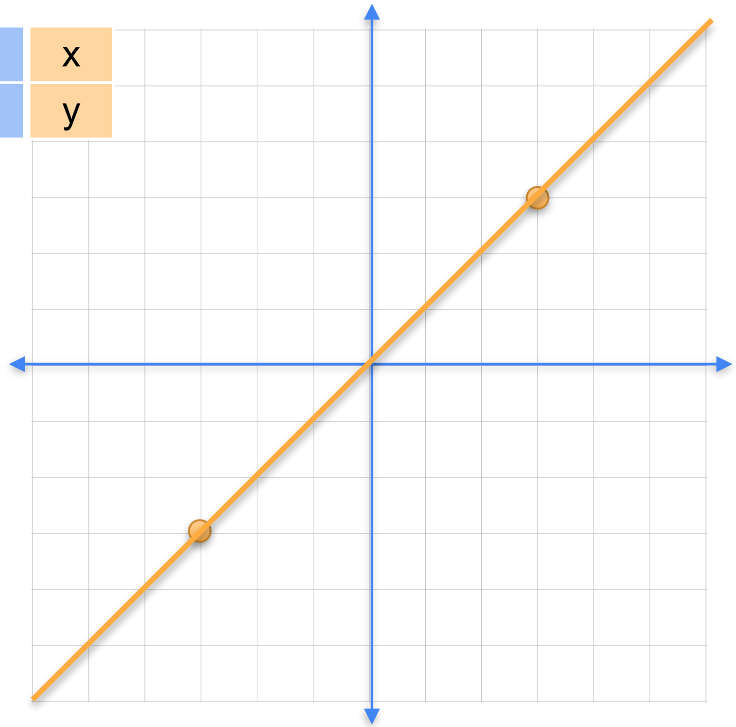


Finding eigenvalues

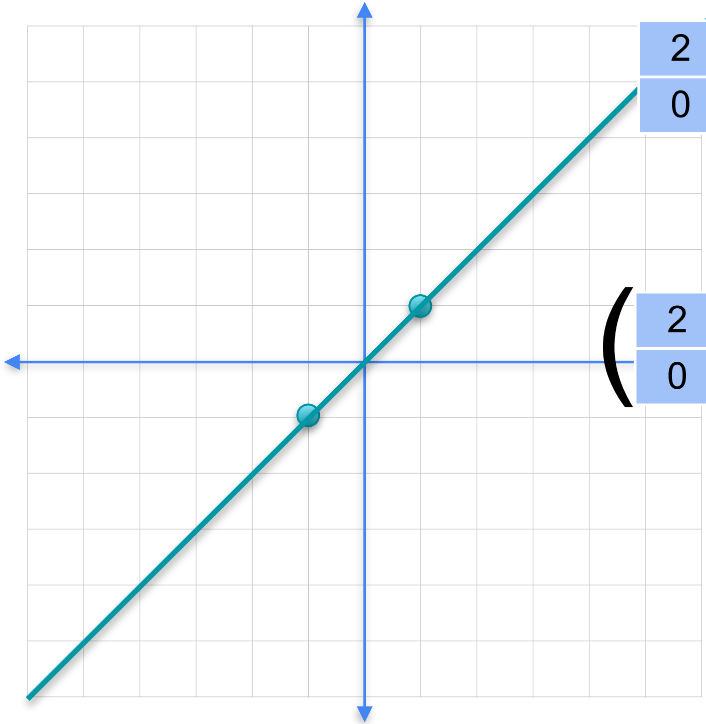


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points



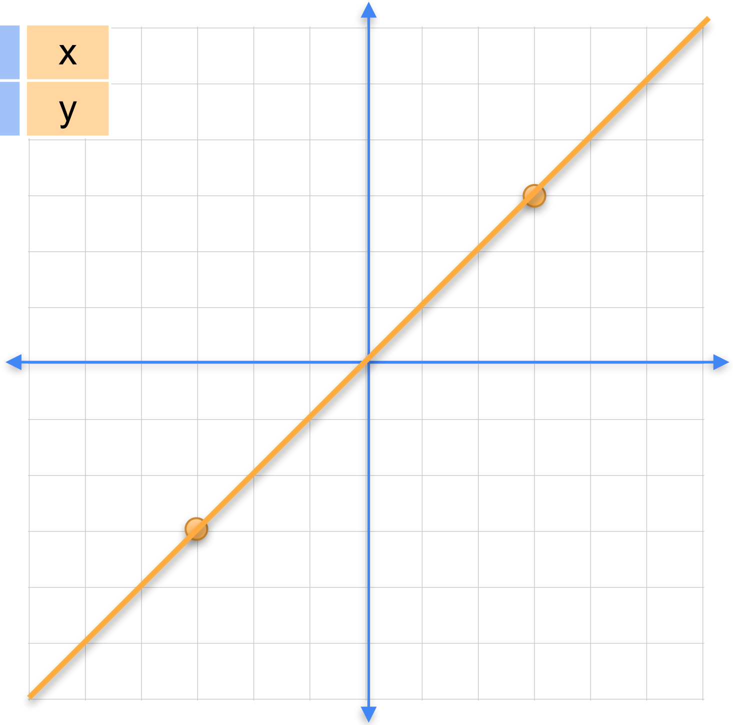
Finding eigenvalues



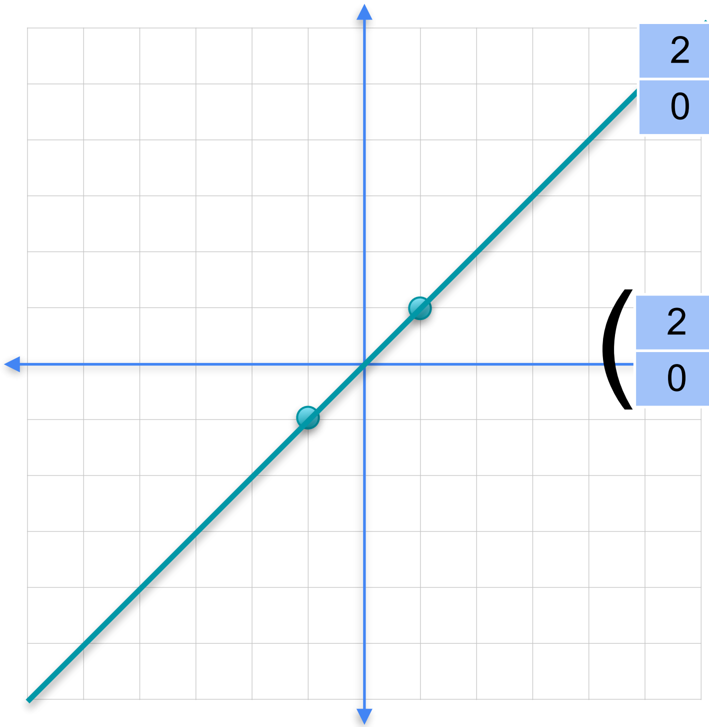
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right)$$



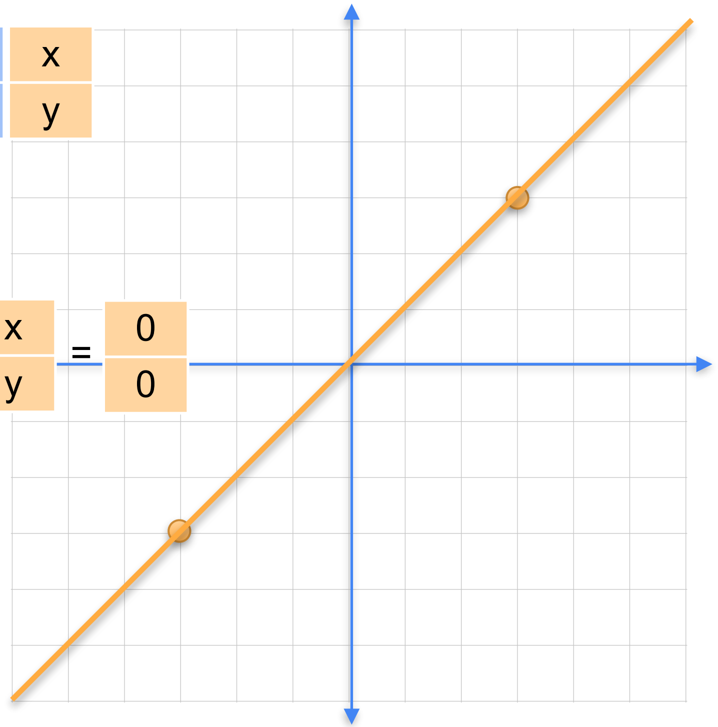
Finding eigenvalues



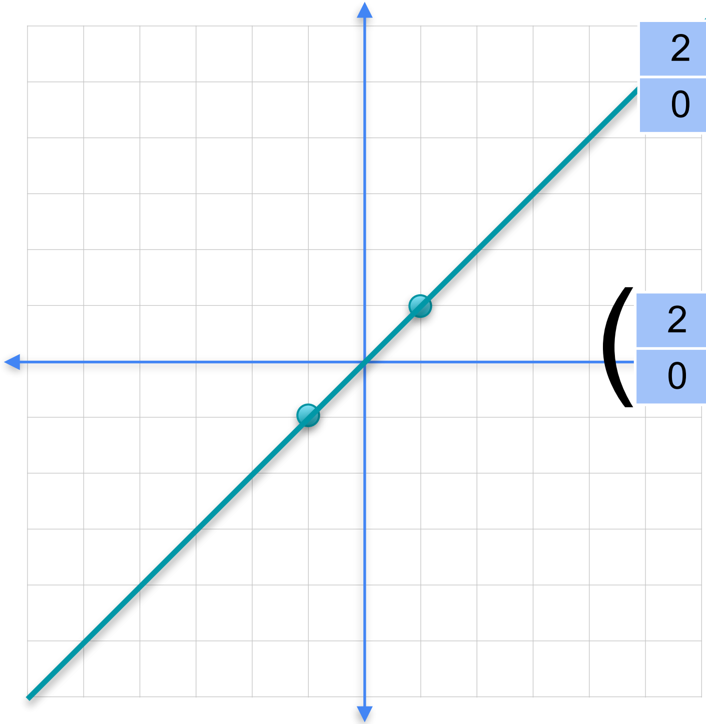
$$\begin{bmatrix}2&1\\0&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3&0\\0&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix}2&1\\0&3\end{bmatrix}-\begin{bmatrix}3&0\\0&3\end{bmatrix}\right)\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$



Finding eigenvalues

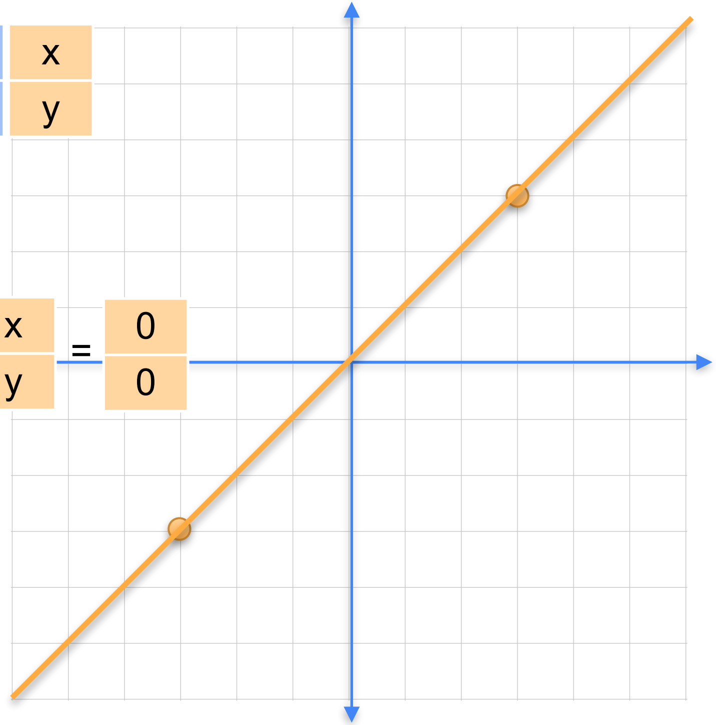


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

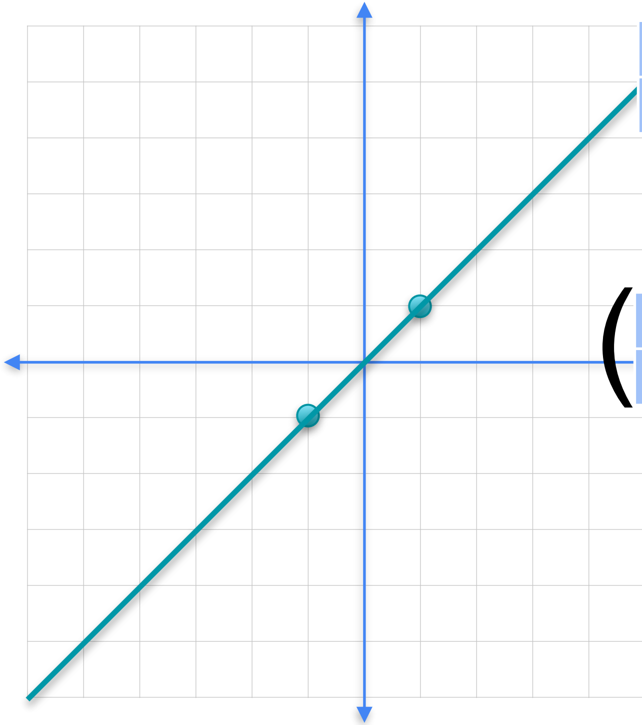
For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Finding eigenvalues



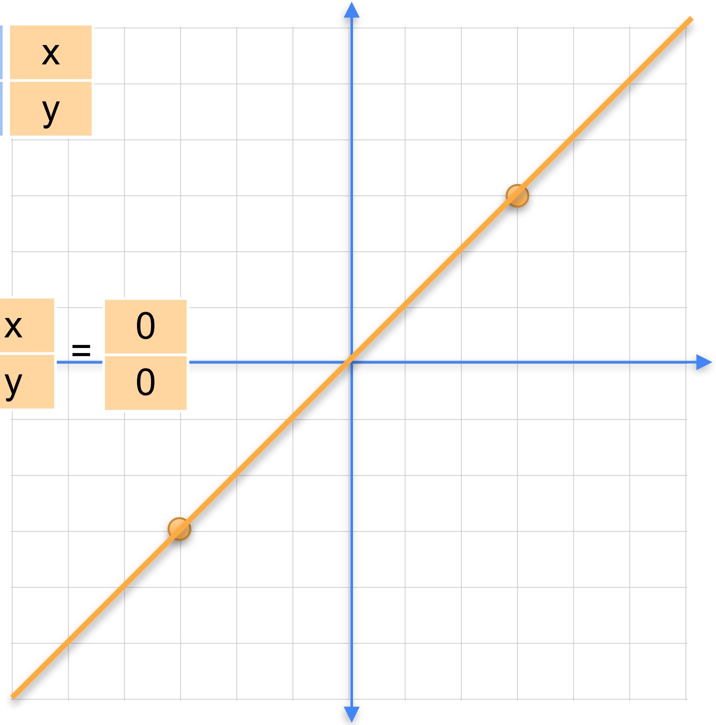
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

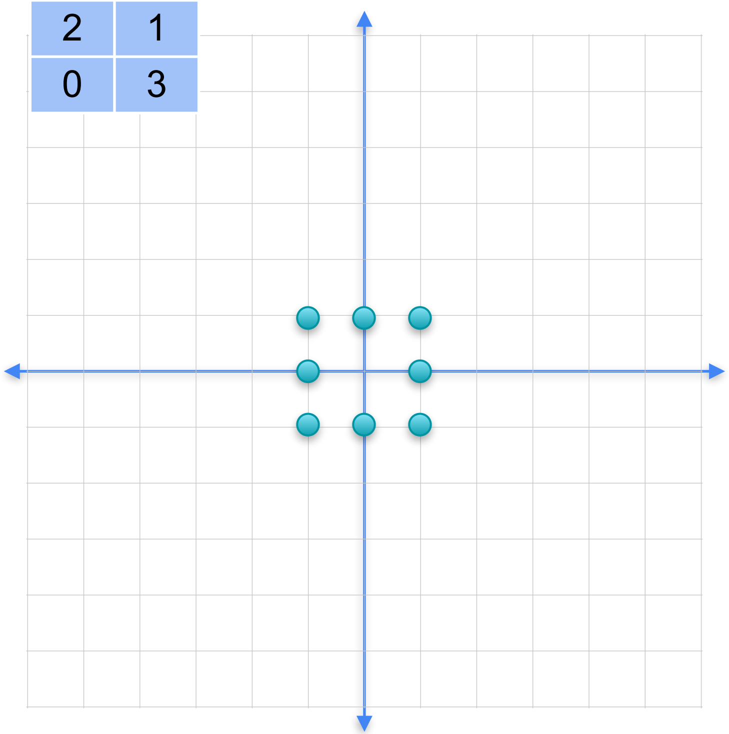
$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = 0$$

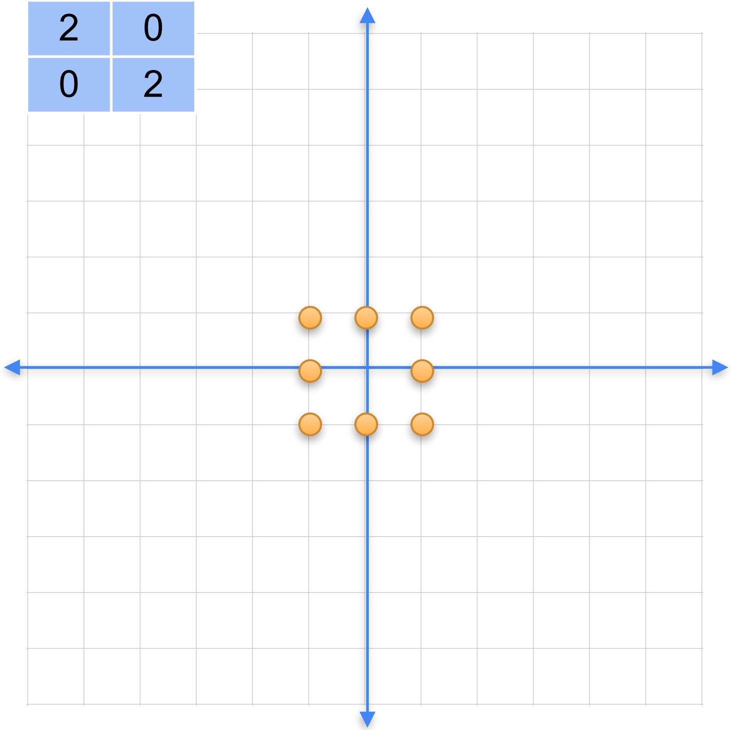


Finding eigenvalues

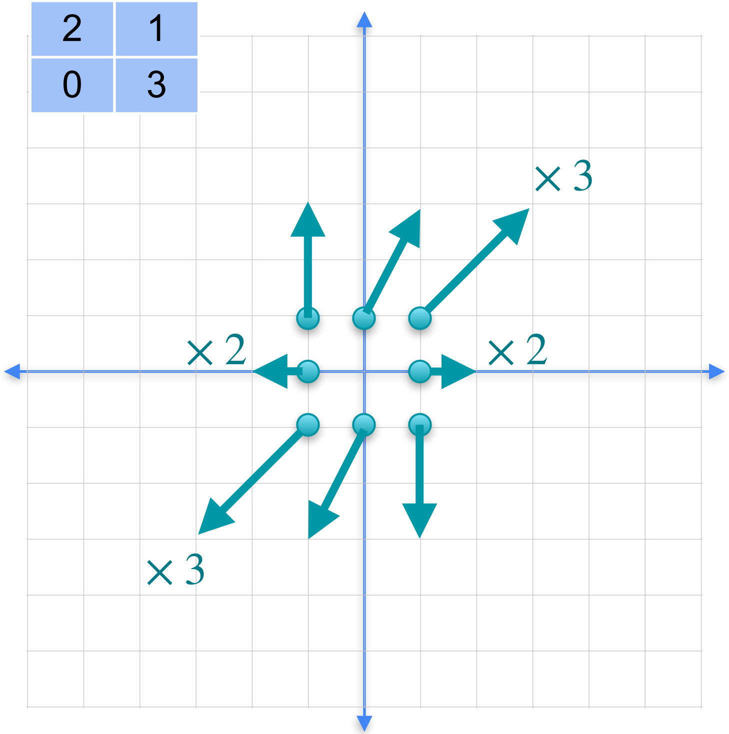
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

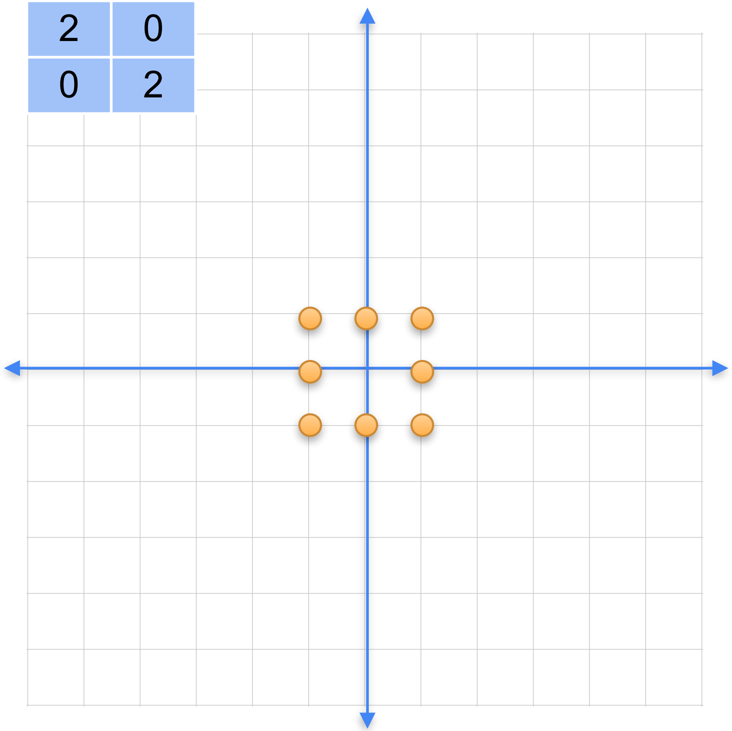


$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

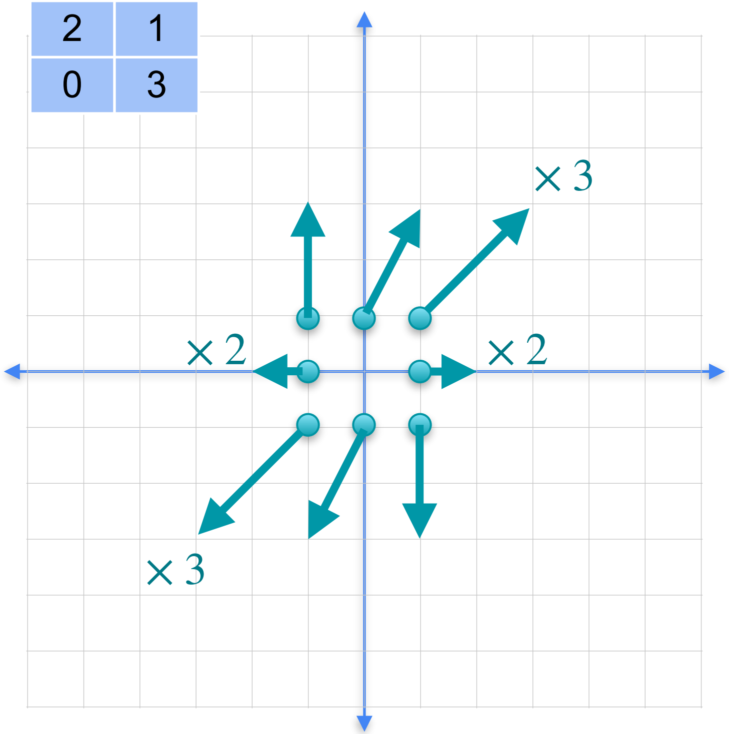


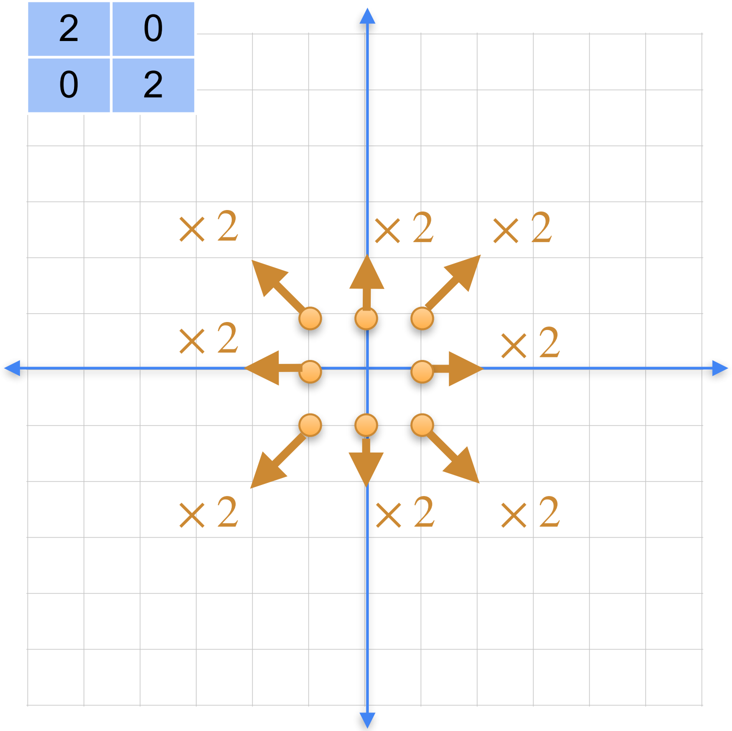
Finding eigenvalues



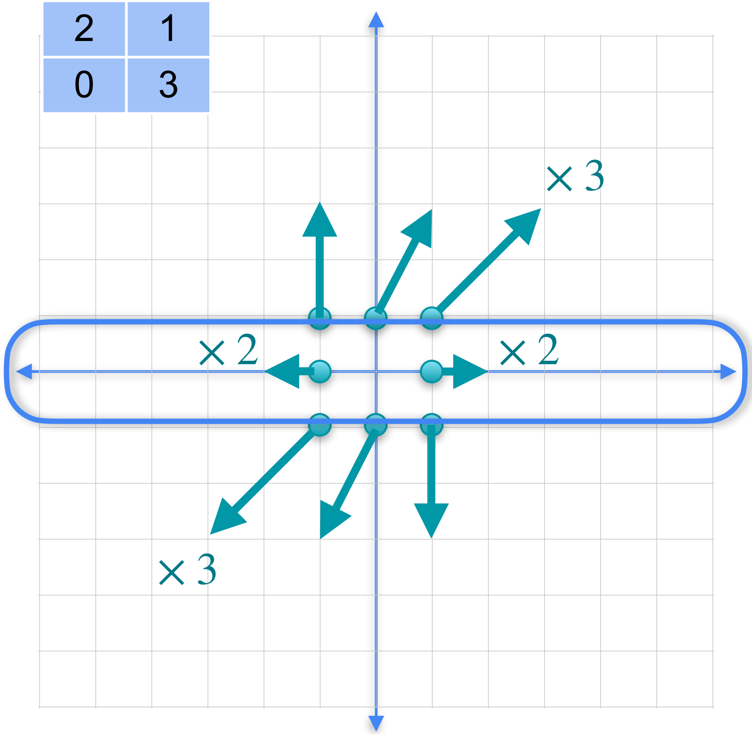


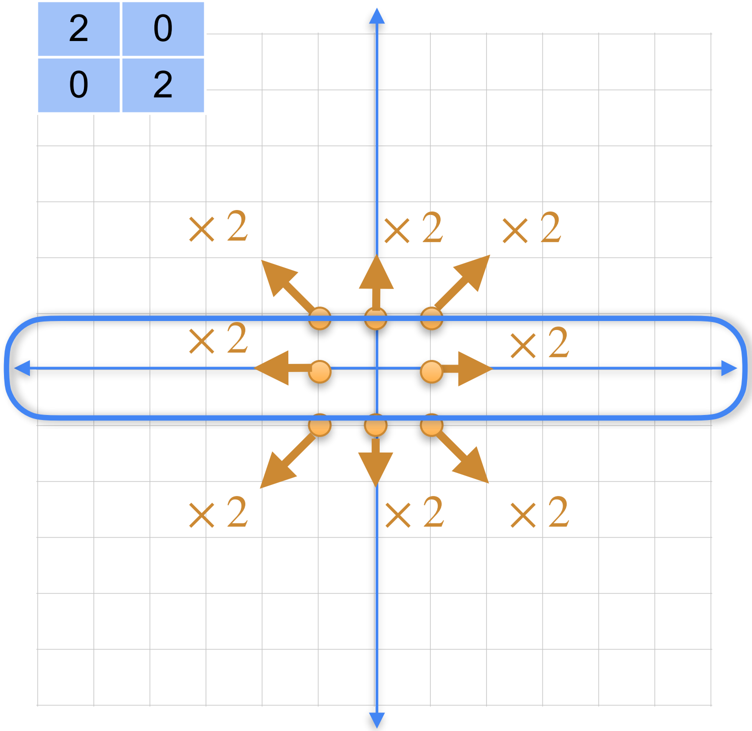
Finding eigenvalues





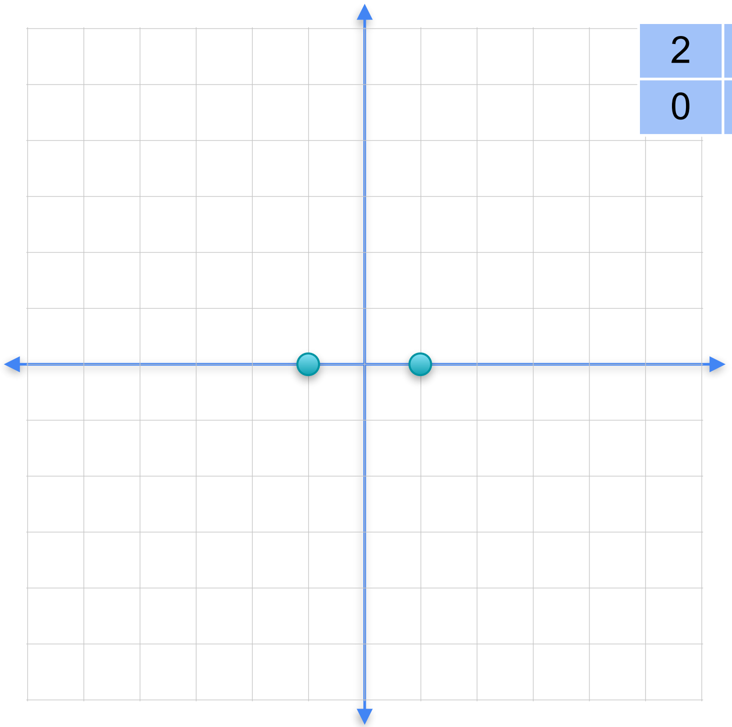
Finding eigenvalues

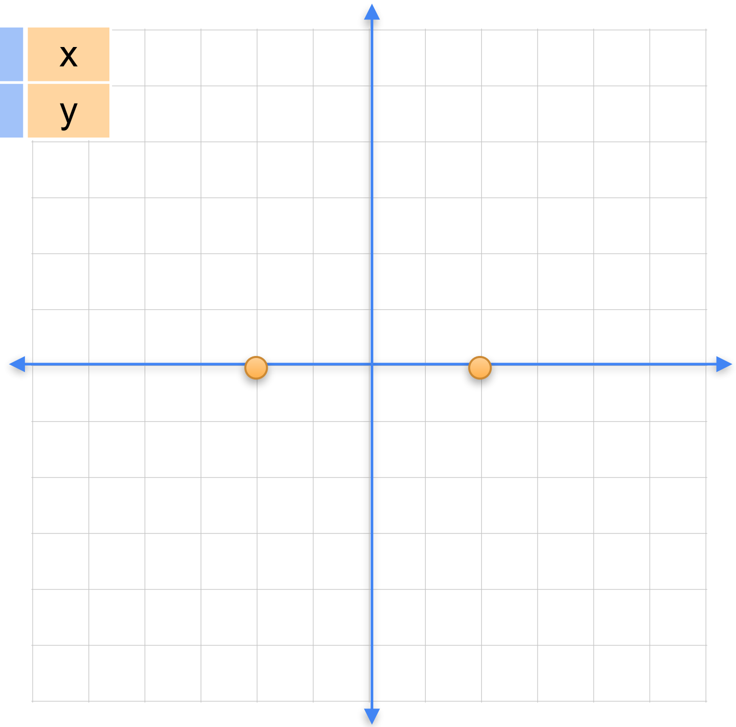




Finding eigenvalues

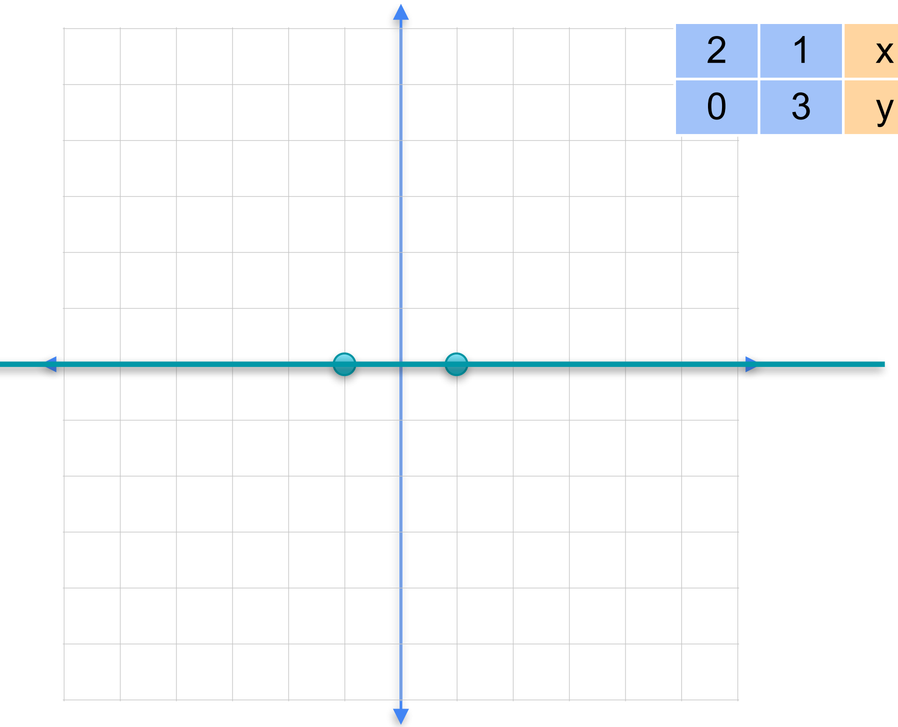
$$\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

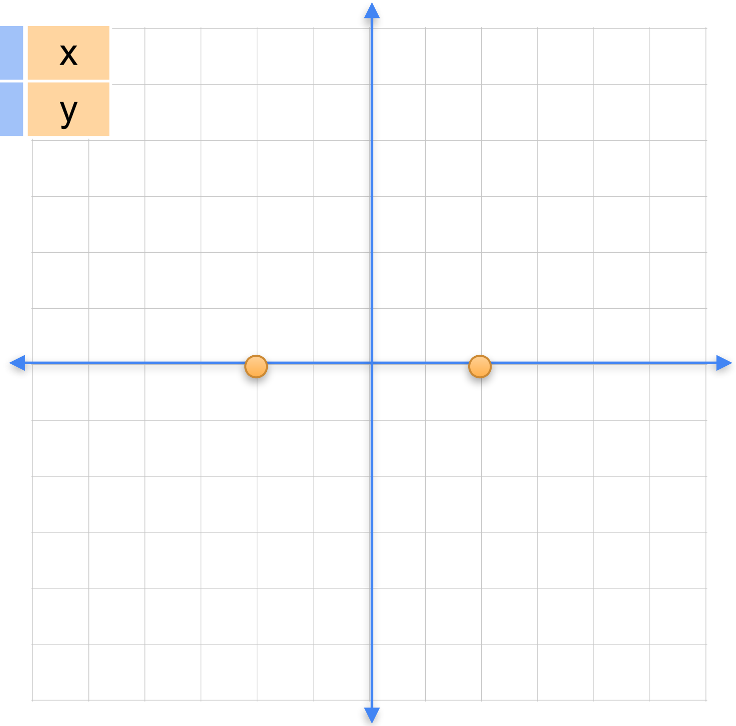




Finding eigenvalues

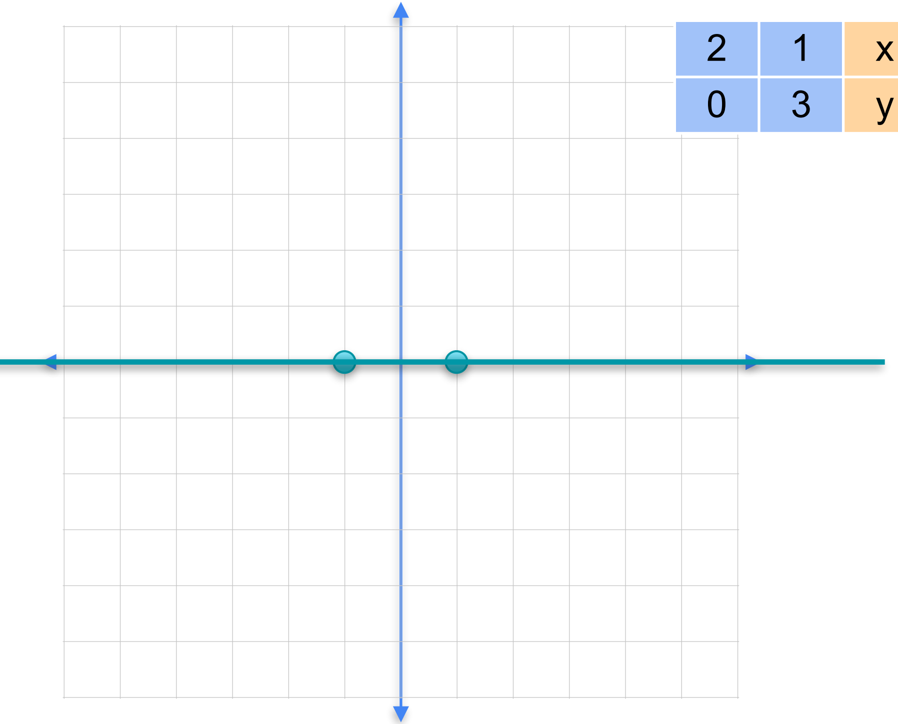
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

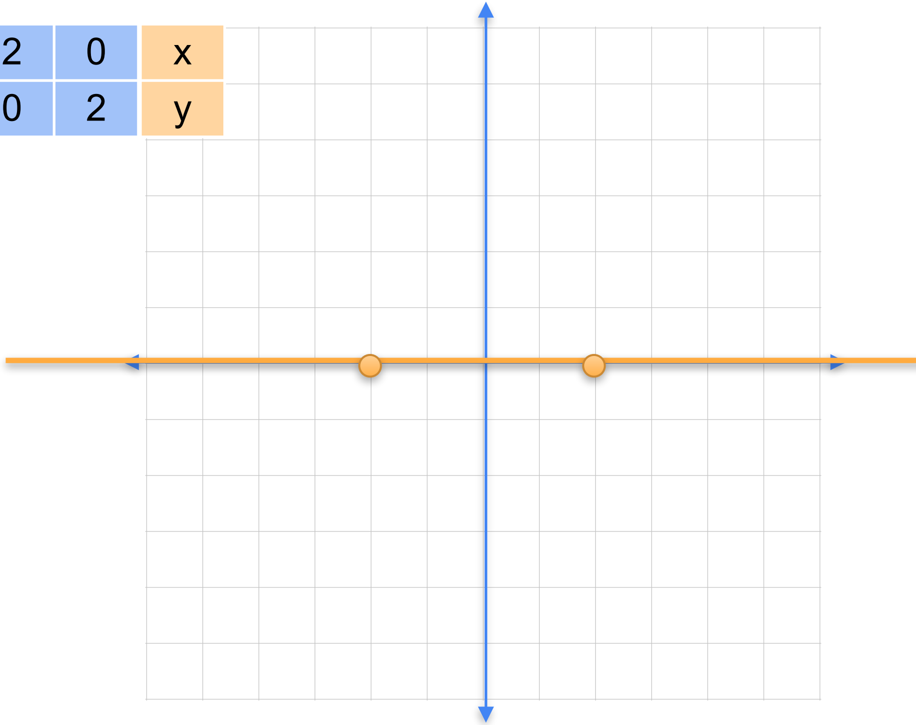




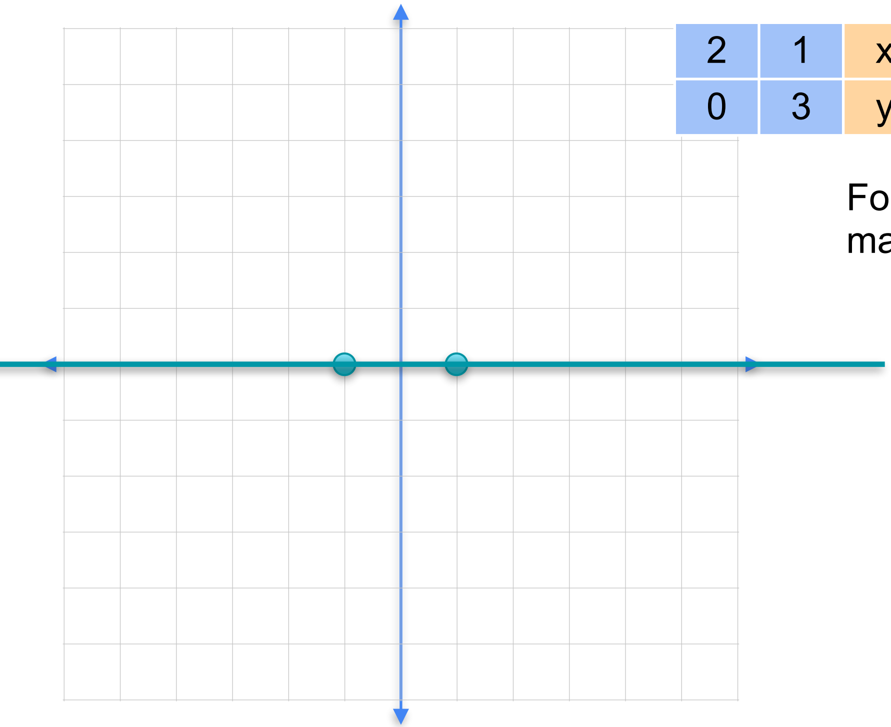
Finding eigenvalues

$$\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$



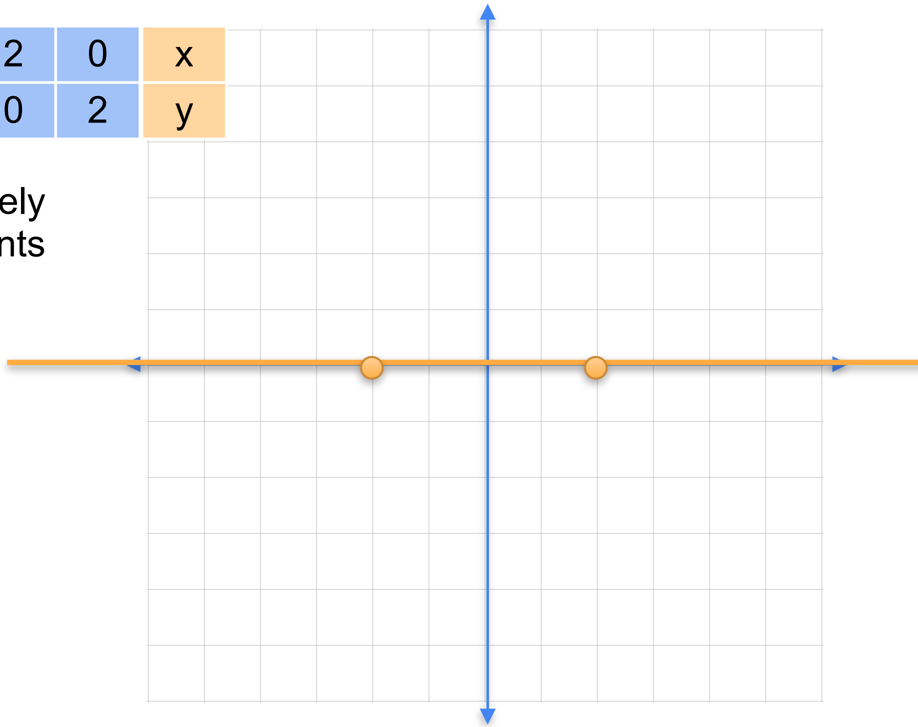


Finding eigenvalues

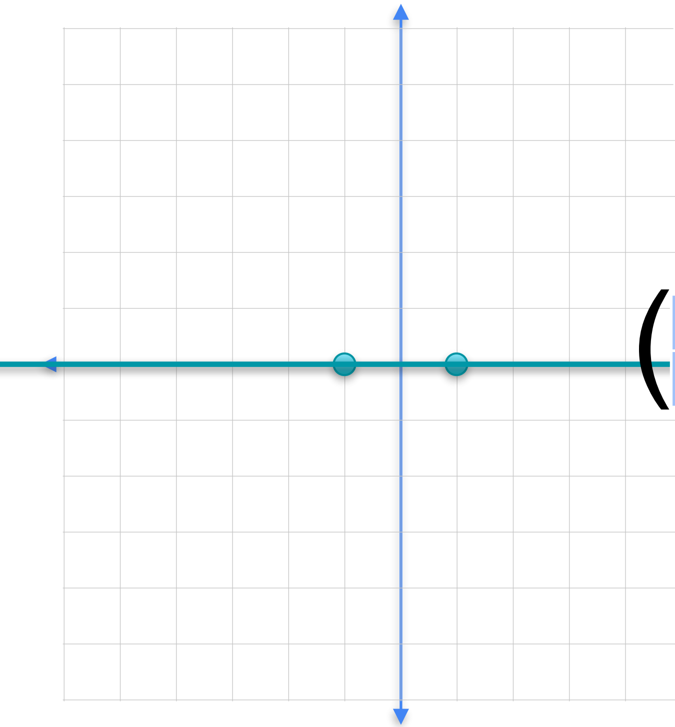


$$\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

For infinitely many points



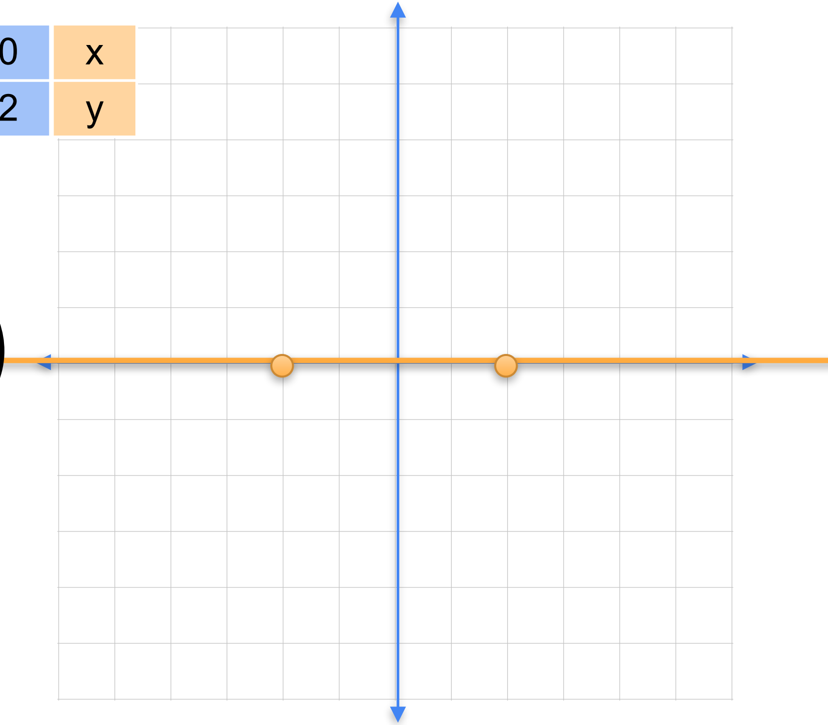
Finding eigenvalues



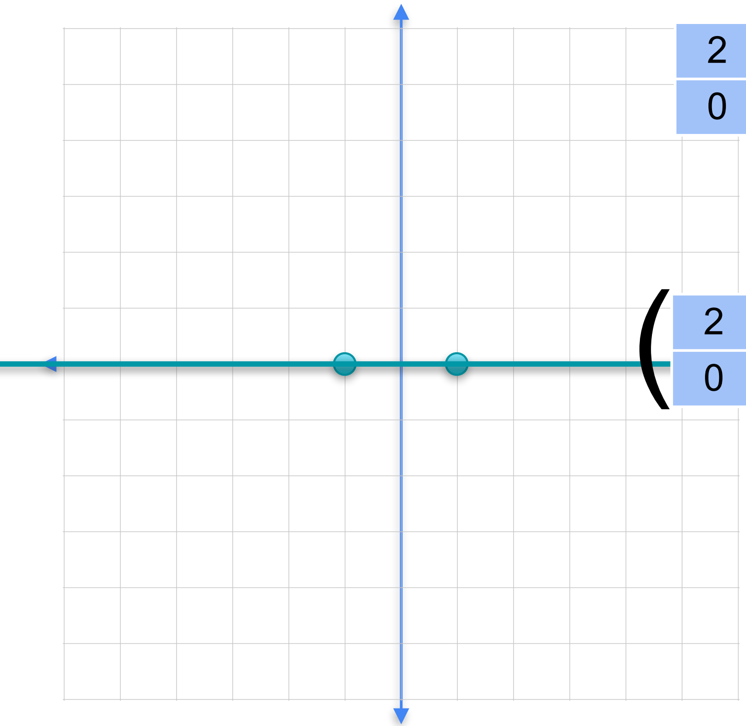
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right)$$



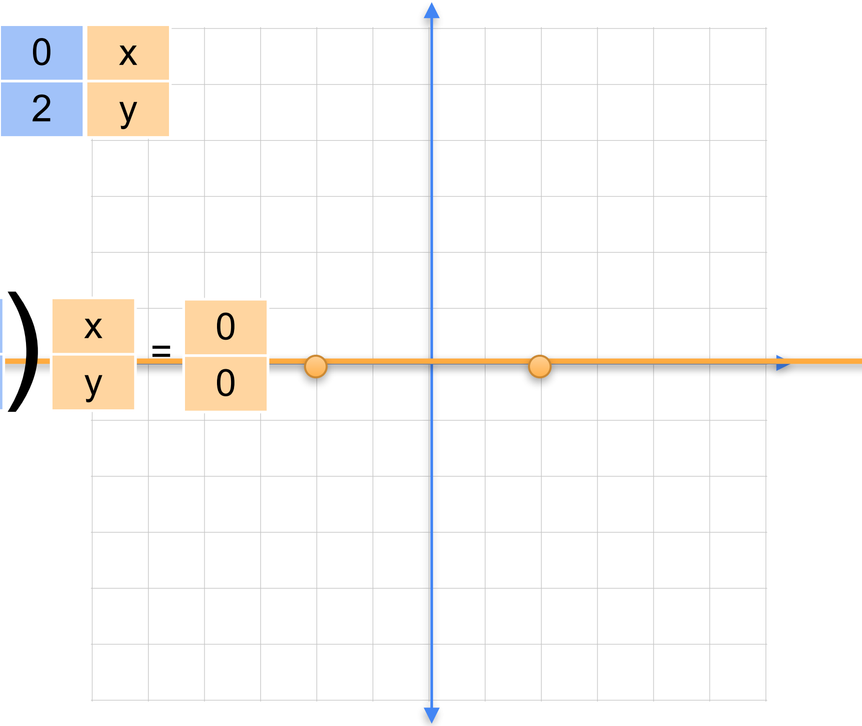
Finding eigenvalues



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



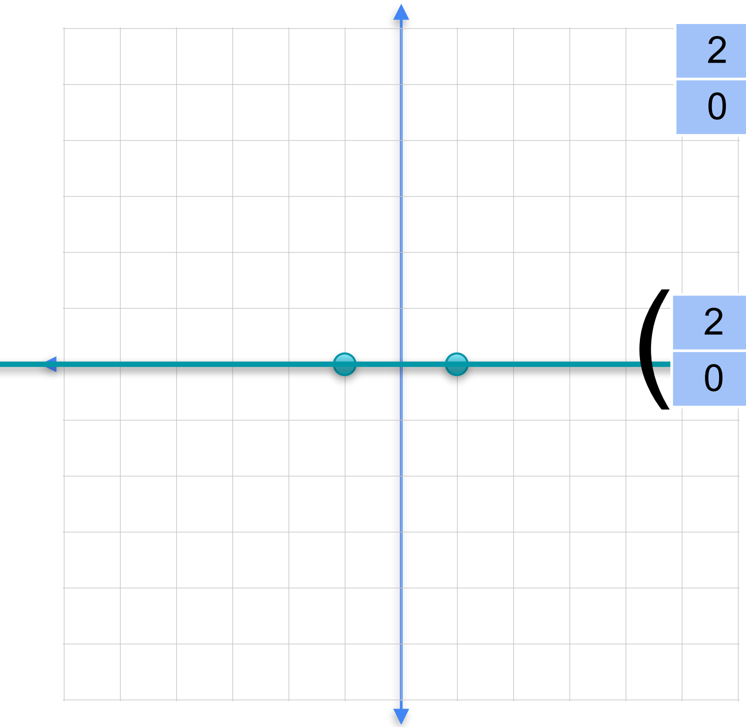
Finding eigenvalues

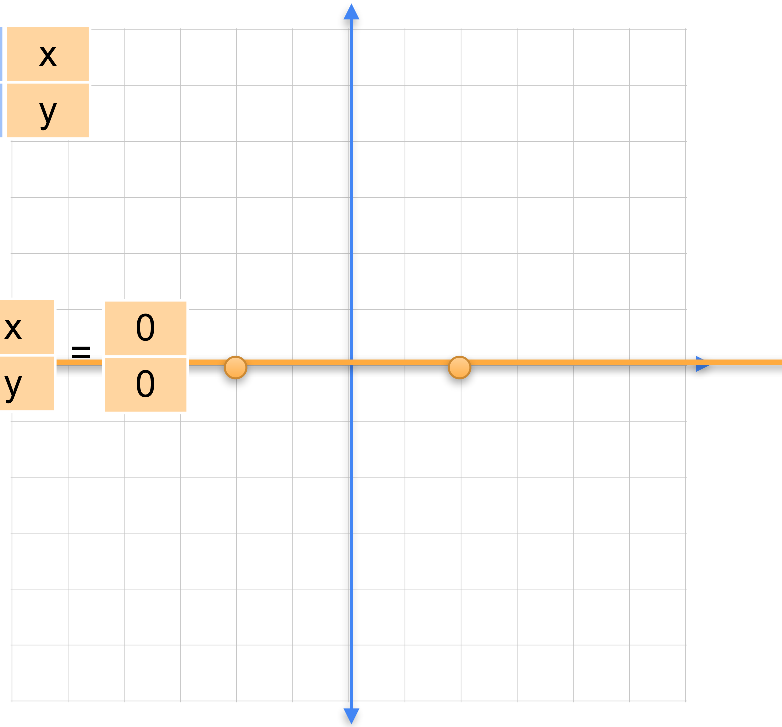
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$





Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = 0$$

Finding eigenvalues

2	1
0	3

Finding eigenvalues

If λ is an eigenvalue:

2	1
0	3

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Has infinitely many solutions

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Has infinitely many solutions

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Has infinitely many solutions

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Has infinitely many solutions

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Has infinitely many solutions

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

$\lambda = 2$

$\lambda = 3$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
$\lambda = 3$

Solve the equations

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 2x \\ 0x + 3y &= 2y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 0 \end{aligned} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 3x \\ 0x + 3y &= 3y \end{aligned}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 2x \\ 0x + 3y &= 2y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 0 \end{aligned} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 3x \\ 0x + 3y &= 3y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 1 \end{aligned}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 2x \\ 0x + 3y &= 2y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 0 \end{aligned} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 3x \\ 0x + 3y &= 3y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 1 \end{aligned} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Quiz

- Find the eigenvalues and eigenvectors of this matrix:

9	4
4	3

Solution

- Eigenvalues: 11, 1
- Eigenvectors: (2,1), (-1,2)

9	4
4	3

- The characteristic polynomial is $\det \begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix} = (9-\lambda)(3-\lambda) - 4 \cdot 4 = 0$

- Which factors as $\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$

- The solutions are $\lambda = 11$, $\lambda = 1$

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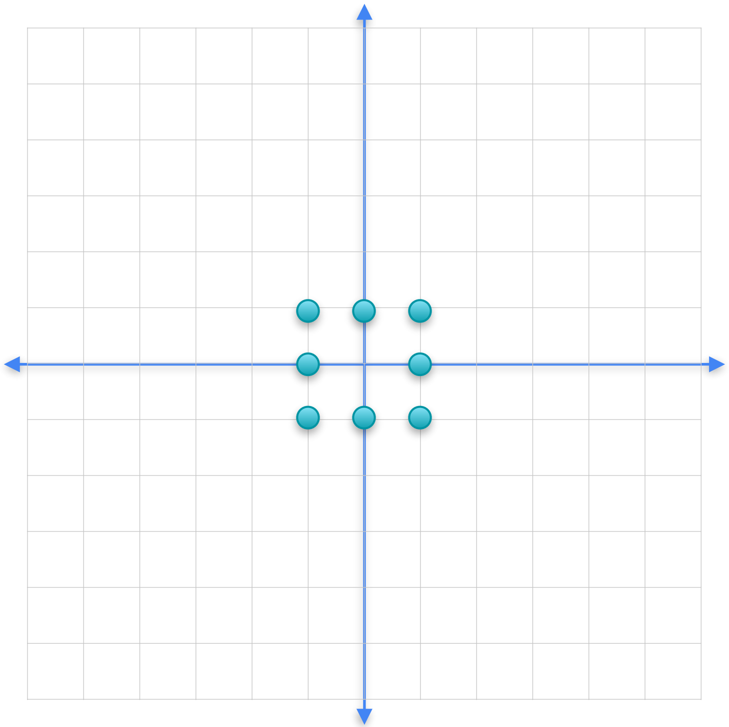
Determinants and Eigenvectors

Conclusion

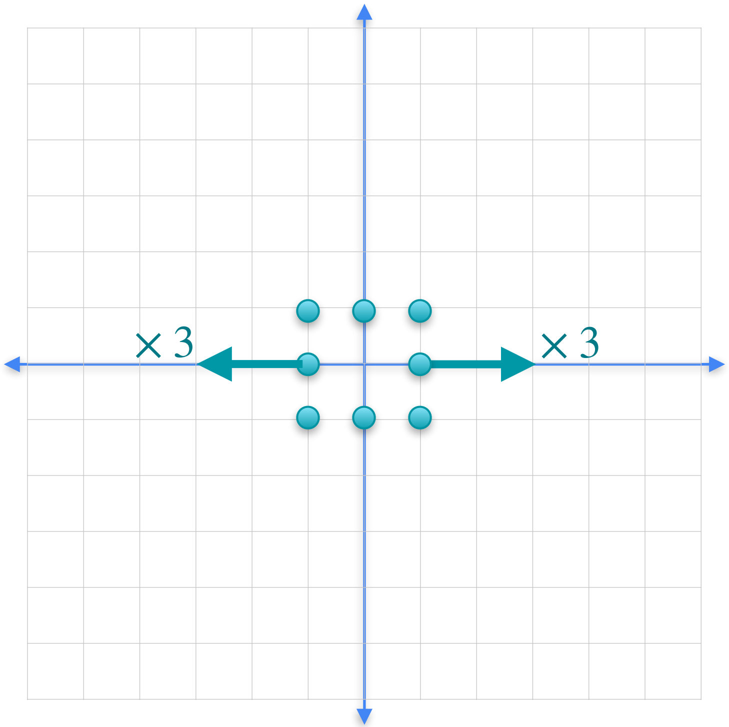
Findir igenvalues

2	1
0	3



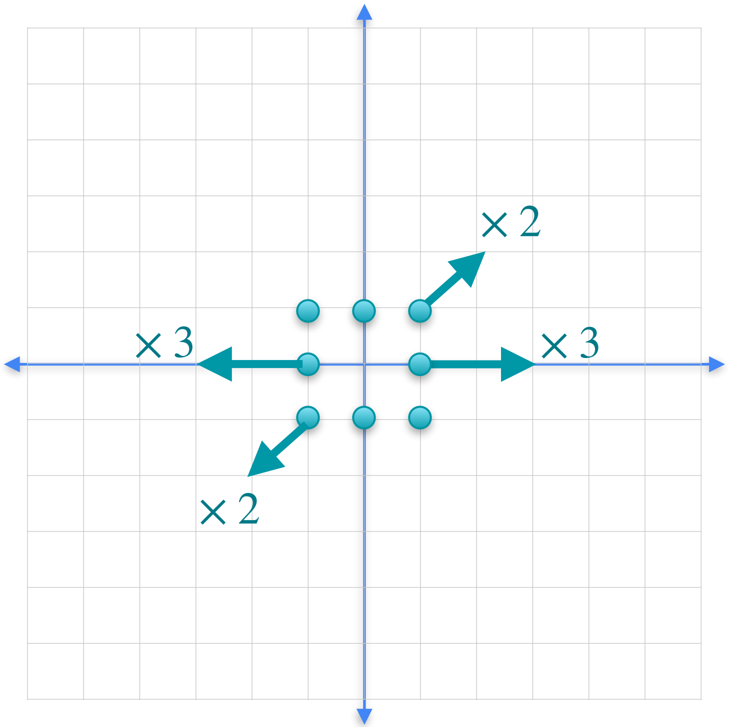
Findir[2 1; 0 3]genvalues

2	1
0	3



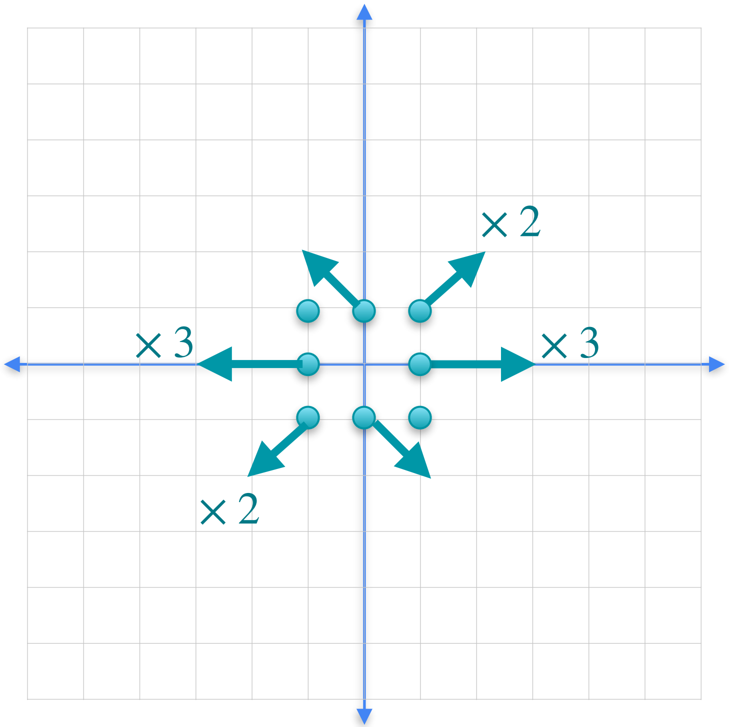
Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

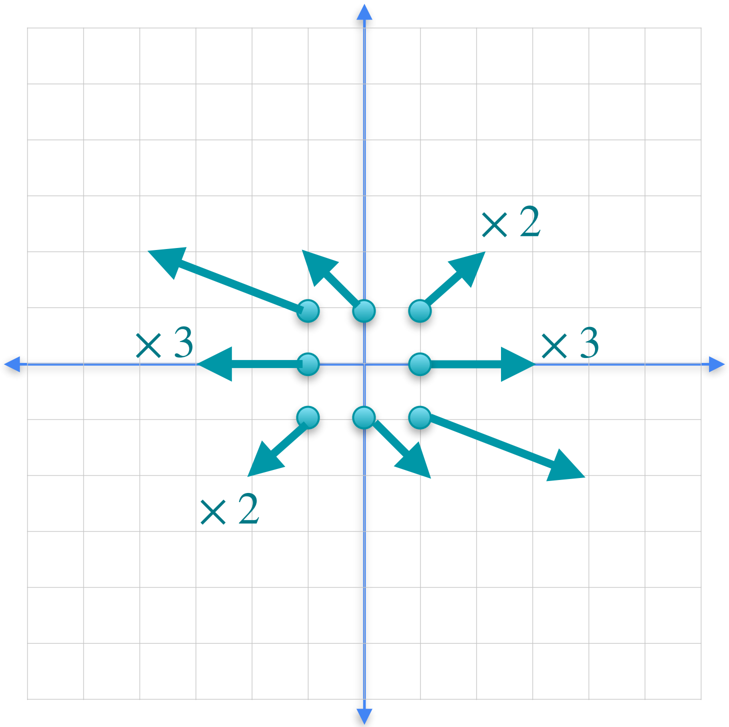


Findir[obscured]igenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

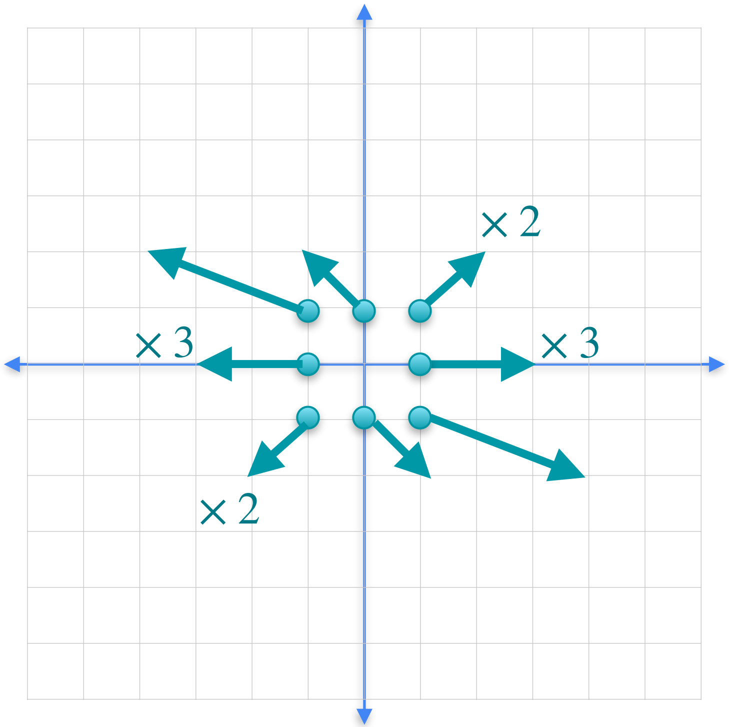


Findir $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ genvalues

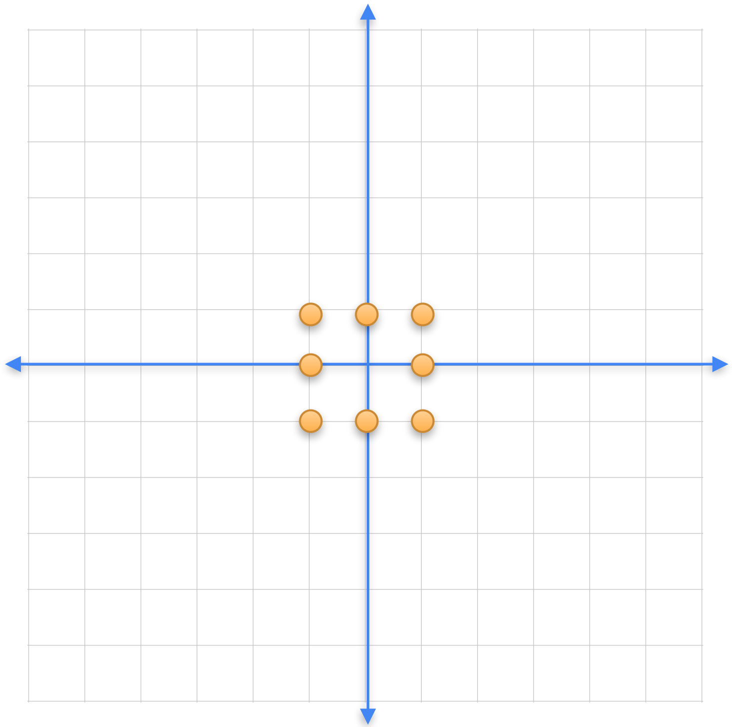


Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

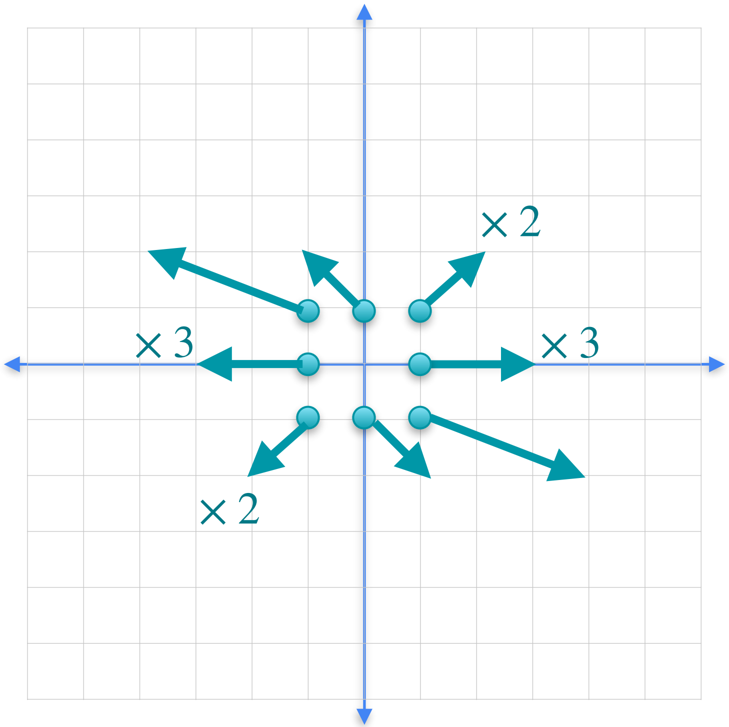


$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

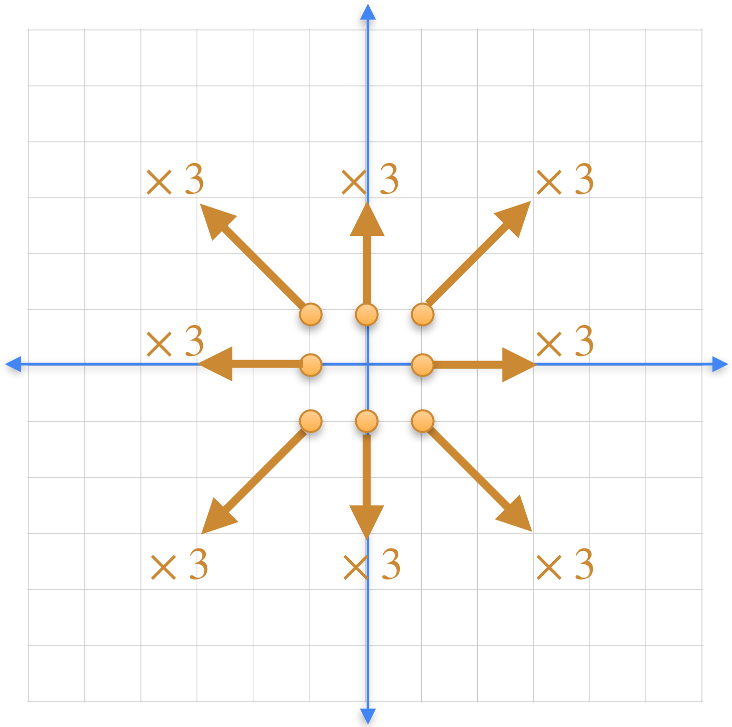


Finding eigenvalues

2	1
0	3

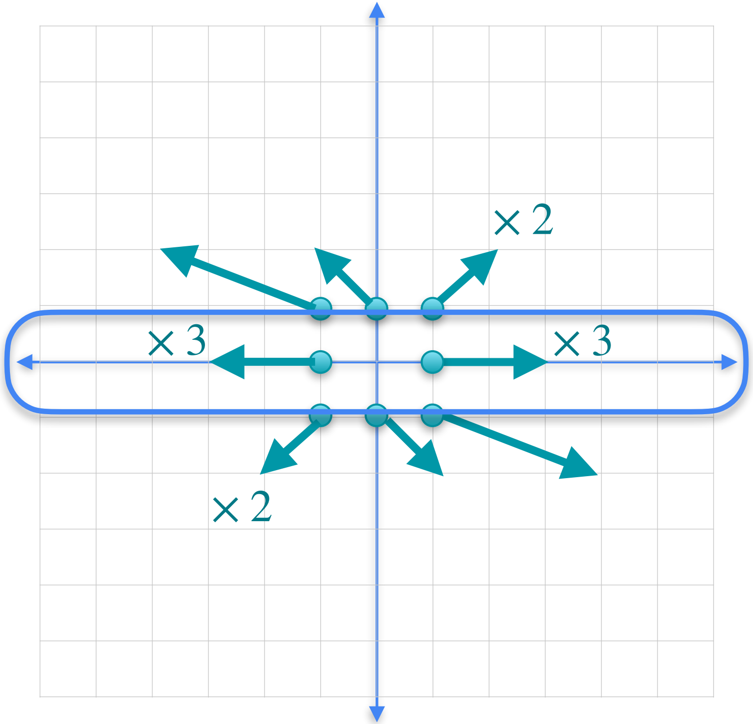


3	0
0	3

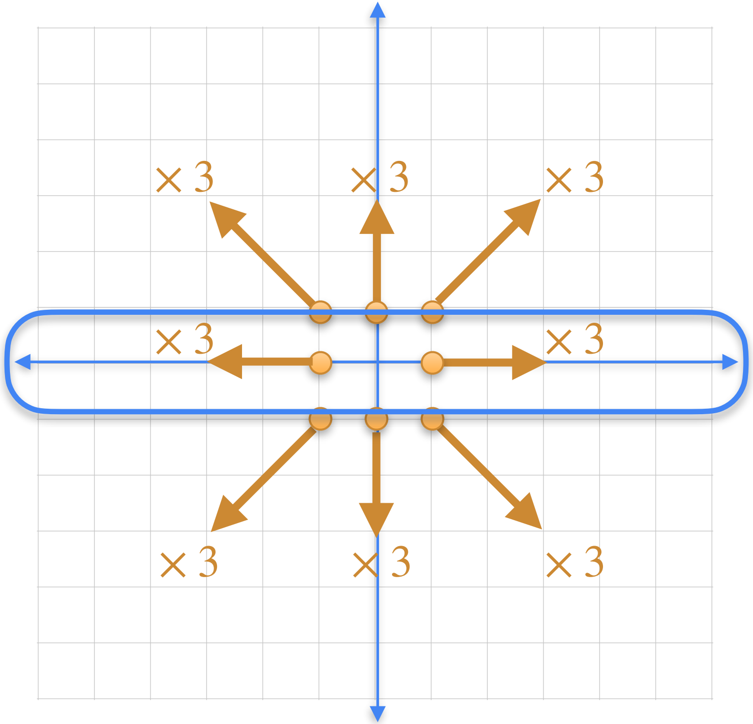


Finding eigenvalues

2	1
0	3

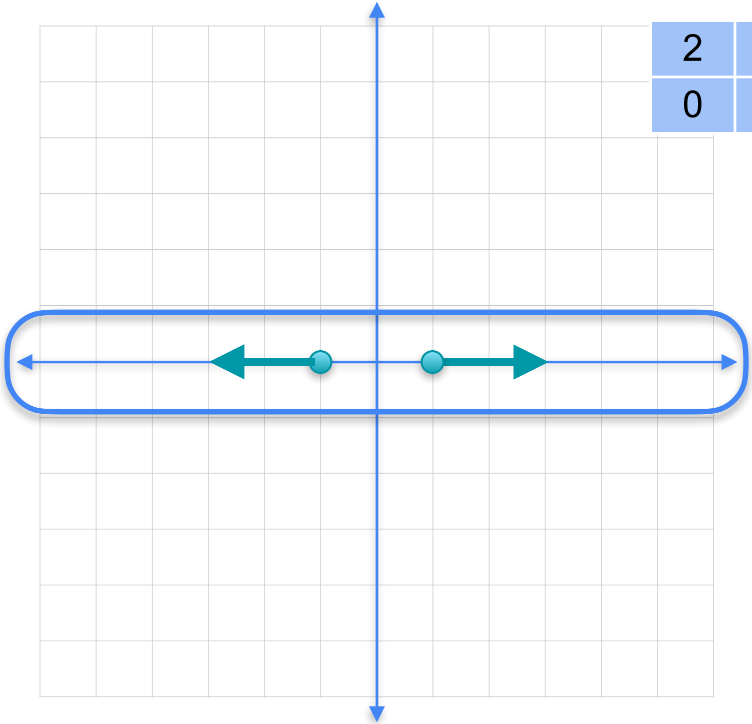


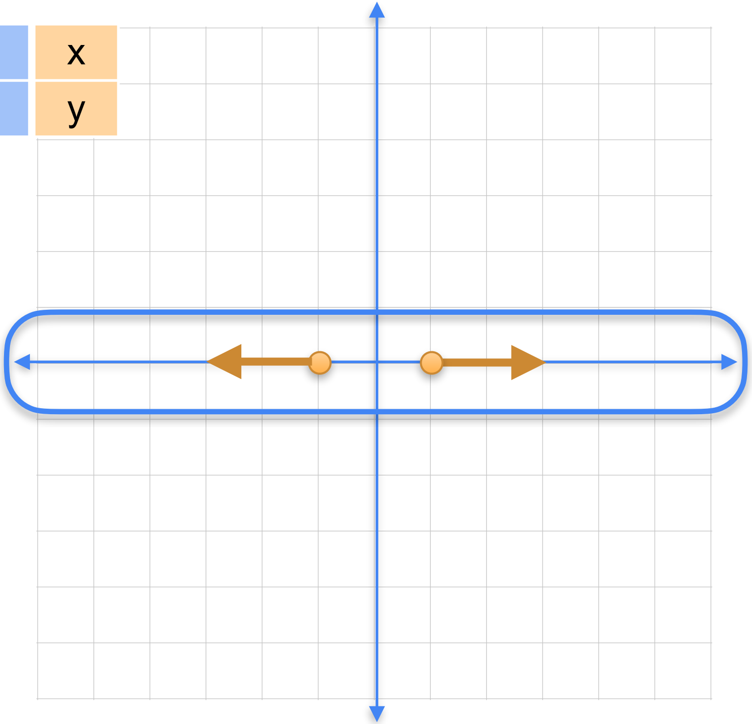
3	0
0	3



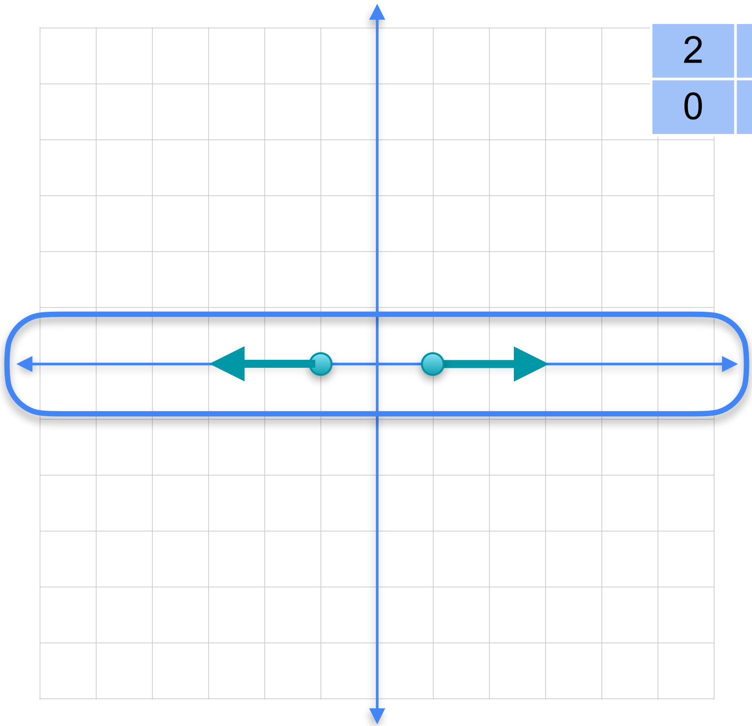
Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



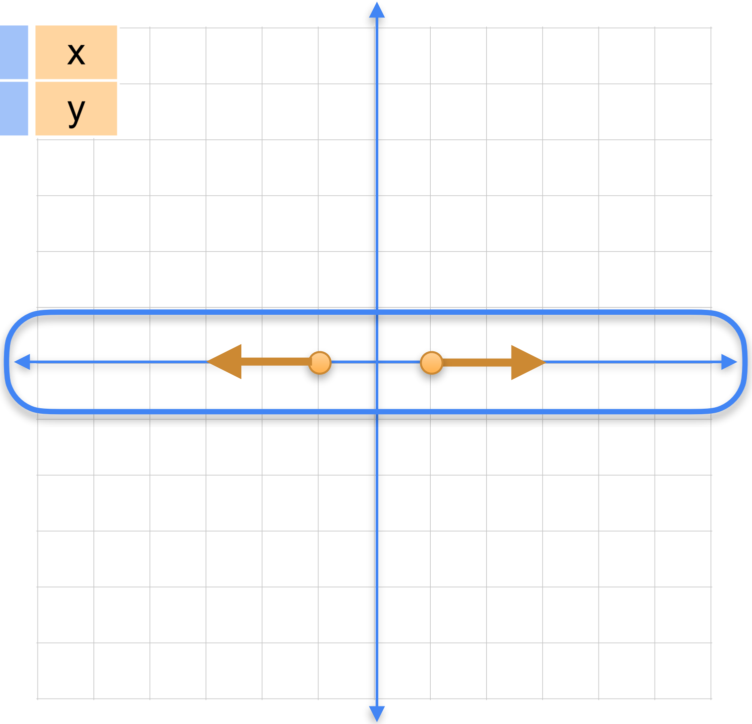


Finding eigenvalues

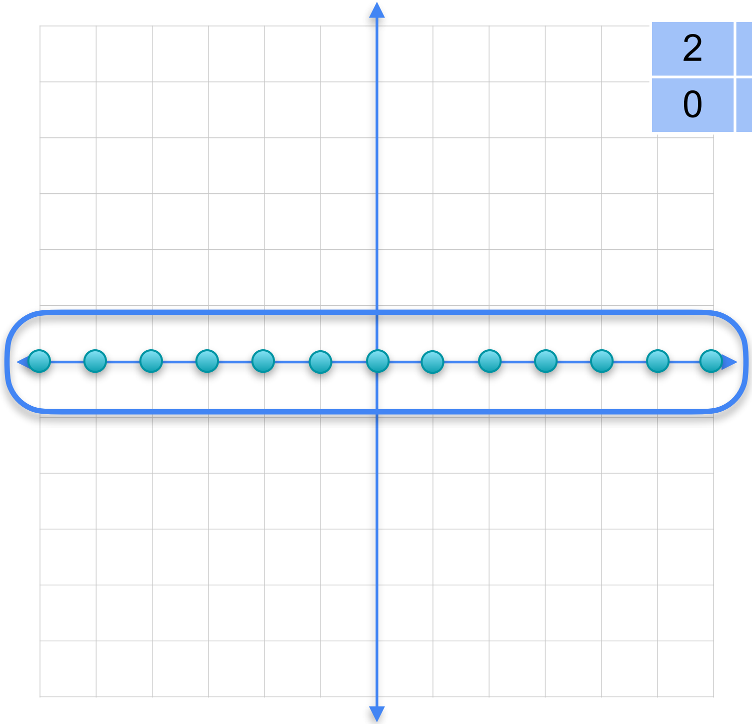


$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

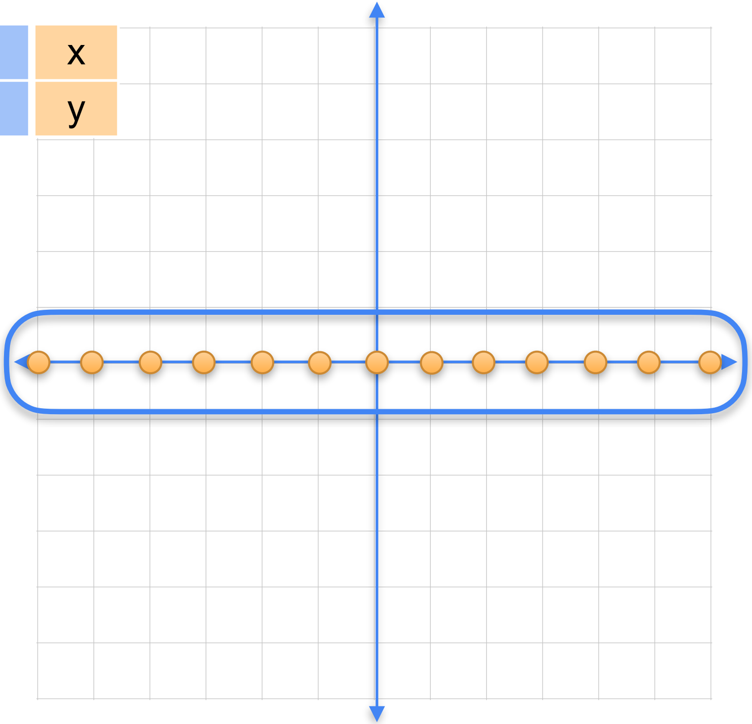


Finding eigenvalues

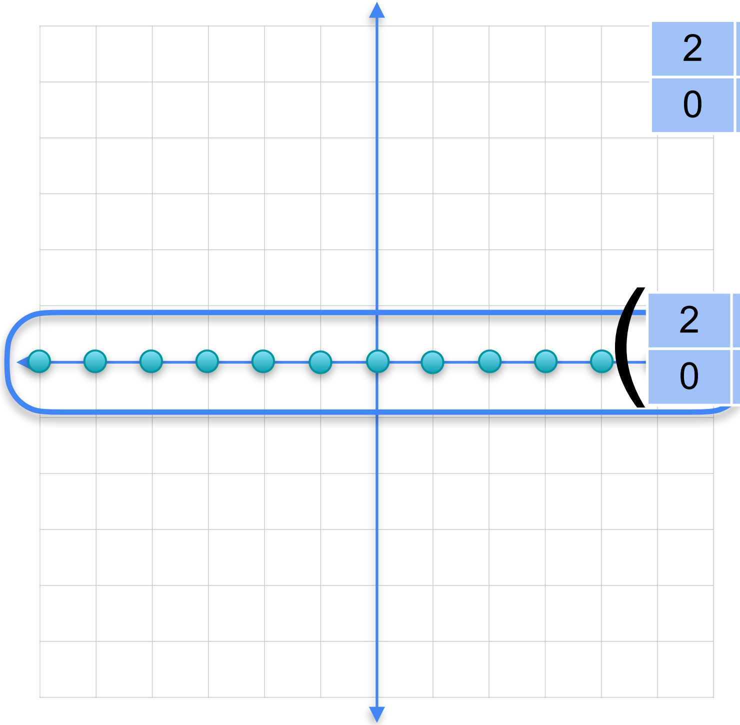


$$\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

For infinitely
many points



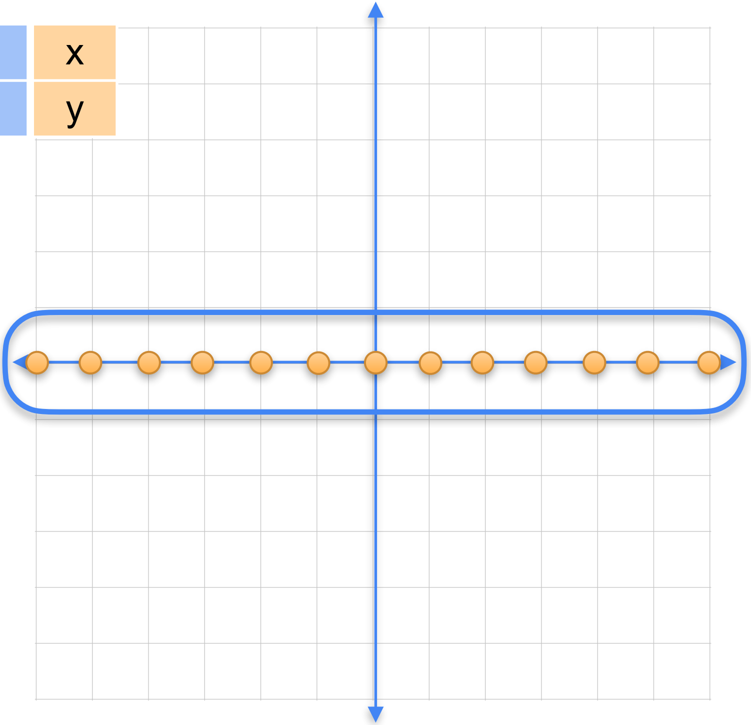
Finding eigenvalues



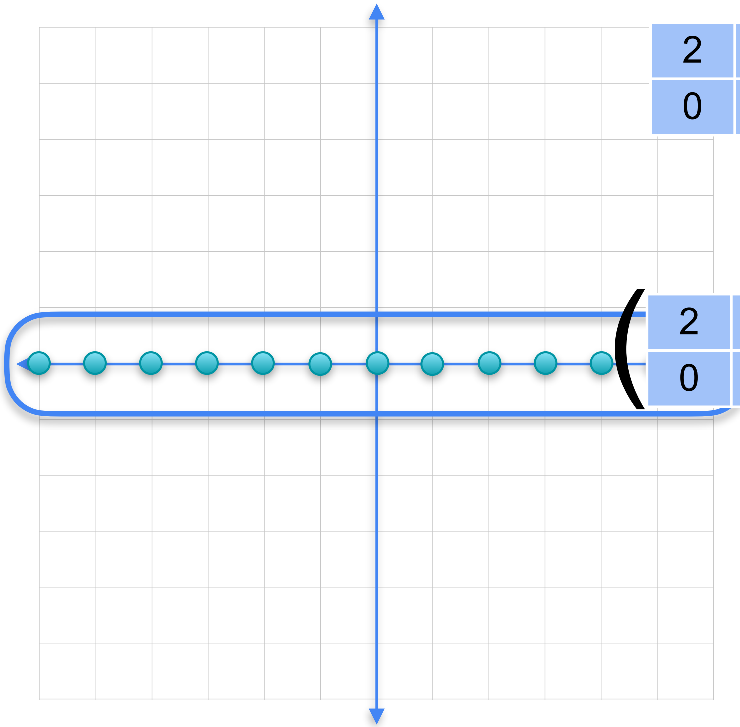
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$$



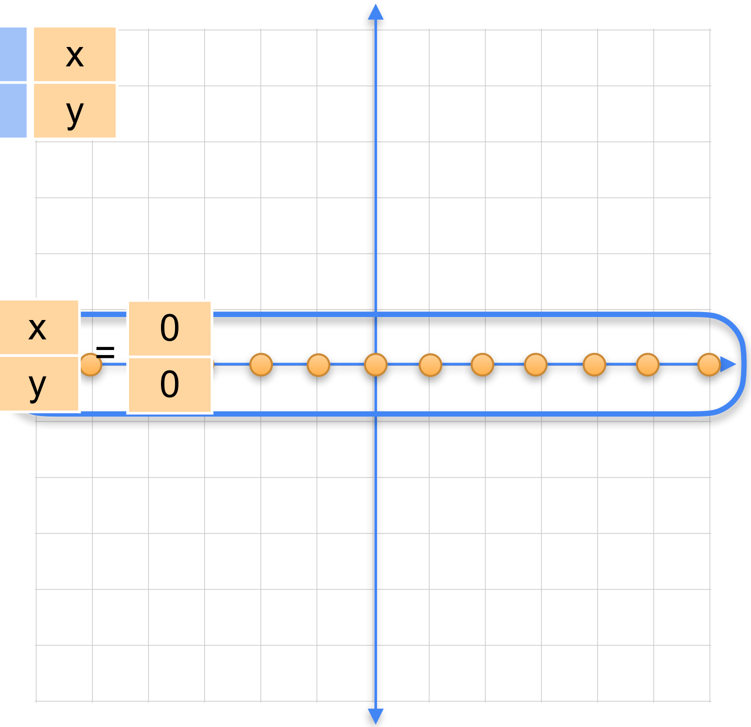
Finding eigenvalues



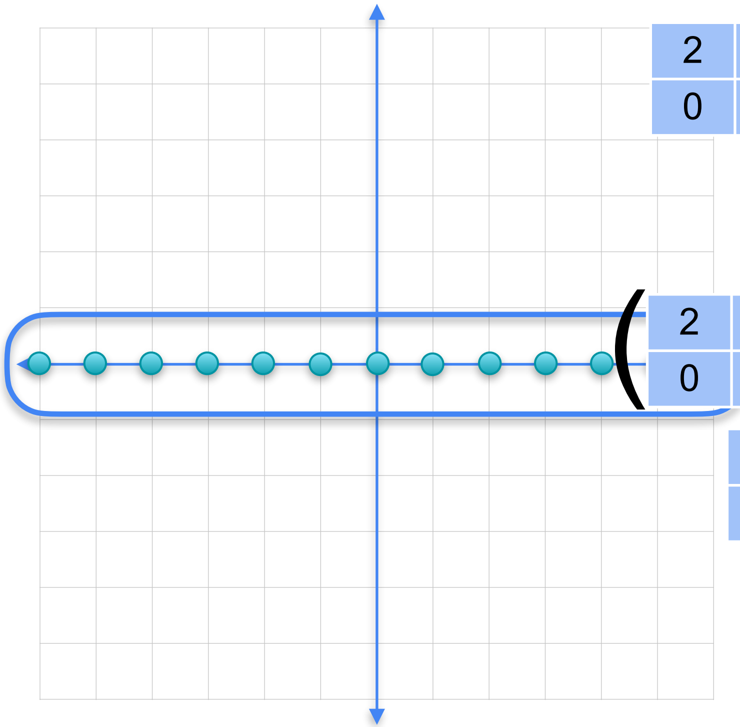
$$\begin{bmatrix}2&1\\0&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3&0\\0&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

For infinitely many points

$$\left(\begin{bmatrix}2&1\\0&3\end{bmatrix}-\begin{bmatrix}3&0\\0&3\end{bmatrix}\right)\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$



Finding eigenvalues

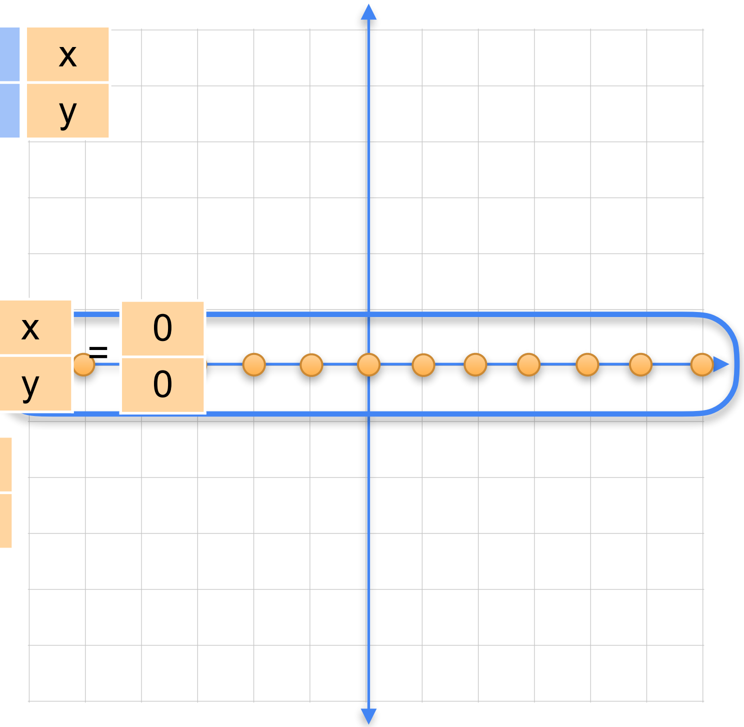


$$\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

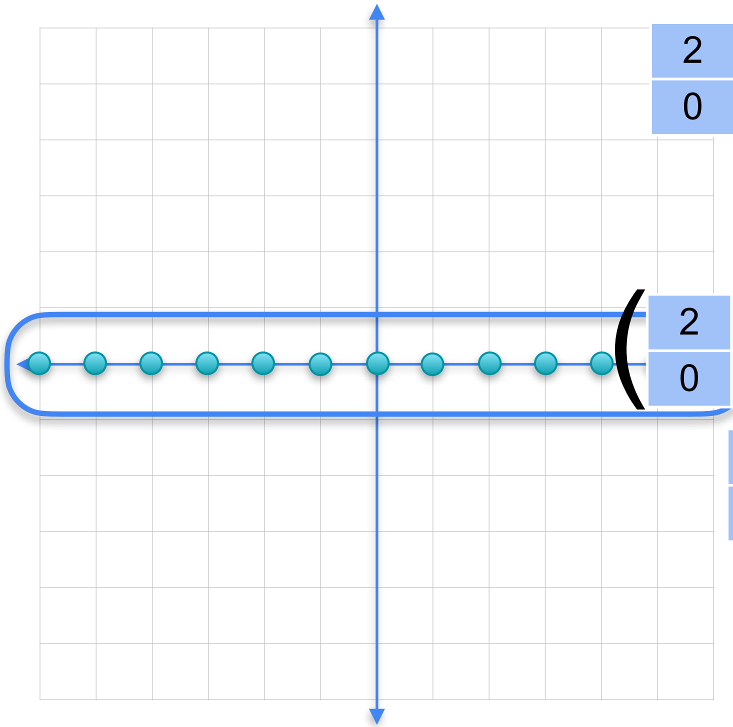
For infinitely many points

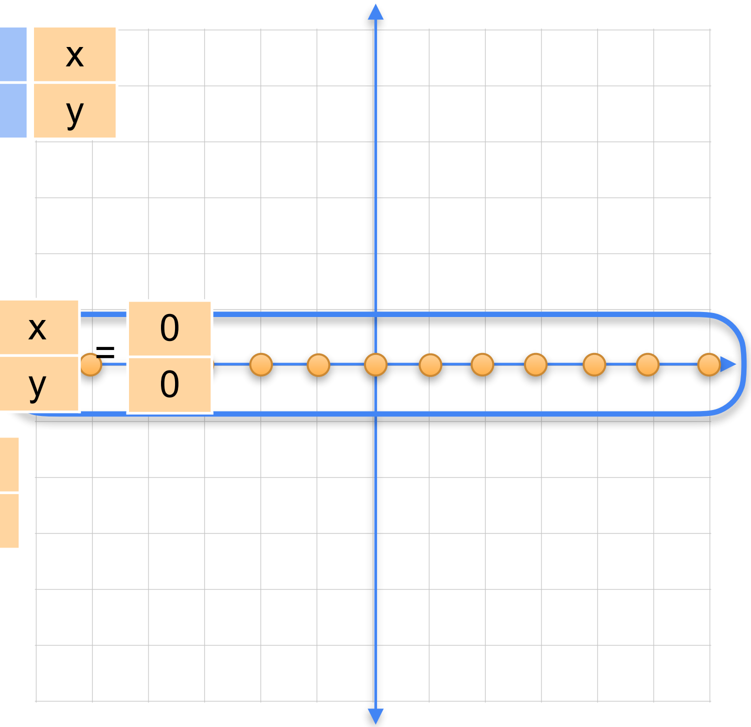
$$\left(\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\right)\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$



Finding eigenvalues





$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

For infinitely many points

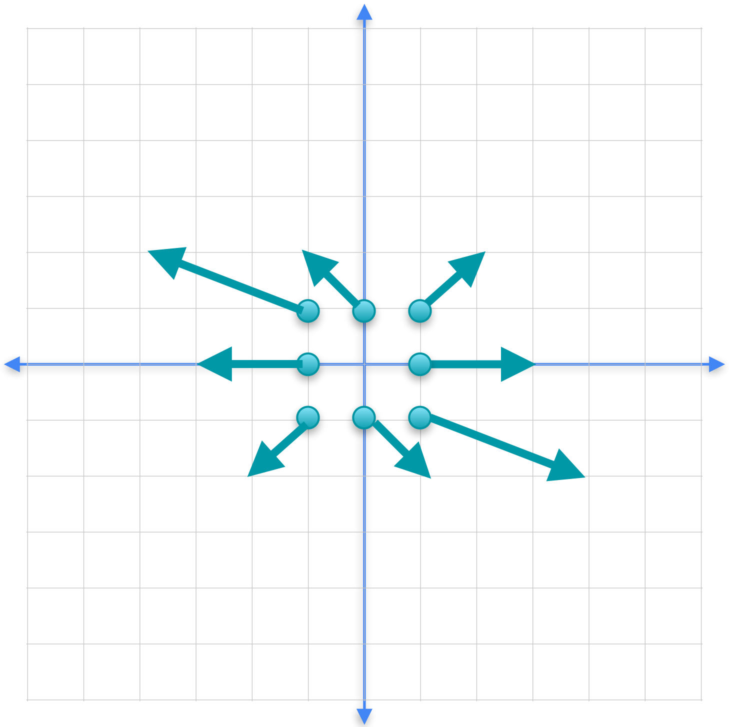
$$\left(\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

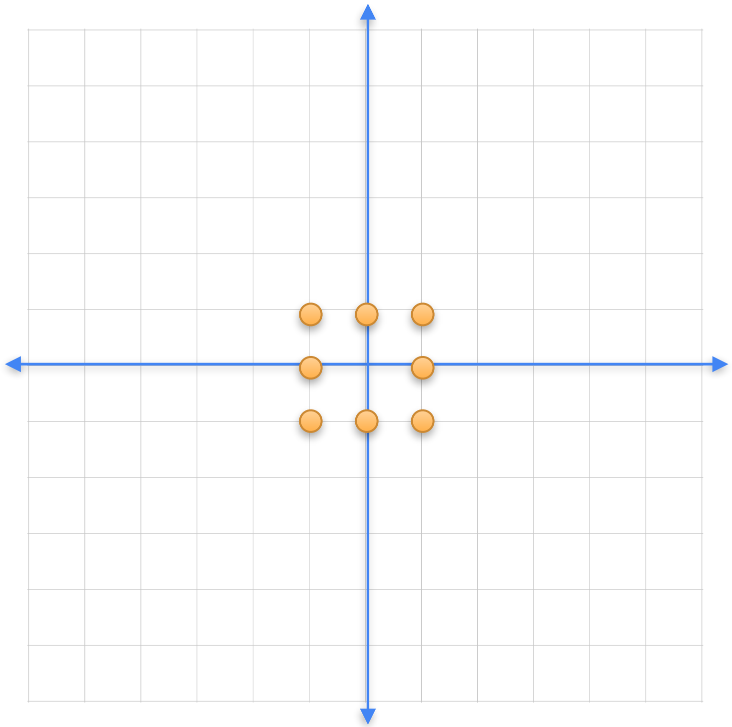
$$\det \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = 0$$

Findir genvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

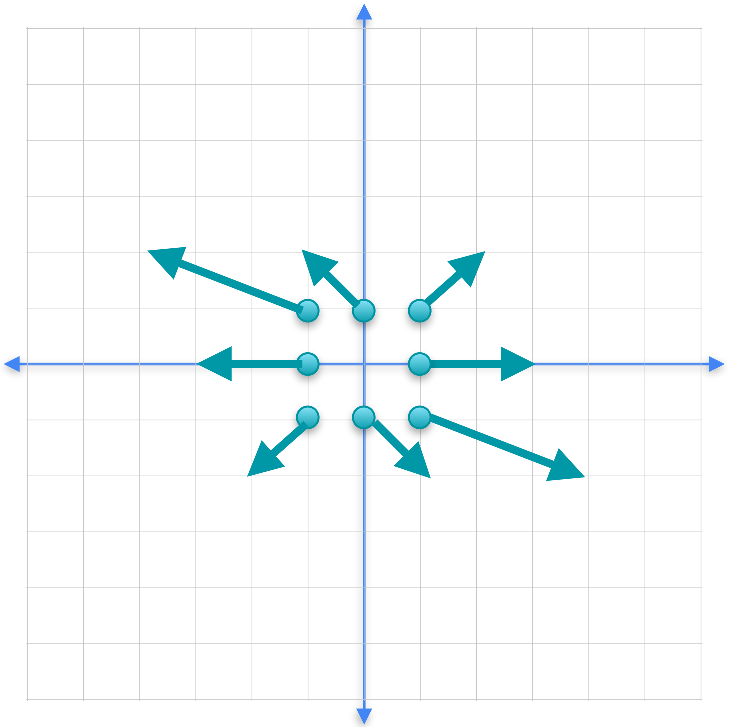


$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

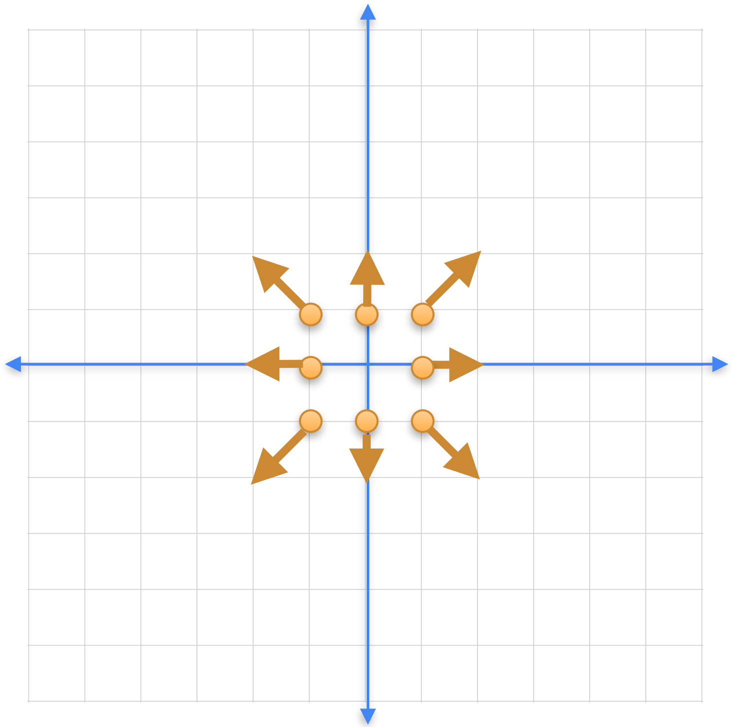


Findir genvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

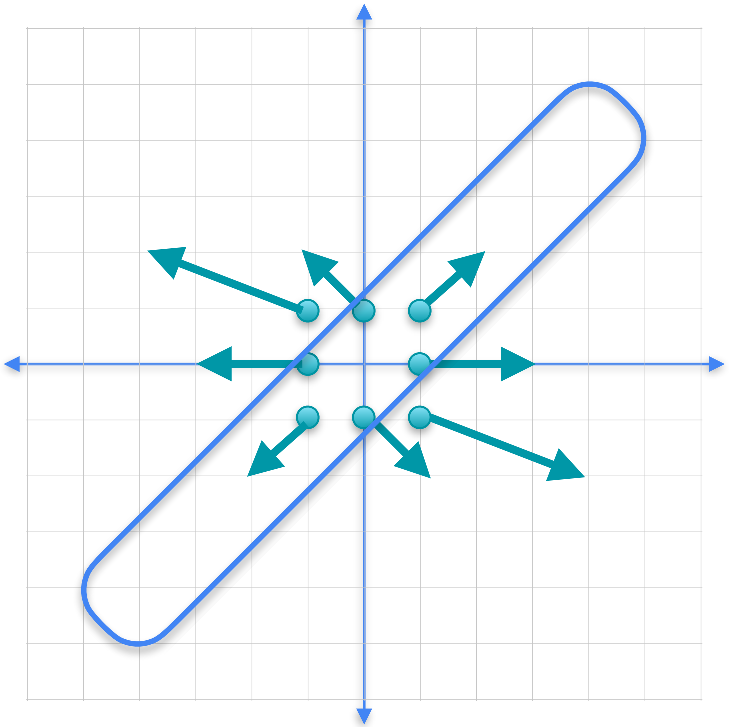


$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

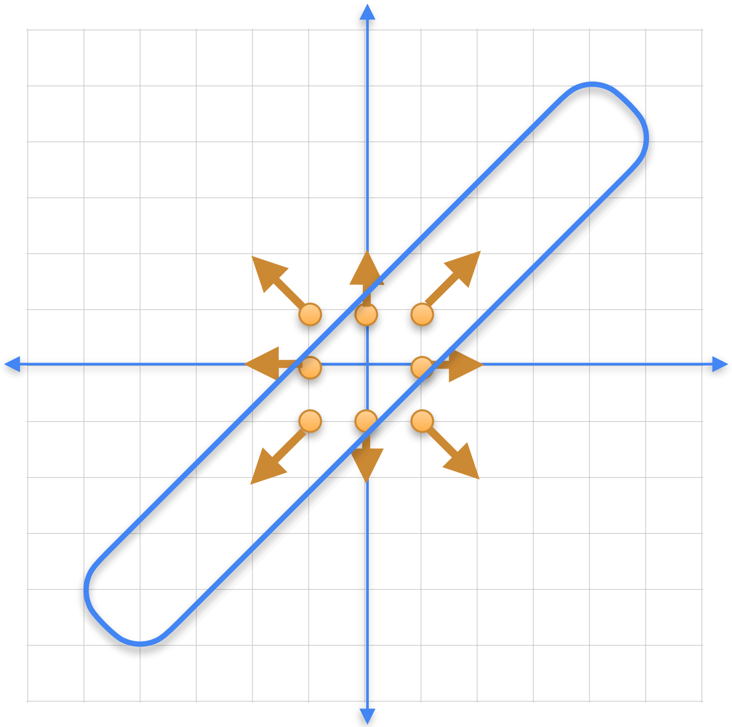


Findir genvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

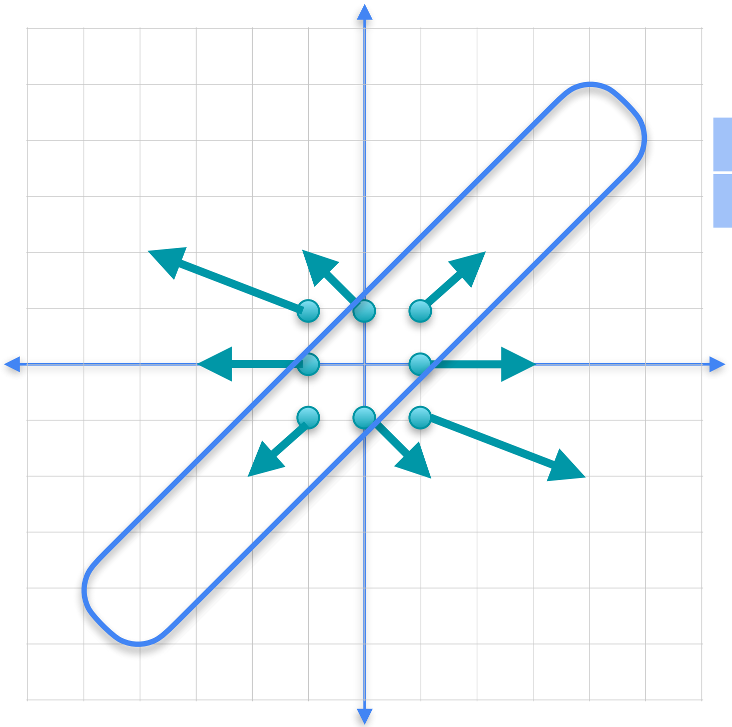


$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$



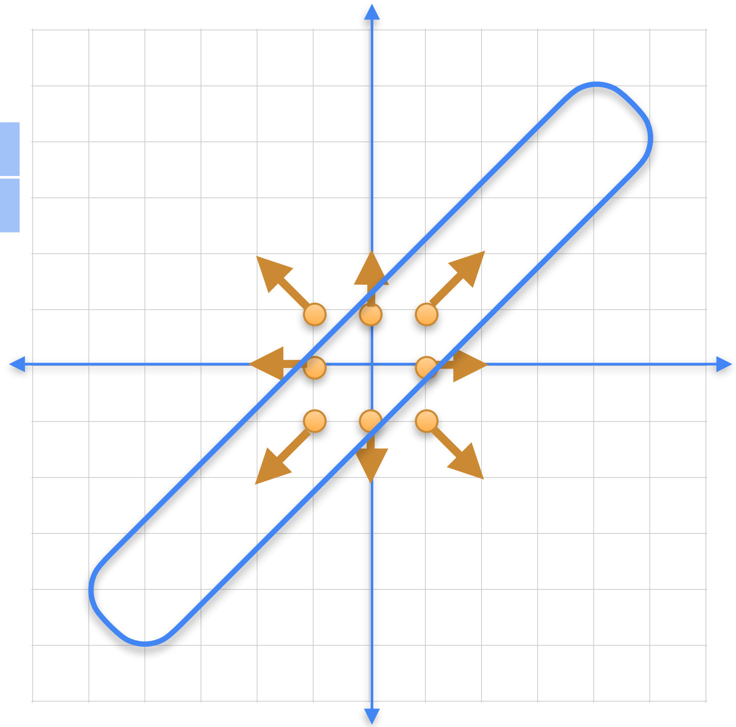
Findir … genvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

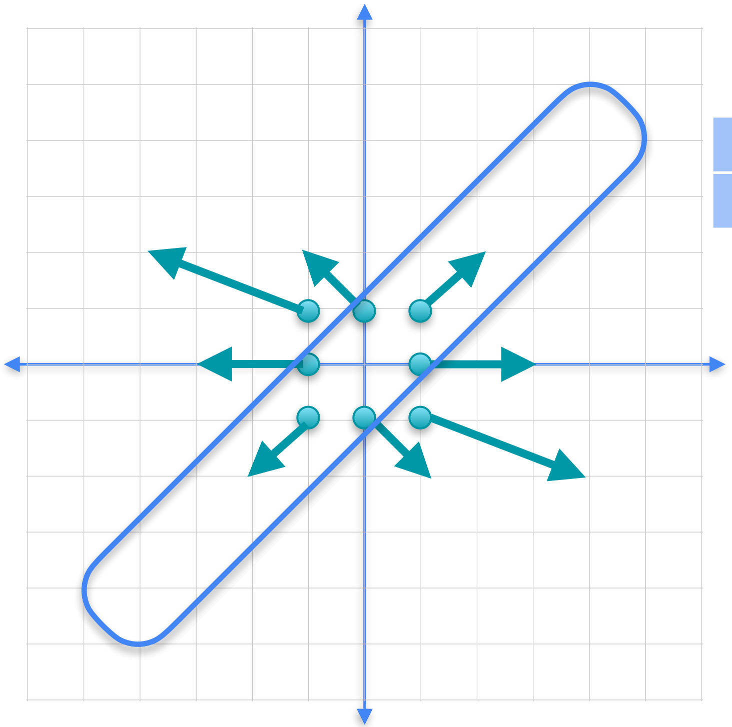
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$



Finding Eigenvalues

2	1
0	3

2	0
0	2

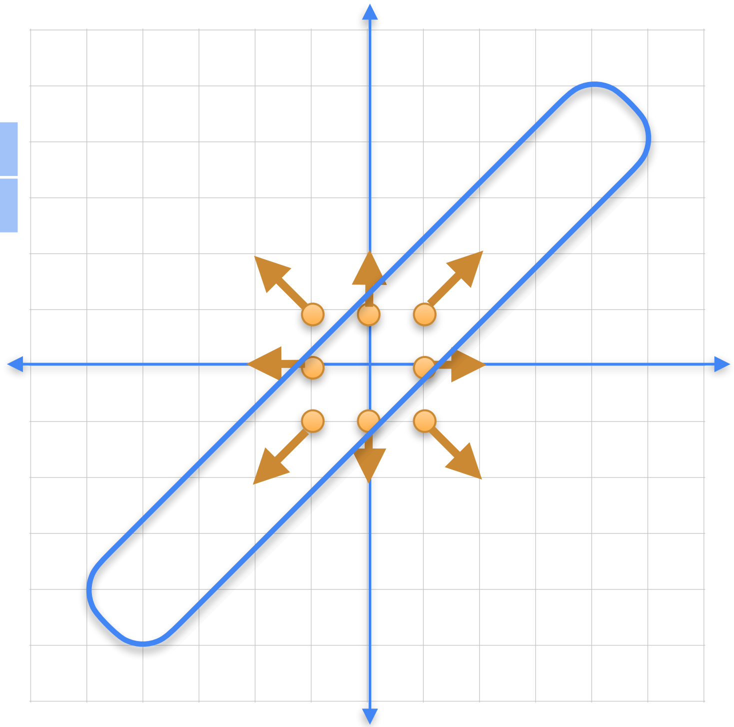


2	1
0	3

-

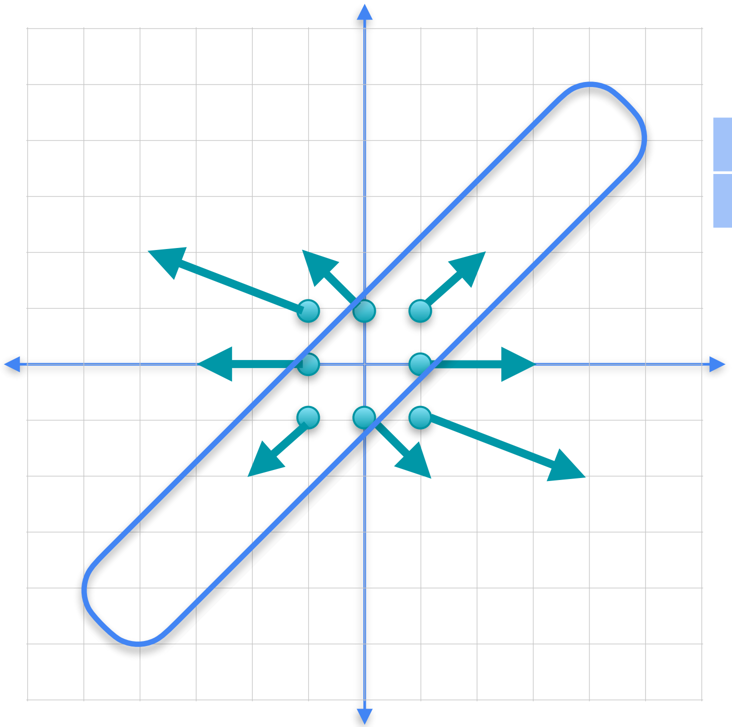
2	0
0	2

0	1
0	1



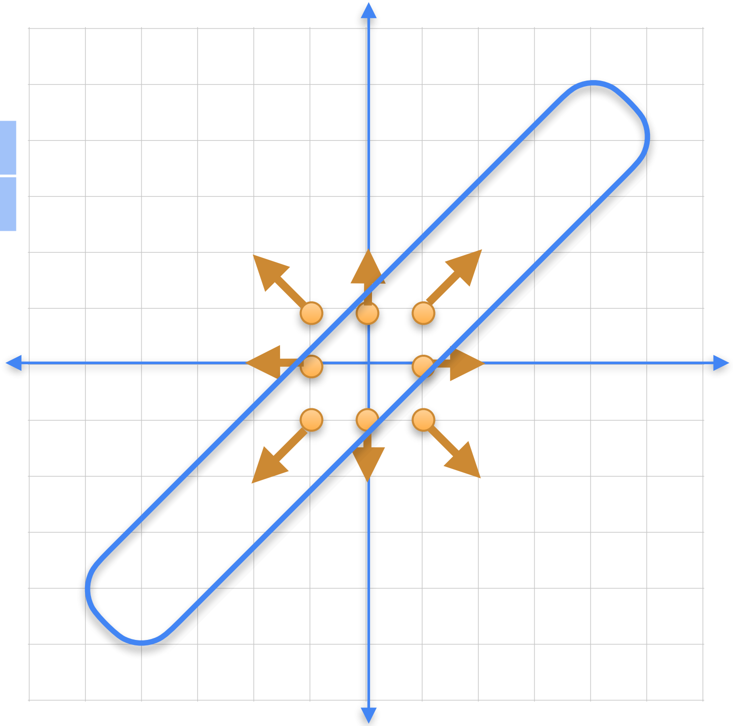
Findir $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ genvalues

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$



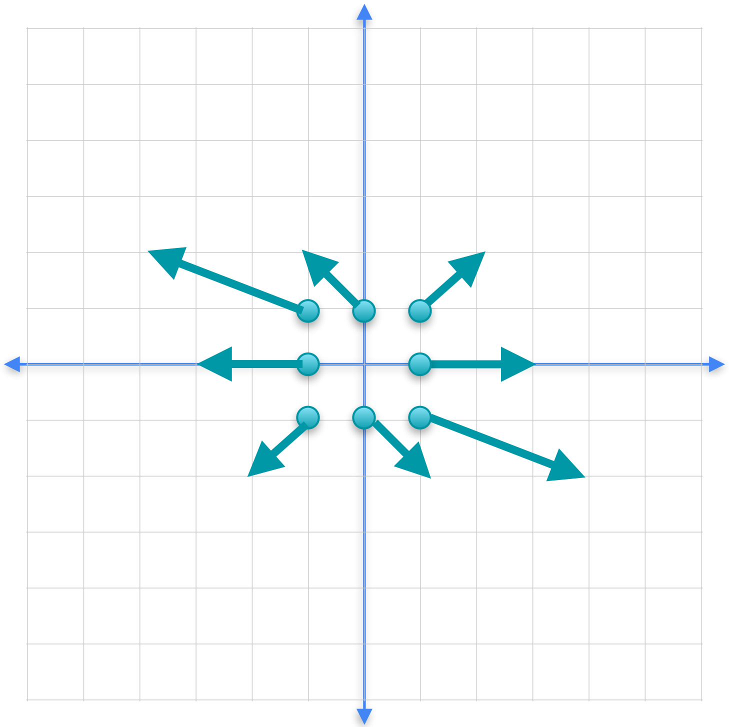
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\det \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = 0$$

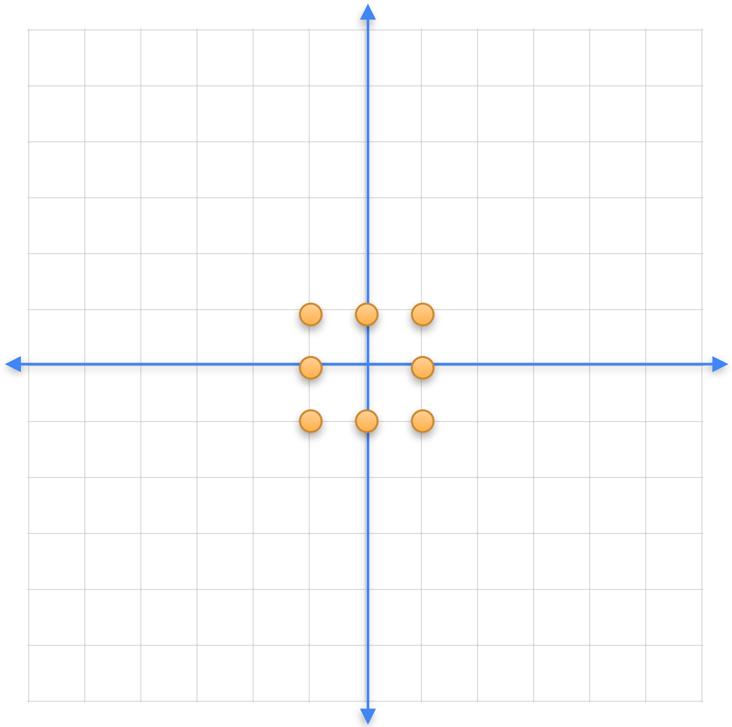


Findir genvalues

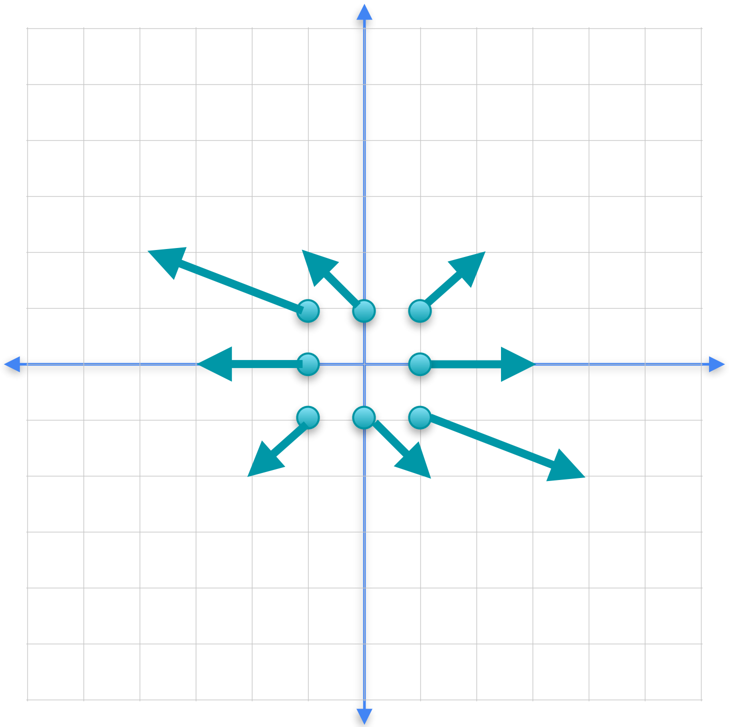
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$



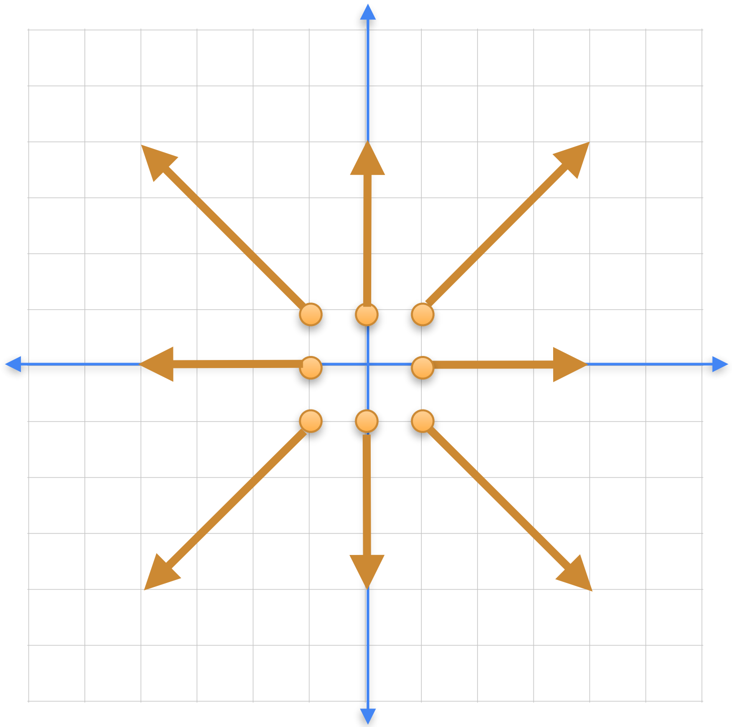
$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$



Findir $\begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}$ genvalues

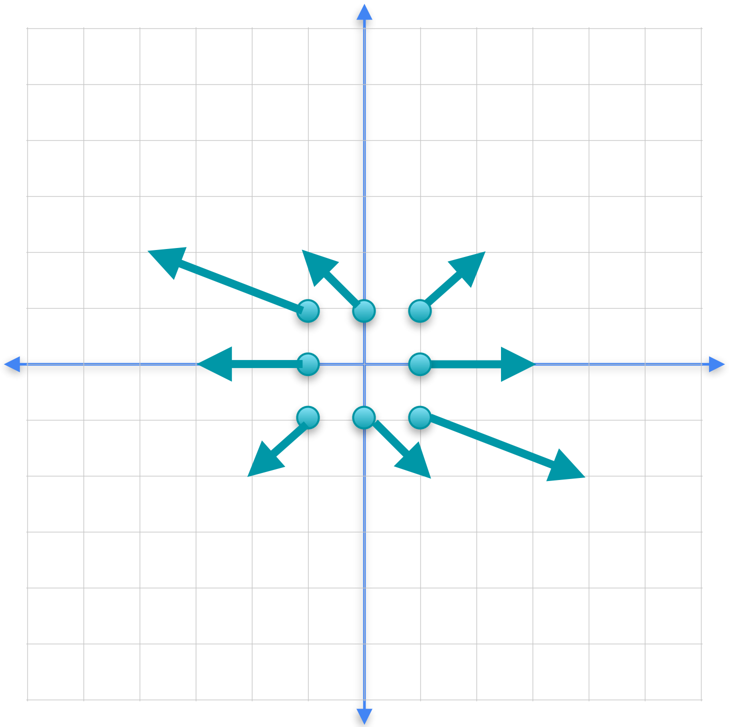


$\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}$

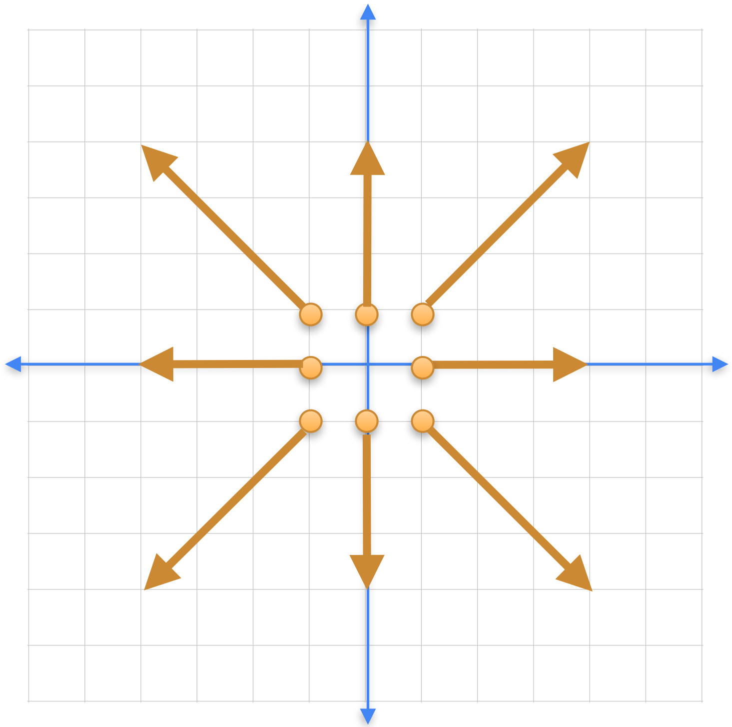


Findir… genvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

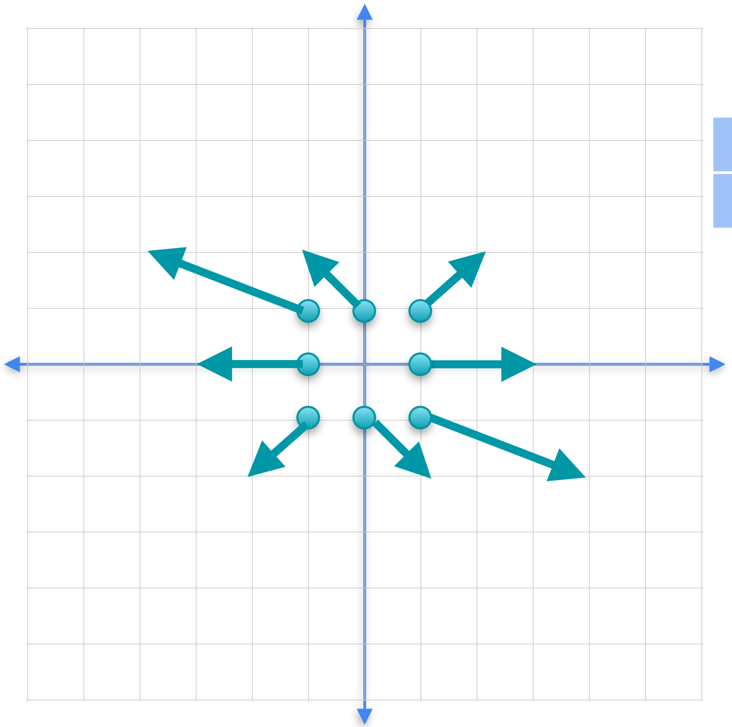


$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$



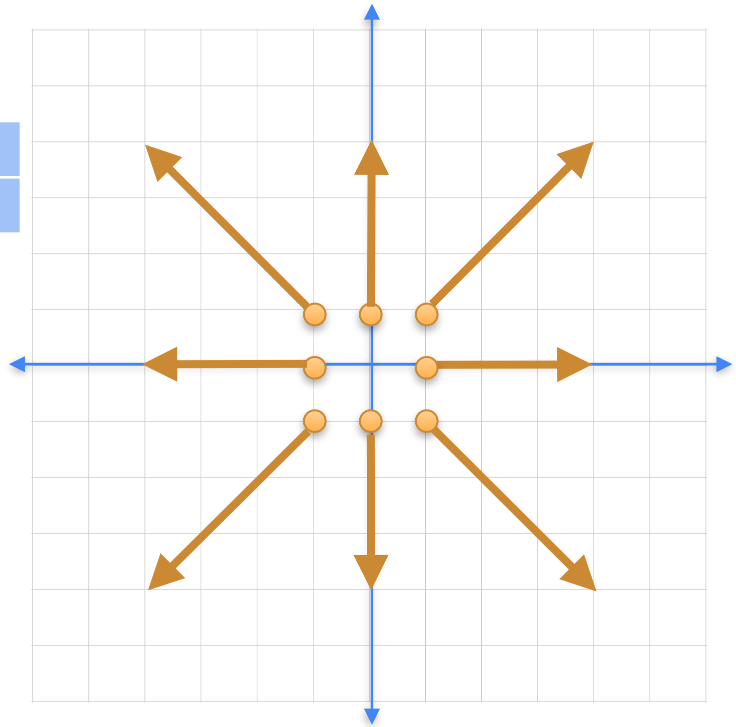
Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

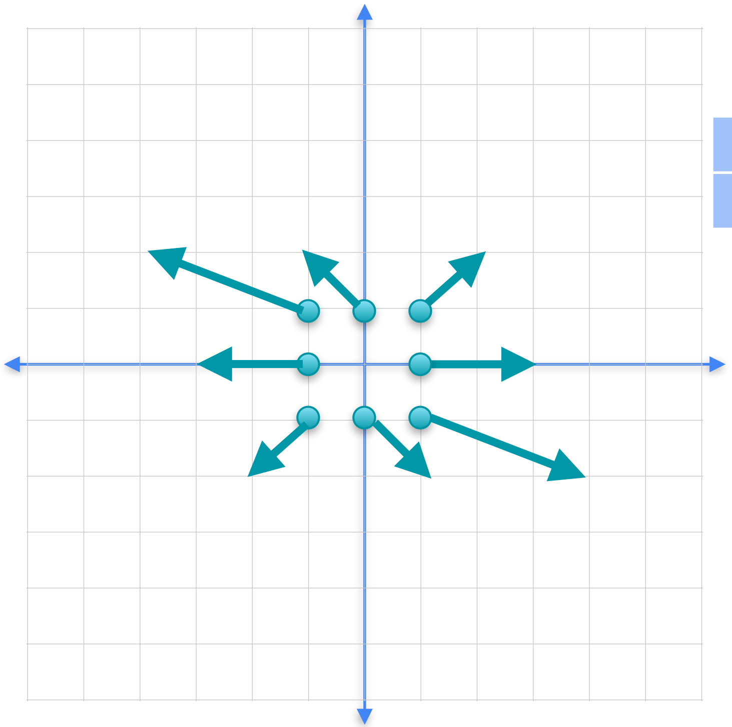
$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$



Findir genvalues

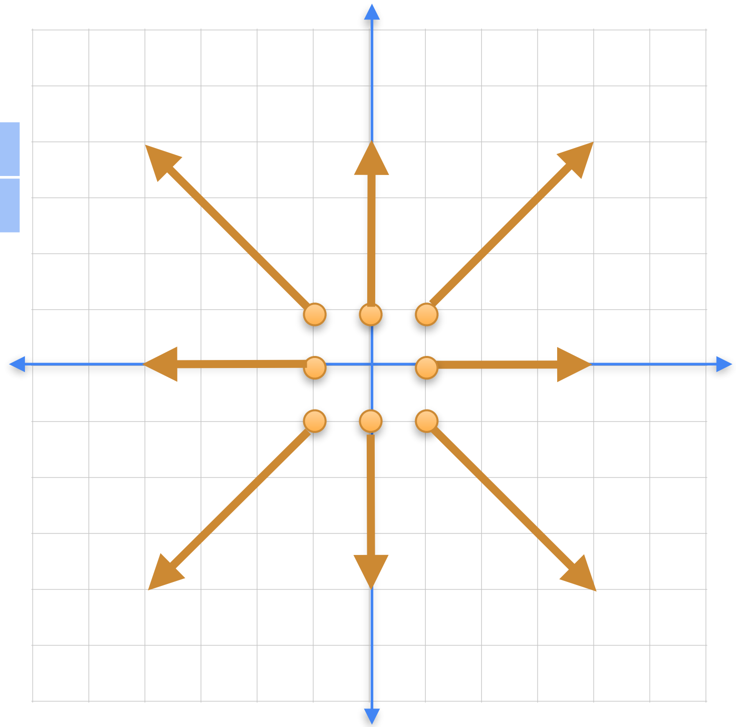
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

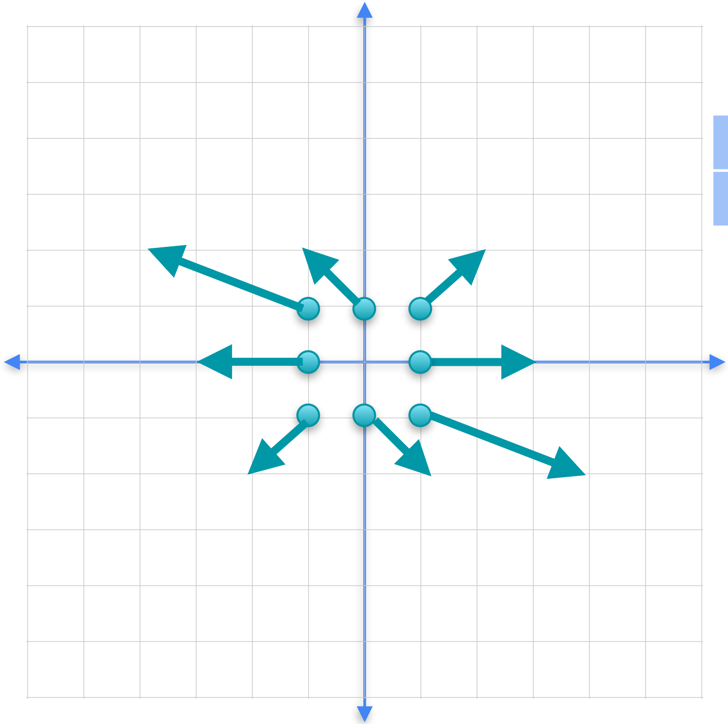
$$\begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$$



Findir genvalues

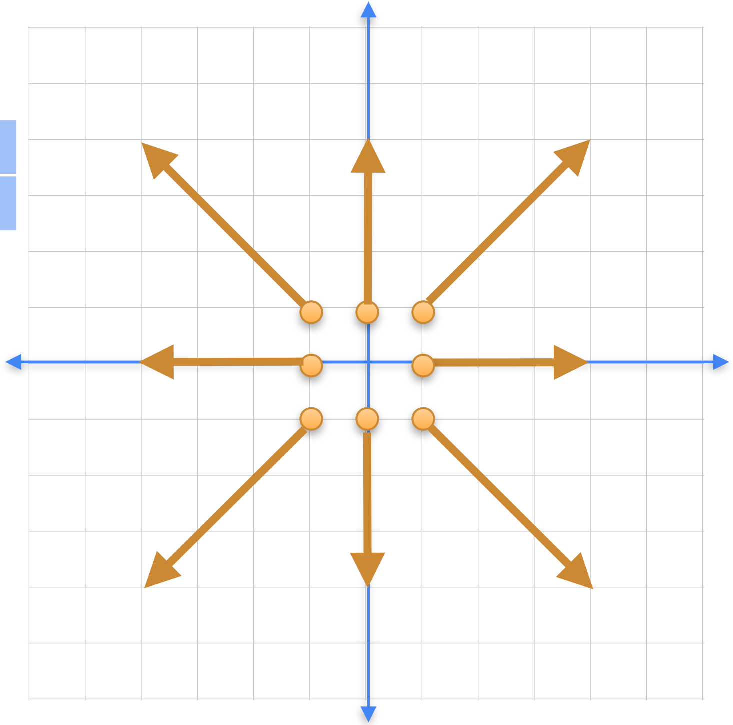
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\det \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \neq 0$$



Finding eigenvalues

2	1
0	3

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

Finding eigenvalues

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

$\lambda = 2$
$\lambda = 3$

Finding eigenvalues

If λ is an eigenvalue:
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

$\lambda = 2$

$\lambda = 3$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$

$$\lambda = 2$$
$$\lambda = 3$$

Finding eigenvalues

If λ is an eigenvalue: $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ For infinitely many (x,y)

$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Has infinitely many solutions

$\det \begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$

Characteristic polynomial $(2-\lambda)(3-\lambda) - 1 \cdot 0 = 0$ $\lambda = 2$ $\lambda = 3$

Finding eigenvectors

Eigenvalues: $$\lambda = 2$$ $$\lambda = 3$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$

$$0x + 3y = 2y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 2x$$
$$0x + 3y = 2y$$

$$x = 1$$
$$y = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2x + y = 3x$$
$$0x + 3y = 3y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$, $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 2x \\ 0x + 3y &= 2y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 0 \end{aligned} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 3x \\ 0x + 3y &= 3y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 1 \end{aligned}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$ $\lambda = 3$

Solve the equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 2x \\ 0x + 3y &= 2y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 0 \end{aligned} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{aligned} 2x + y &= 3x \\ 0x + 3y &= 3y \end{aligned} \qquad \begin{aligned} x &= 1 \\ y &= 1 \end{aligned} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Quiz

- Find the eigenvalues and eigenvectors of this matrix:

9	4
4	3

Solution

- Eigenvalues: 11, 1
- Eigenvectors: (2,1), (-1,2)

$$\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix}$$

- The characteristic polynomial is $\det \begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix} = (9-\lambda)(3-\lambda) - 4 \cdot 4 = 0$

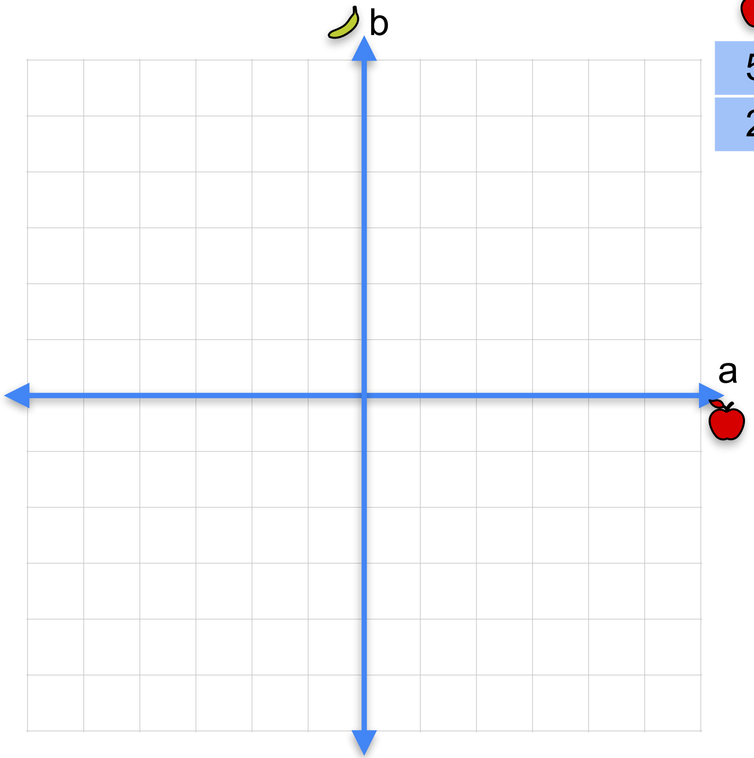
- Which factors as $\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$

- The solutions are $\lambda = 11$, $\lambda = 1$

Matrices as linear transformations

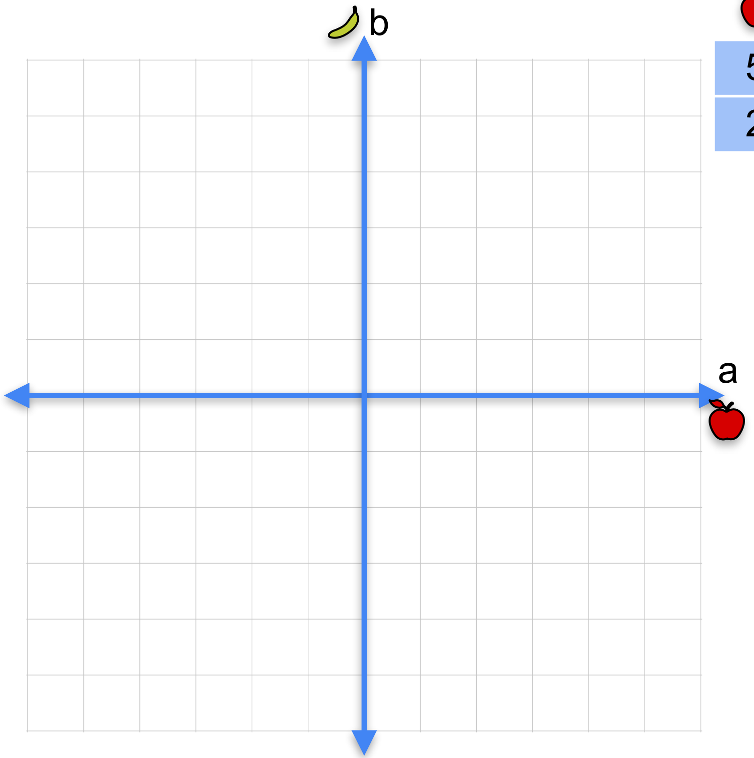
🍎	🍌
5	2
2	2

Matrices as linear transformations

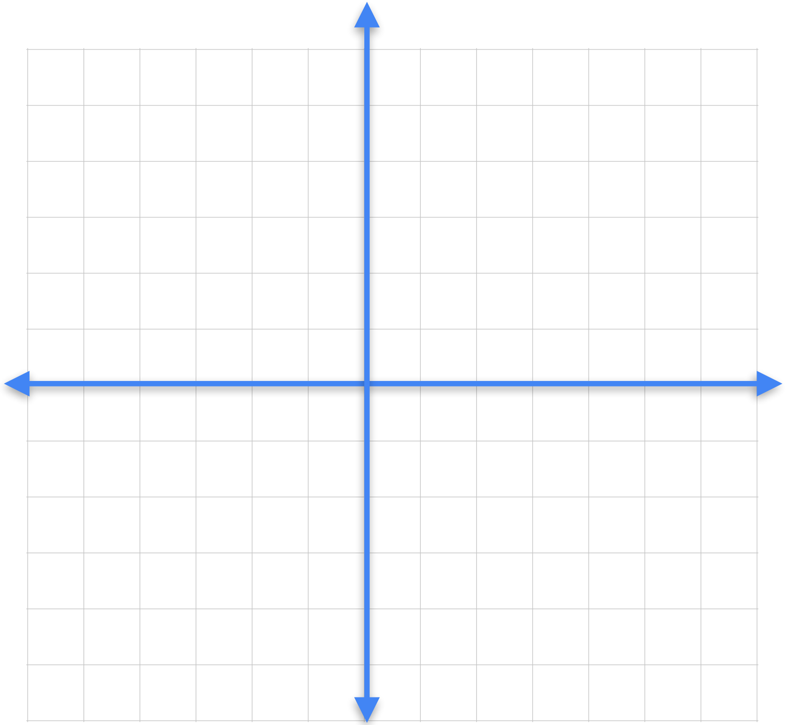


5	2
2	2

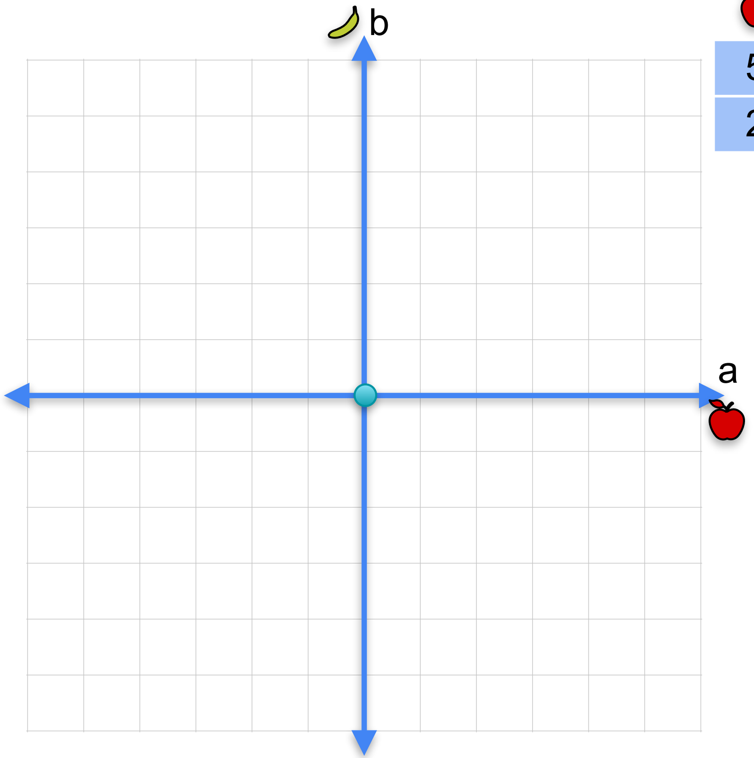
Matrices as linear transformations



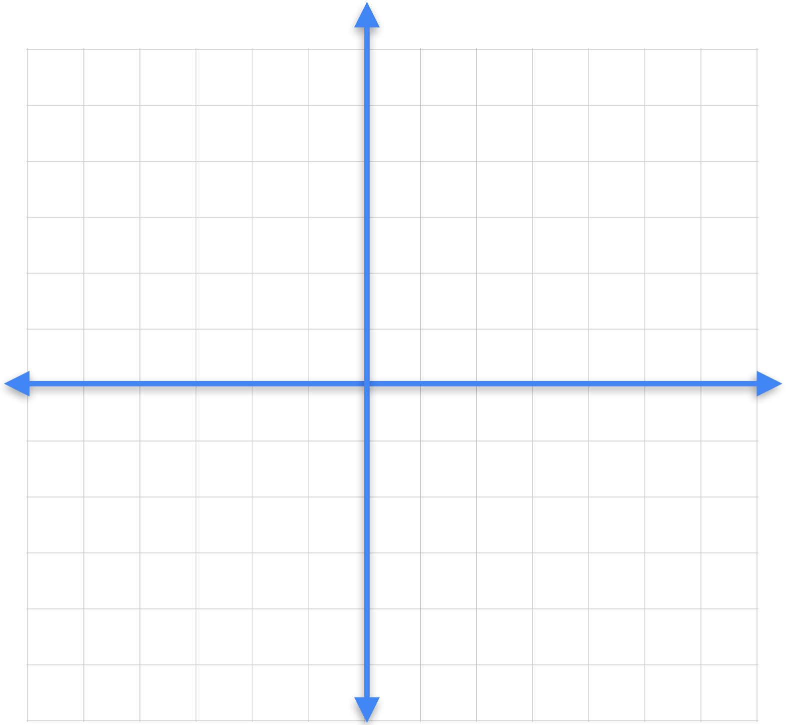
5	2
2	2



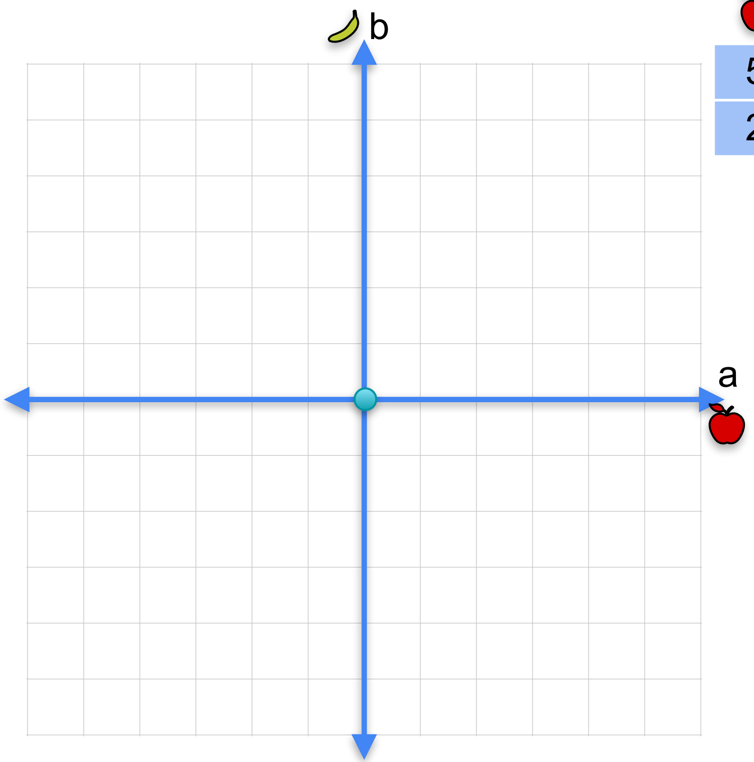
Matrices as linear transformations



🍎	🍌
5	2
2	2

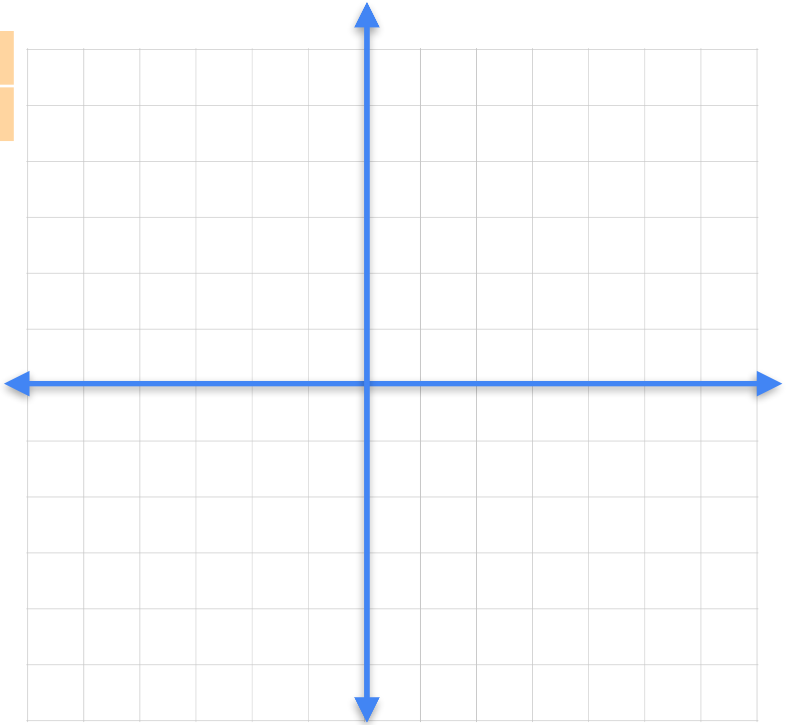


Matrices as linear transformations

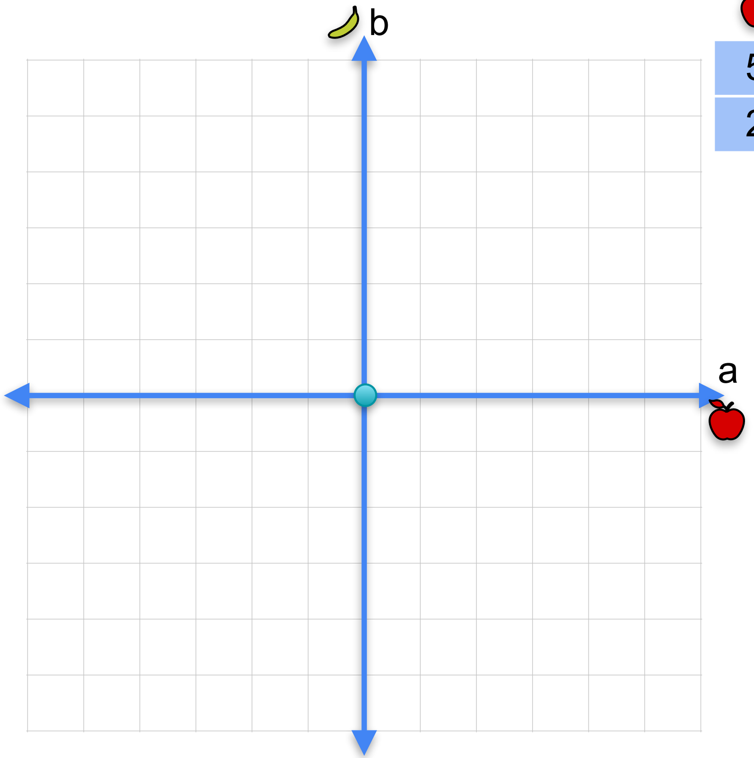


$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(0,0) \rightarrow (0,0)$



Matrices as linear transformations



$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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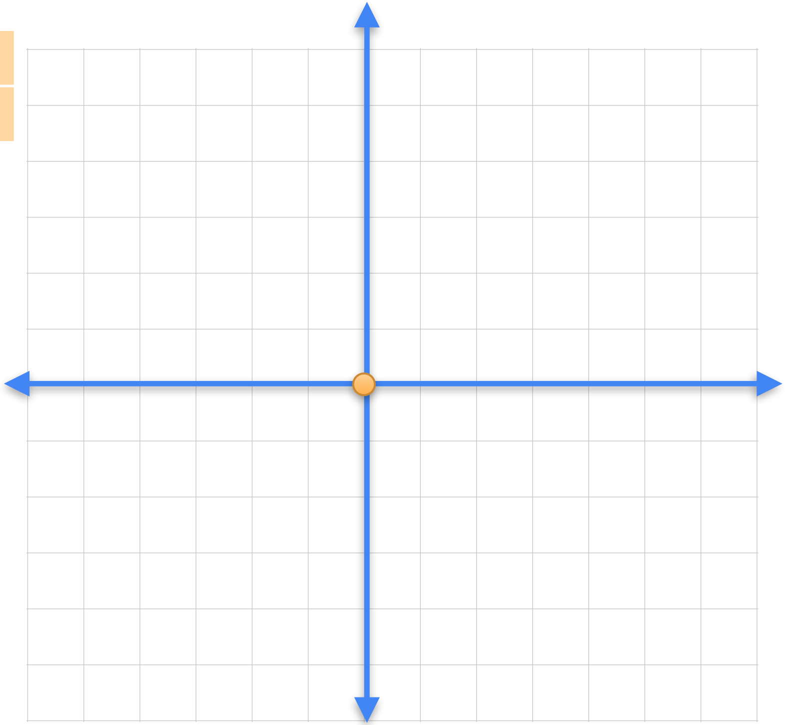
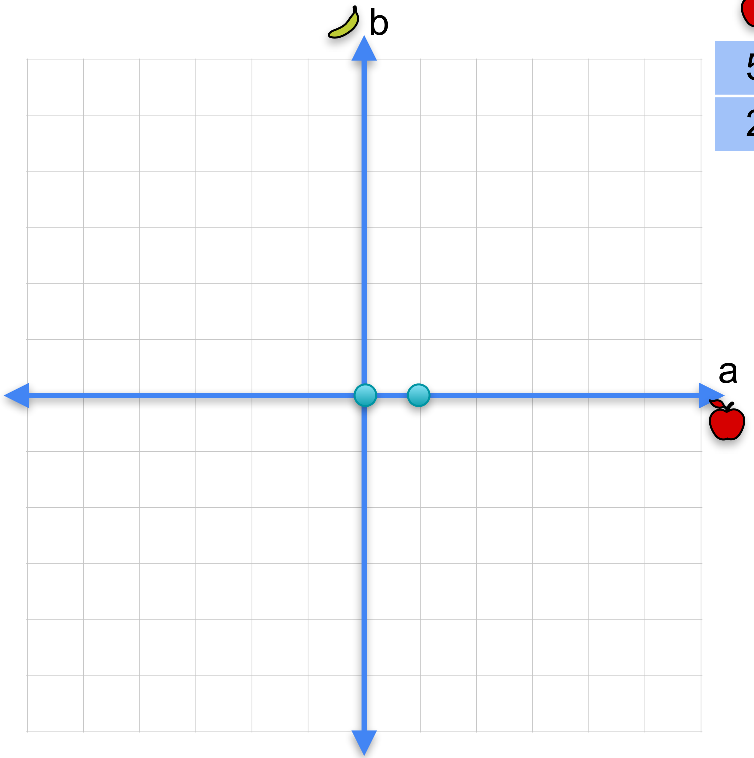
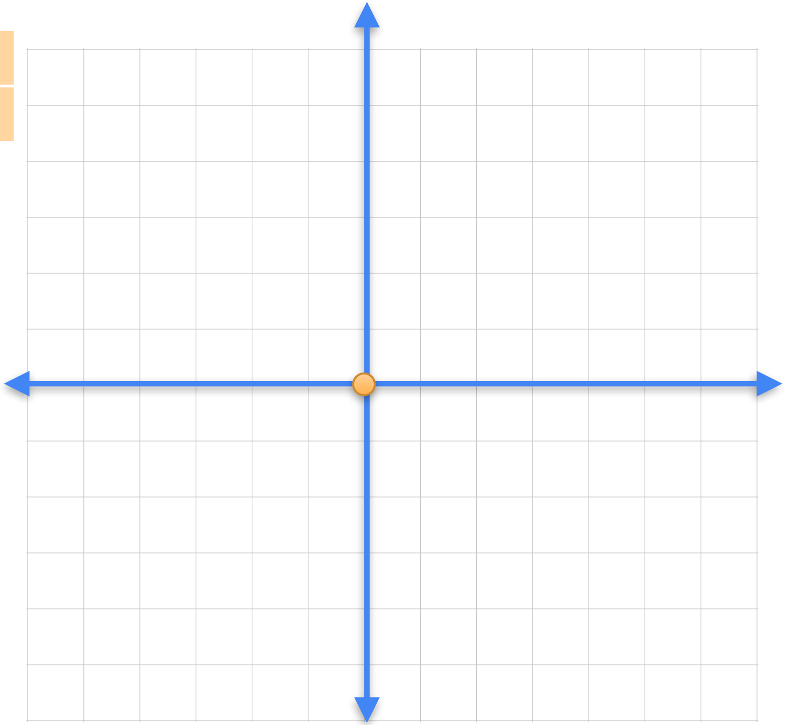


Matrices as linear transformations

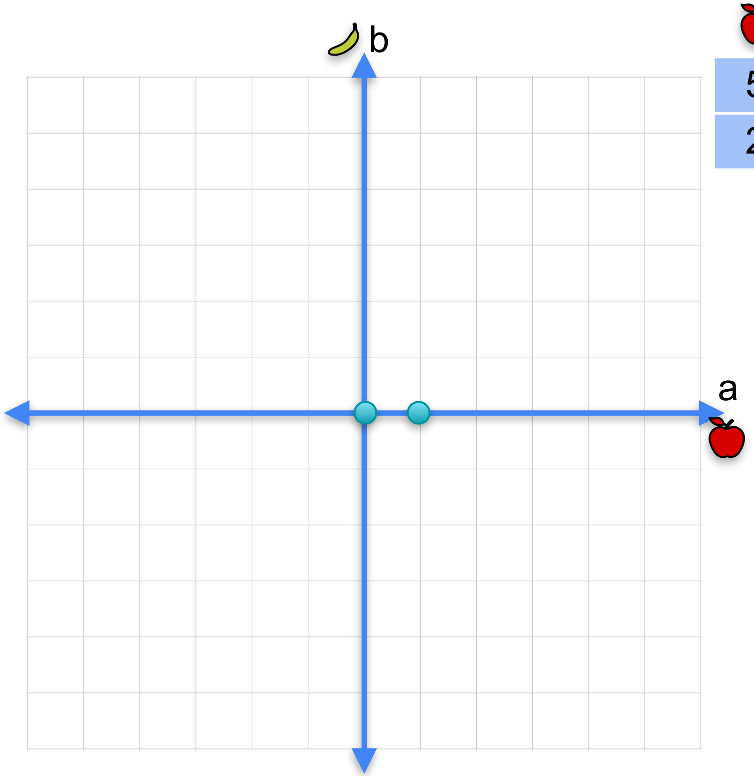


$$\begin{bmatrix}5 & 2\\2 & 2\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

(0,0) → (0,0)



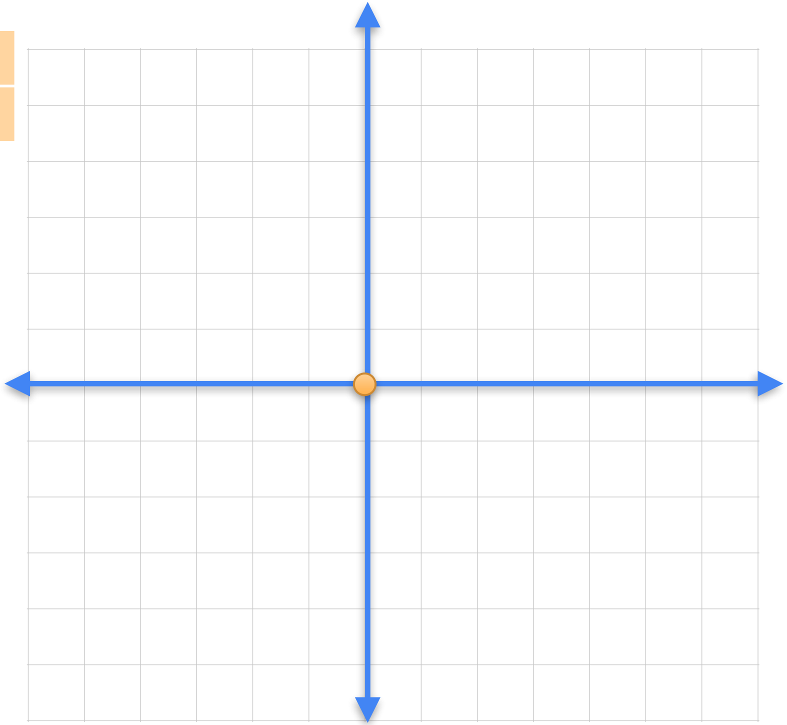
Matrices as linear transformations



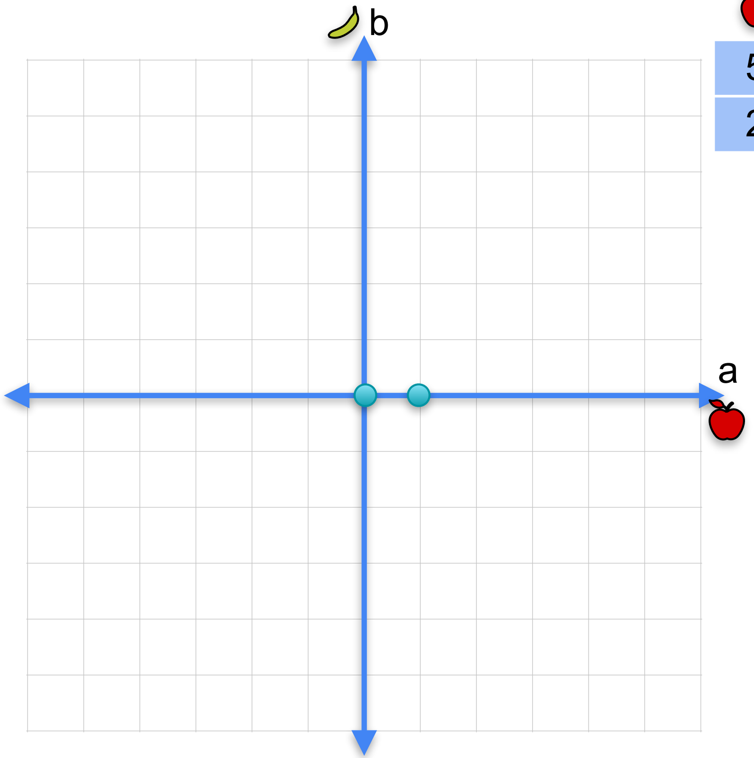
$$\begin{bmatrix}5 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}5\\ 2\end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



Matrices as linear transformations

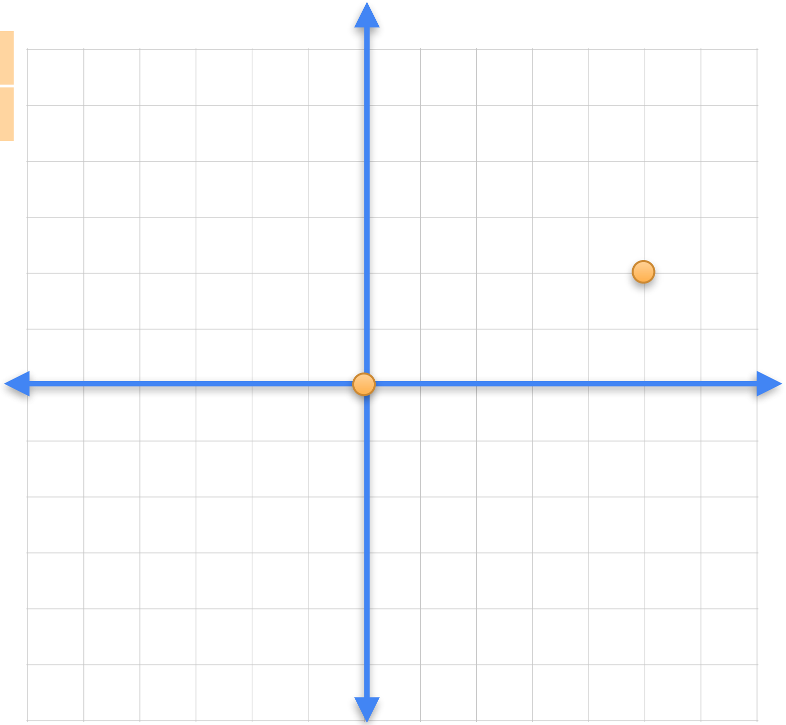


$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



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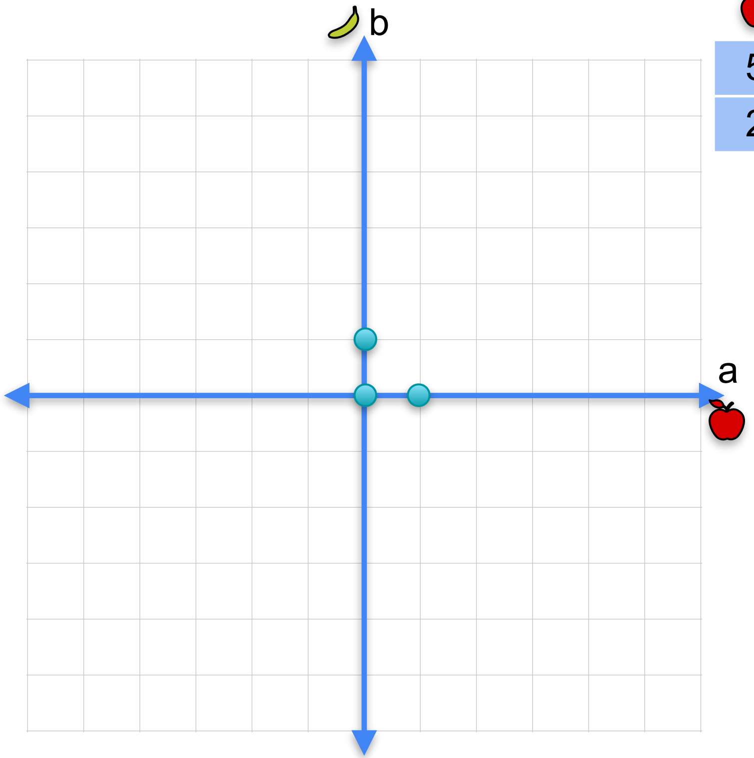
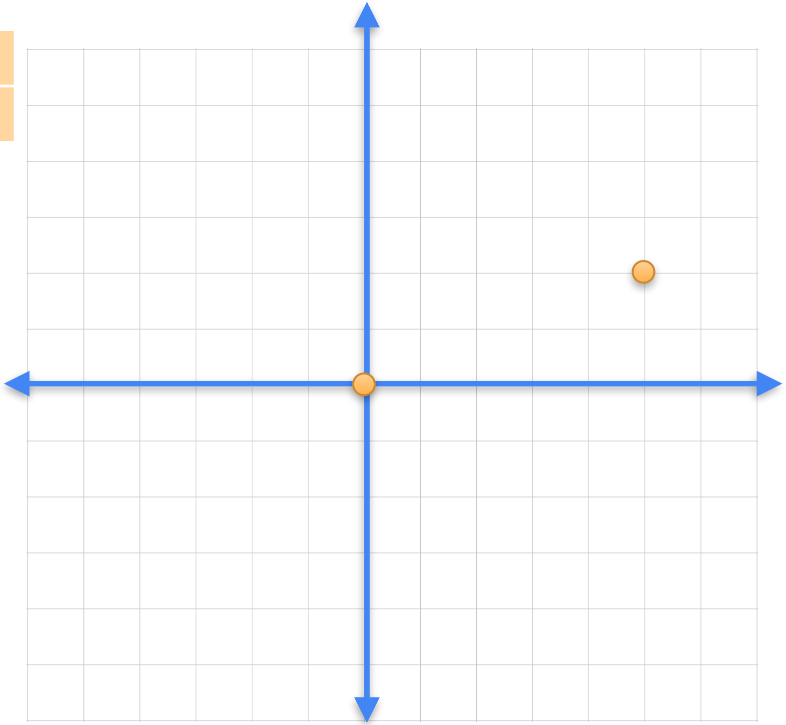
Matrices as linear transformations



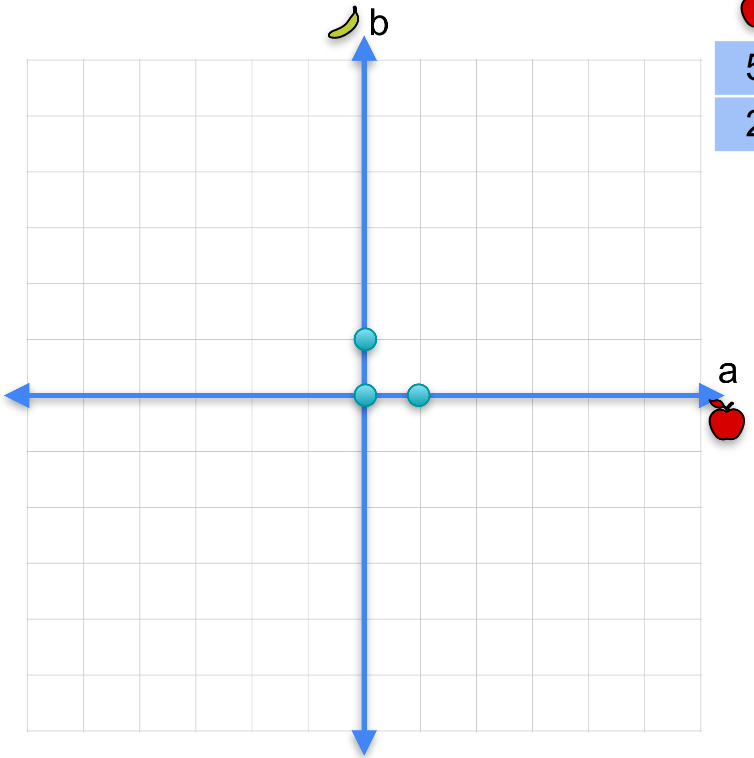
$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



Matrices as linear transformations

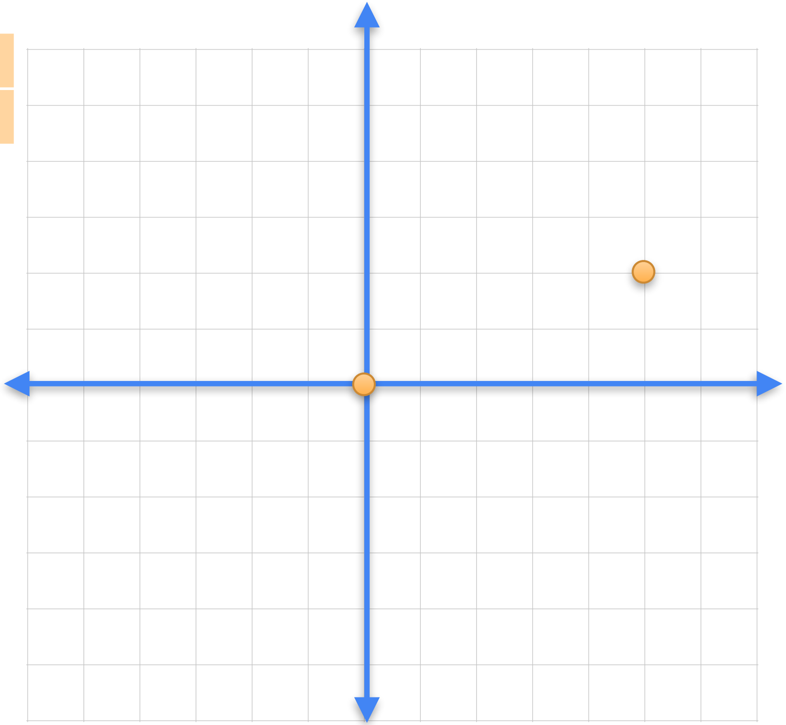


$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

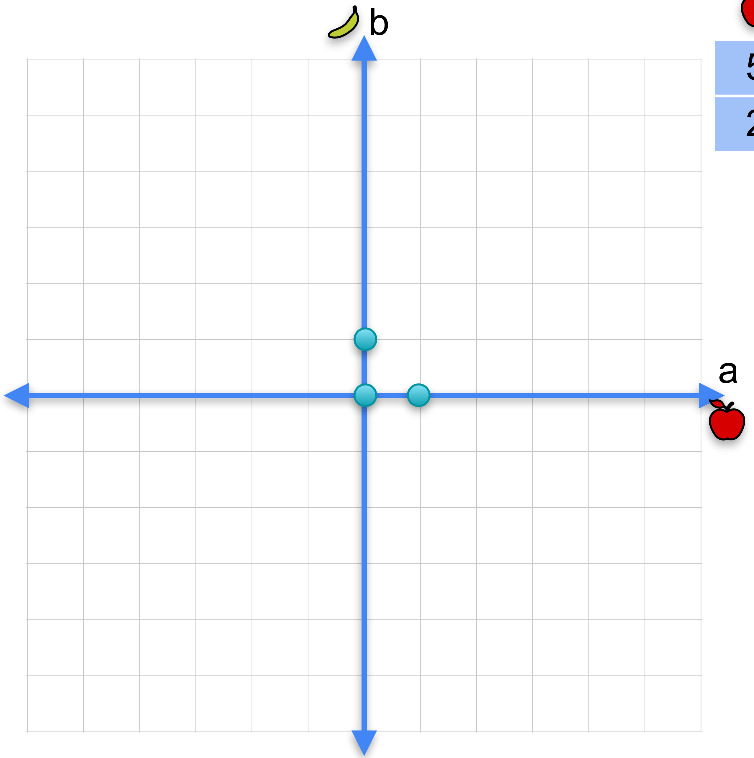
$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (3,1)$

$(0,1) \rightarrow (1,2)$



Matrices as linear transformations



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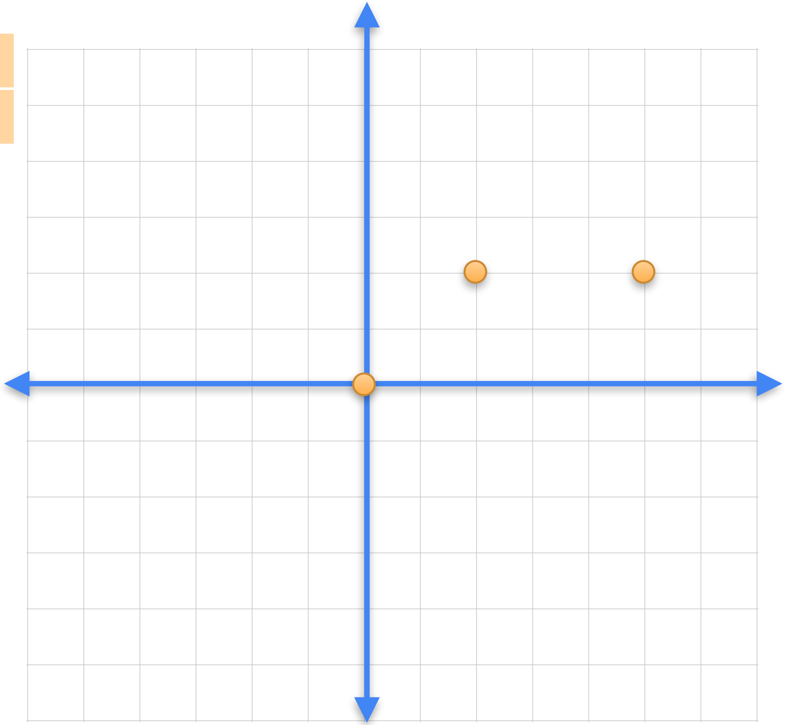
$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

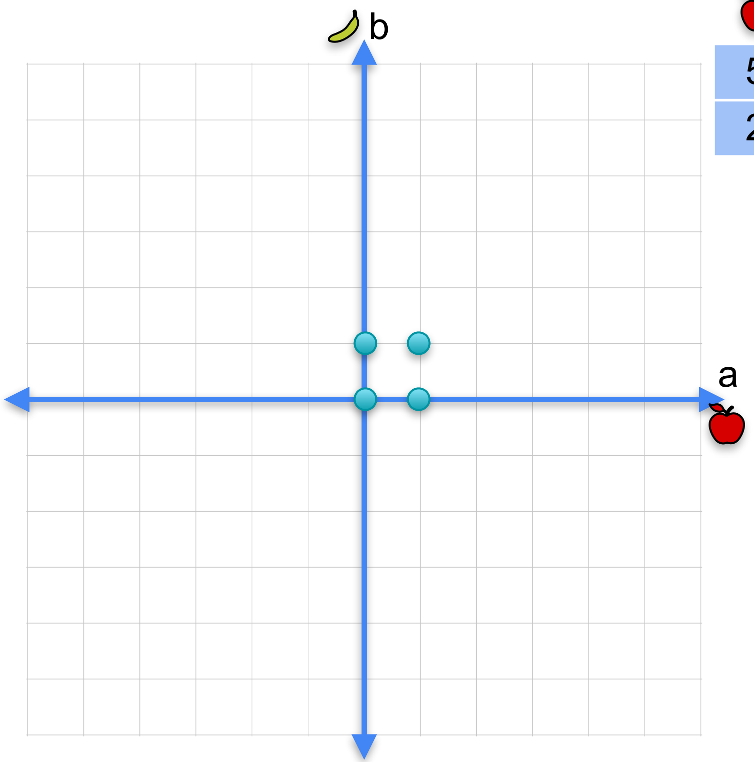
$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (3,1)$

$(0,1) \rightarrow (1,2)$

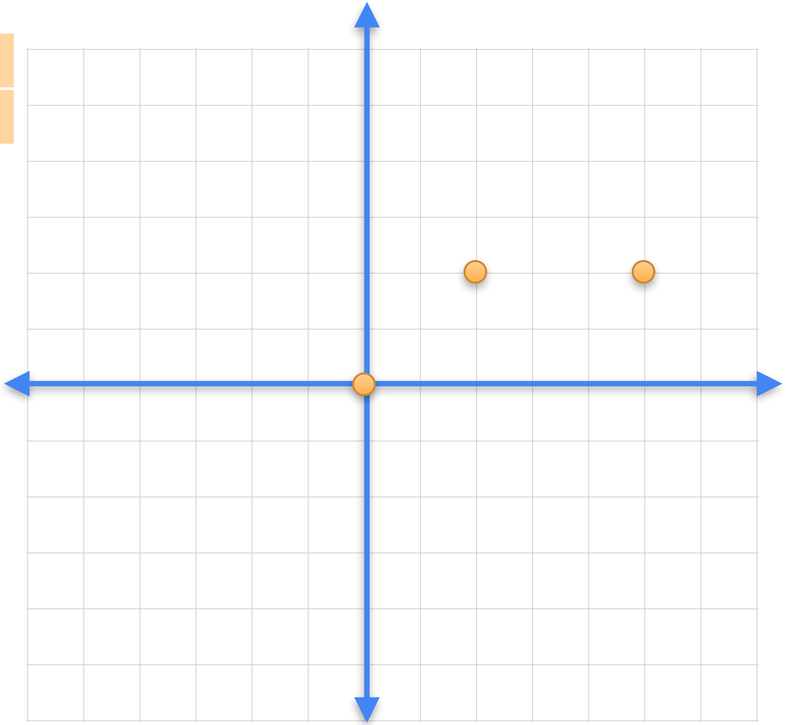


Matrices as linear transformations

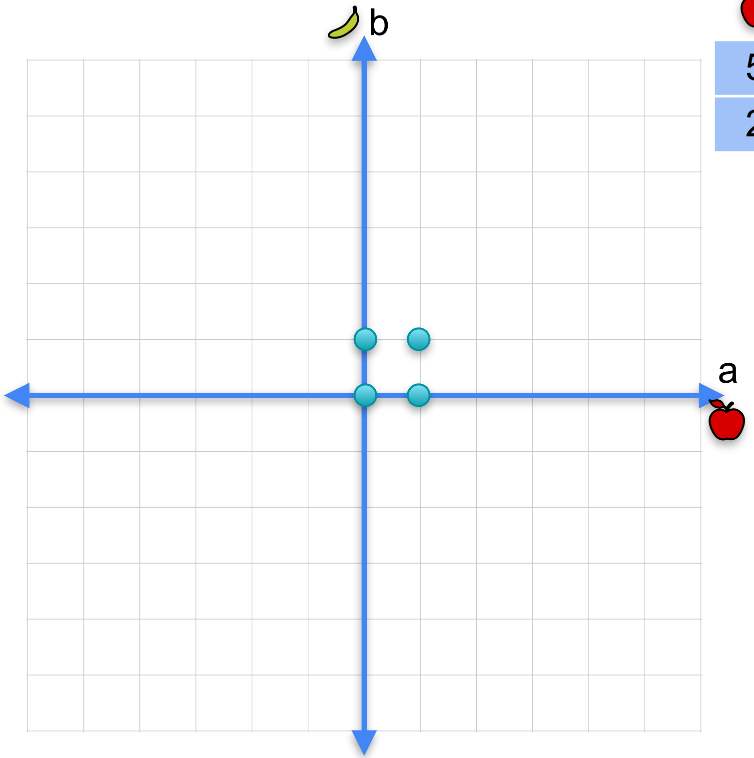


$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)



Matrices as linear transformations



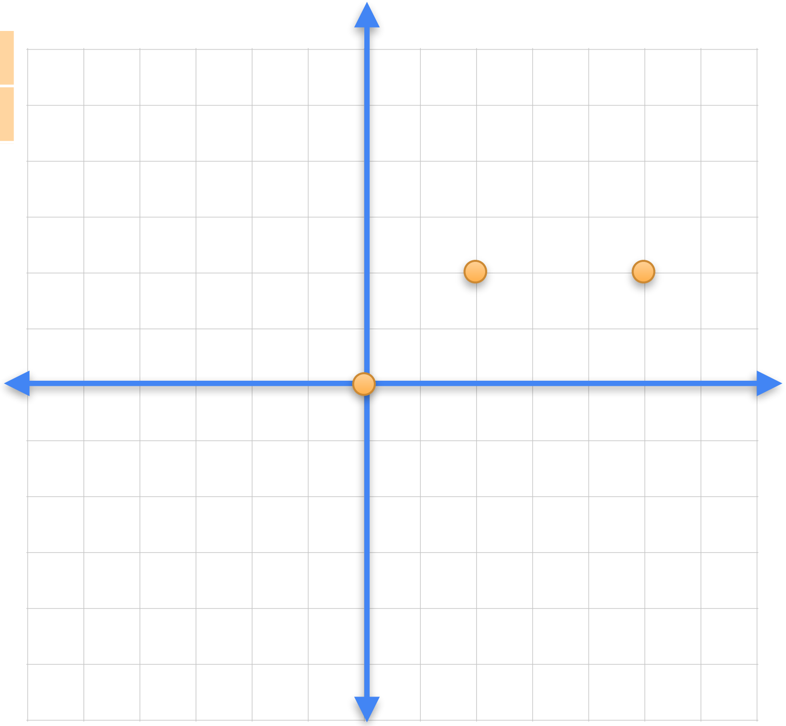
$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

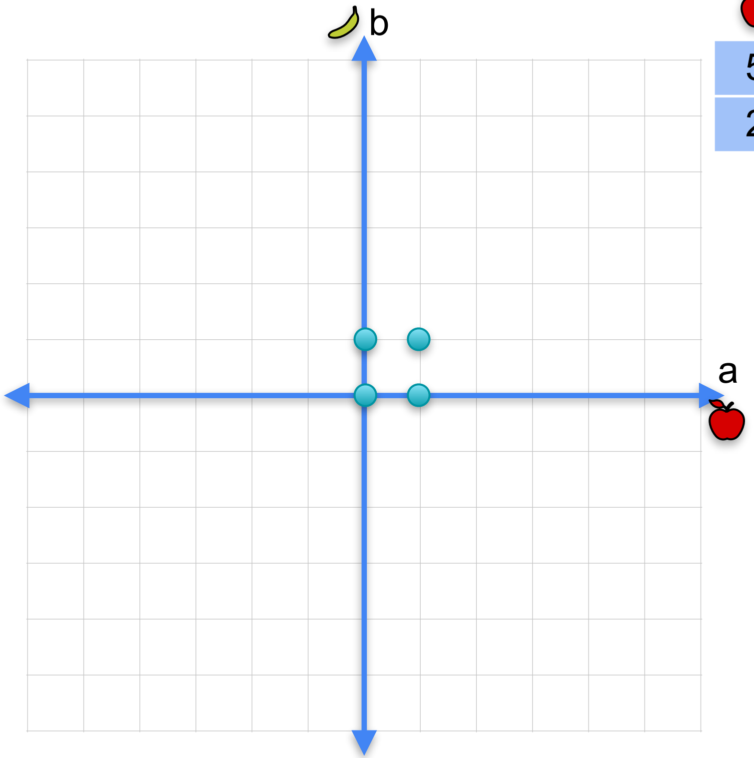
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Matrices as linear transformations



$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$

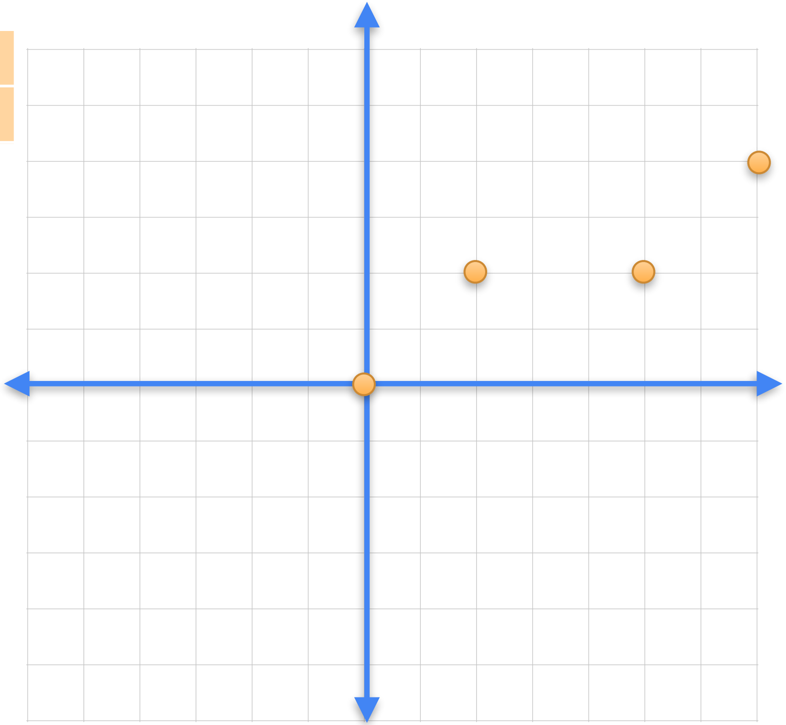
$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

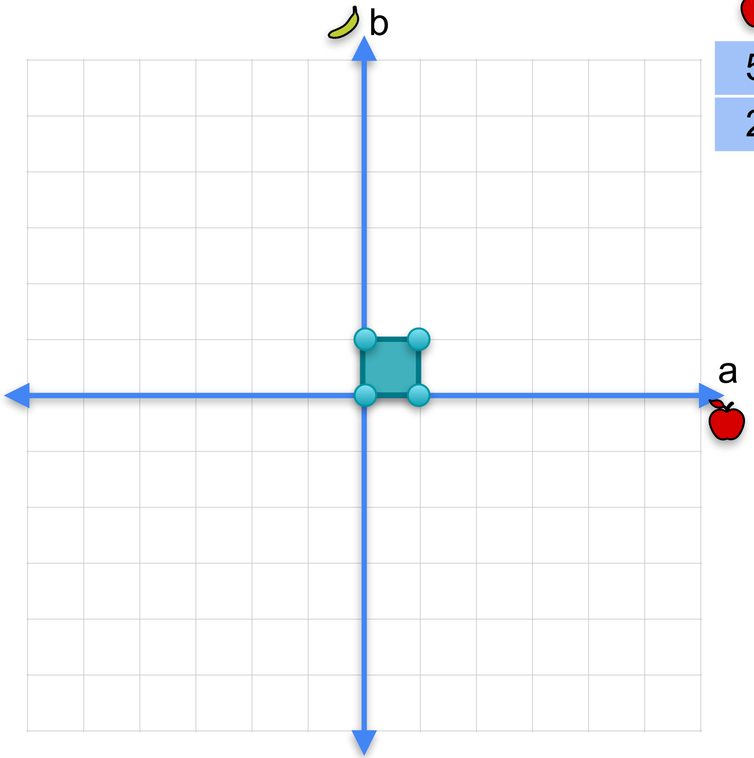
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Matrices as linear transformations



$$\begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
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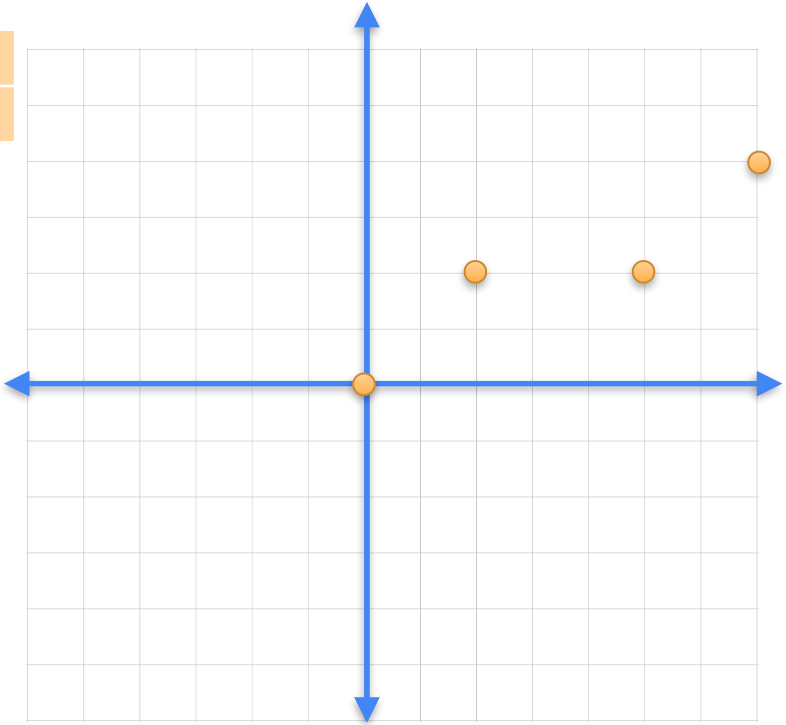
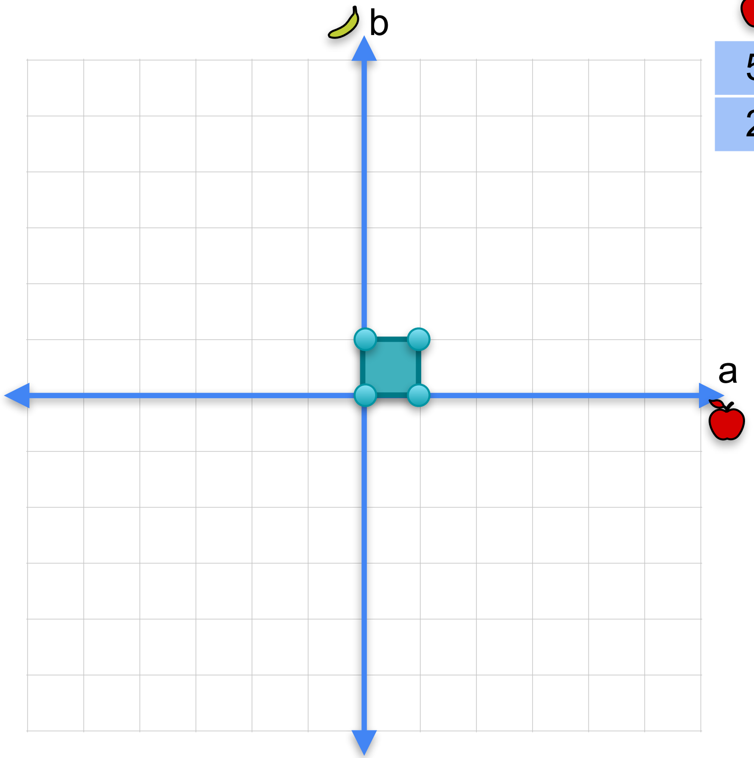
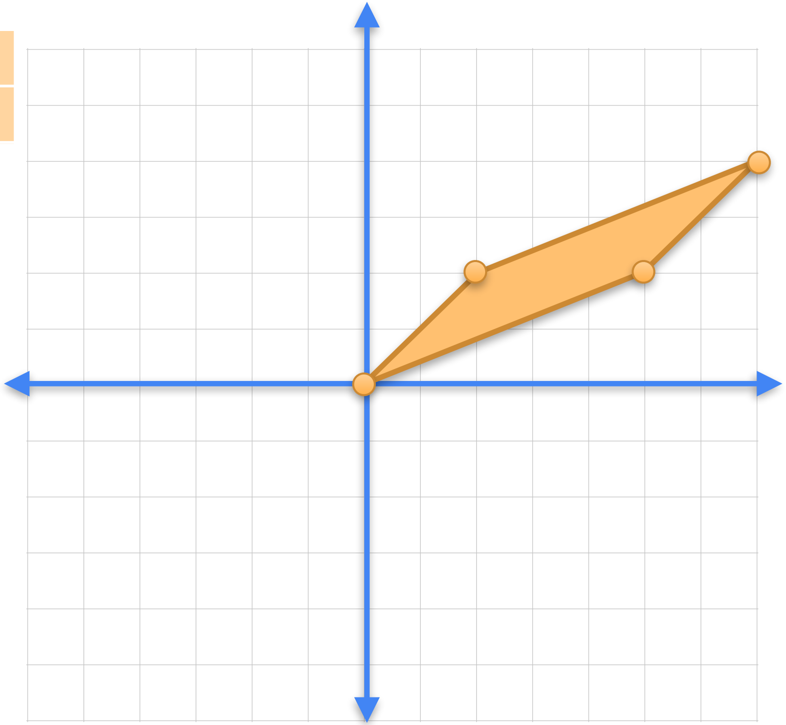


Matrices as linear transformations



$$\begin{bmatrix}5 & 2\\2 & 2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}7\\2\end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)



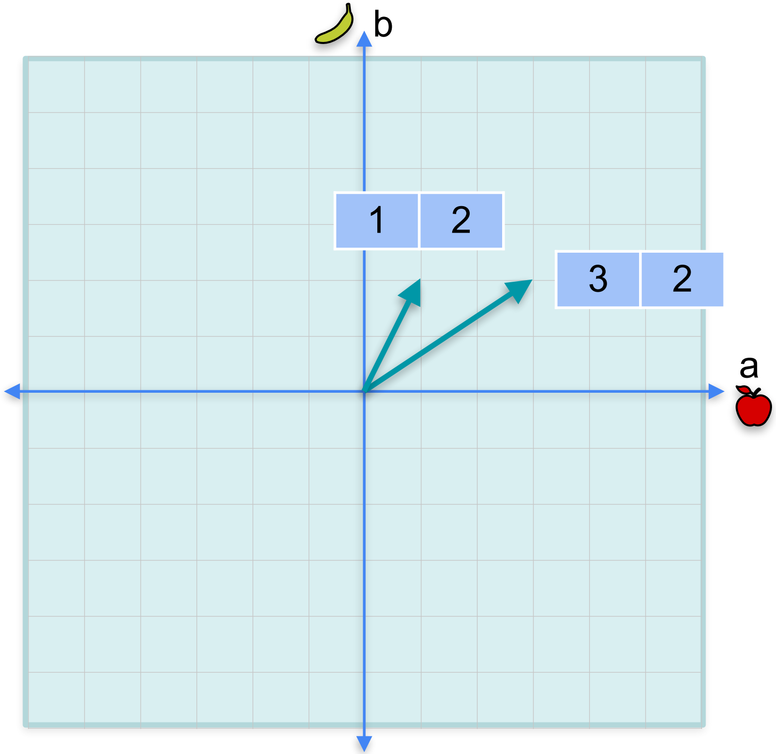
Row span of a matrix

3	2
1	2

Rows

3	2

1	2



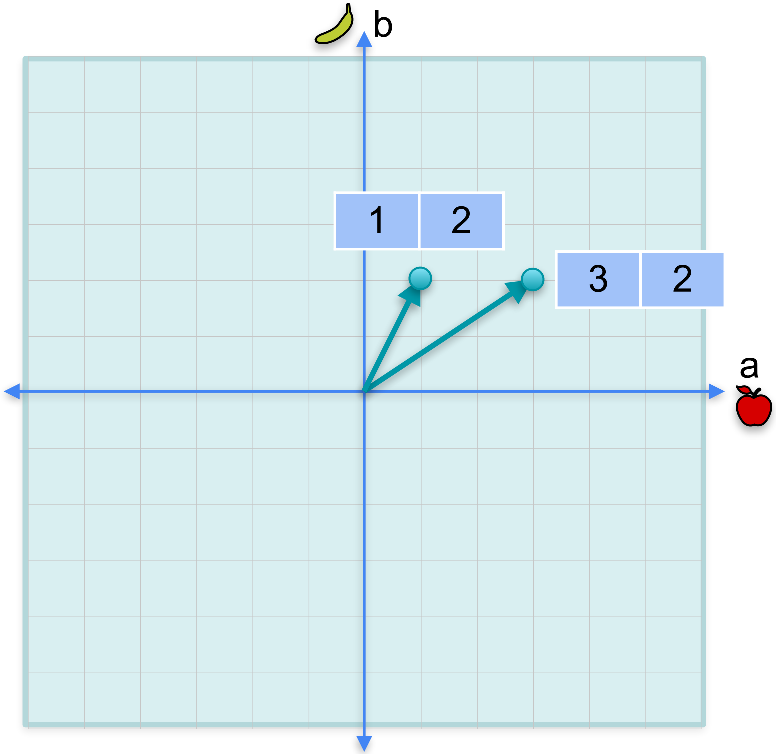
Row span of a matrix

3	2
1	2

Rows

3	2

1	2

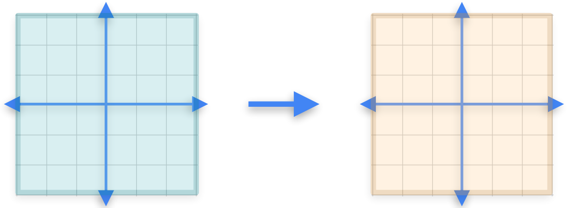


Span of the rows

Non-singular

3	1
1	2

Rank = 2

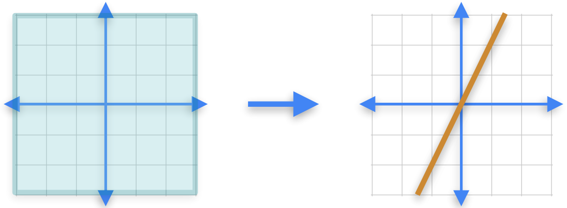


The whole plane

Singular

1	1
2	2

Rank = 1

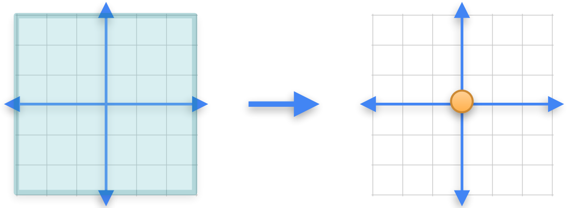


A line

Singular

0	0
0	0

Rank = 0



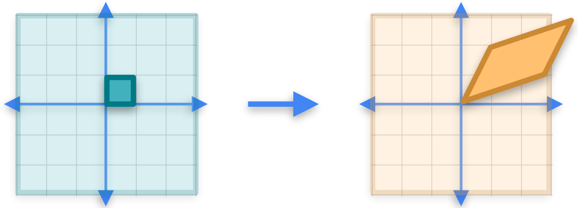
A point

Basis vectors

Non-singular

3	1
1	2

Rank = 2

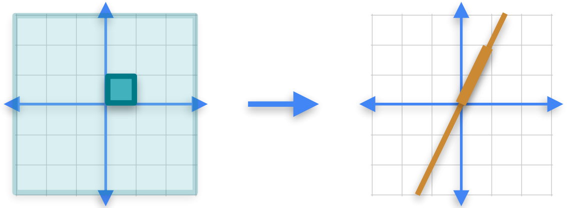


The whole plane

Singular

1	1
2	2

Rank = 1

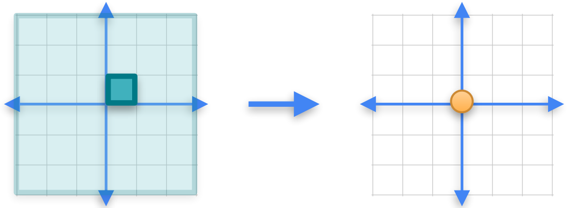


A line

Singular

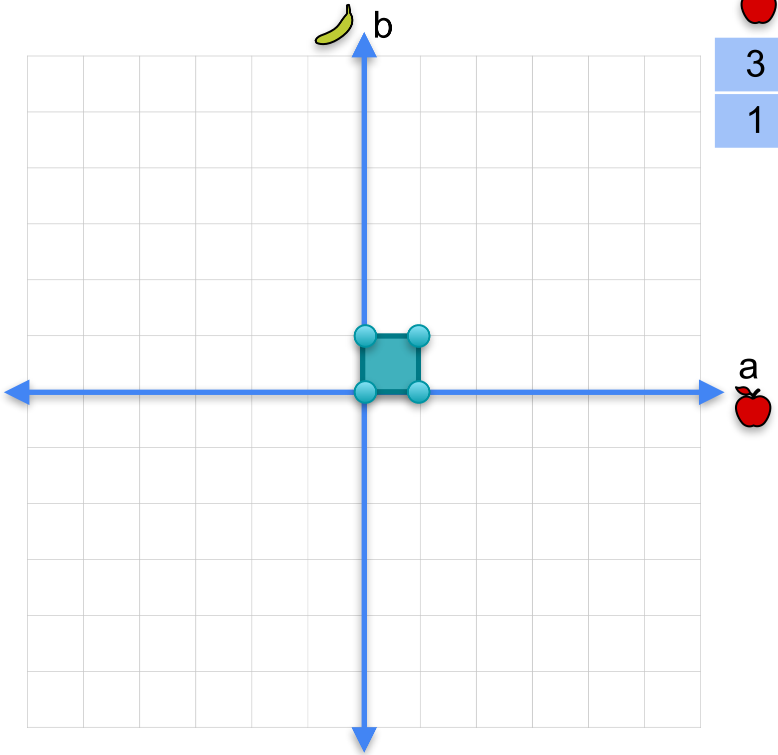
0	0
0	0

Rank = 0



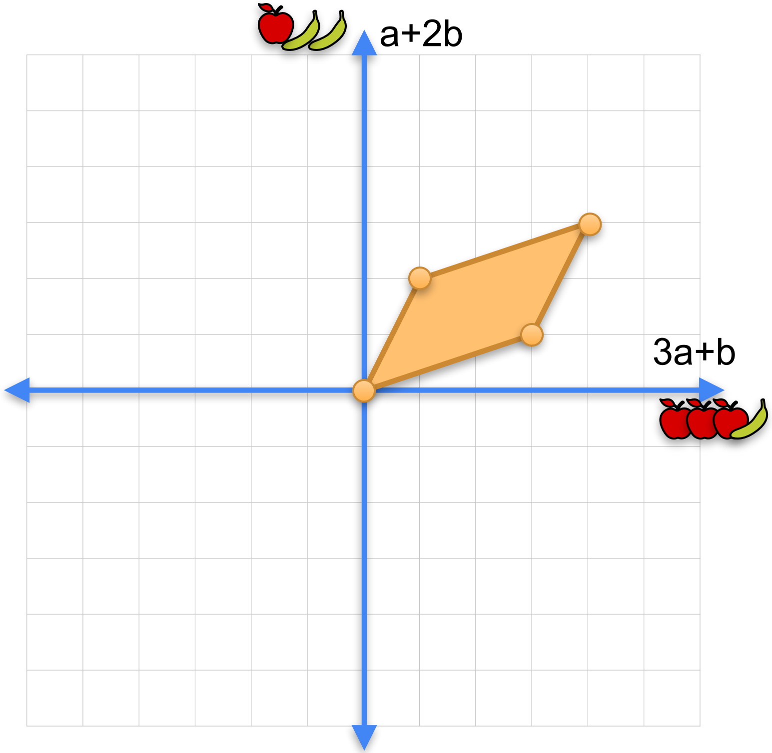
A point

Linear transformation

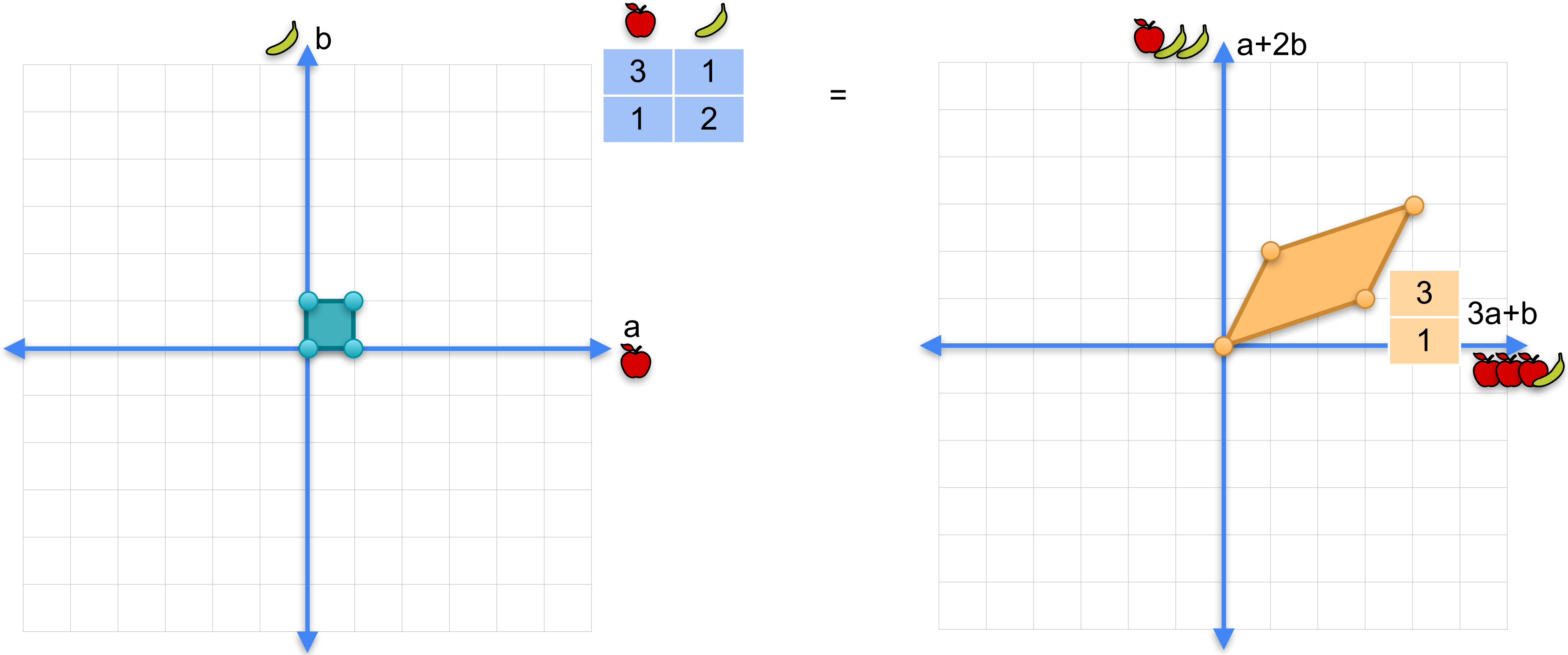


3	1
1	2

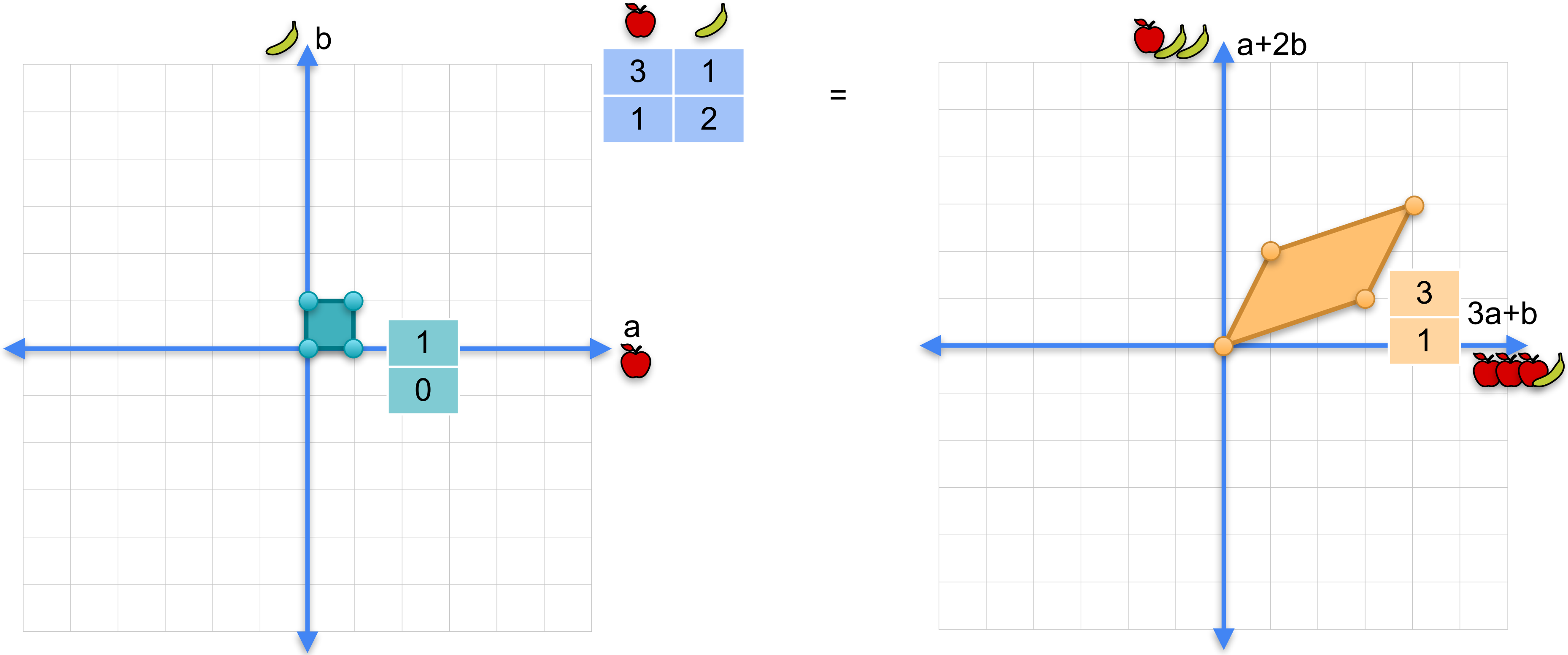
=



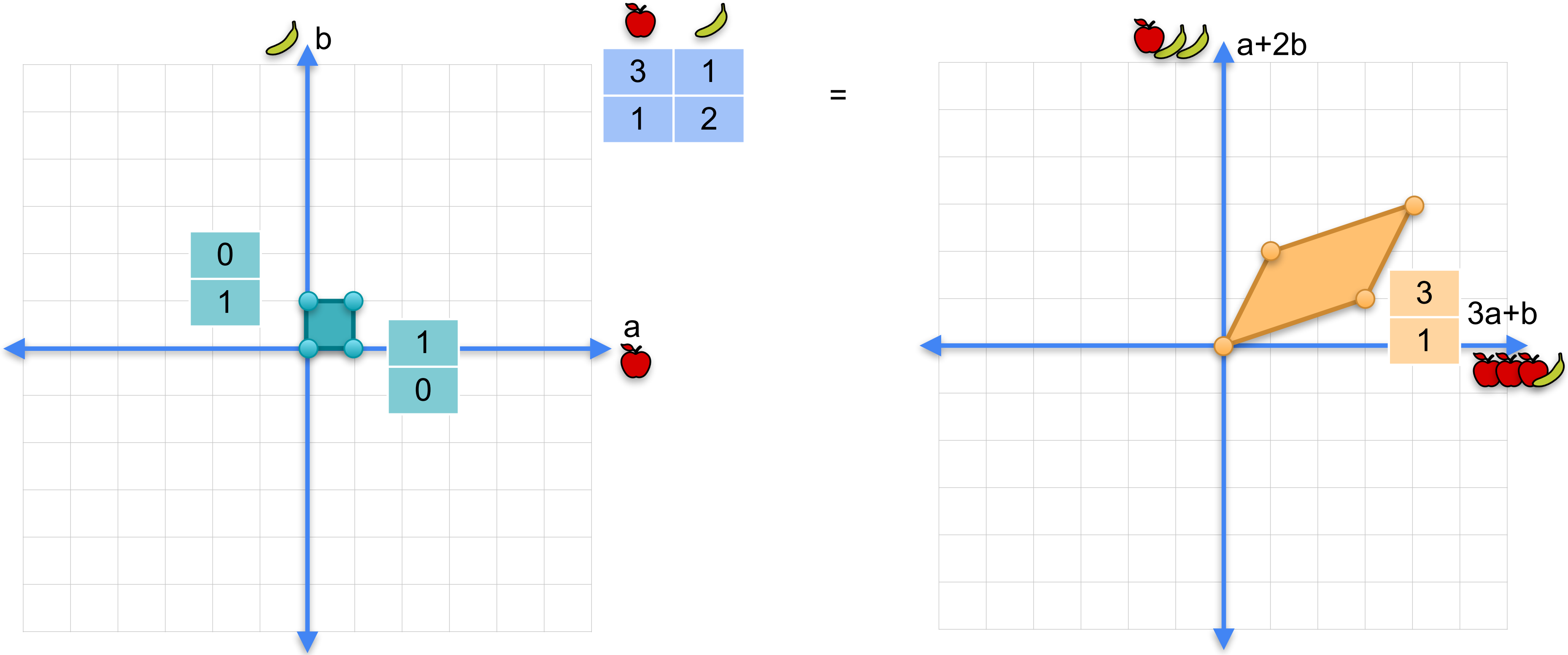
Linear transformation



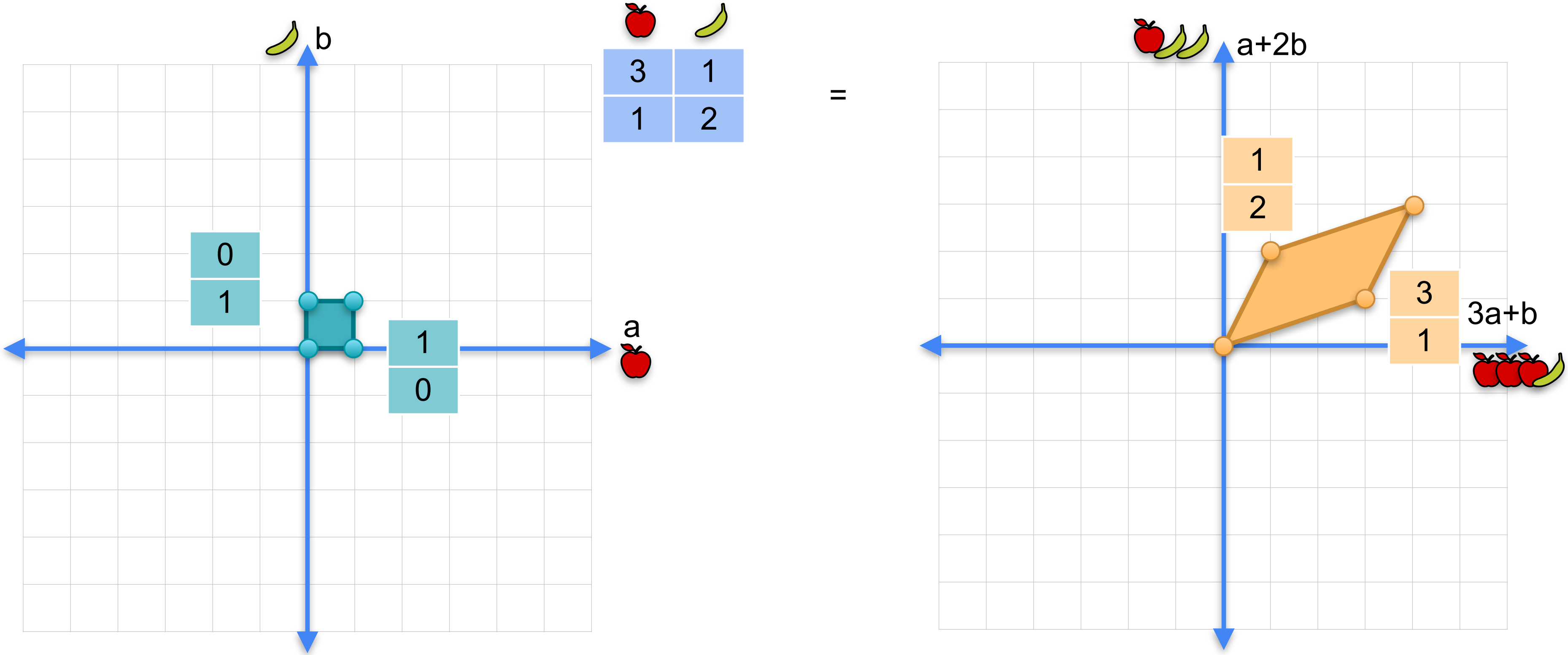
Linear transformation



Linear transformation

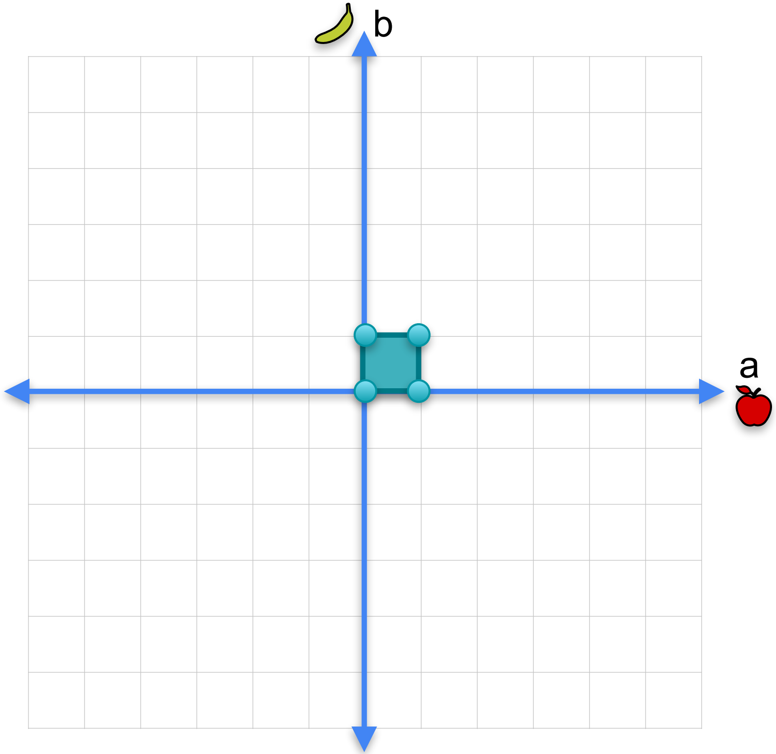


Linear transformation

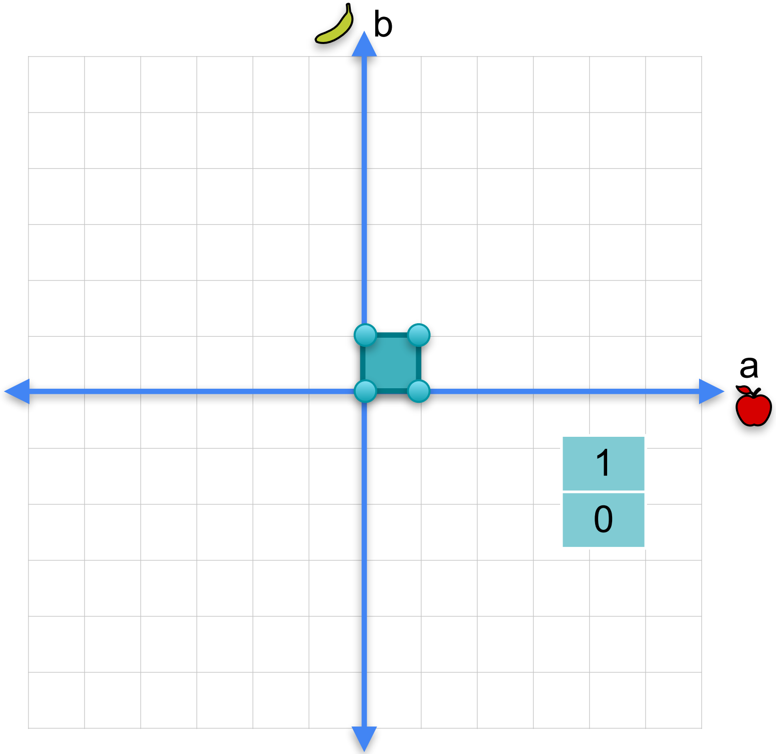


DeepLearning.AI

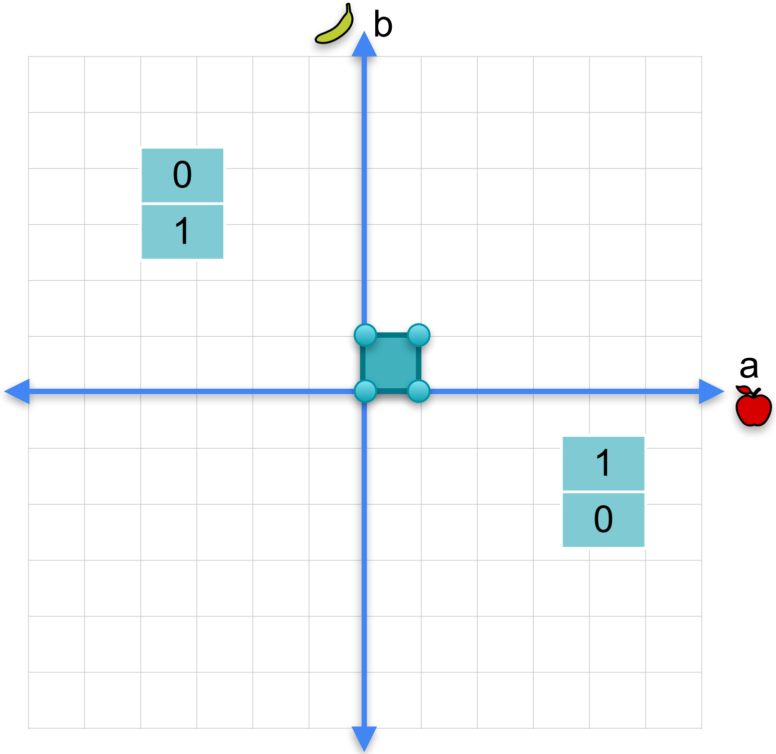
Linear transformation



Linear transformation



Linear transformation



3D

DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 4

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations

A matrix and its corresponding system of equations

A matrix and its corresponding system of equations

	
1	1
1	2

A matrix and its corresponding system of equations

🍎	🍌
1	1
1	2

🍎	🍌
1	1
2	2

A matrix and its corresponding system of equations

🍎	🍌
1	1
1	2

🍎	🍌
1	1
2	2

🍎	🍌
0	0
0	0

A matrix and its corresponding system of equations

System 1

- a + b = **0**
- a + 2b = **0**

1	1
1	2

1	1
2	2

0	0
0	0

A matrix and its corresponding system of equations

System 1

- a + b = **0**
- a + 2b = **0**

1	1
1	2

System 2

- a + b = **0**
- 2a + 2b = **0**

1	1
2	2

0	0
0	0

A matrix and its corresponding system of equations

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

0	0
0	0

A matrix and its corresponding system of equations

System 1

- a + b = **0**
- a + 2b = **0**

1	1
1	2

System 2

- a + b = **0**
- 2a + 2b = **0**

1	1
2	2

System 3

- 0a + 0b = **0**
- 0a + 0b = **0**

0	0
0	0

The only two numbers a, b, such that
- a+b = 0

and
- a+2b = 0

are:
a=0 and b=0

A matrix and its corresponding system of equations

System 1

- $a + b = 0$
- $a + 2b = 0$

1	1
1	2

The only two numbers a, b, such that
- $a+b = 0$

and
- $a+2b = 0$

are:
$a=0$ and $b=0$

System 2

- $a + b = 0$
- $2a + 2b = 0$

1	1
2	2

Any pair $(x, -x)$ satisfies that
- $a+b = 0$

and
- $a+2b = 0$

For example:
$(1,-1)$, $(2,-2)$, $(-8,8)$, etc.

System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

0	0
0	0

A matrix and its corresponding system of equations

System 1

- a + b = **0**
- a + 2b = **0**

1	1
1	2

System 2

- a + b = **0**
- 2a + 2b = **0**

1	1
2	2

System 3

- 0a + 0b = **0**
- 0a + 0b = **0**

0	0
0	0

The only two numbers a, b, such that
- a+b = 0

and
- a+2b = 0

are:
a=0 and b=0

Any pair (x, -x) satisfies that
- a+b = 0

and
- a+2b = 0

For example:
(1,-1), (2,-2), (-8,8), etc.

Any pair of numbers satisfies that
- 0a+0b = 0

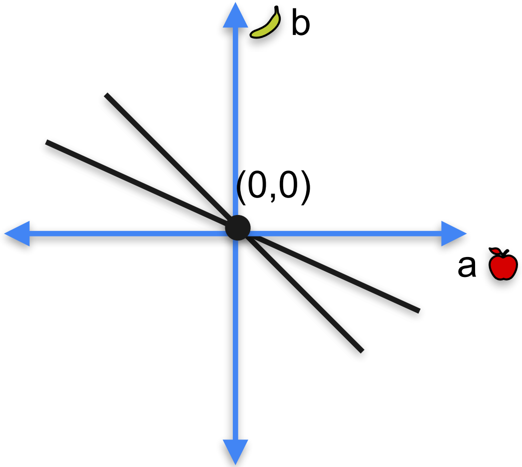
and
- 0a+0b = 0

For example:
(1,2), (3,-9), (-90,8.34), etc.

The set of solutions of a system of equations

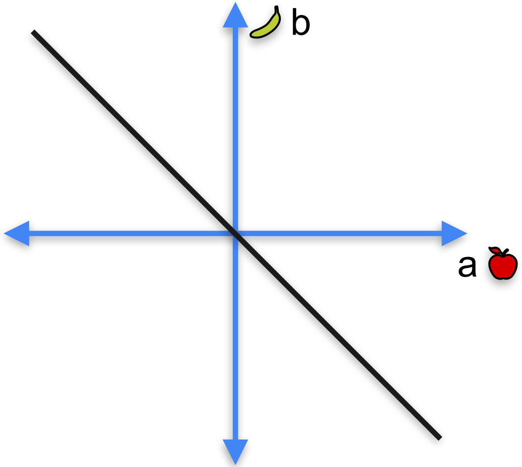
System 1

- $a + b = 0$
- $a + 2b = 0$



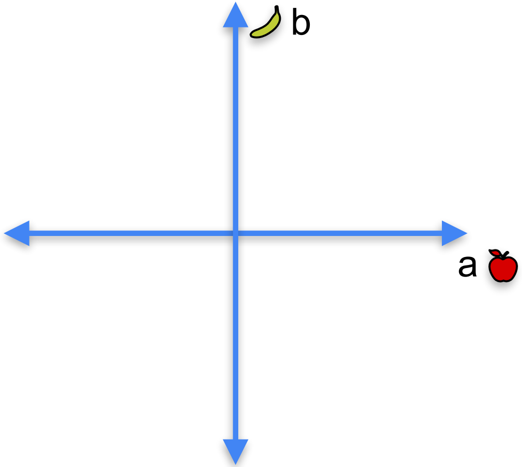
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$



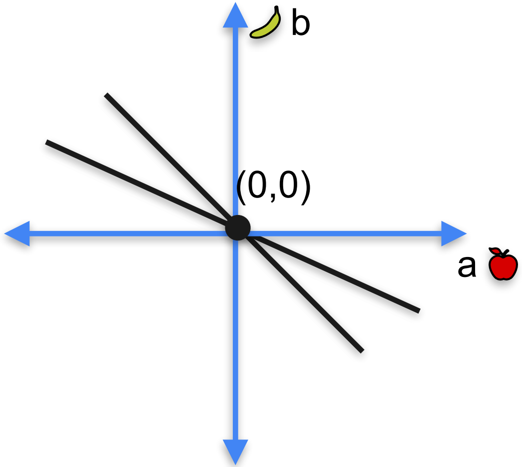
The set of solutions of a system of equations

System 1

- a + b = **0**
- a + 2b = **0**

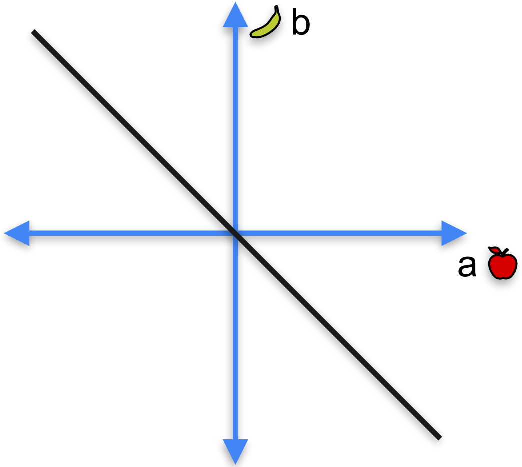
Solution

- $a = 0$
- $b = 0$



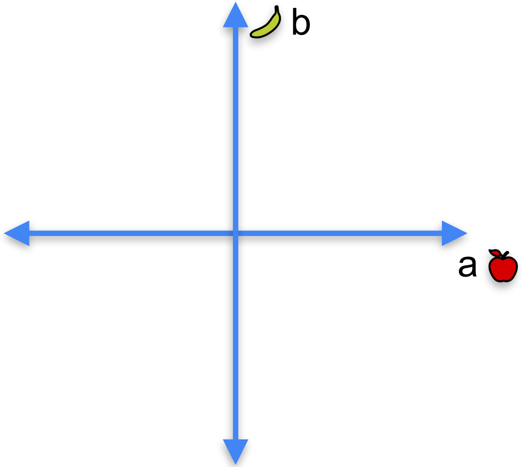
System 2

- a + b = **0**
- 2a + 2b = **0**



System 3

- 0a + 0b = **0**
- 0a + 0b = **0**



The set of solutions of a system of equations

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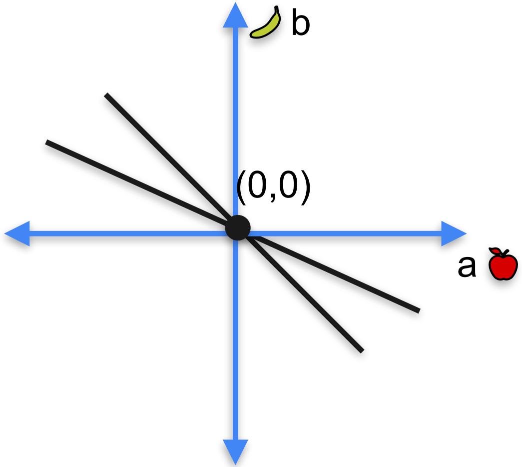

System 2

- a + b = **0**
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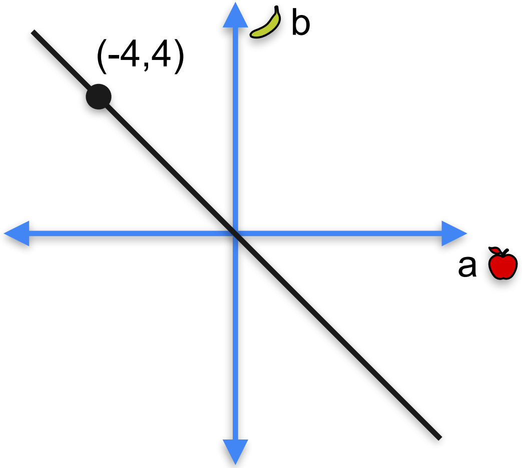
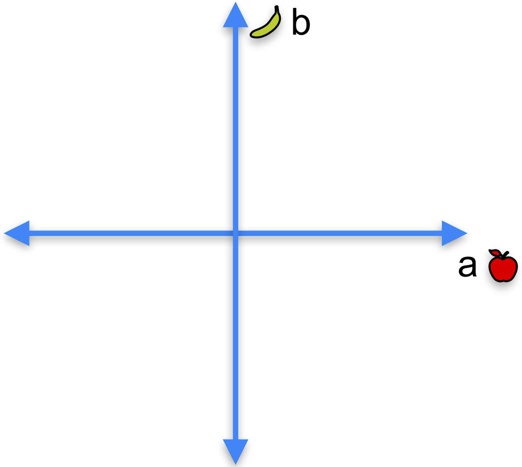


System 3

- 0a + 0b = **0**
- 0a + 0b = **0**



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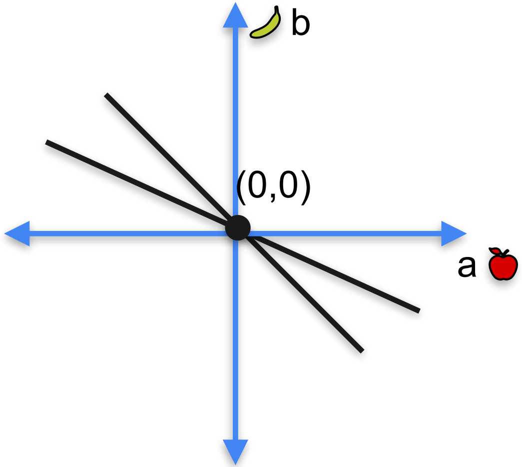
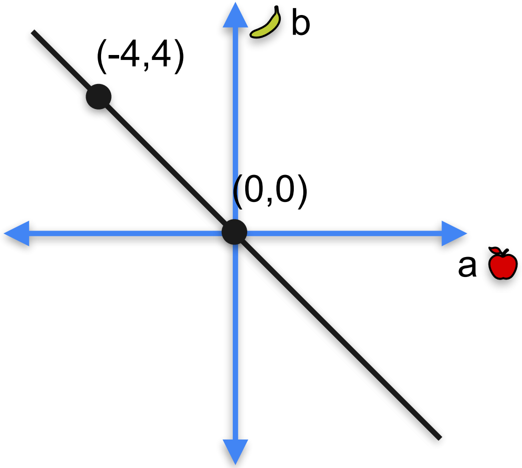


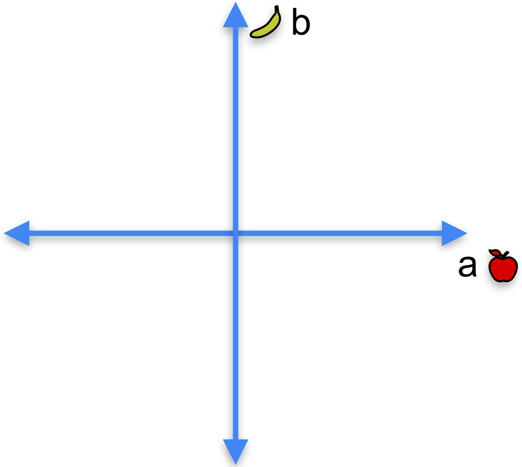
System 2

- a + b = **0**
- 2a + 2b = **0**



System 3

- 0a + 0b = **0**
- 0a + 0b = **0**



The set of solutions of a system of equations

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- a + b = 0
- a + 2b = 0

Solution

- $a = 0$
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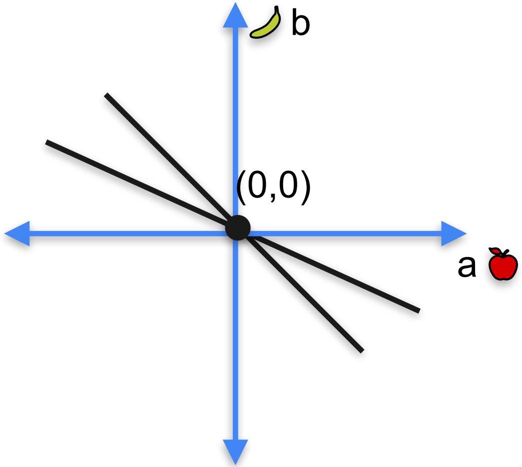
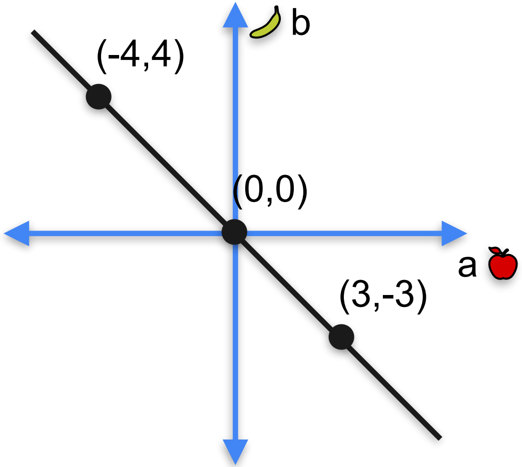


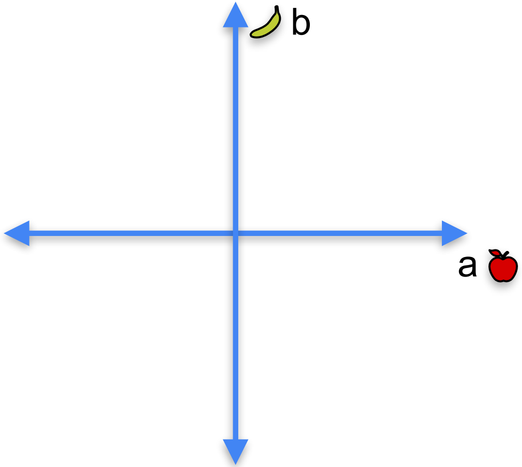
System 2

- a + b = 0
- 2a + 2b = 0



System 3

- 0a + 0b = 0
- 0a + 0b = 0



The set of solutions of a system of equations

System 1

- a + b = **0**
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Solution

- $a = 0$
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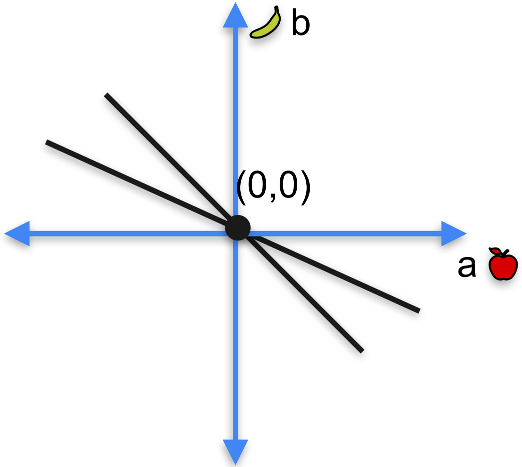
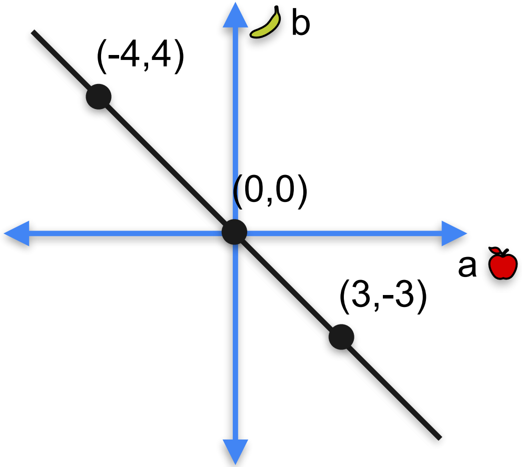


System 2

- a + b = **0**
- 2a + 2b = **0**

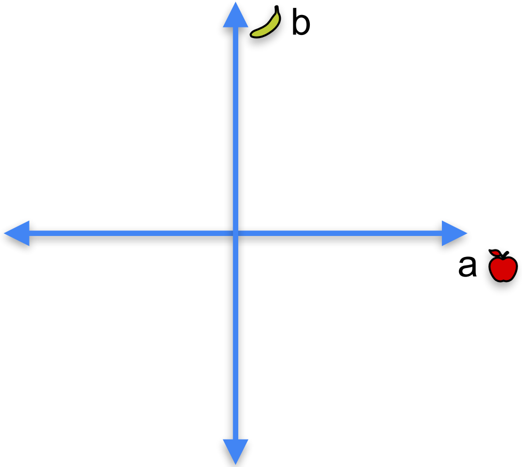
Solutions

- any a
- $b = -a$



System 3

- 0a + 0b = **0**
- 0a + 0b = **0**



The set of solutions of a system of equations

System 1

- a + b = **0**
- a + 2b = **0**

Solution

- $a = 0$
- $b = 0$

System 2

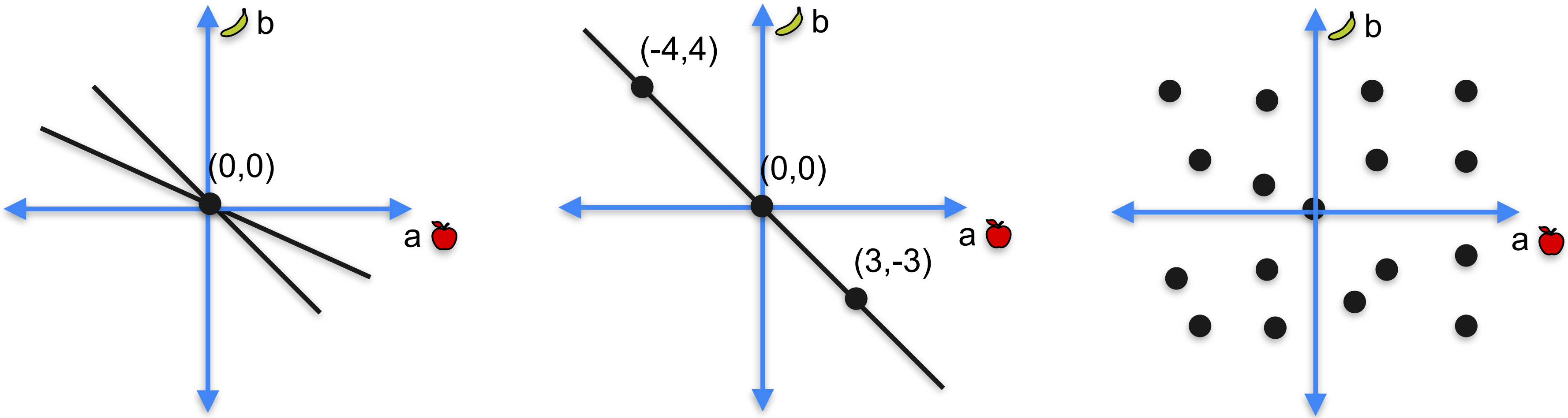
- a + b = **0**
- 2a + 2b = **0**

Solutions

- any a
- $b = -a$

System 3

- 0a + 0b = **0**
- 0a + 0b = **0**



The set of solutions of a system of equations

System 1

- a + b = **0**
- a + 2b = **0**

Solution

- $a = 0$
- $b = 0$

System 2

- a + b = **0**
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Solutions

- any a
- $b = -a$

System 3

- 0a + 0b = **0**
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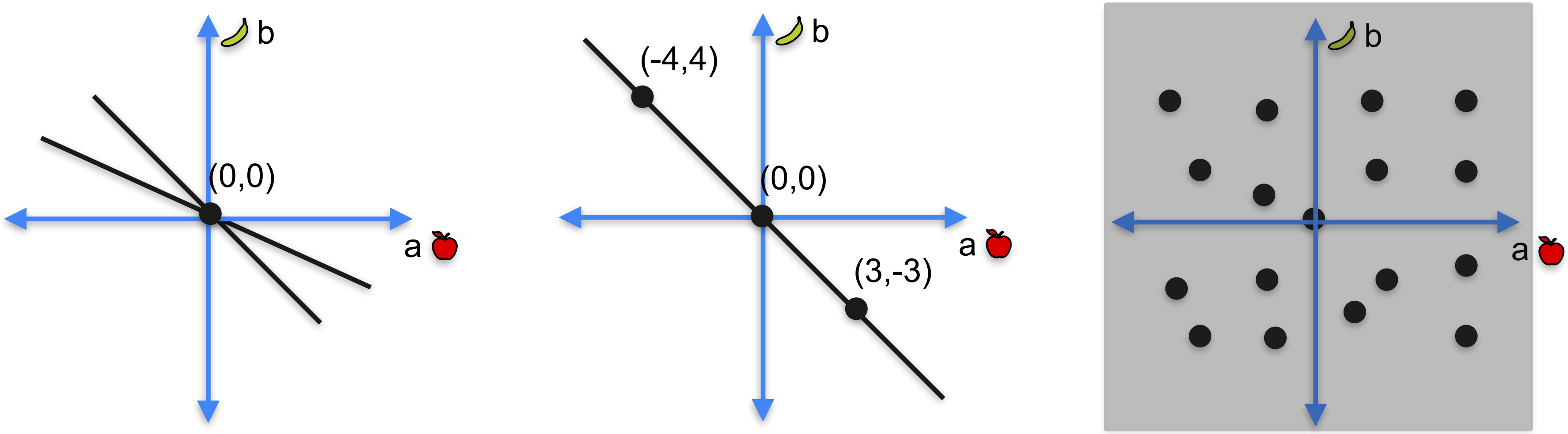

The set of solutions of a system of equations

System 1

- a + b = **0**
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Solution

- $a = 0$
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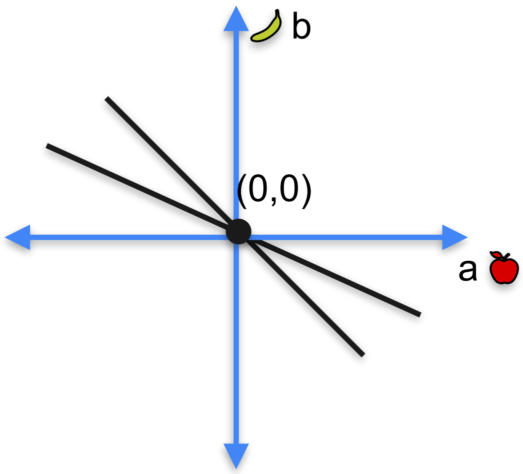
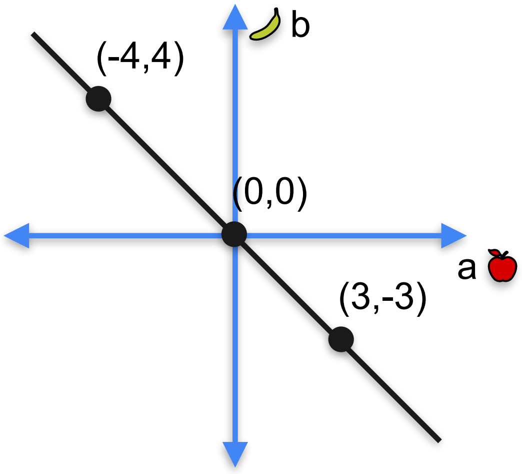


System 2

- a + b = **0**
- 2a + 2b = **0**

Solutions

- any a
- $b = -a$

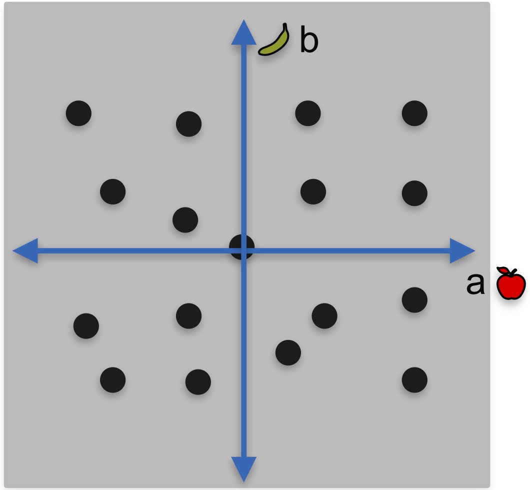


System 3

- 0a + 0b = **0**
- 0a + 0b = **0**

Solutions

- any a
- any b



The null space of a matrix

1	1
1	2

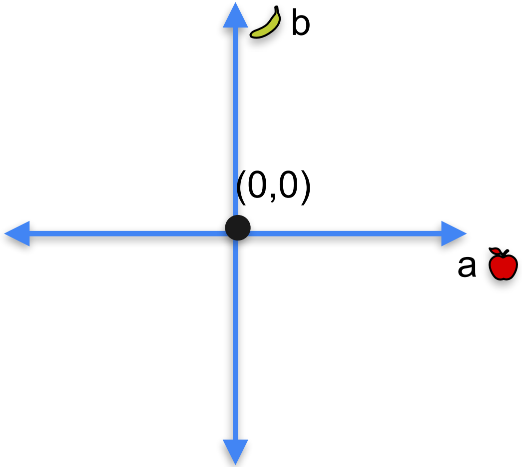
Null space
- $a = 0$
- $b = 0$

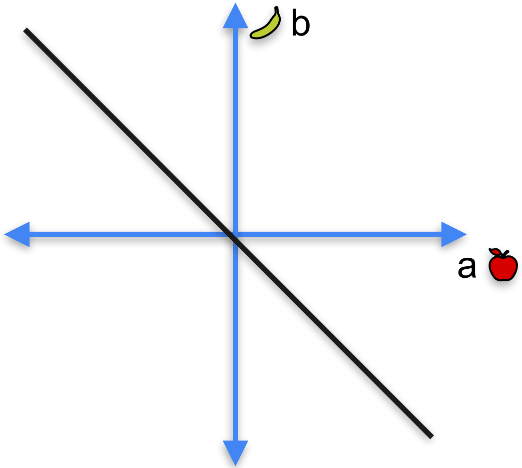
1	1
2	2

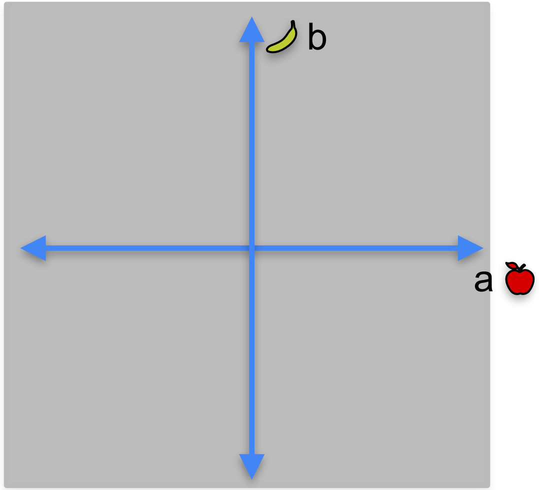
Null space
- any a
- $b = -a$

0	0
0	0

Null space
- any a
- any b





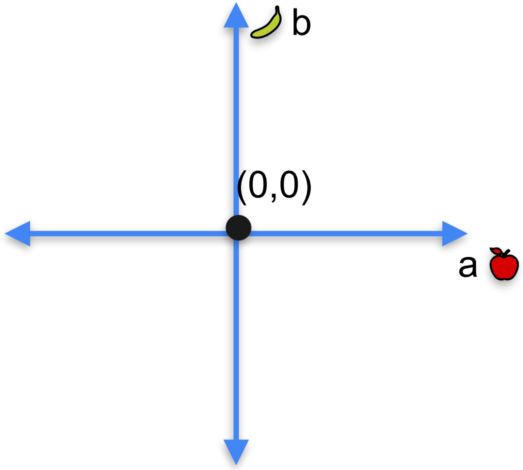


The null space of a matrix

🍎	🍌
1	1
1	2

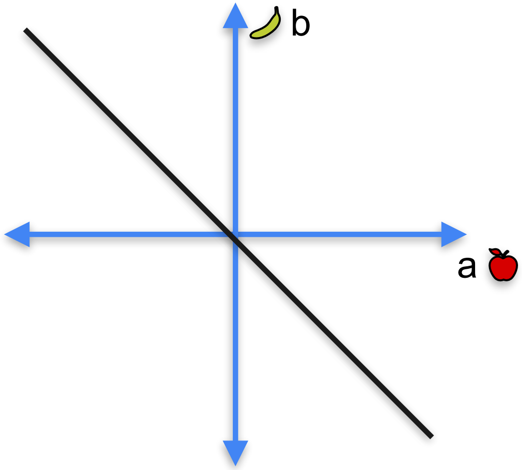
Null space
- $a = 0$
- $b = 0$

Dimension = 0



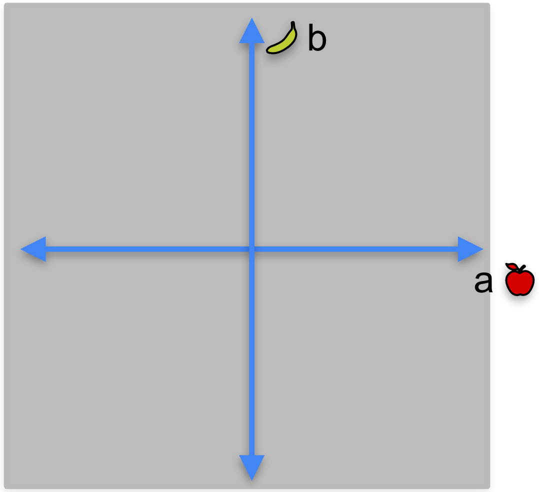
🍎	🍌
1	1
2	2

Null space
- any a
- $b = -a$



🍎	🍌
0	0
0	0

Null space
- any a
- any b

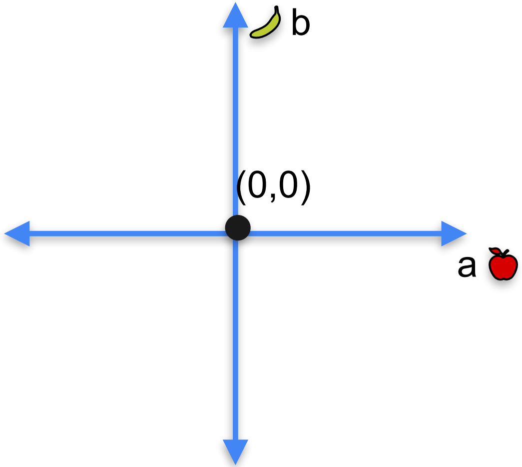


The null space of a matrix

1	1
1	2

Null space
- $a = 0$
- $b = 0$

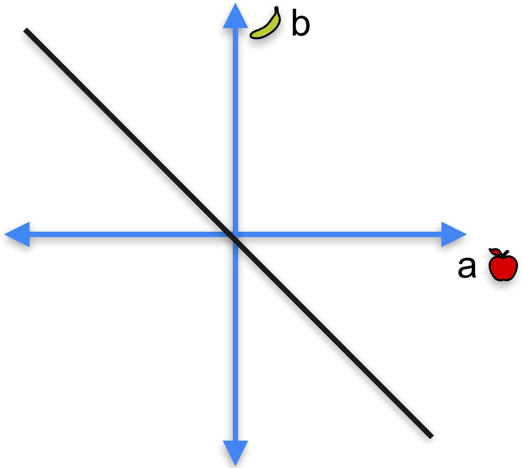
Dimension = 0



1	1
2	2

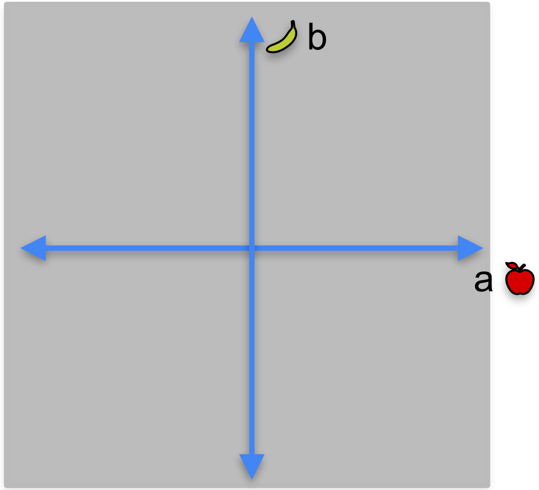
Null space
- any a
- $b = -a$

Dimension = 1



0	0
0	0

Null space
- any a
- any b

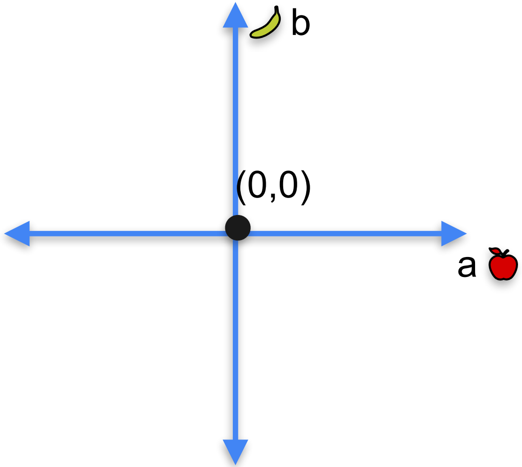


The null space of a matrix

🍎	🍌
1	1
1	2

Null space
- $a = 0$
- $b = 0$

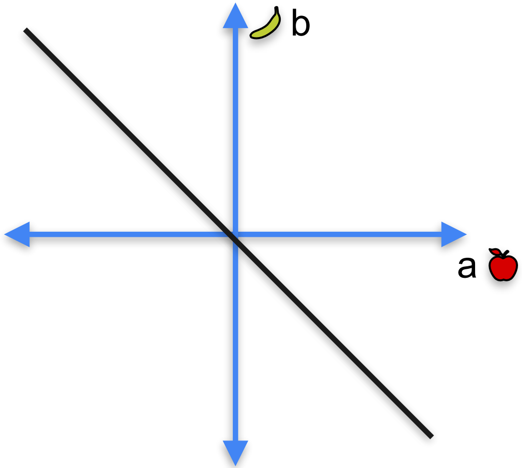
Dimension = 0



🍎	🍌
1	1
2	2

Null space
- any a
- $b = -a$

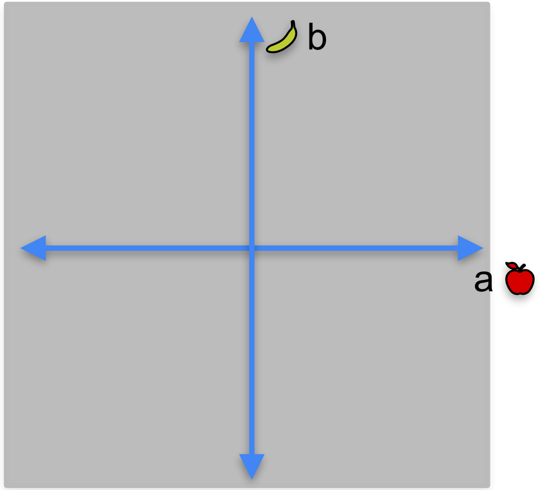
Dimension = 1



🍎	🍌
0	0
0	0

Null space
- any a
- any b

Dimension = 2



The null space of a matrix

🍎	🍌
1	1
1	2

Null space
- $a = 0$
- $b = 0$

Dimension = 0

🍎	🍌
1	1
2	2

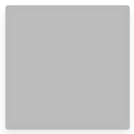
Null space
- any a
- $b = -a$

Dimension = 1

🍎	🍌
0	0
0	0

Null space
- any a
- any b

Dimension = 2



The null space of a matrix

$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$
$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

Null space
- $a = 0$
- $b = 0$

Dimension = 0

Non-singular

$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

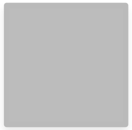
Null space
- any a
- $b = -a$

Dimension = 1

$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Null space
- any a
- any b

Dimension = 2



The null space of a matrix

🍎	🍌
1	1
1	2

Null space

- $a = 0$
- $b = 0$

Dimension = 0

Non-singular

🍎	🍌
1	1
2	2

Null space

- any a
- $b = -a$

Dimension = 1

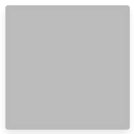
Singular

🍎	🍌
0	0
0	0

Null space

- any a
- any b

Dimension = 2



The null space of a matrix

🍎	🍌
1	1
1	2

Null space
- $a = 0$
- $b = 0$

Dimension = 0

Non-singular

🍎	🍌
1	1
2	2

Null space
- any a
- $b = -a$

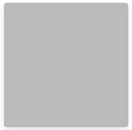
Dimension = 1

Singular

🍎	🍌
0	0
0	0

Null space
- any a
- any b

Dimension = 2



Singular

The null space of a matrix

🍎	🍌
1	1
1	2

Null space
- $a = 0$
- $b = 0$

Dimension = 0

Non-singular

🍎	🍌
1	1
2	2

Null space
- any a
- $b = -a$

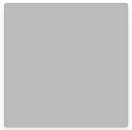
Dimension = 1

Singular

🍎	🍌
0	0
0	0

Null space
- any a
- any b

Dimension = 2



Singular

More conceptual explanation of the null space

- Elaborate here

Quiz: Null space of a matrix

Problem: Determine the dimension of the null space of the following two matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Solutions: Null space of a matrix

Matrix 1: Notice that this is a non-singular matrix, since the determinant is 16. Therefore, the null space is only the point (0,0). The dimension is 0.

5	1
-1	3

Matrix 2: The corresponding system of equation has the equations 2a-b=0 and -6a+3b=0. Some inspection shows that the first equation has the points (1,2), (2,4), (3,6), etc. as solutions. All of them are also solutions to the second equation, -6a+3b=0. Therefore the null space is all the points of the form (x, 2x). The dimension of this null space is 1, and the matrix is singular.

2	-1
-6	3

Systems of linear equations

Systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

1	1	1
1	2	1
1	1	2

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Systems of linear equations

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

1	1	1
1	2	1
1	1	2

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

1	1	1
1	1	2
1	1	3

System 3

- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

System 4

- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

1	1	1
1	1	2
1	1	3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

1	1	1
2	2	2
3	3	3

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

1	1	1
1	2	1
1	1	2

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

1	1	1
1	1	2
1	1	3

System 3

- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

1	1	1
2	2	2
3	3	3

System 4

- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**

0	0	0
0	0	0
0	0	0

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Null space for systems of linear equations

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

System 3

- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

System 4

- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**

Solution space

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

Solution space

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space

System 3

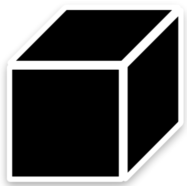
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

Solution space

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

Solution space

Dimension = 0

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

Solution space

System 3

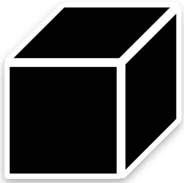
- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

Solution space

System 4

- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**

Solution space



Null space for systems of linear equations

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

Solution space

Dimension = 0

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

Solution space

Dimension = 1

System 3

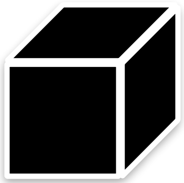
- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

Solution space

System 4

- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**
- 0a + 0b + 0c = **0**

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space

Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space

Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

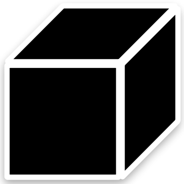
Solution space

Dimension = 2

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space

Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space

Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

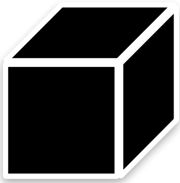
Solution space

Dimension = 2

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Dimension = 3

Null space for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Null space

Dimension = 0

Matrix 2

1	1	1
1	1	2
1	1	3

Null space

Dimension = 1

Matrix 3

1	1	1
2	2	2
3	3	3

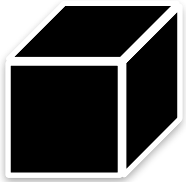
Null space

Dimension = 2

Matrix 4

0	0	0
0	0	0
0	0	0

Null space



Dimension = 3

Quiz: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

1	1	1
1	1	2
0	0	-1

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

1	1	1
0	2	2
0	0	3

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = **0**
- b = **0**
- 3a + 2b + 3c = **0**

All points of the form

$(x, 0, -x)$

1	1	1
1	1	2
0	0	-1

- a + b + c = **0**
- a + b + 2c = **0**
- c = **0**

1	1	1
0	2	2
0	0	3

- a + b + c = **0**
- 2b + 2c = **0**
- 3c = **0**

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = **0**
- b = **0**
- 3a + 2b + 3c = **0**

All points of the form

$(x, 0, -x)$

Dimension = 1

1	1	1
1	1	2
0	0	-1

- a + b + c = **0**
- a + b + 2c = **0**
- c = **0**

1	1	1
0	2	2
0	0	3

- a + b + c = **0**
- 2b + 2c = **0**
- 3c = **0**

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = **0**
- b = **0**
- 3a + 2b + 3c = **0**

All points of the form
$(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- a + b + c = **0**
- a + b + 2c = **0**
- c = **0**

All points of the form
$(x, -x, 0)$

1	1	1
0	2	2
0	0	3

- a + b + c = **0**
- 2b + 2c = **0**
- 3c = **0**

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = **0**
- b = **0**
- 3a + 2b + 3c = **0**

All points of the form
$(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- a + b + c = **0**
- a + b + 2c = **0**
- c = **0**

All points of the form
$(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

- a + b + c = **0**
- 2b + 2c = **0**
- 3c = **0**

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = **0**
- b = **0**
- 3a + 2b + 3c = **0**

All points of the form
$(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- a + b + c = **0**
- a + b + 2c = **0**
- c = **0**

All points of the form
$(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

- a + b + c = **0**
- 2b + 2c = **0**
- 3c = **0**

The point
$(0,0,0)$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- a + c = **0**
- b = **0**
- 3a + 2b + 3c = **0**

All points of the form
$(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- a + b + c = **0**
- a + b + 2c = **0**
- c = **0**

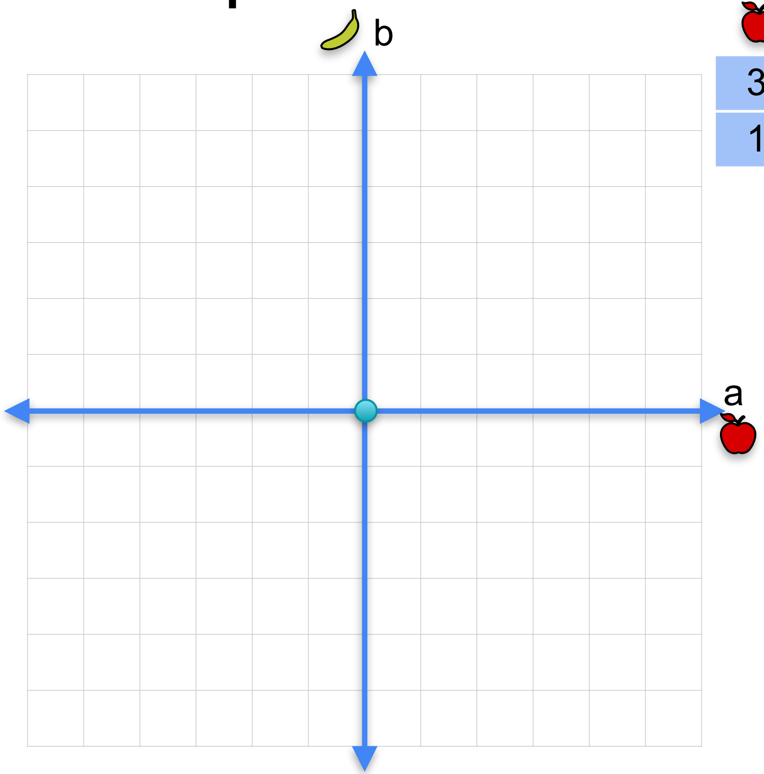
All points of the form
$(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

- a + b + c = **0**
- 2b + 2c = **0**
- 3c = **0**

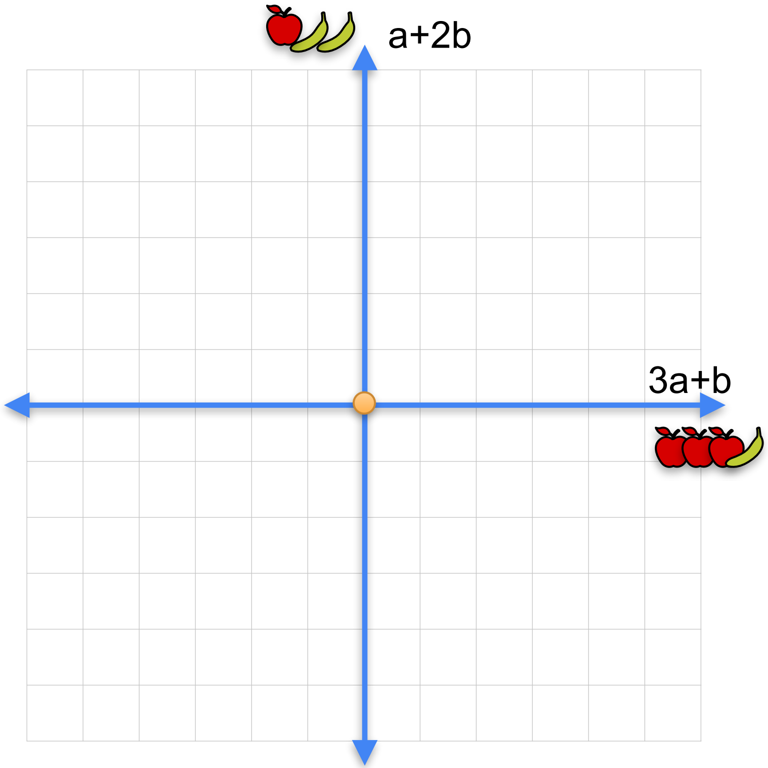
The point
$(0, 0, 0)$
Dimension = 0

Null space

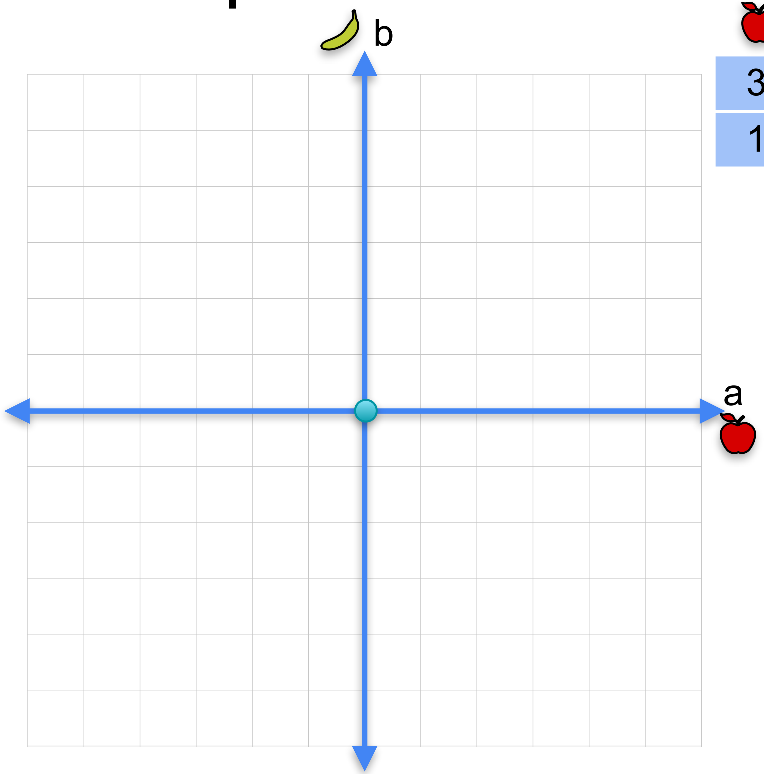


3	1
1	2

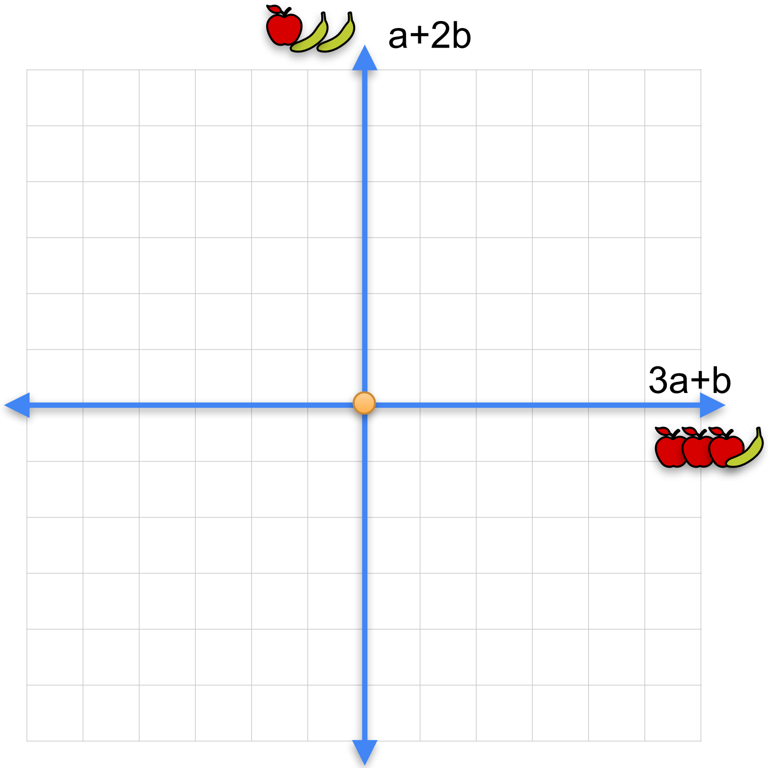
=



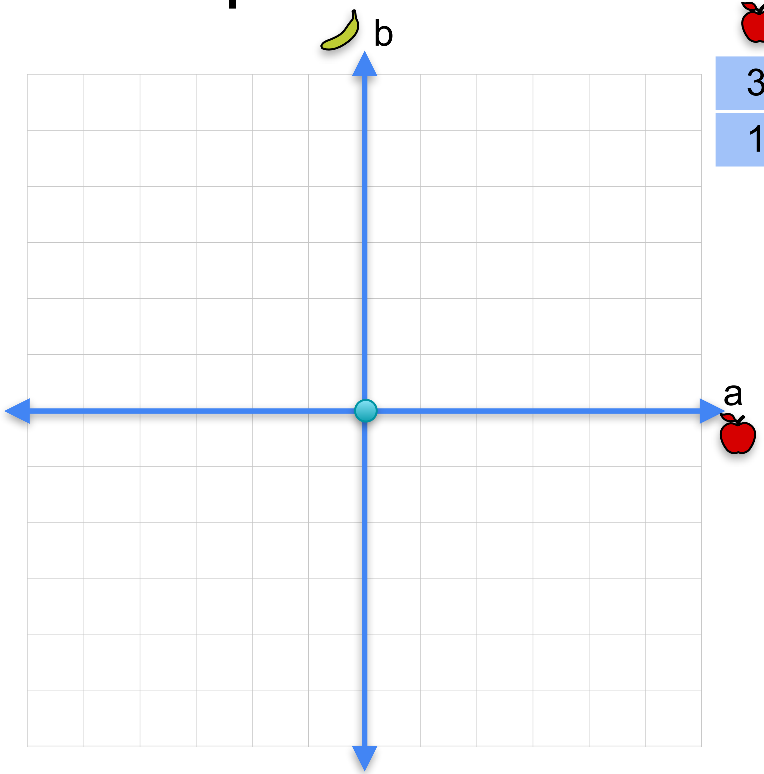
Null space



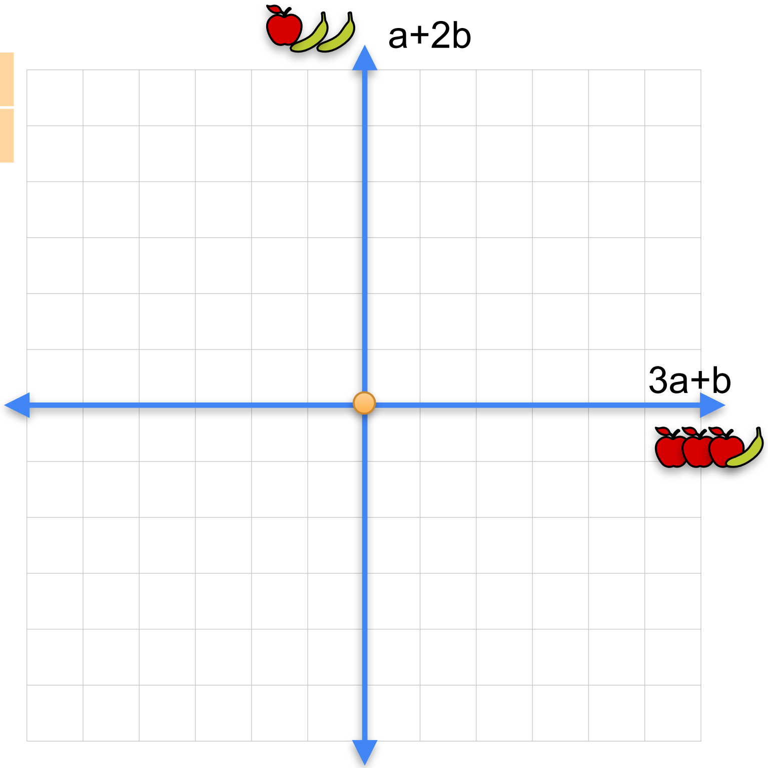
$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} =$$



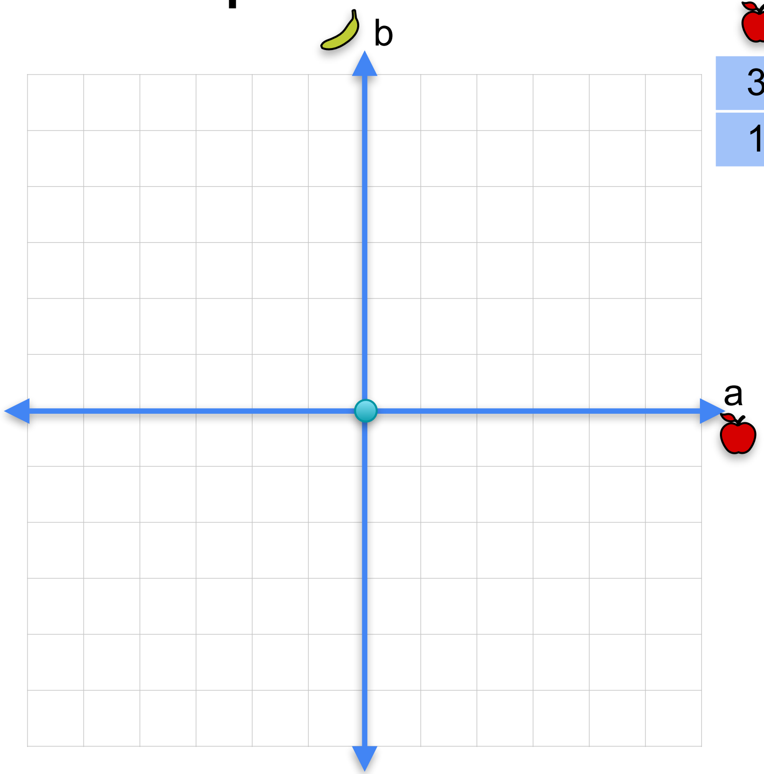
Null space



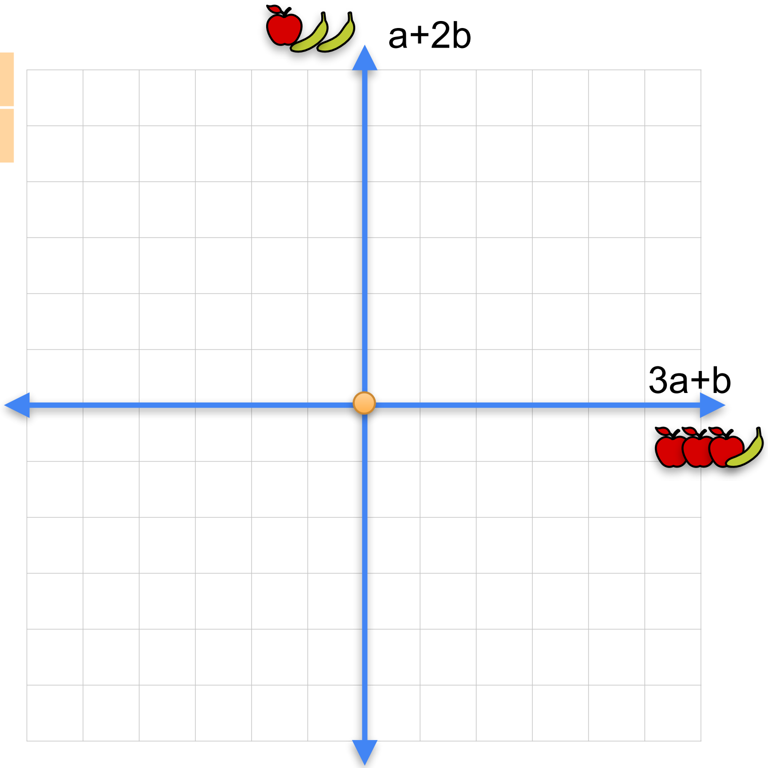
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



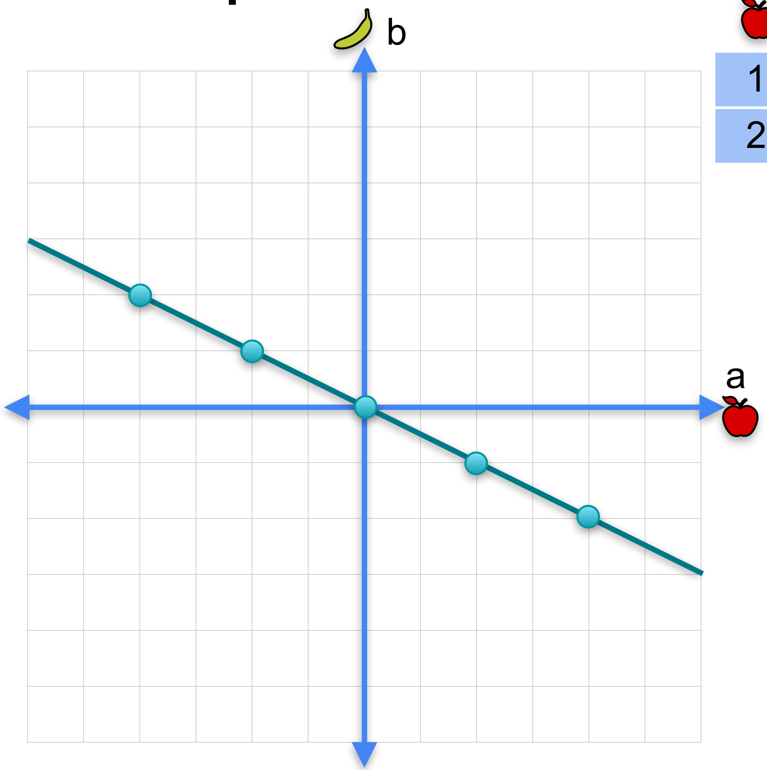
Null space



$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

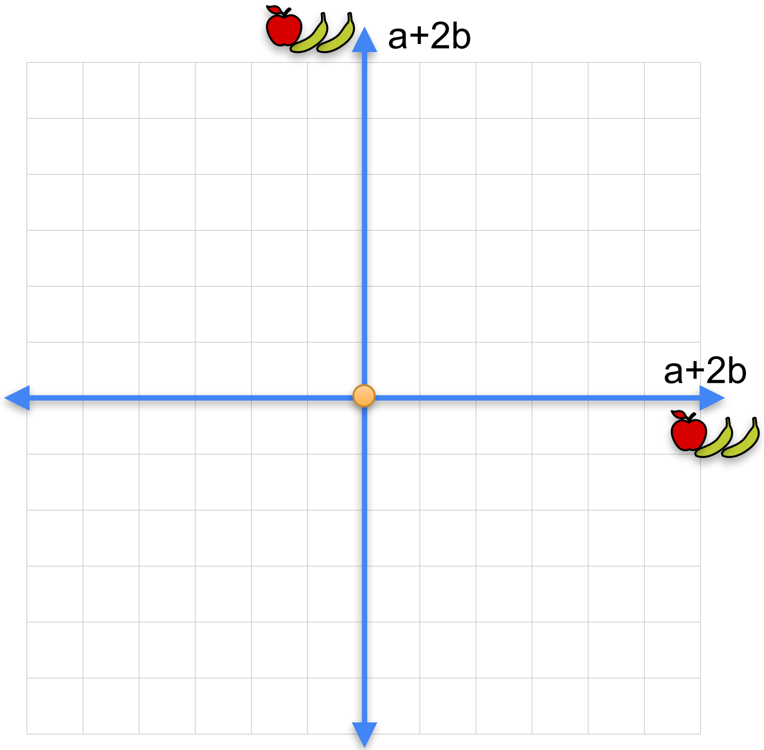


Null space

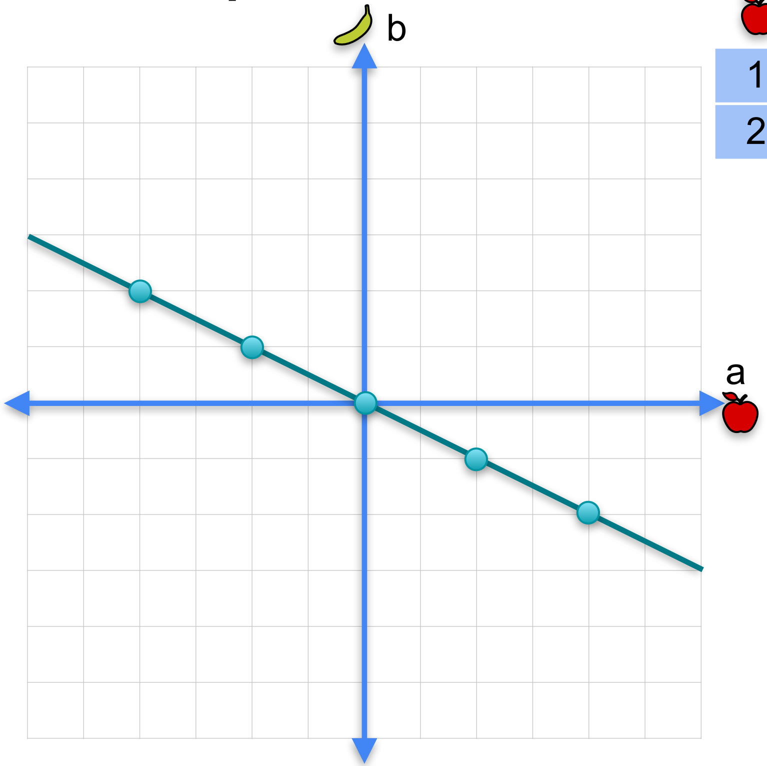


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

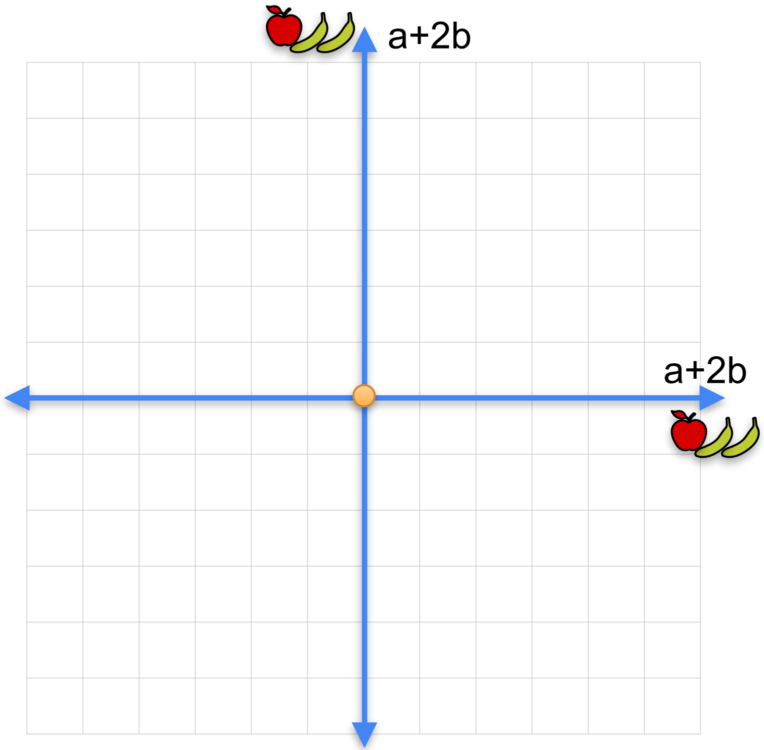
=



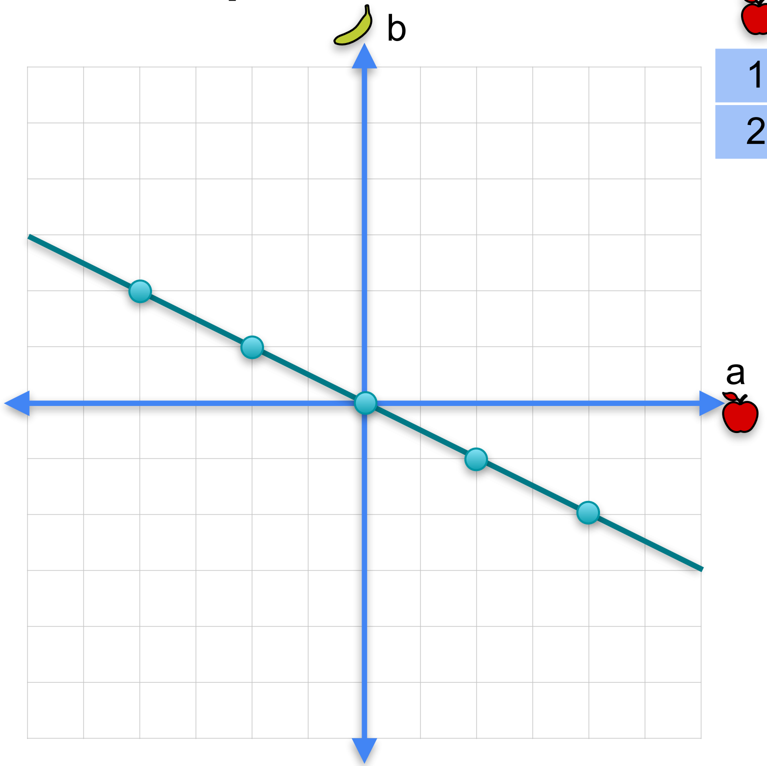
Null space



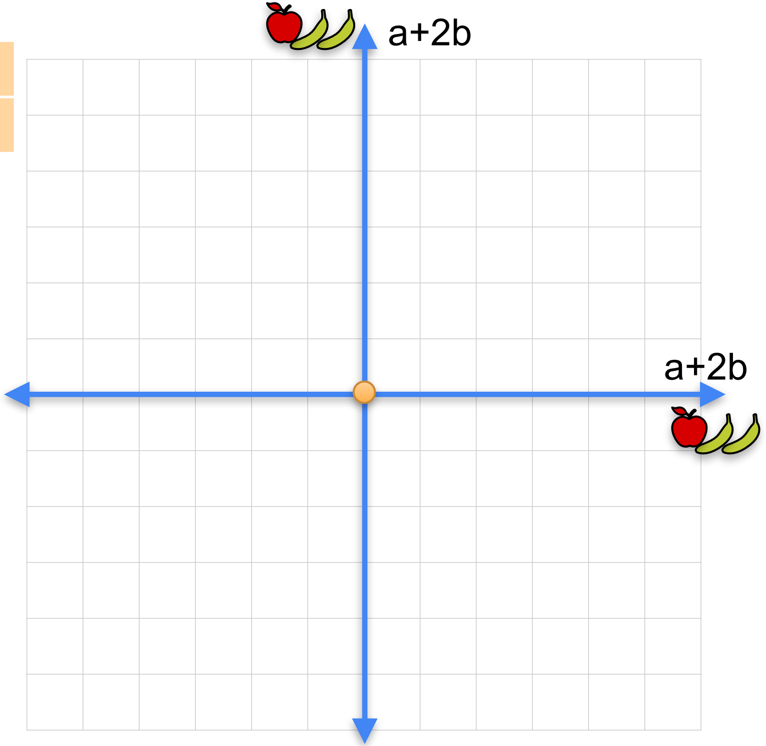
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} =$$



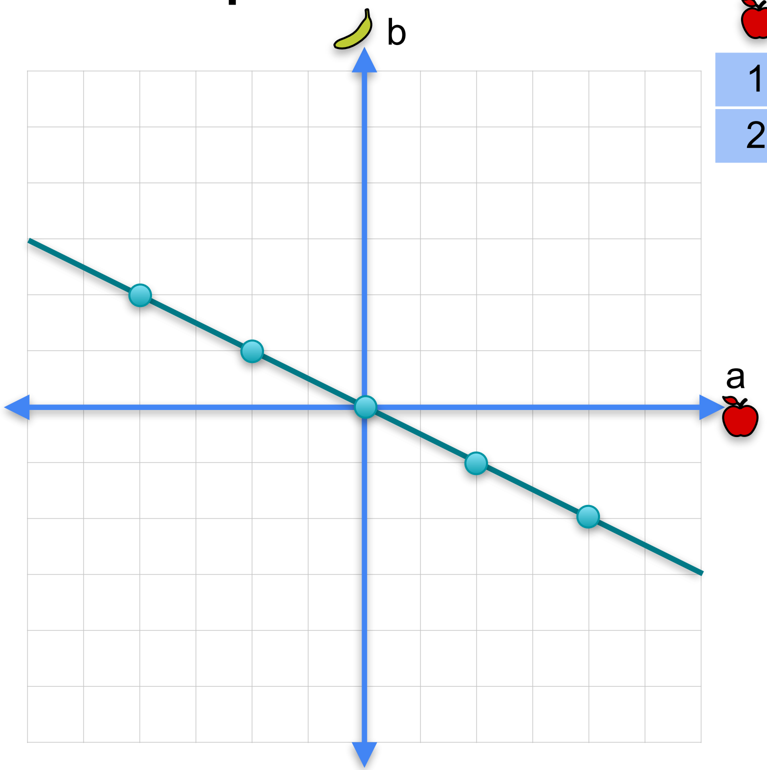
Null space



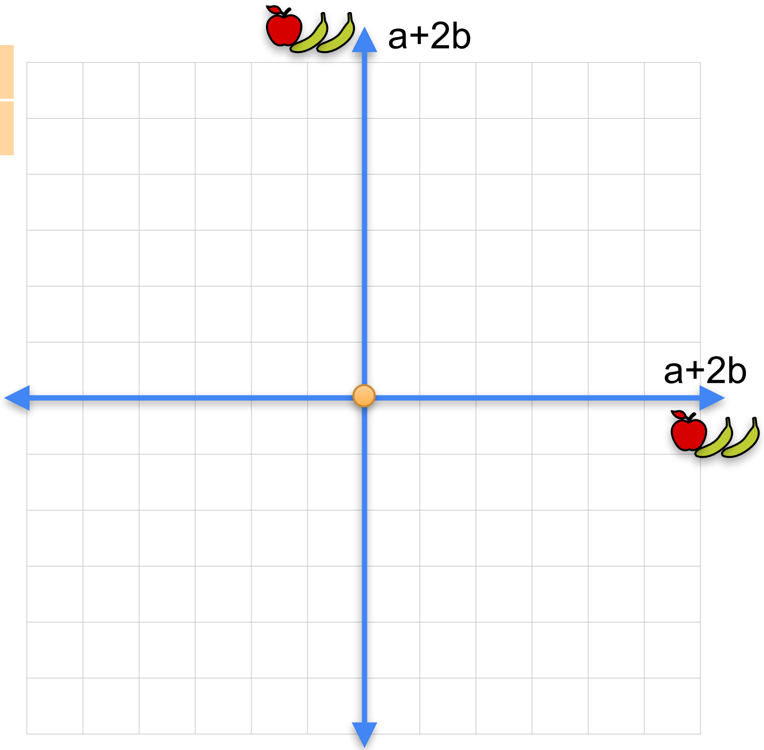
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



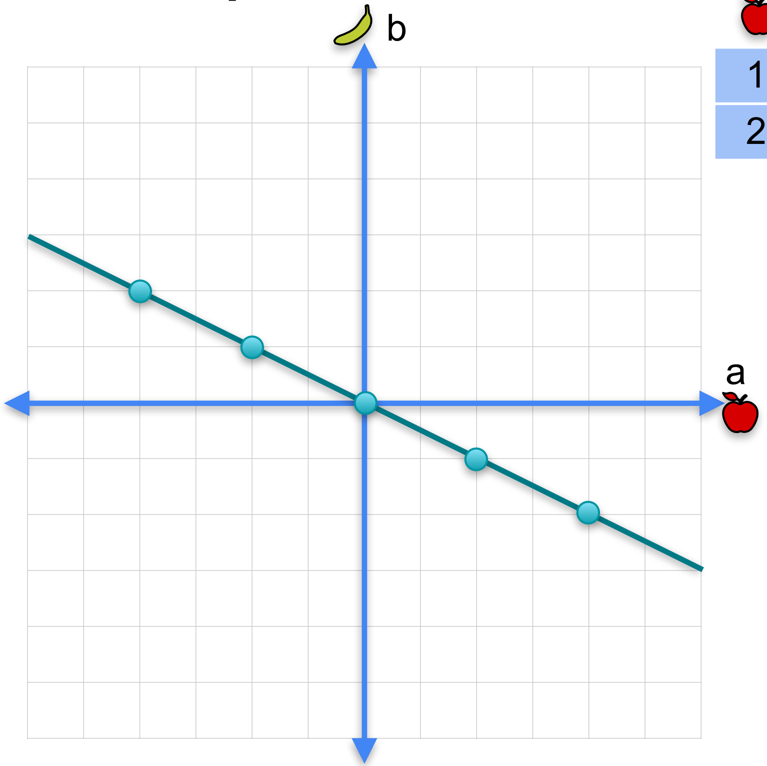
Null space



$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

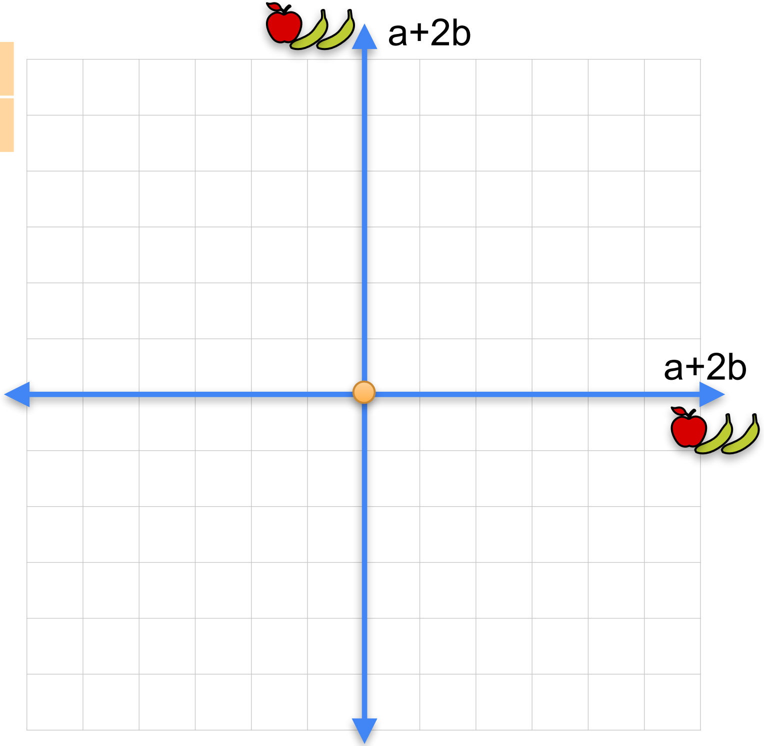


Null space

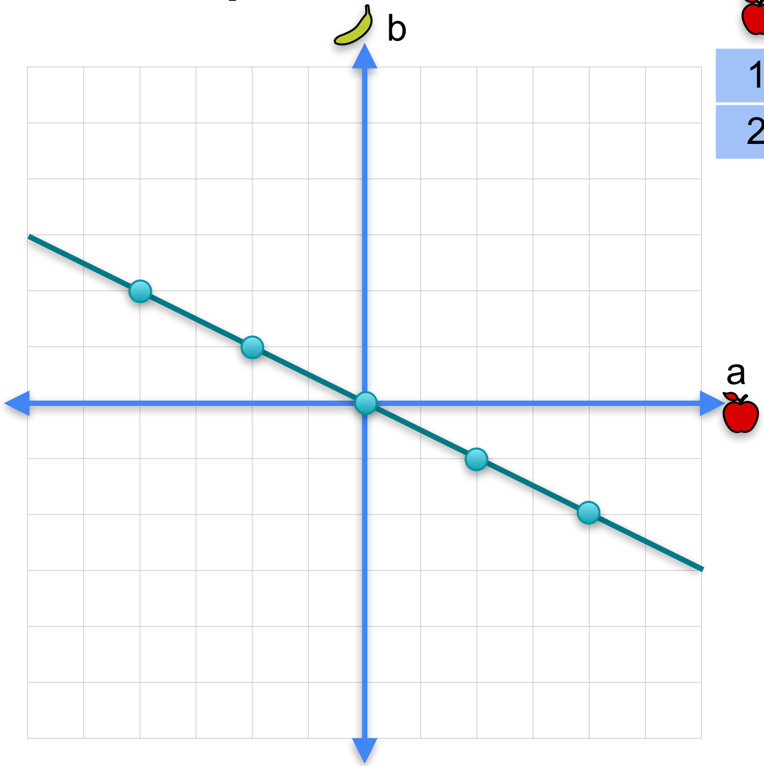


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

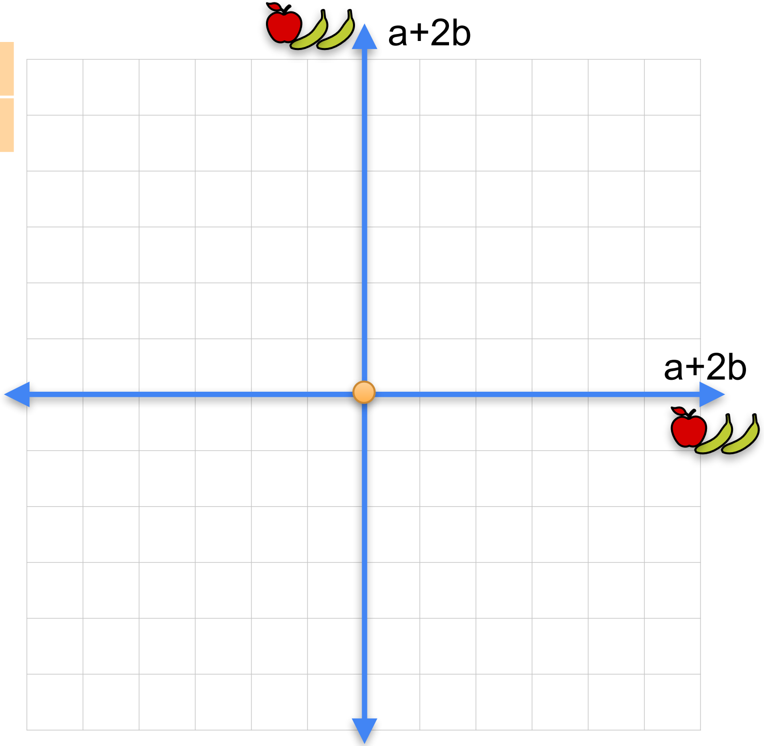


Null space

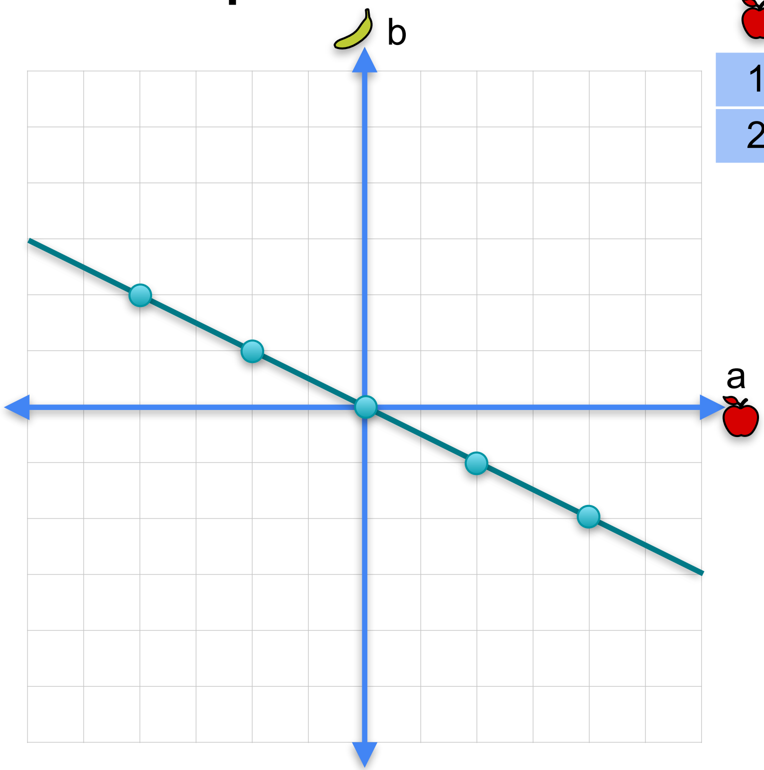


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$



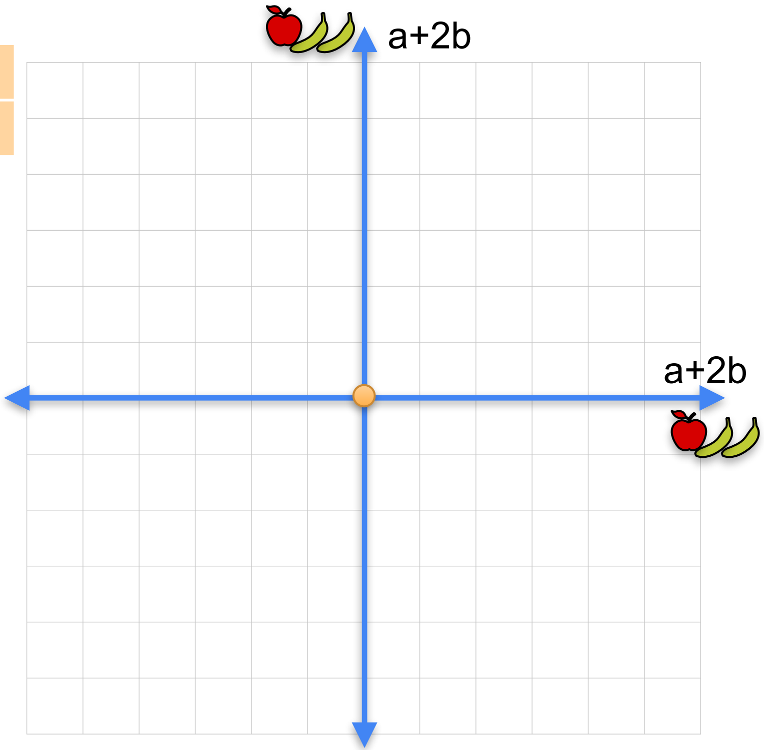
Null space



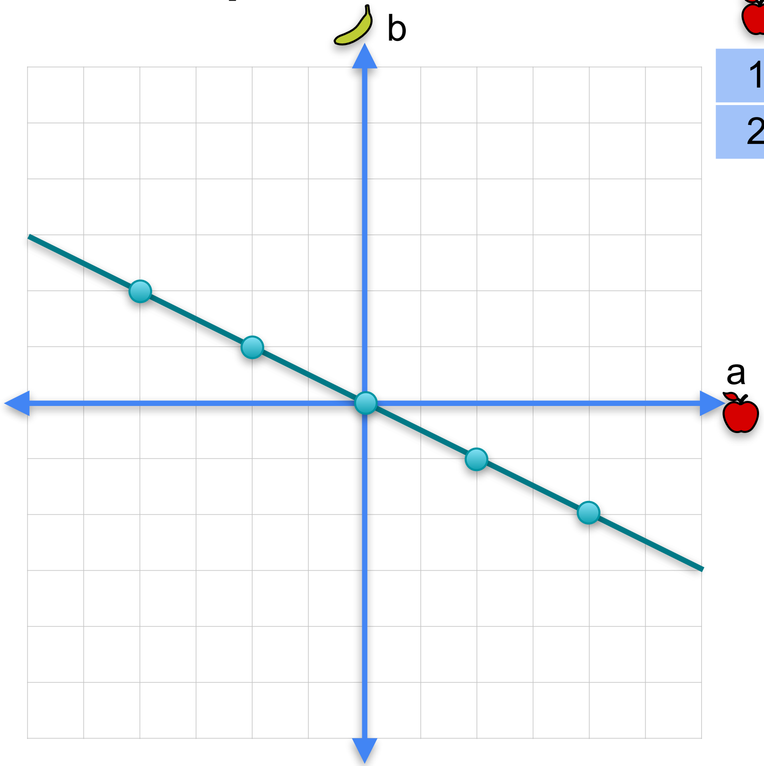
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$



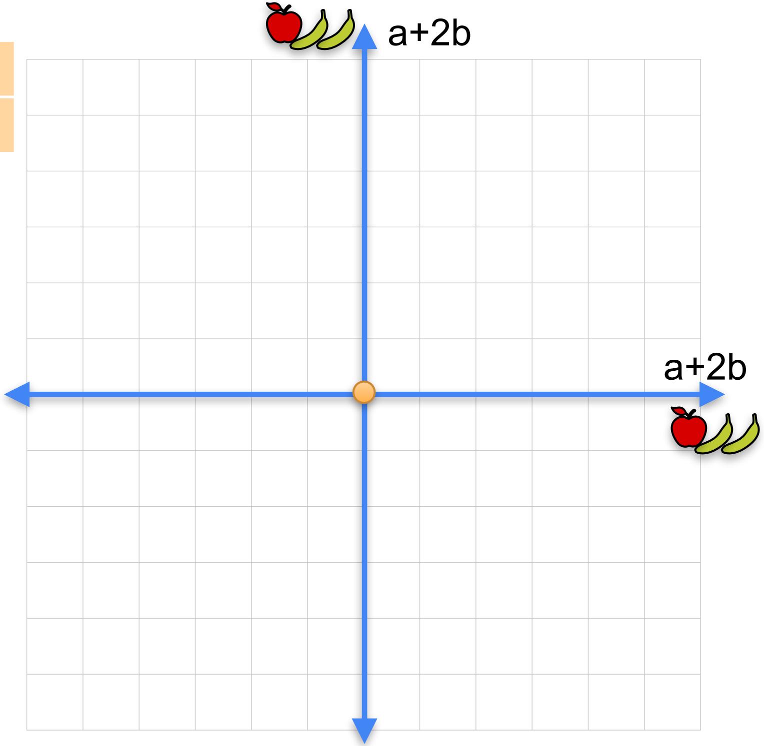
Null space



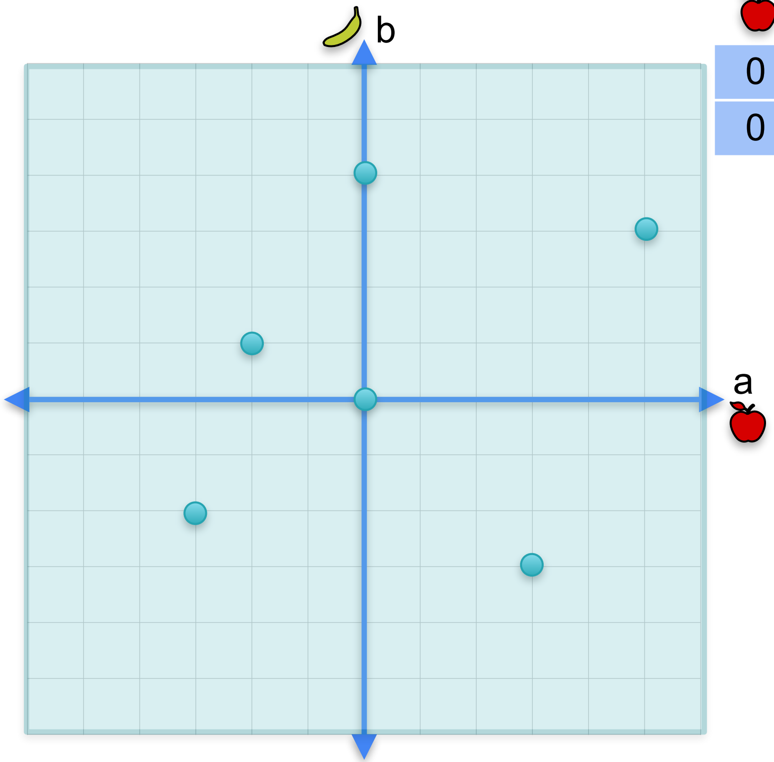
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

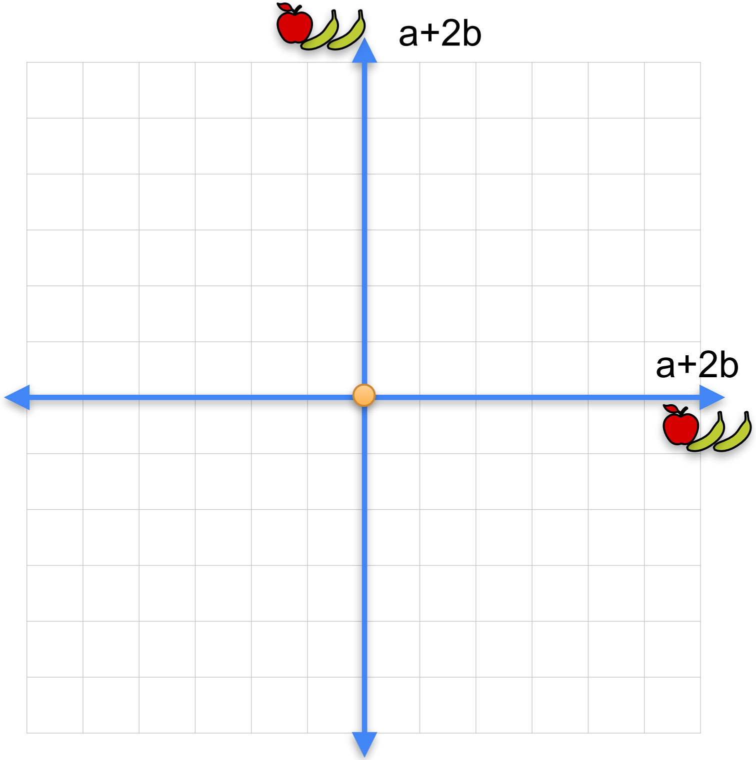


Null space

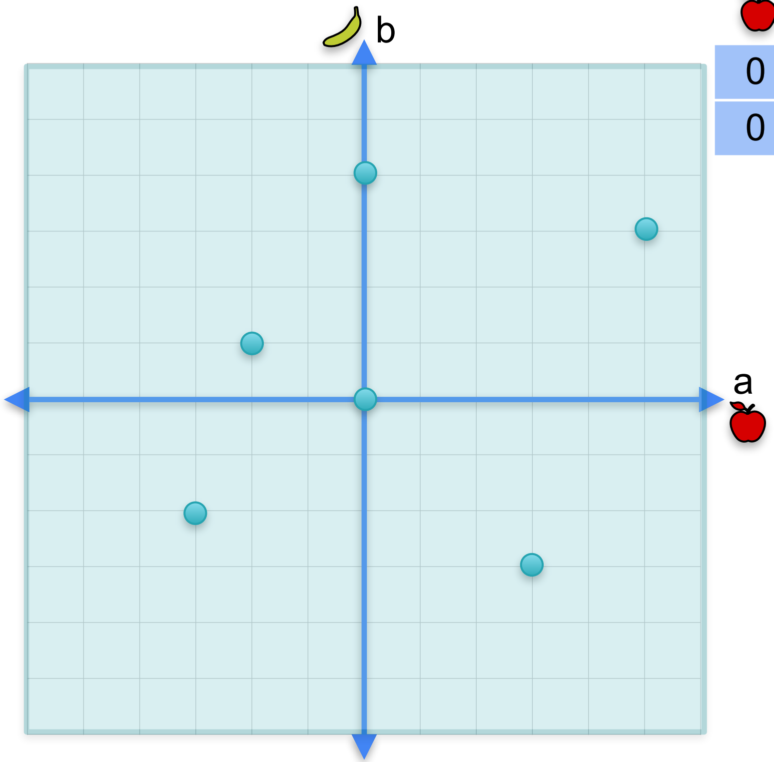


$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

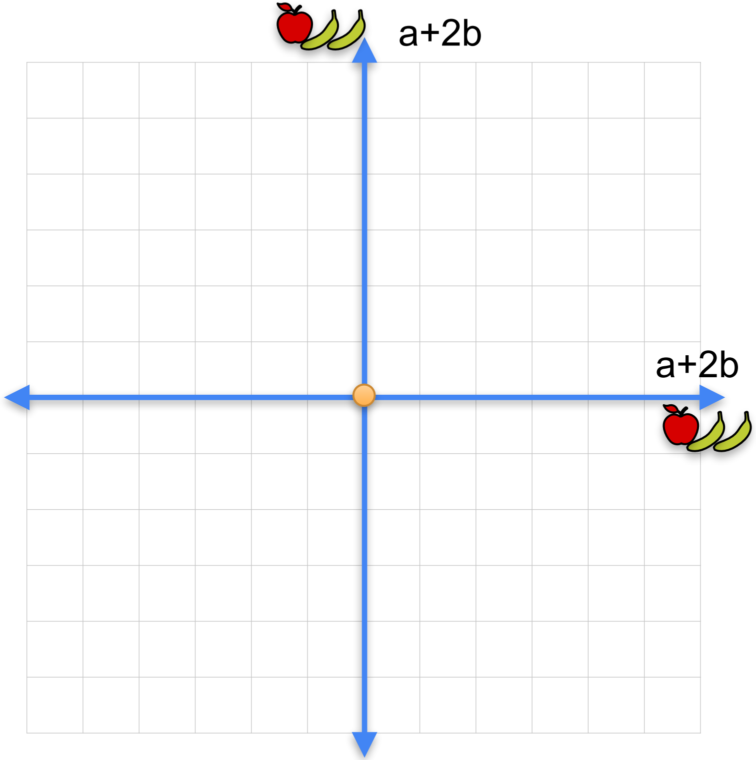
=



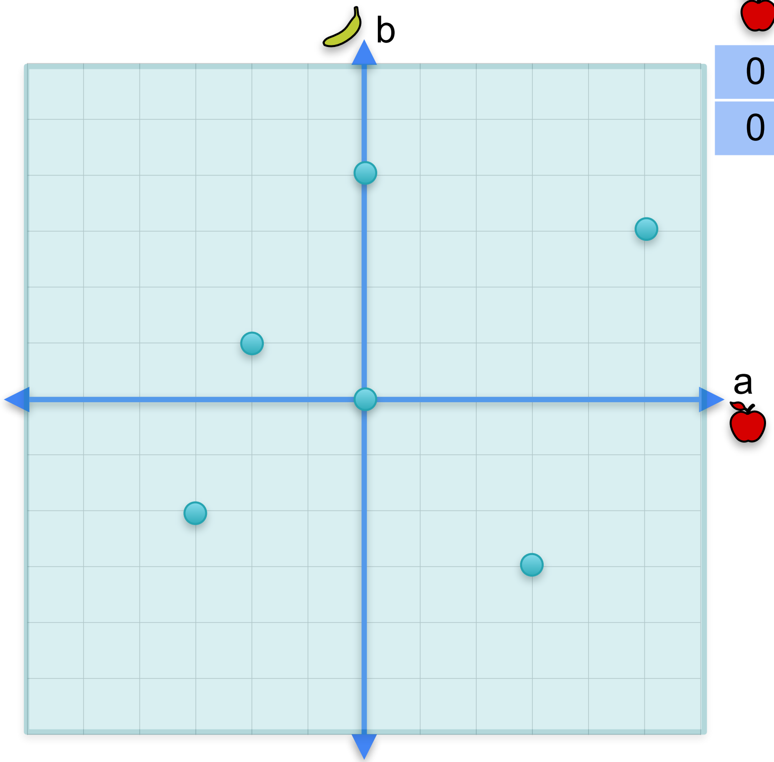
Null space



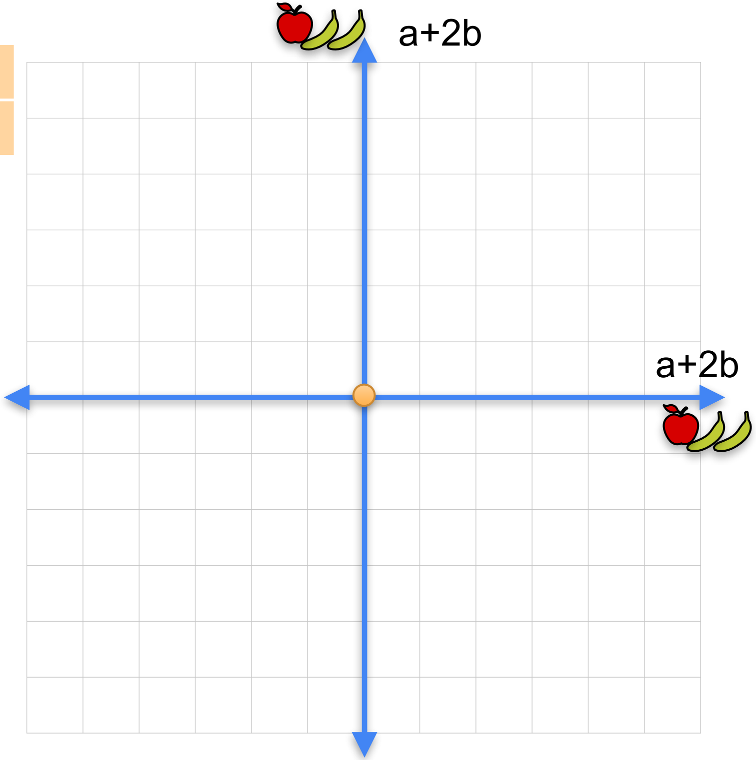
$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} =$$



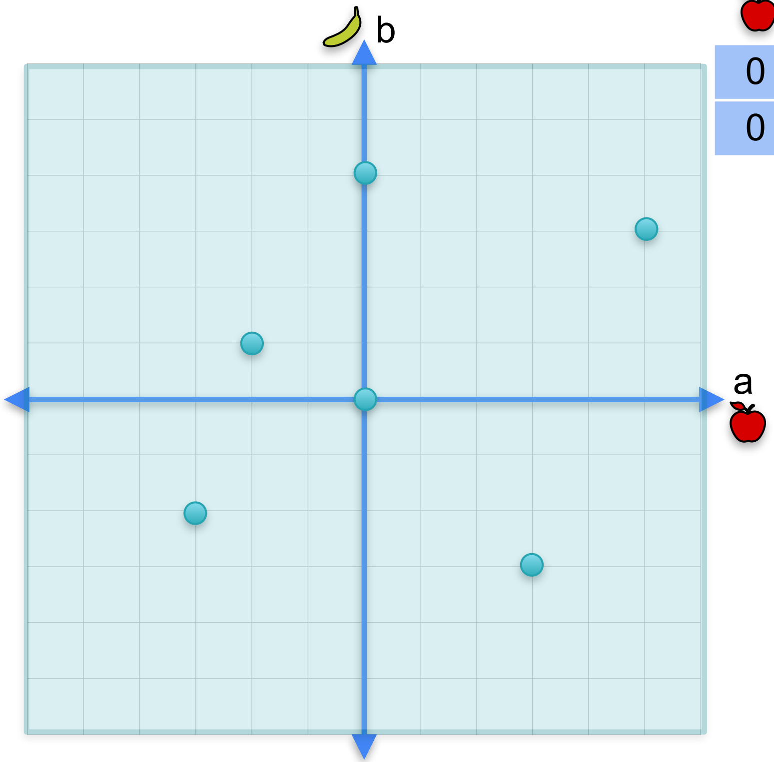
Null space



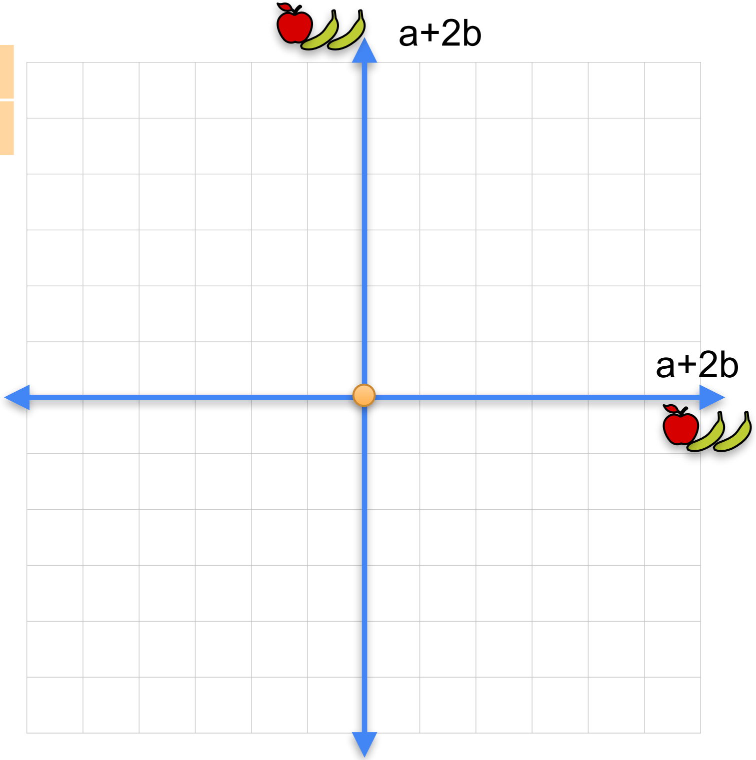
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$



Null space



$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$



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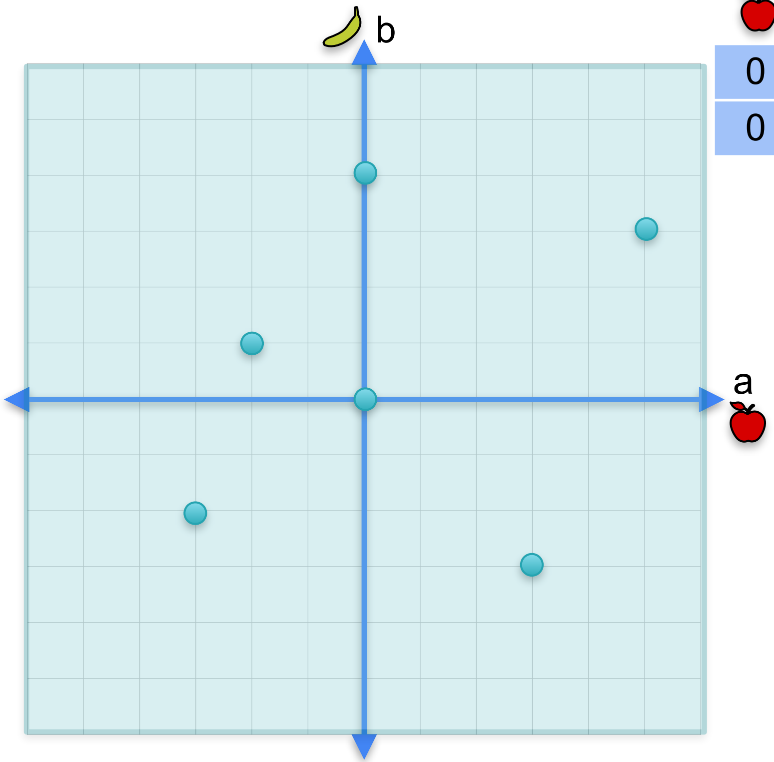
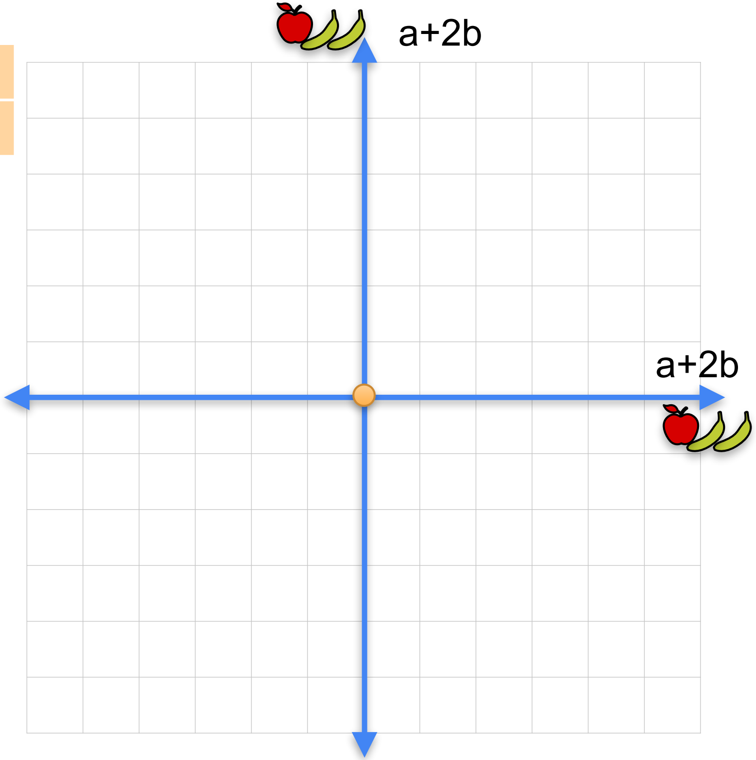
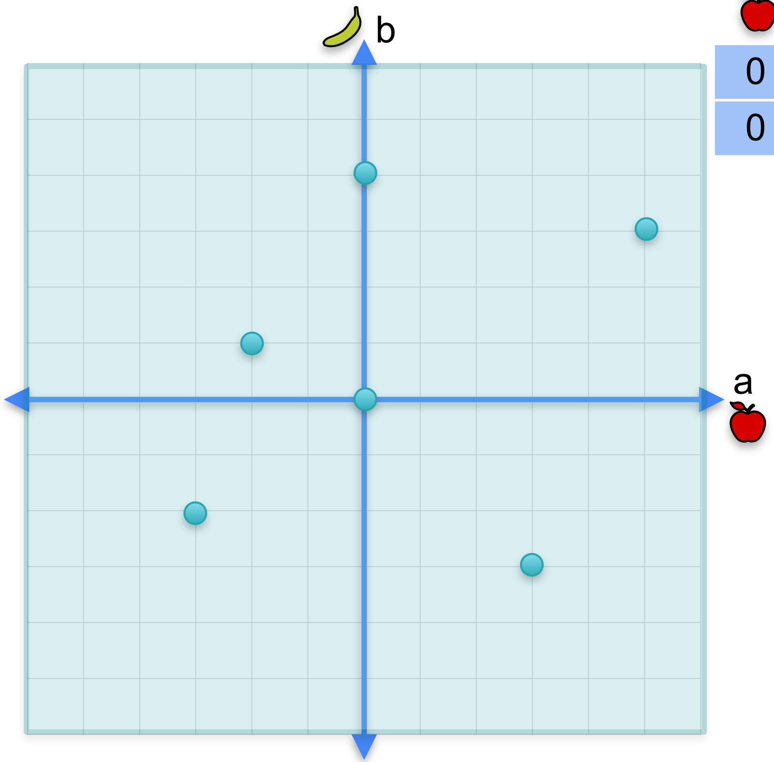


$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

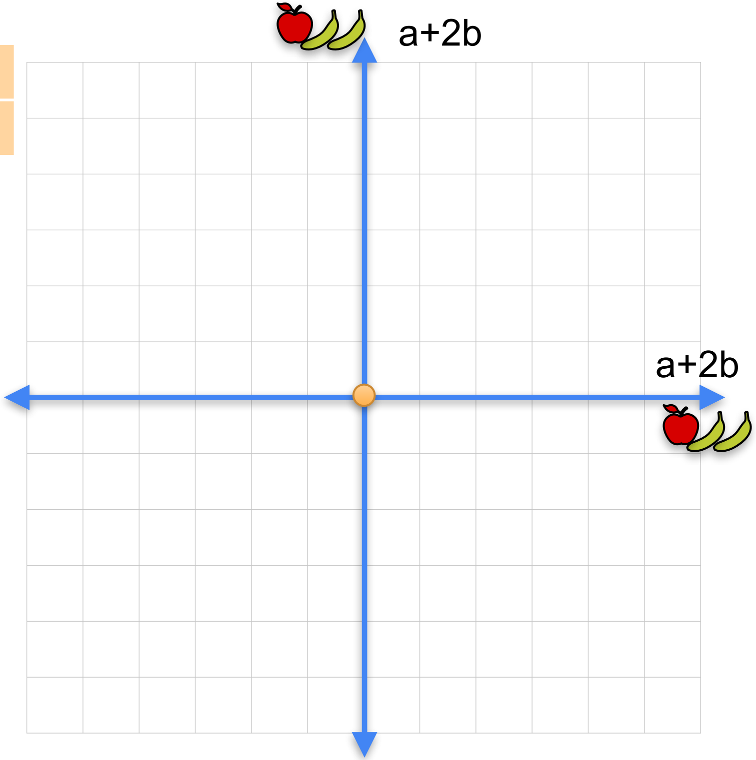


Null space



$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$



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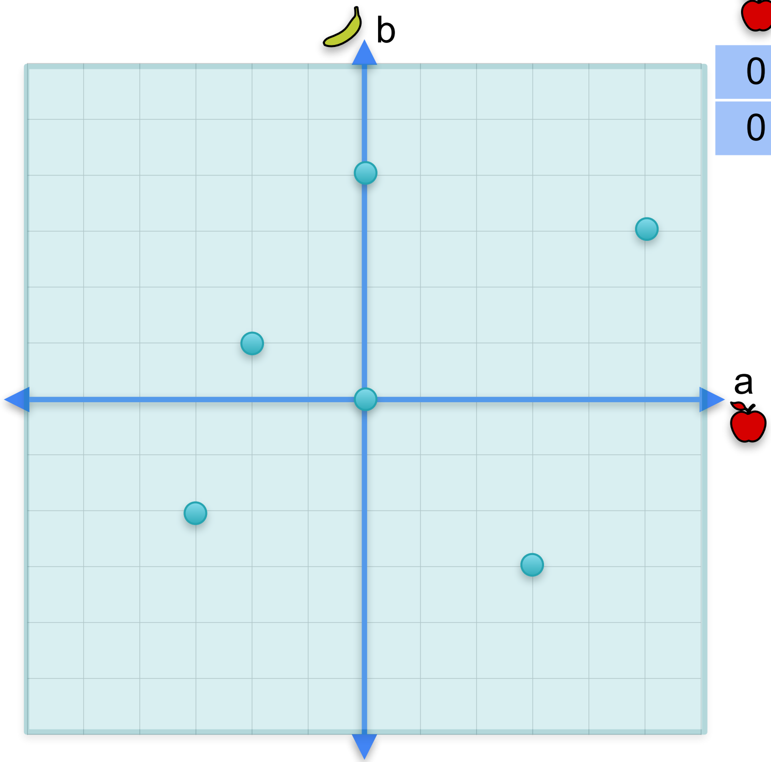
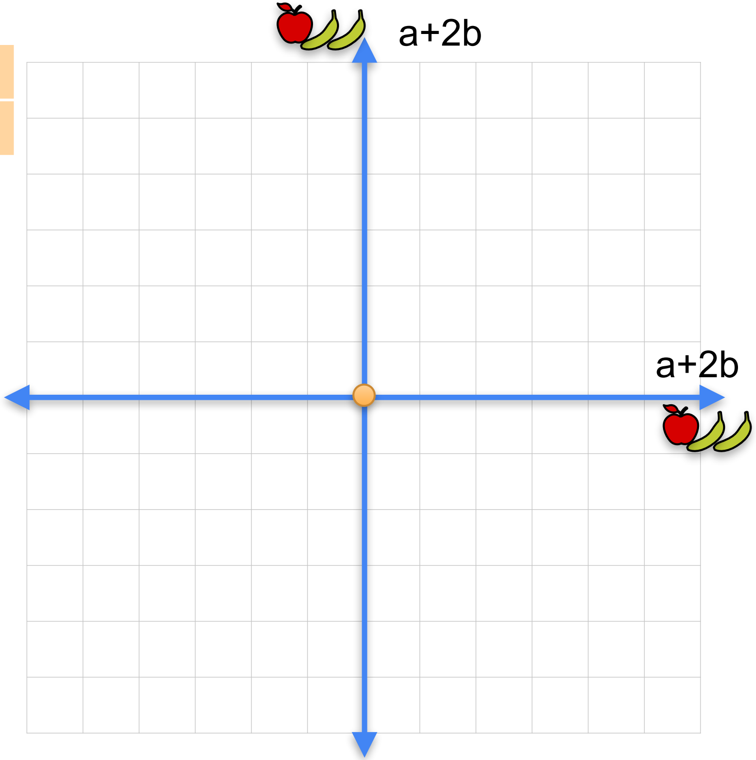


0	0
0	0

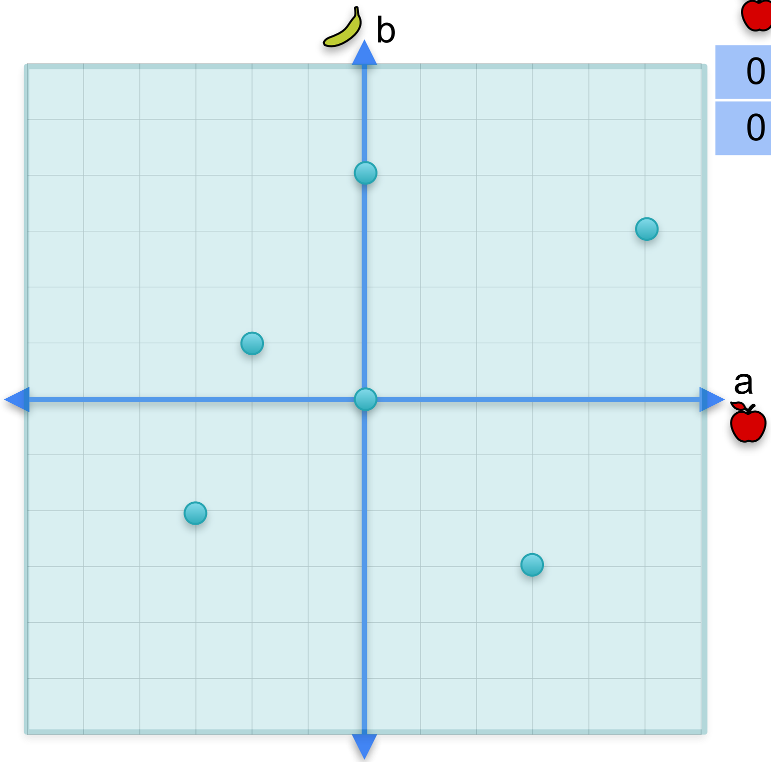
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$

$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 4 \end{bmatrix}$



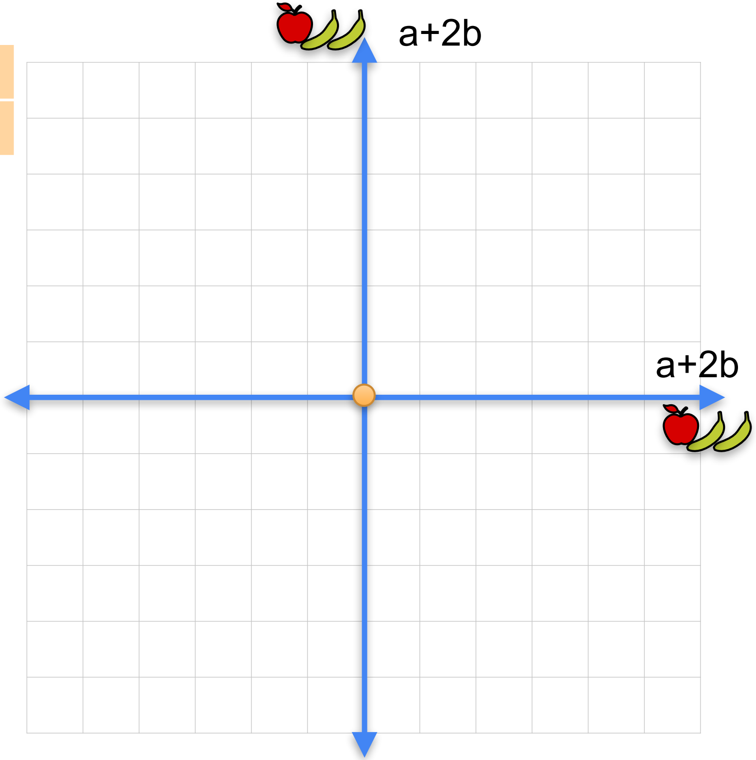
Null space



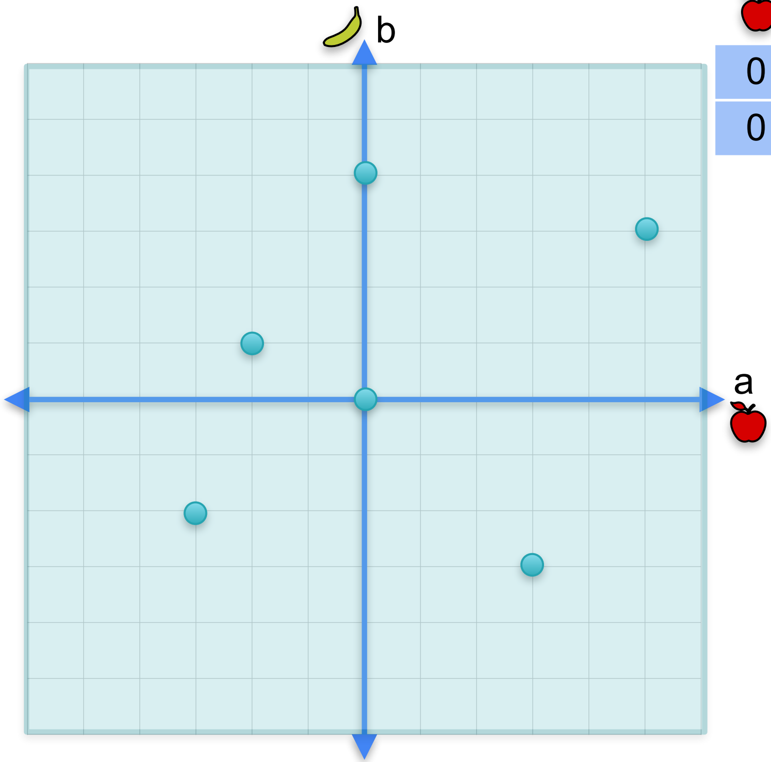
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 4 \end{bmatrix} \quad \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$



Null space



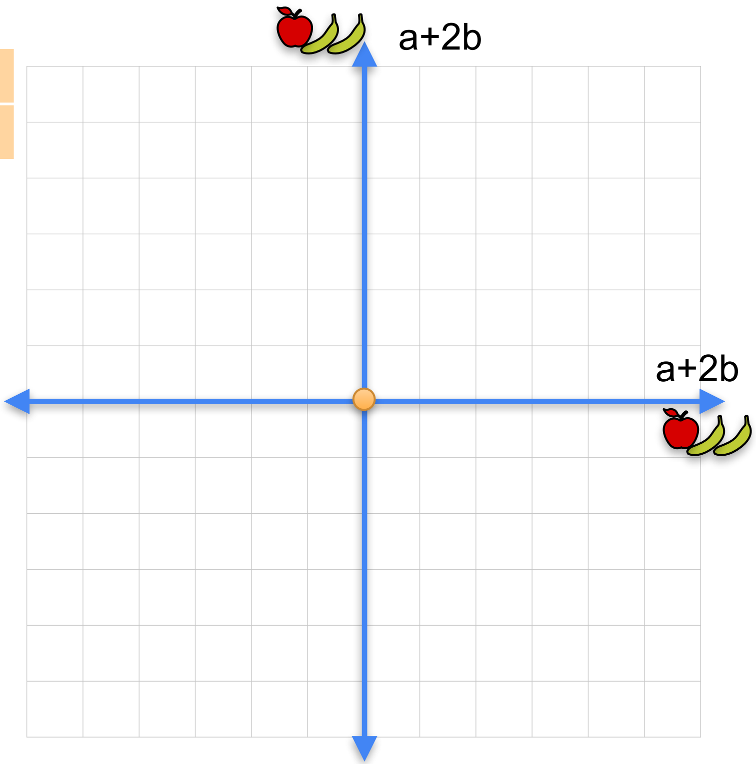
$$\begin{array}{cc} 🍎 & 🍌 \end{array}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 4 \end{bmatrix} \quad \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

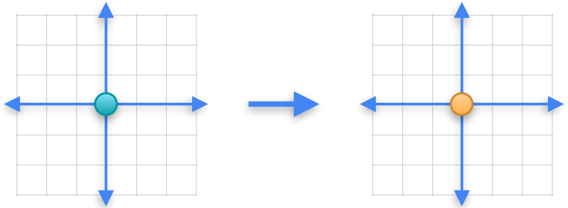


Null space

Non-singular

3	1
1	2

Rank = 2

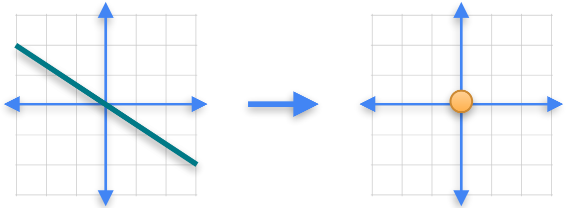


Dimension = 0

Singular

1	1
2	2

Rank = 1

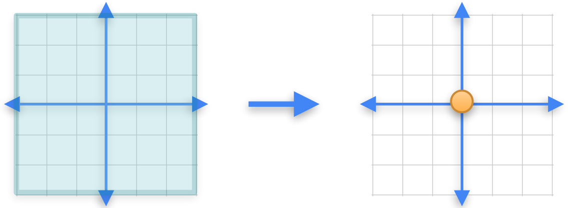


Dimension = 1

Singular

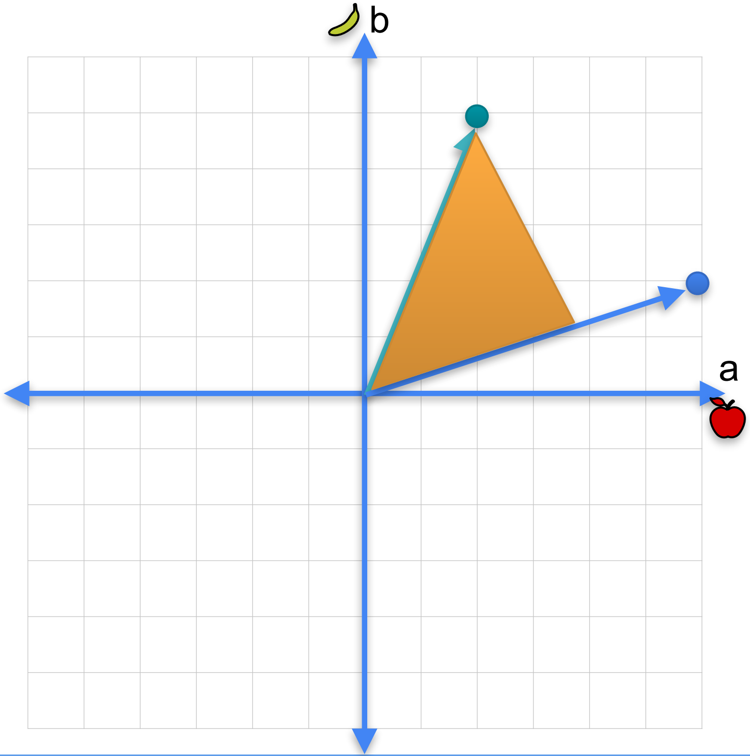
0	0
0	0

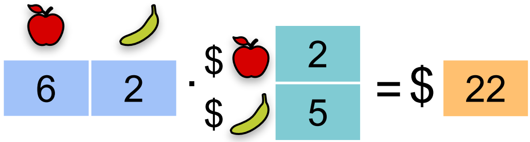
Rank = 0



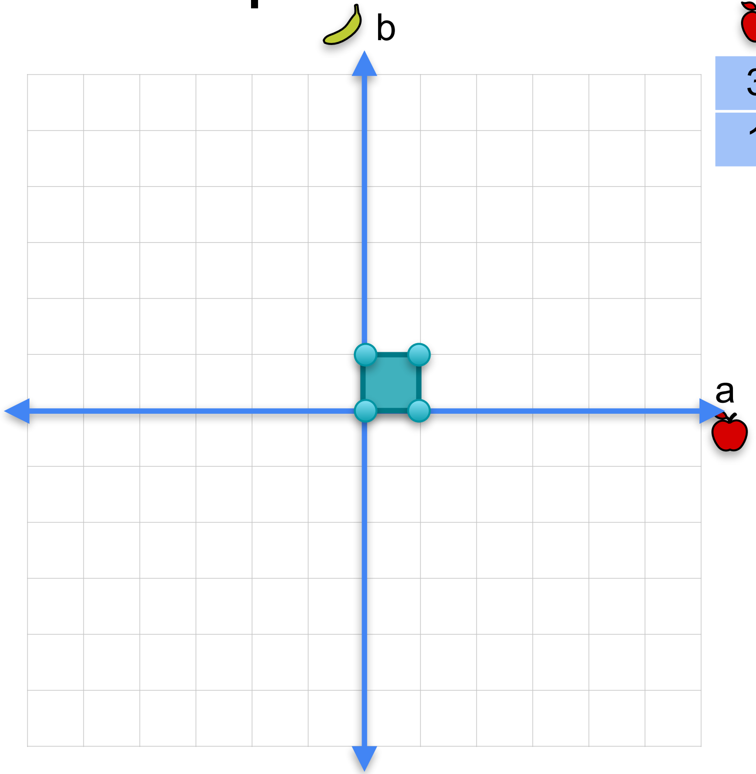
Dimension = 2

Dot product as an area



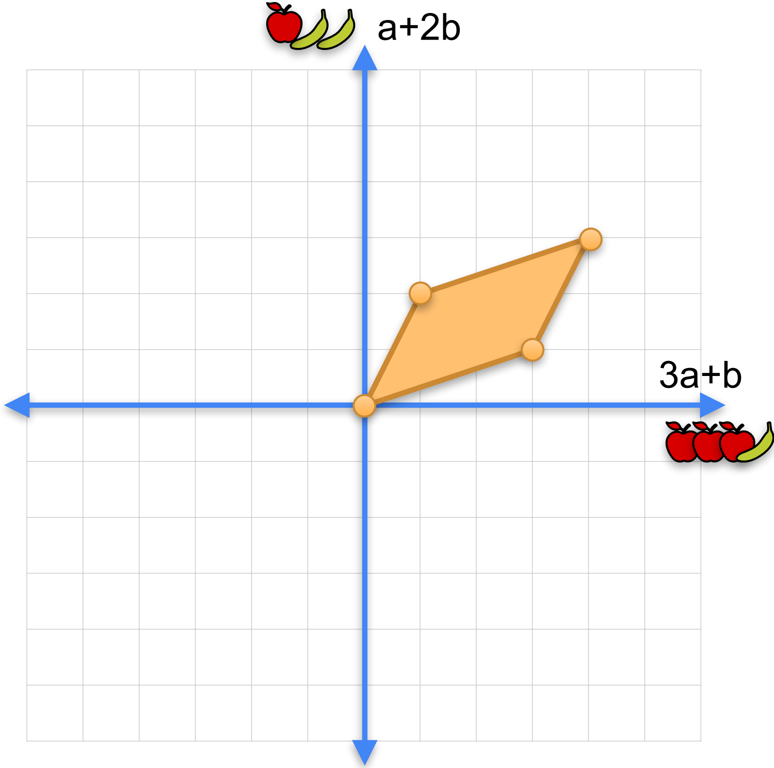


Row space



3	1
1	2

=



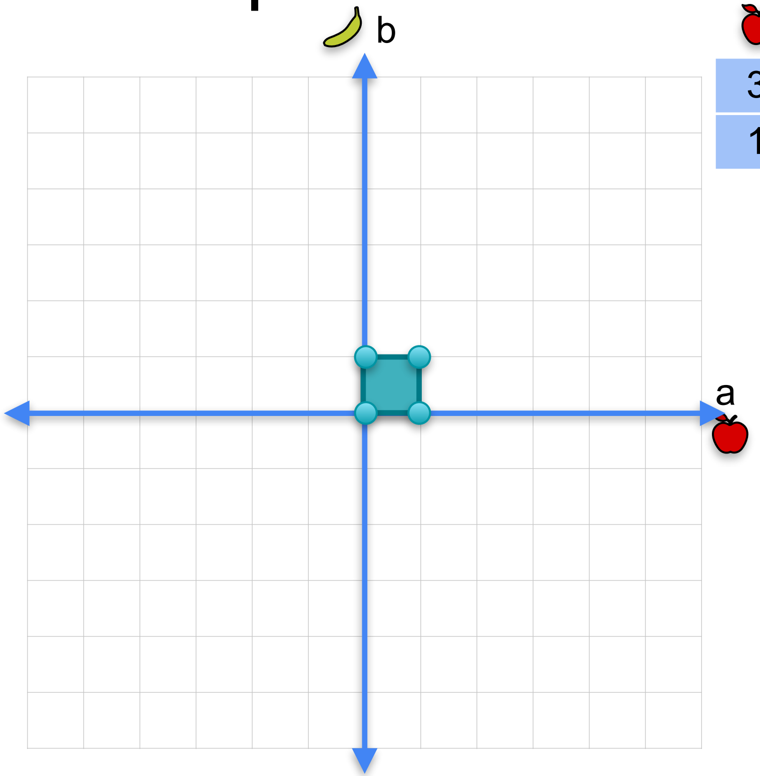
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)

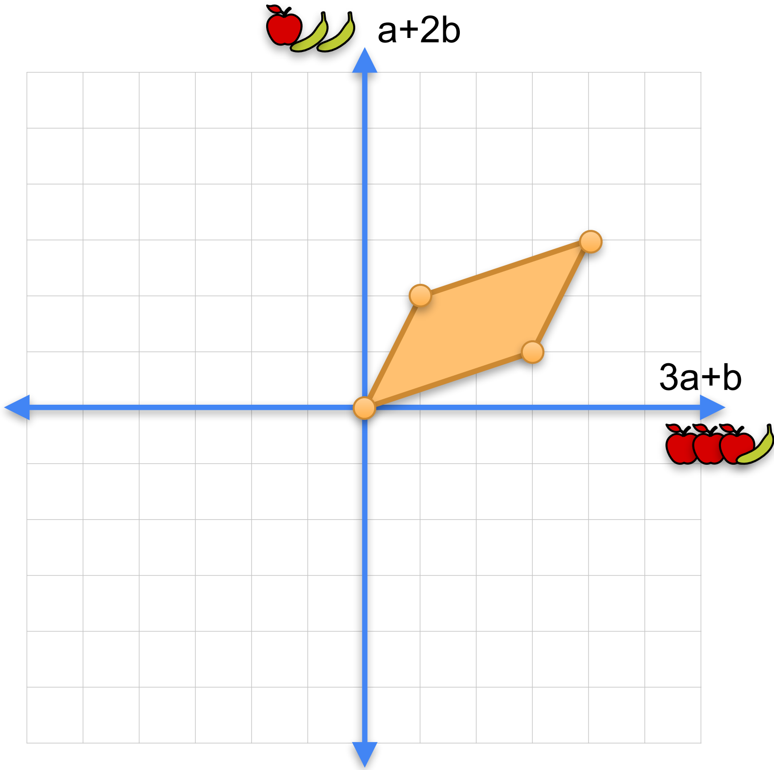
(1,1) → (4,3)

Row space

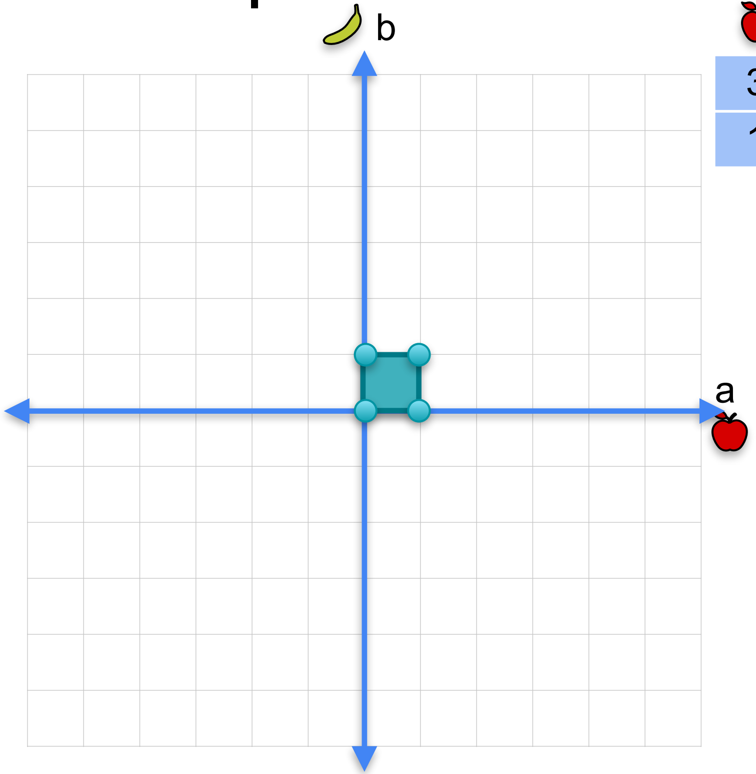


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} =$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)

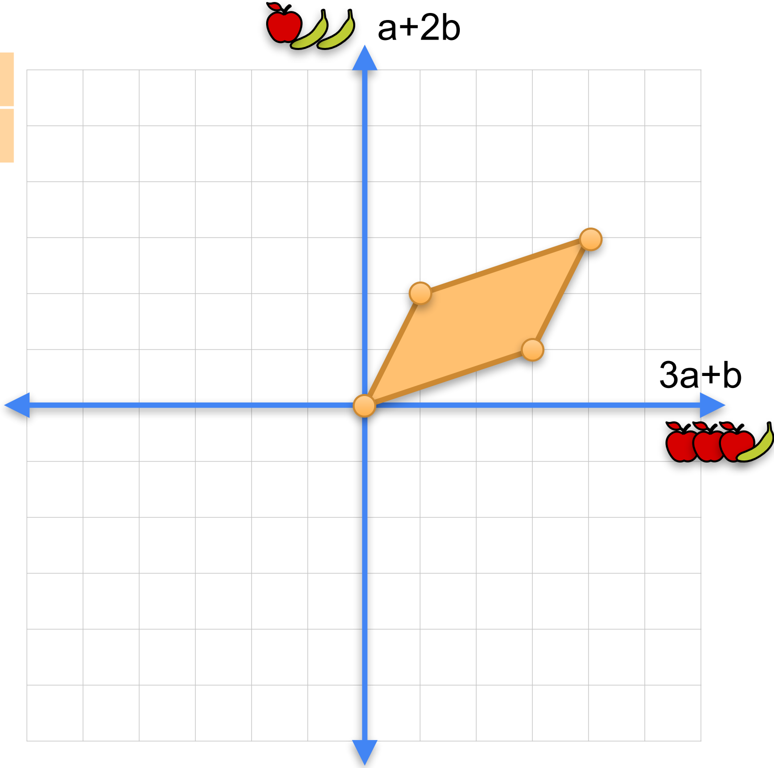


Row space

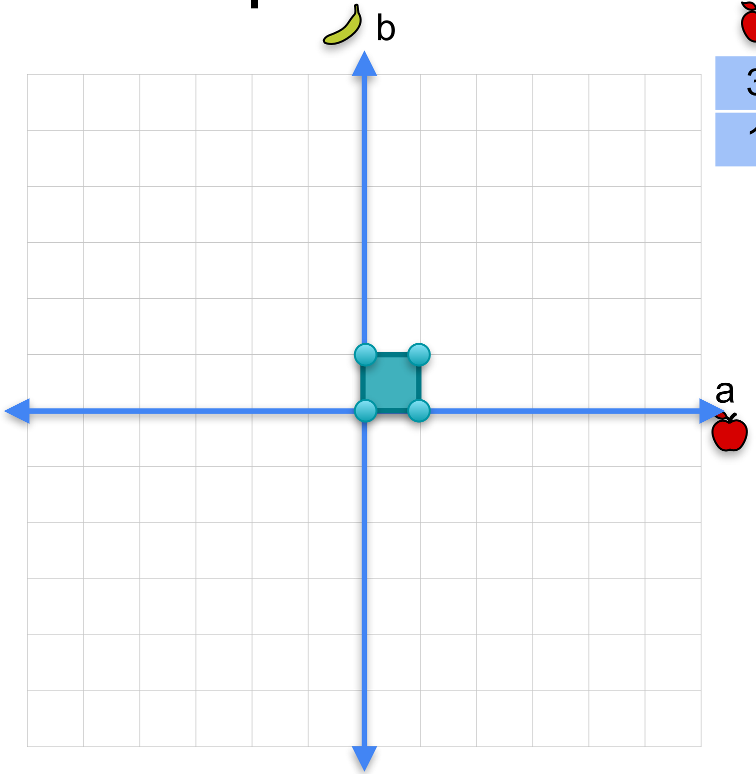


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)

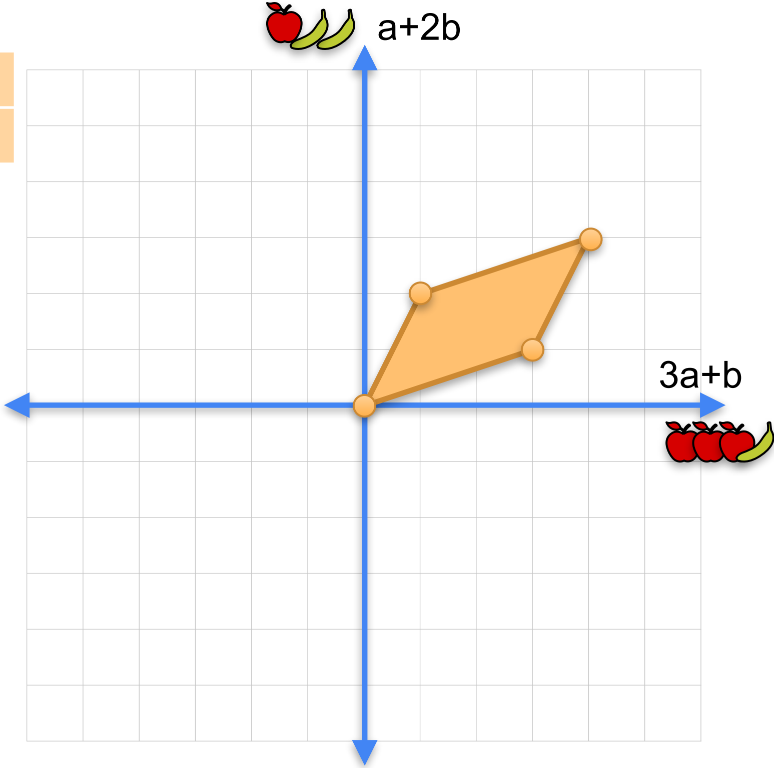


Row space

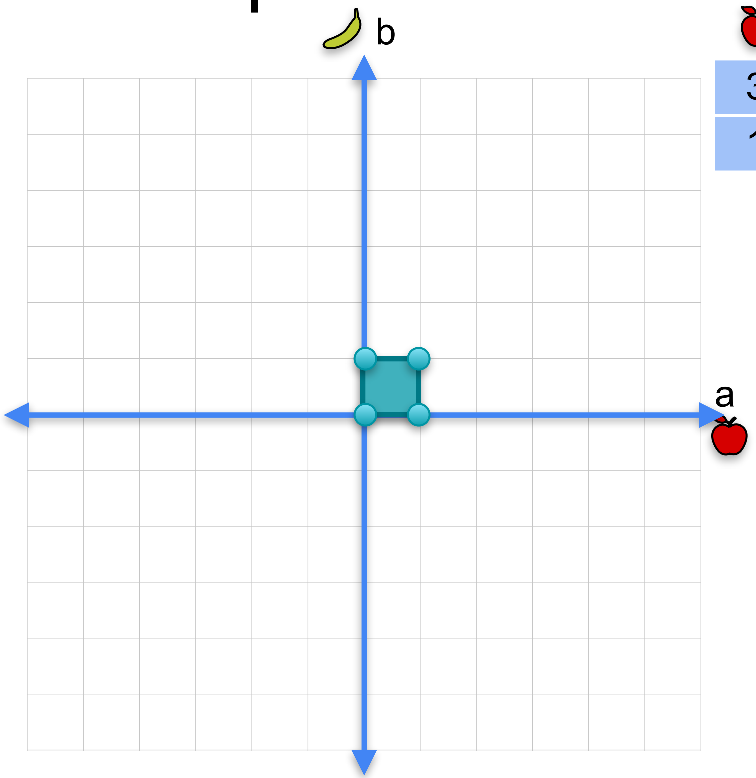


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)



Row space



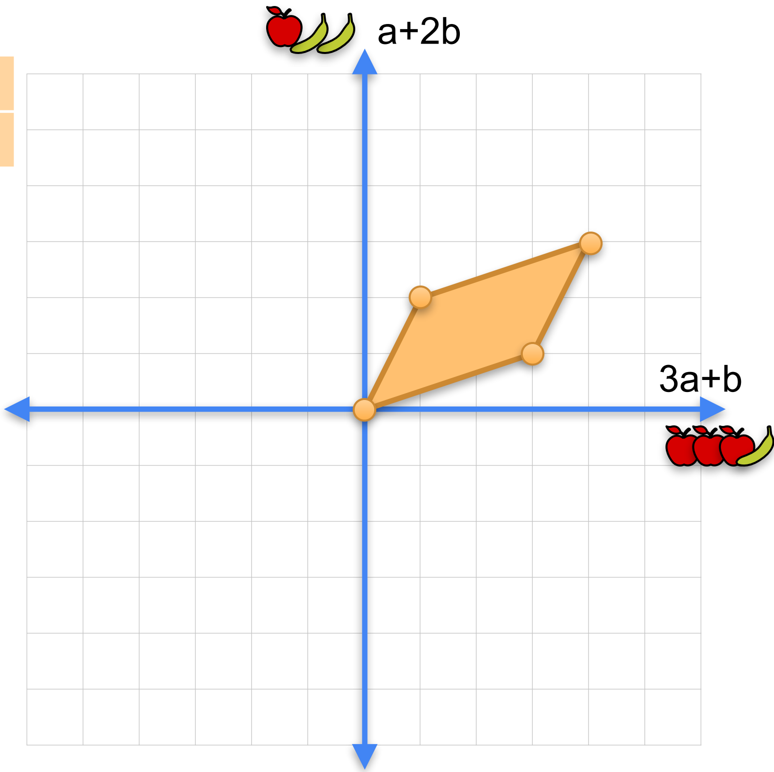
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)

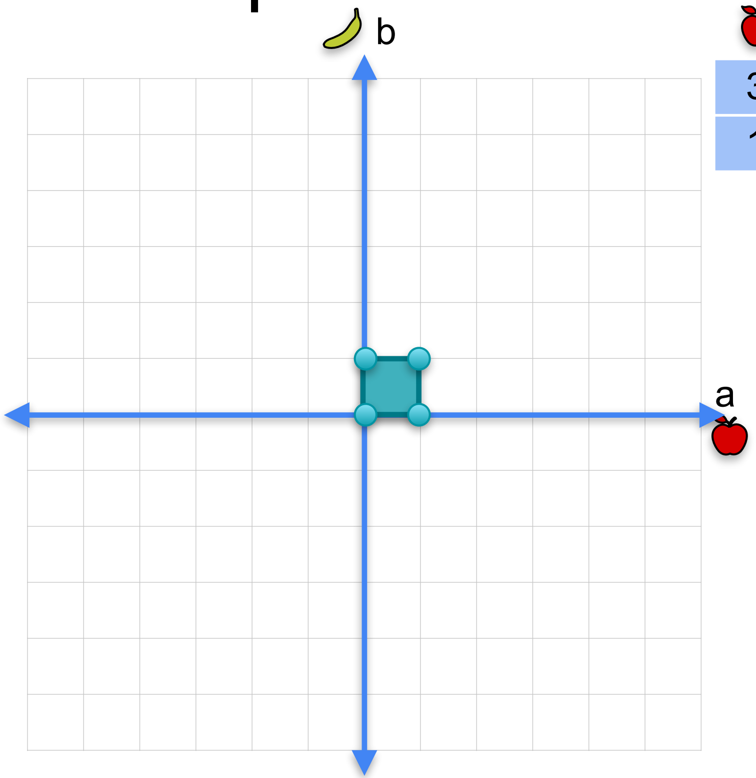
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)

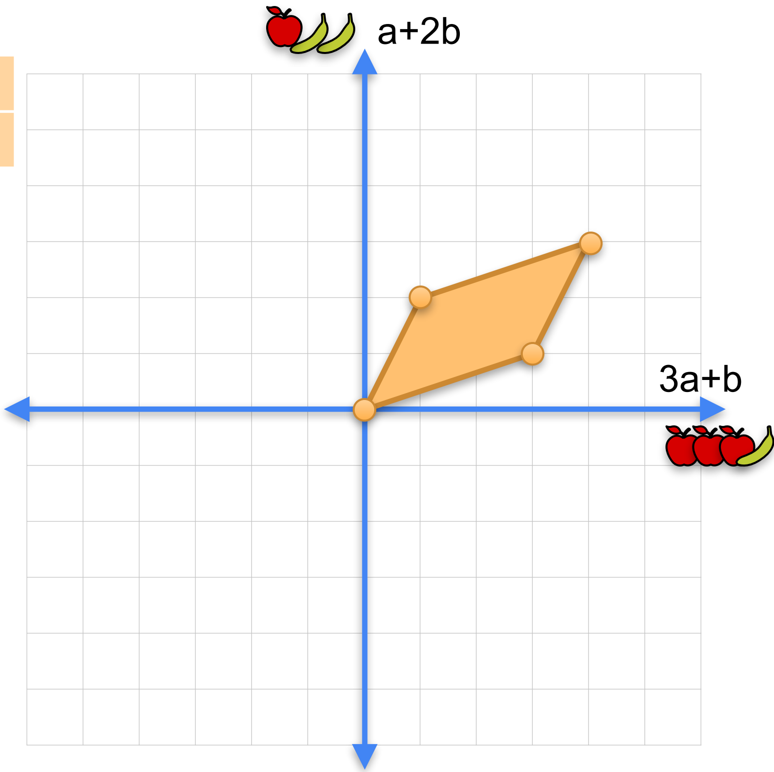


Row space

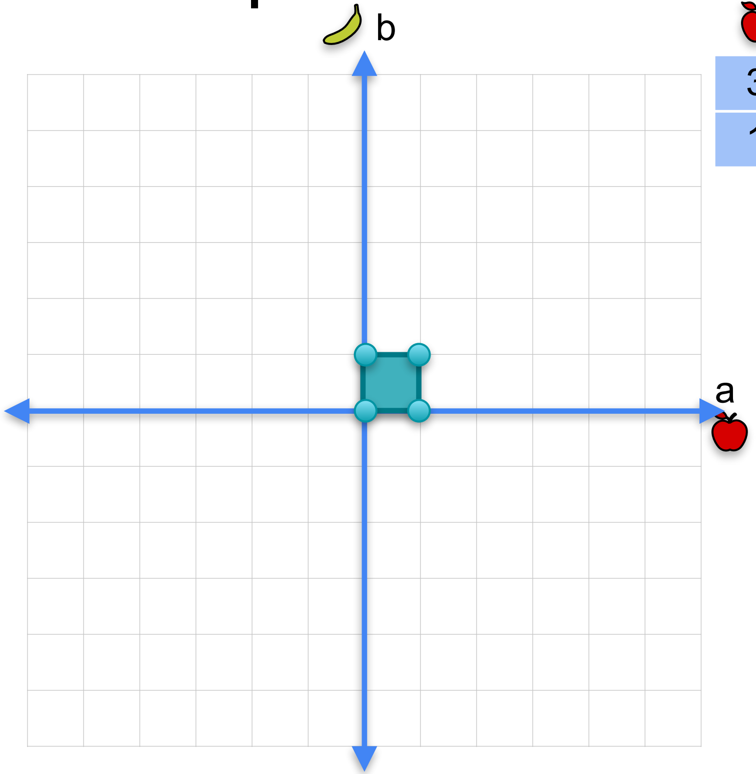


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)



Row space



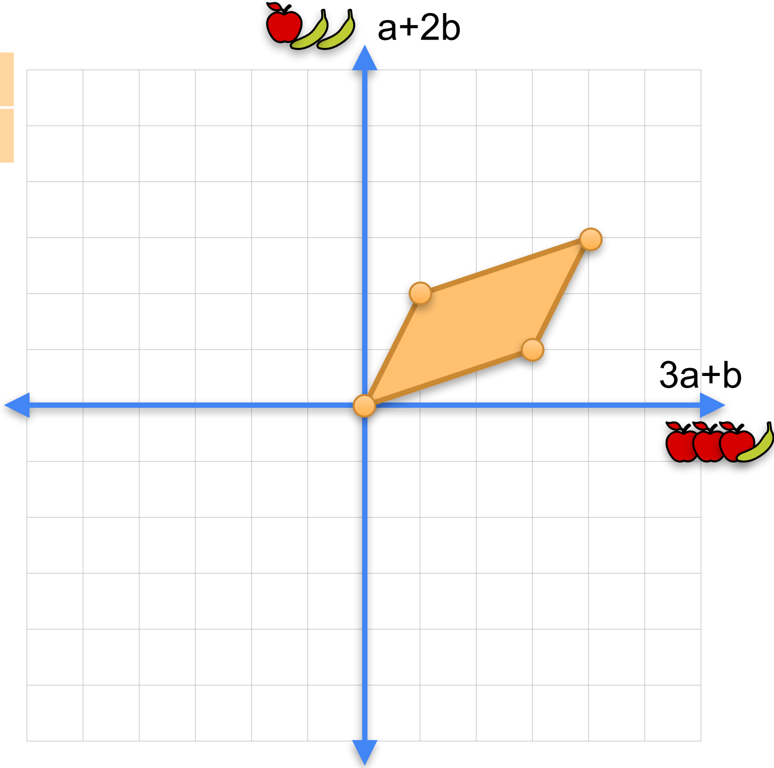
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

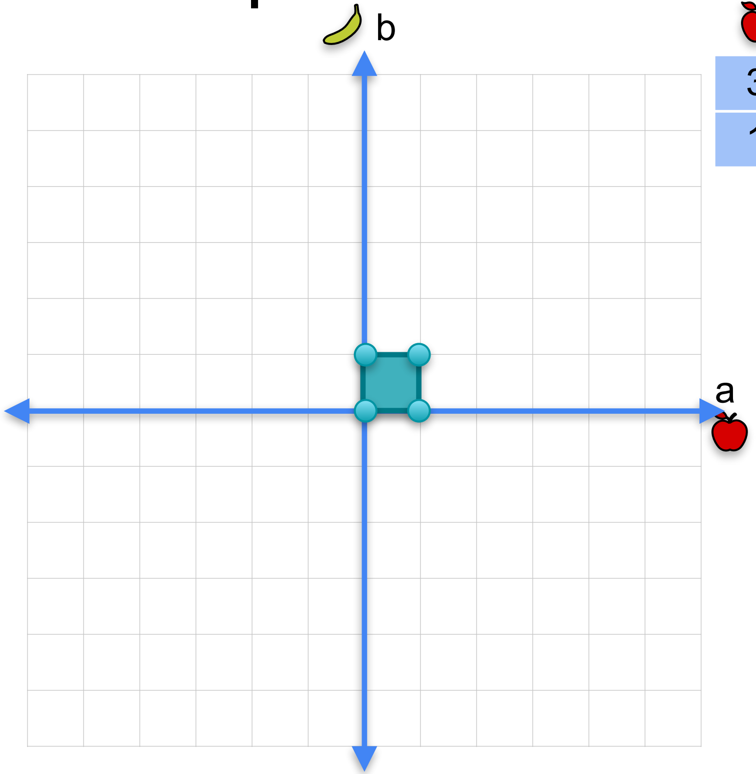
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Row space



$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

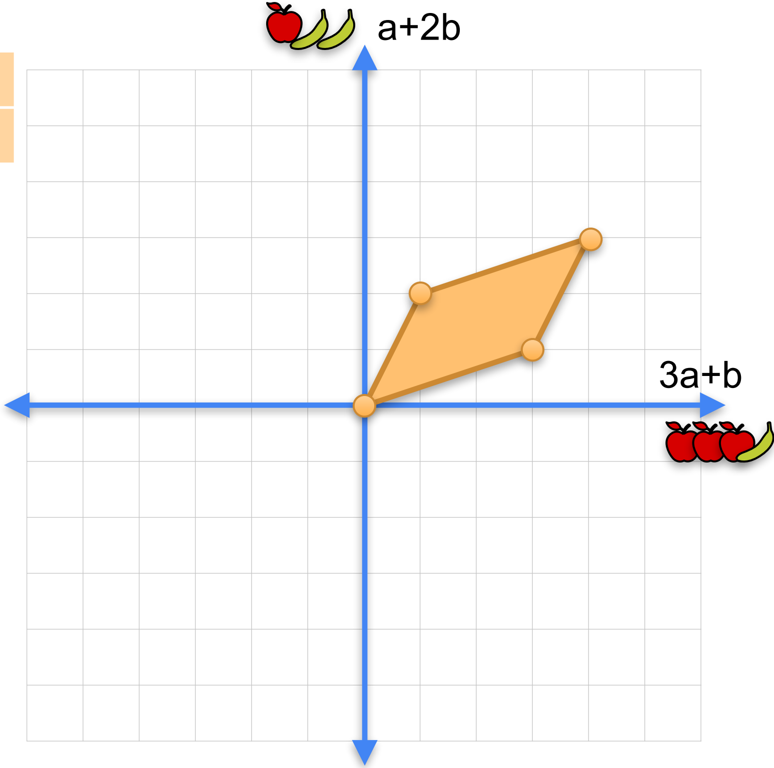
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

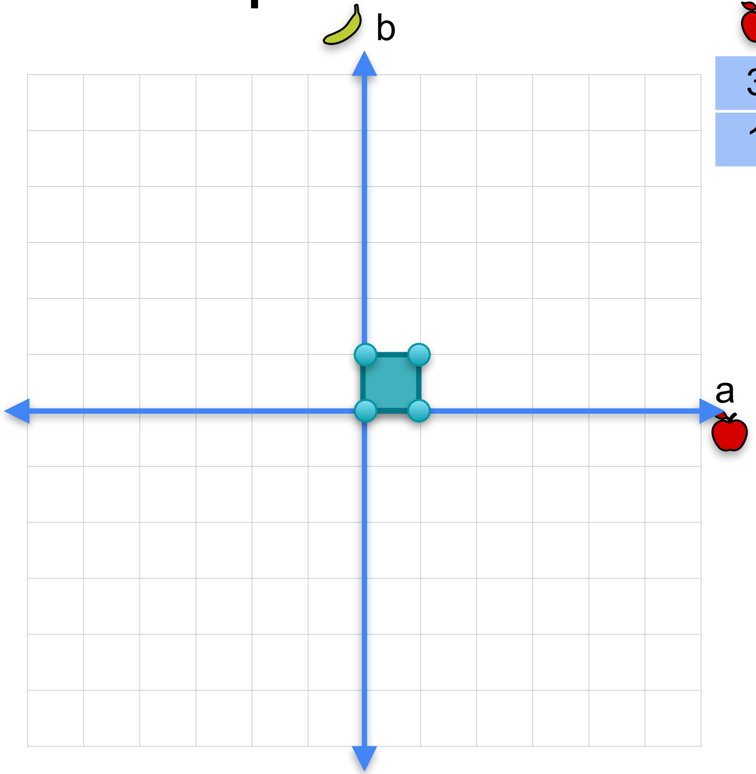
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Row space



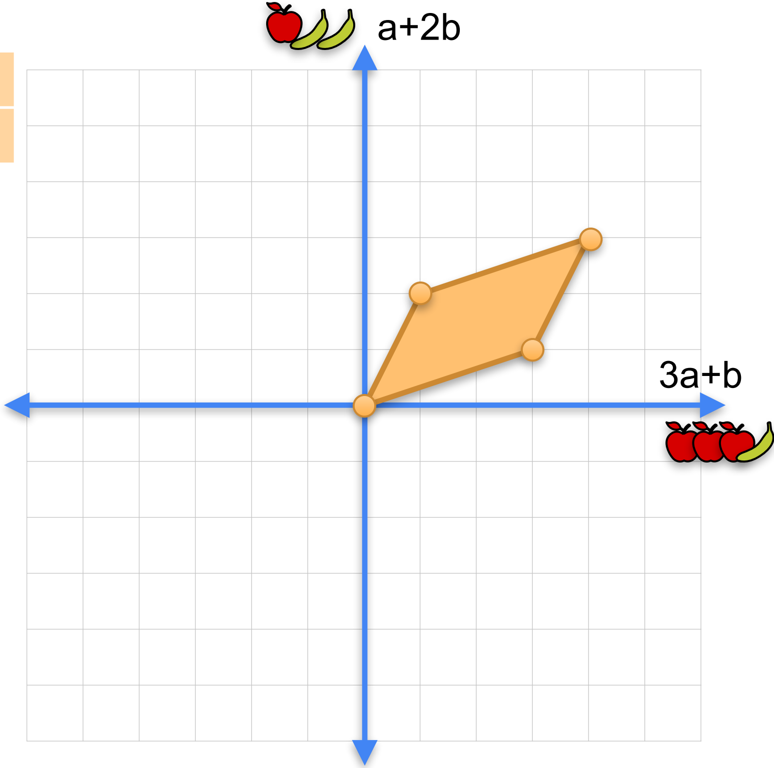
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

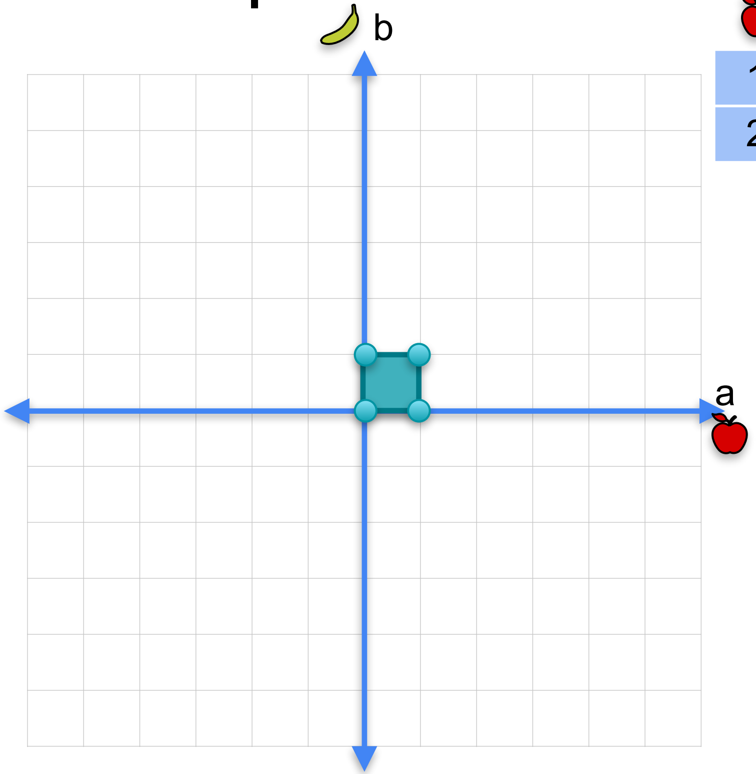
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Row space



1	1
2	2

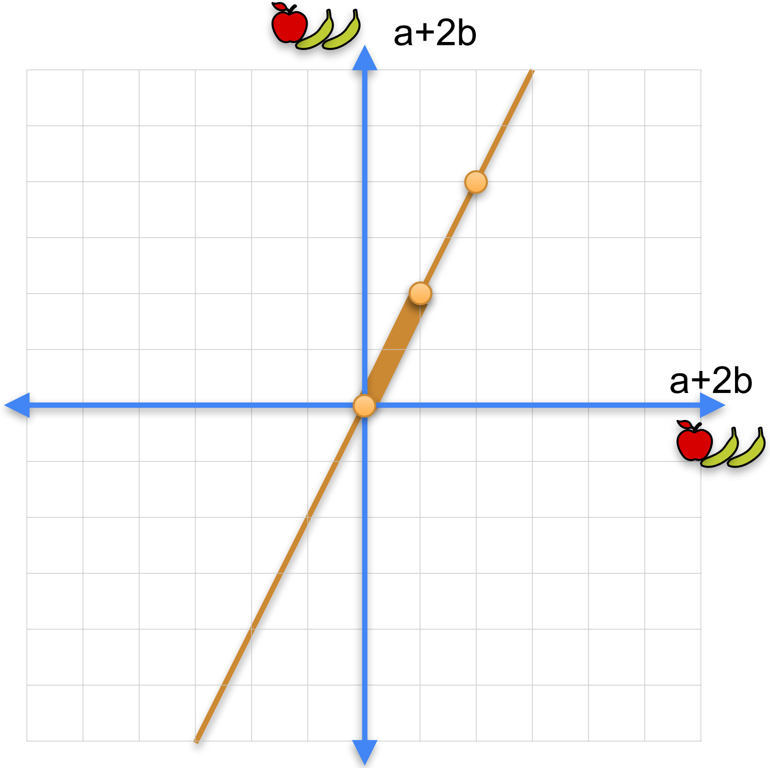
=

(0,0) → (0,0)

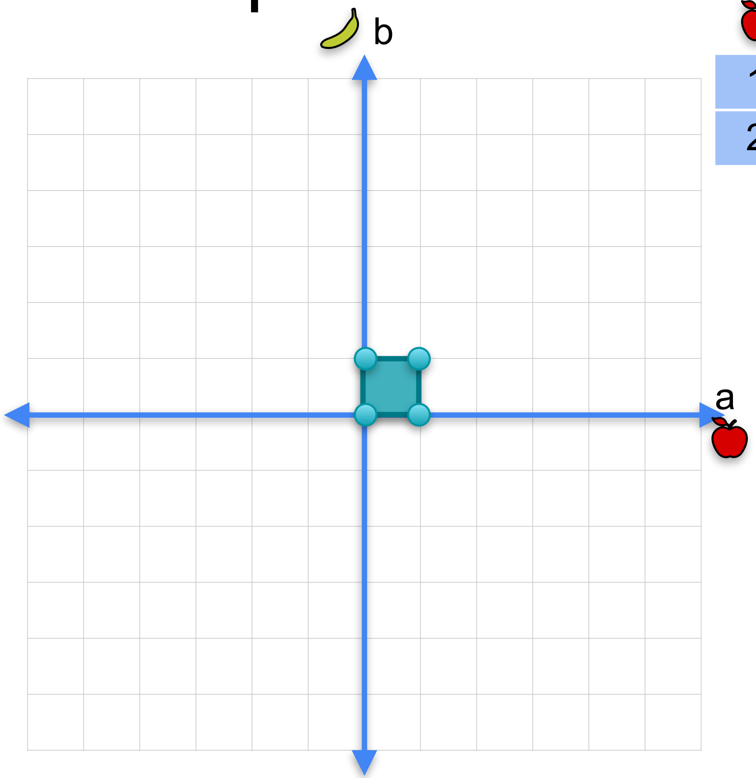
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)



Row space



1	1	0
2	2	0

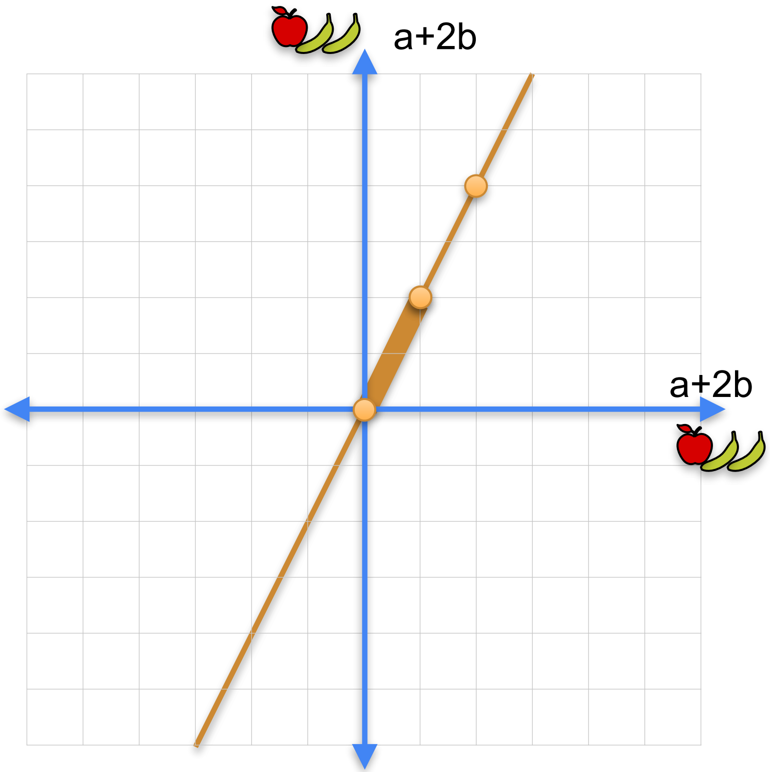
=

(0,0) → (0,0)

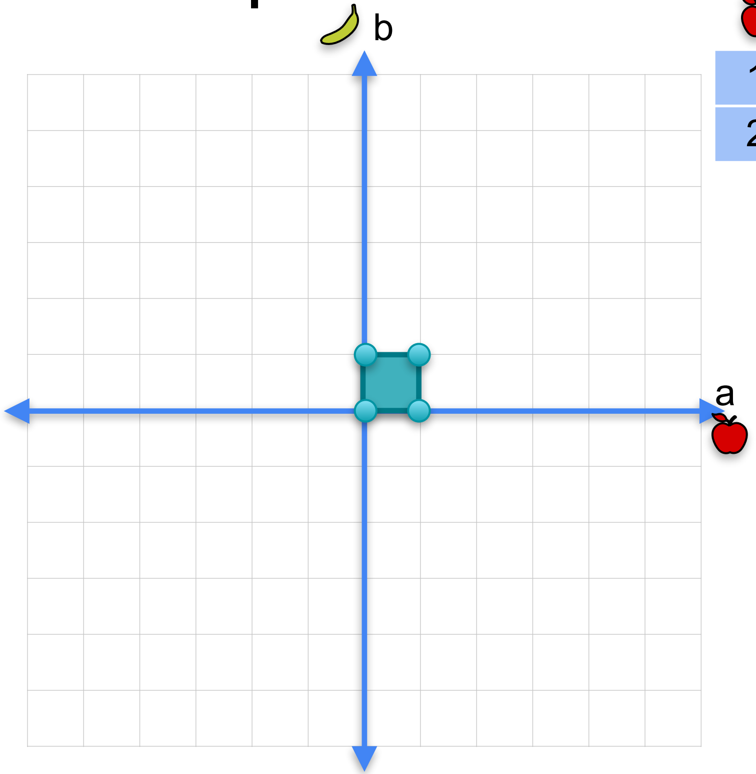
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)



Row space



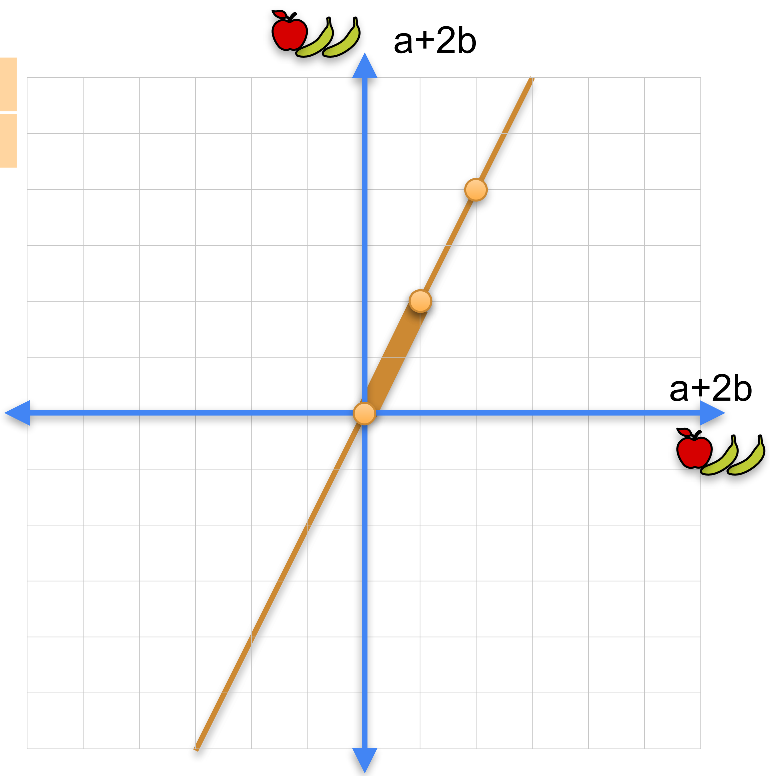
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)

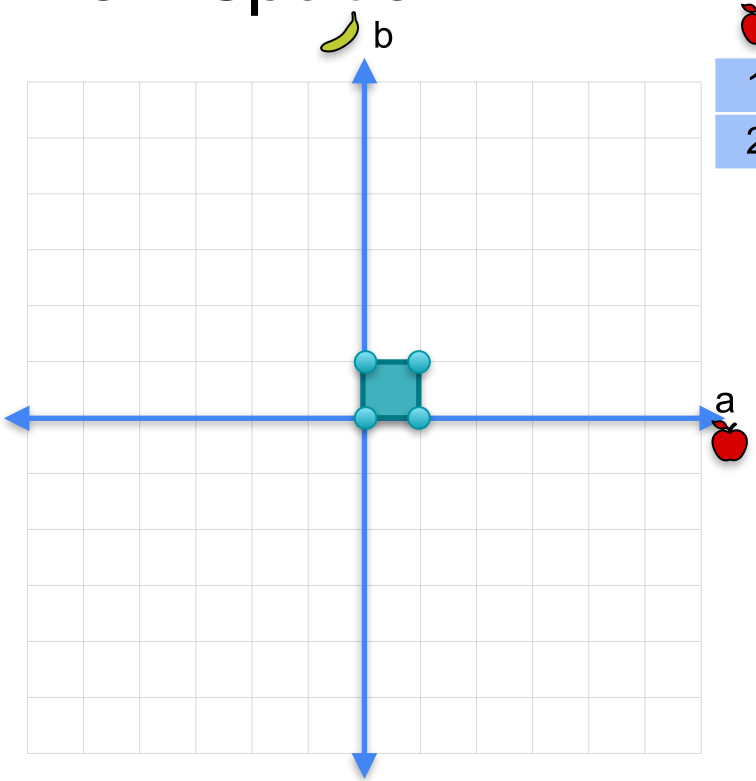
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)

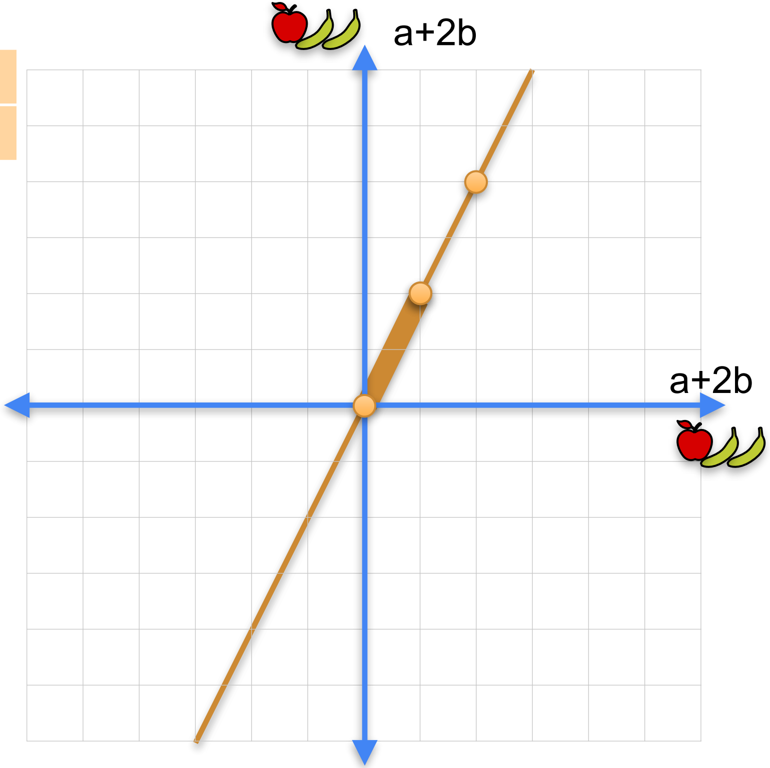


Row space

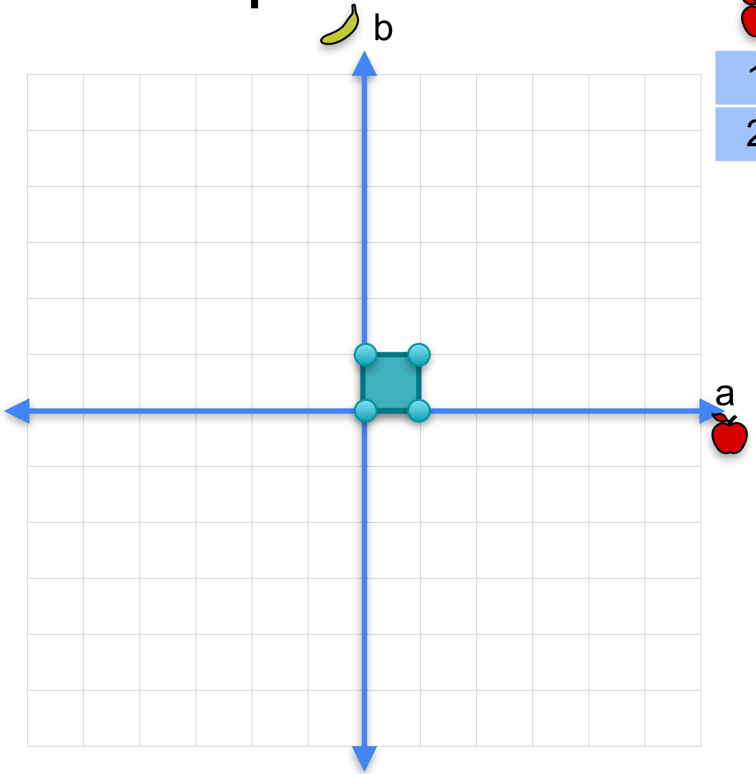


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (1,2)
(0,1) → (1,2)
(1,1) → (2,4)



Row space



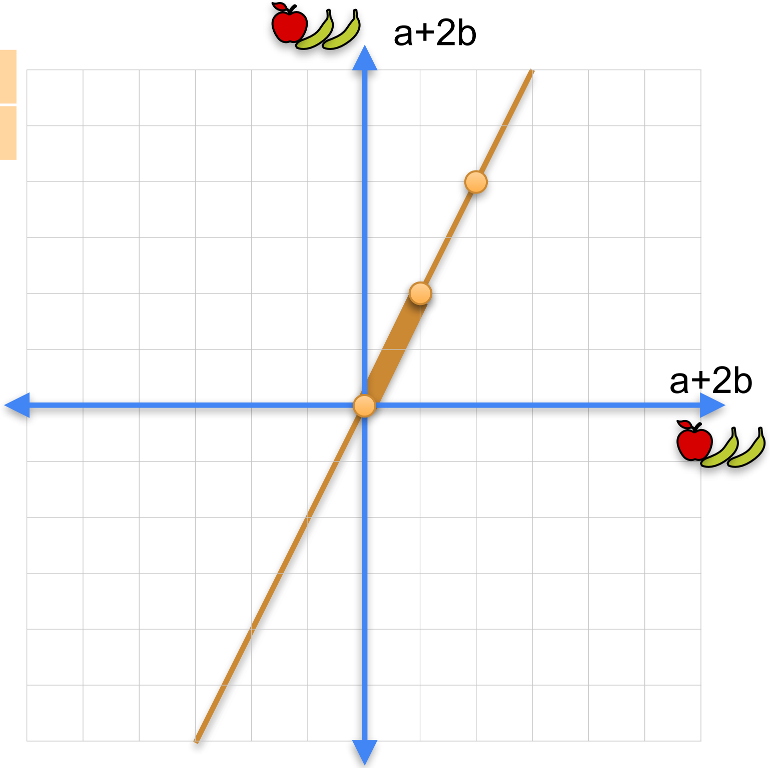
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)

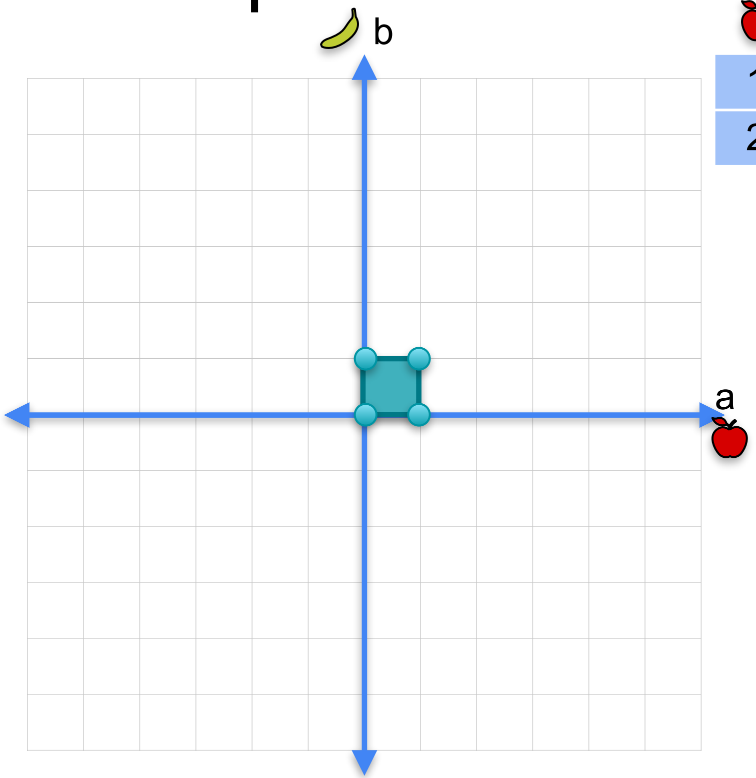
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)



Row space



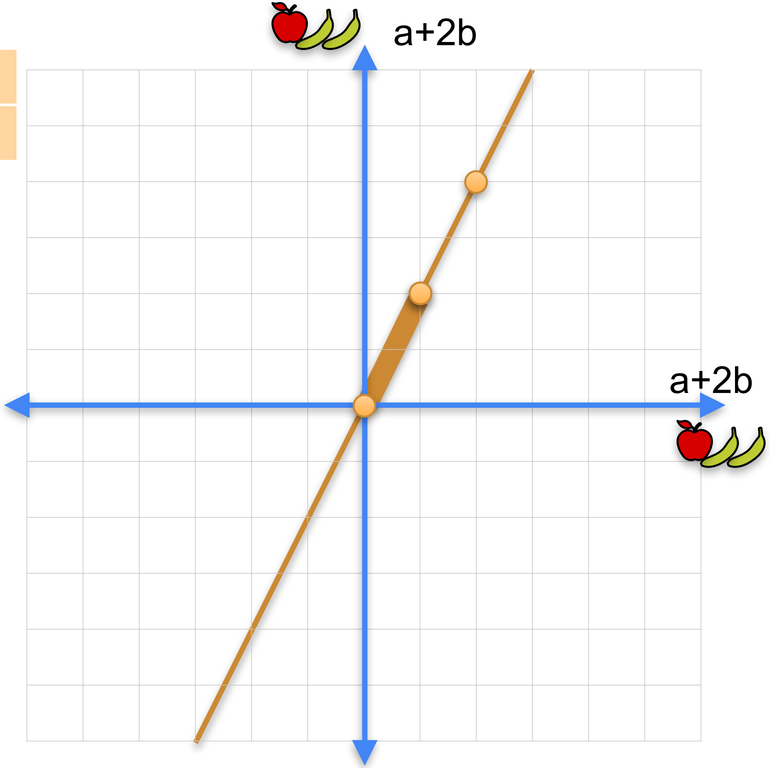
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)

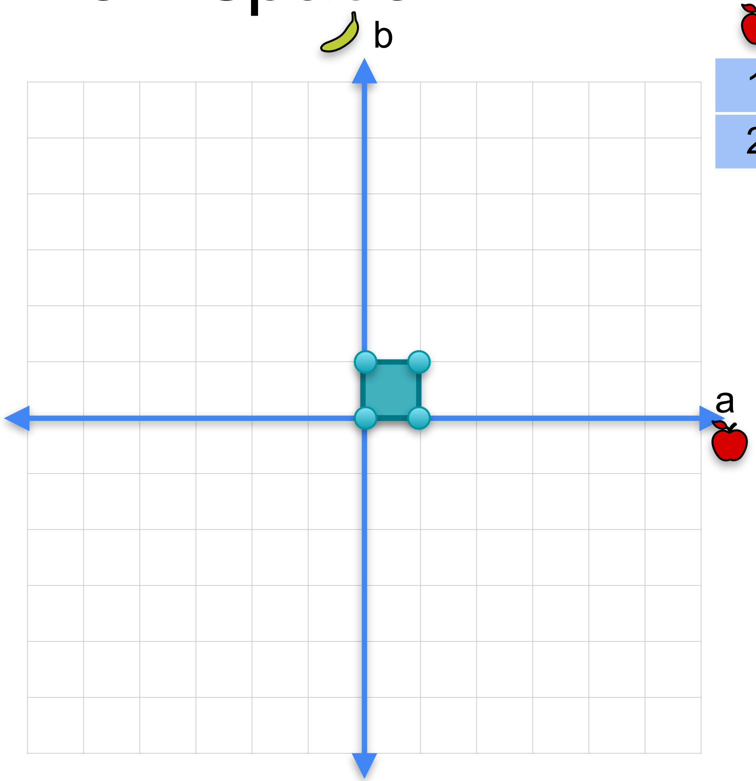
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)



Row space



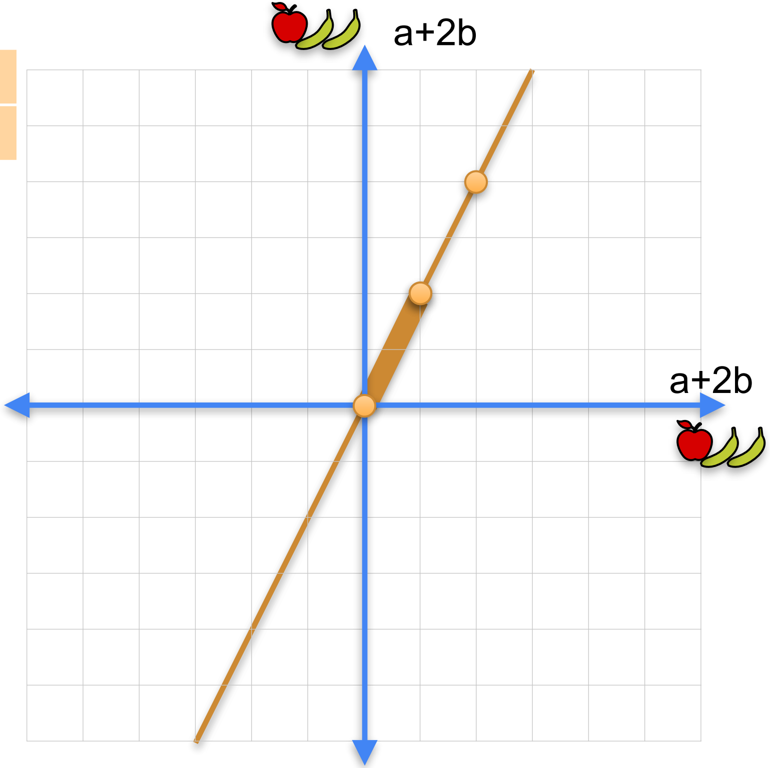
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$(0,0) \rightarrow (0,0)$

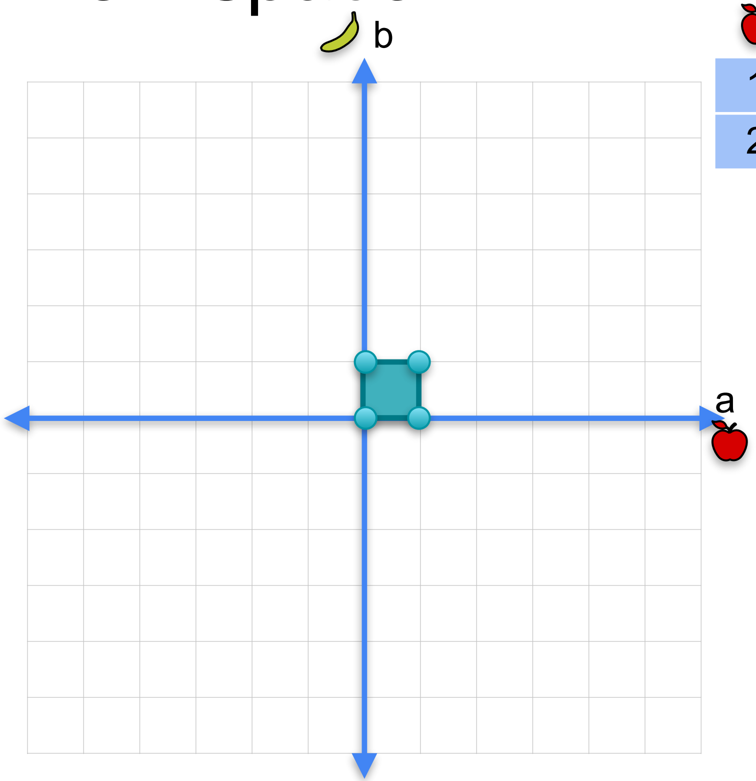
$(1,0) \rightarrow (1,2)$

$(0,1) \rightarrow (1,2)$

$(1,1) \rightarrow (2,4)$



Row space



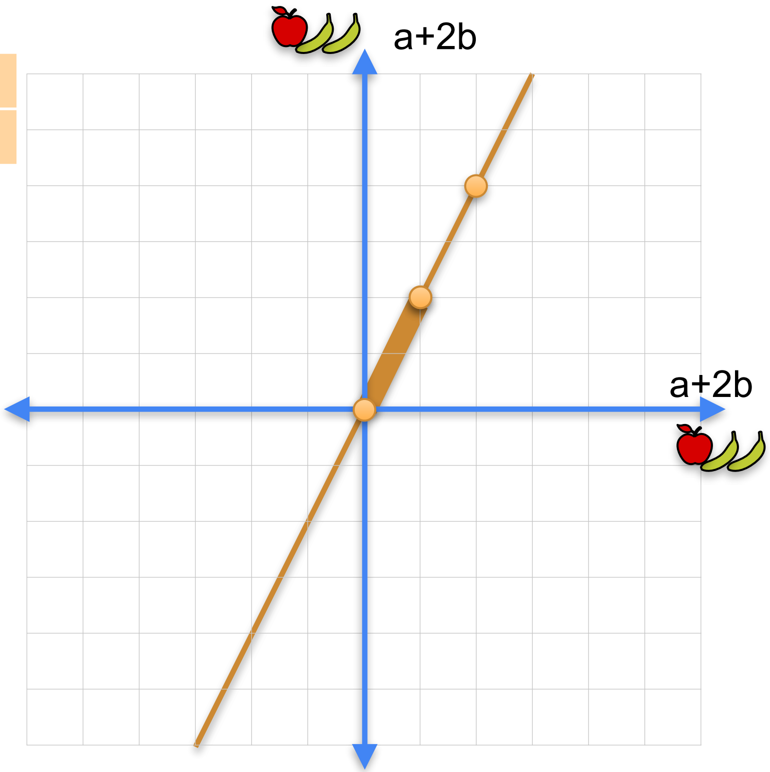
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(0,0) → (0,0)

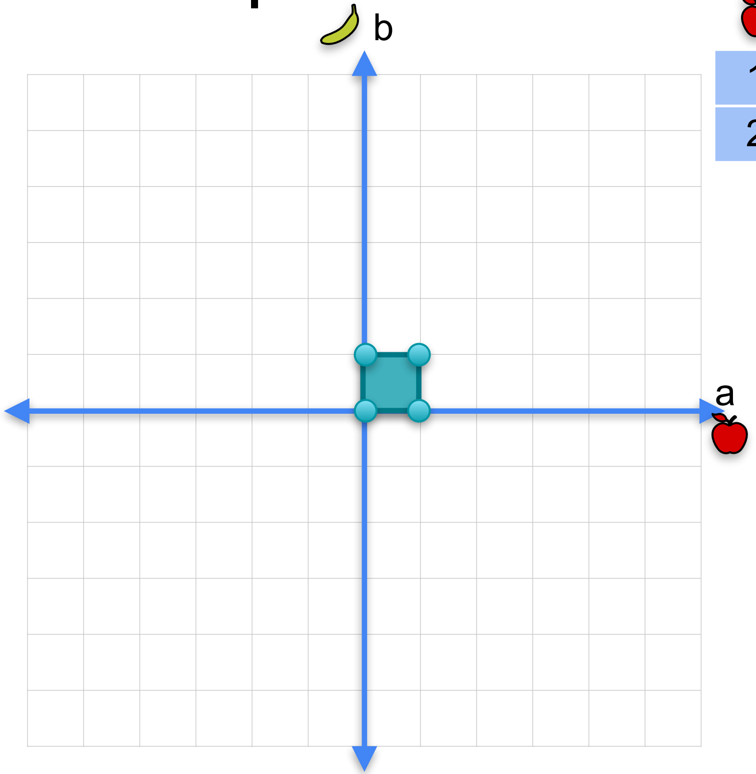
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)



Row space



$$\begin{array}{cc} 🍎 & 🍌 \\ \end{array}$$

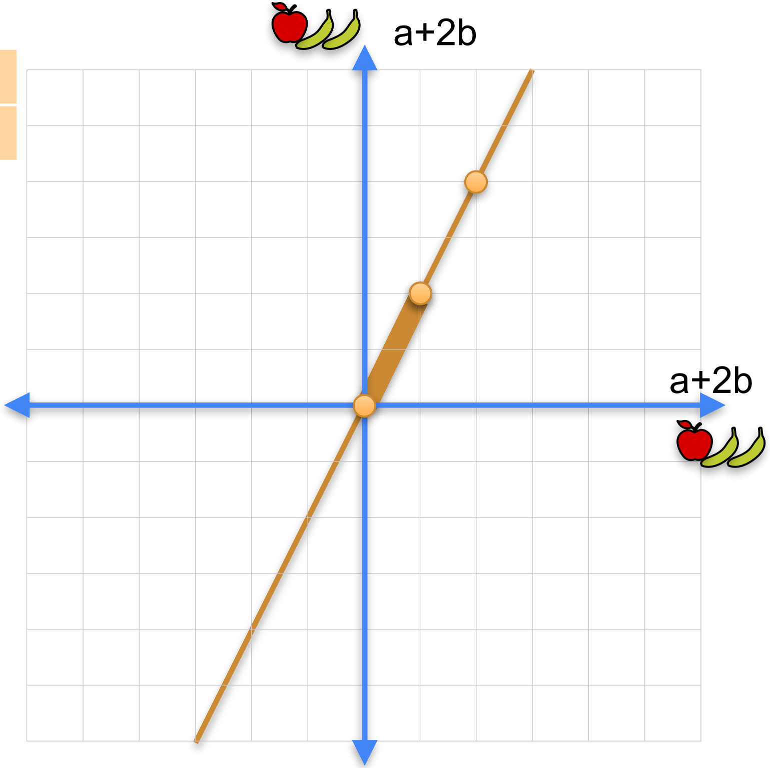
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

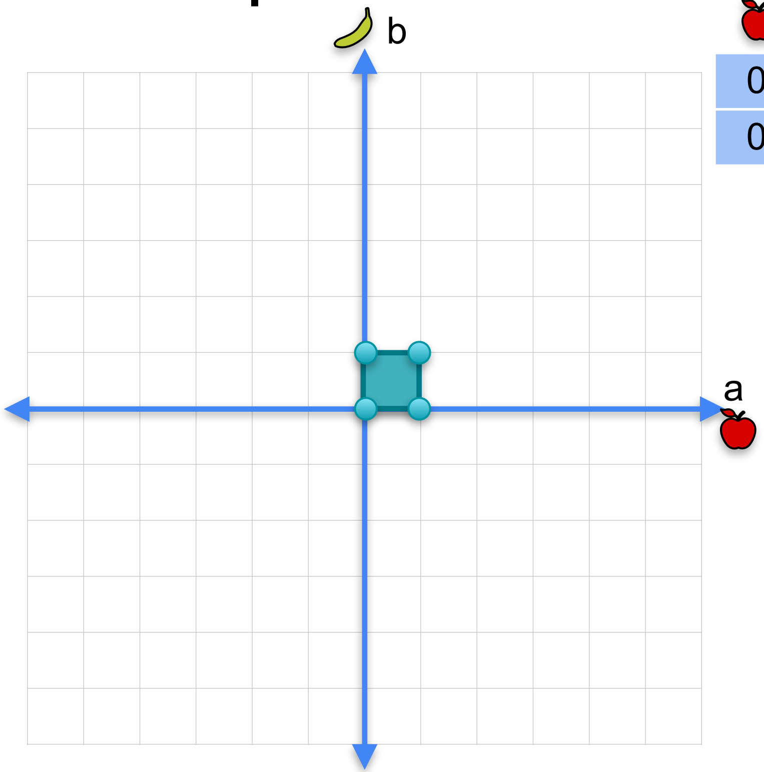
(1,0) → (1,2)

(0,1) → (1,2)

(1,1) → (2,4)

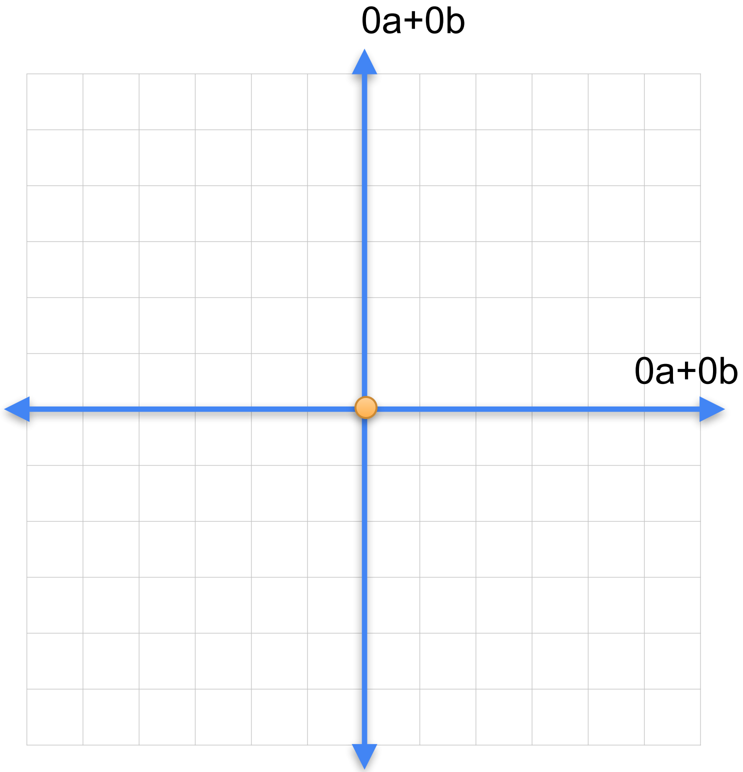


Row space



🍎	🍌
0	0
0	0

=



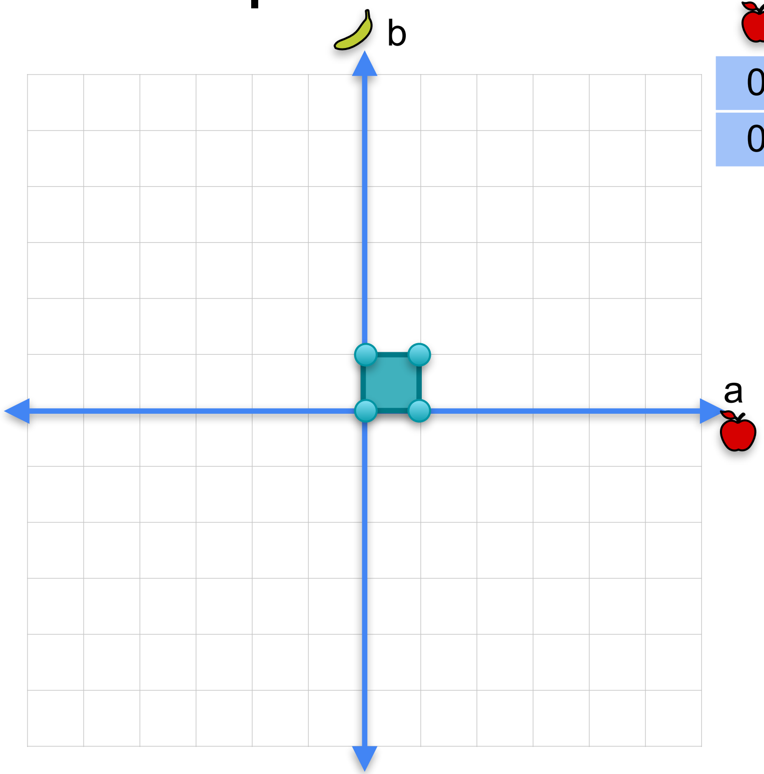
(0,0) → (0,0)

(1,0) → (0,0)

(0,1) → (0,0)

(1,1) → (0,0)

Row space



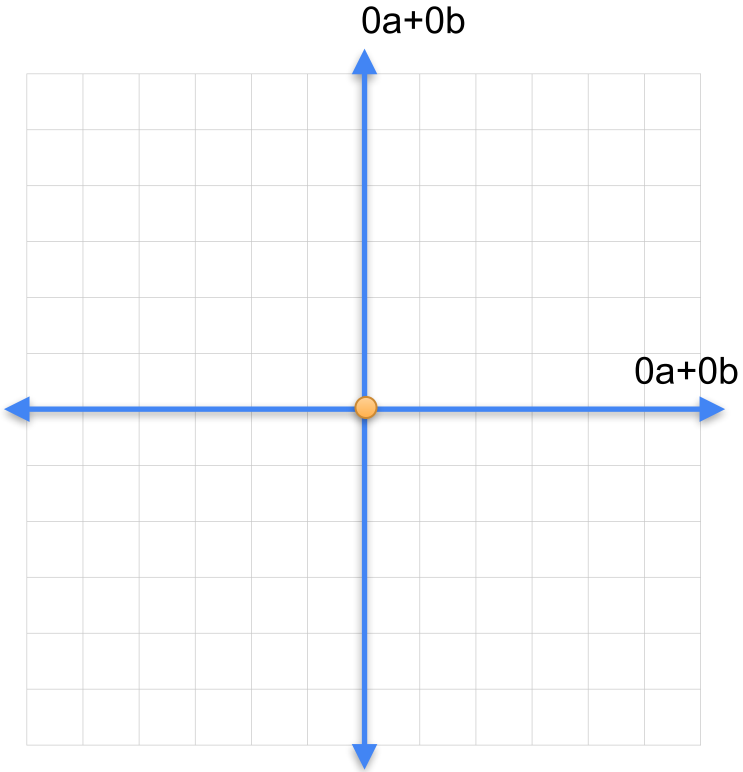
$$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} =$$

(0,0) → (0,0)

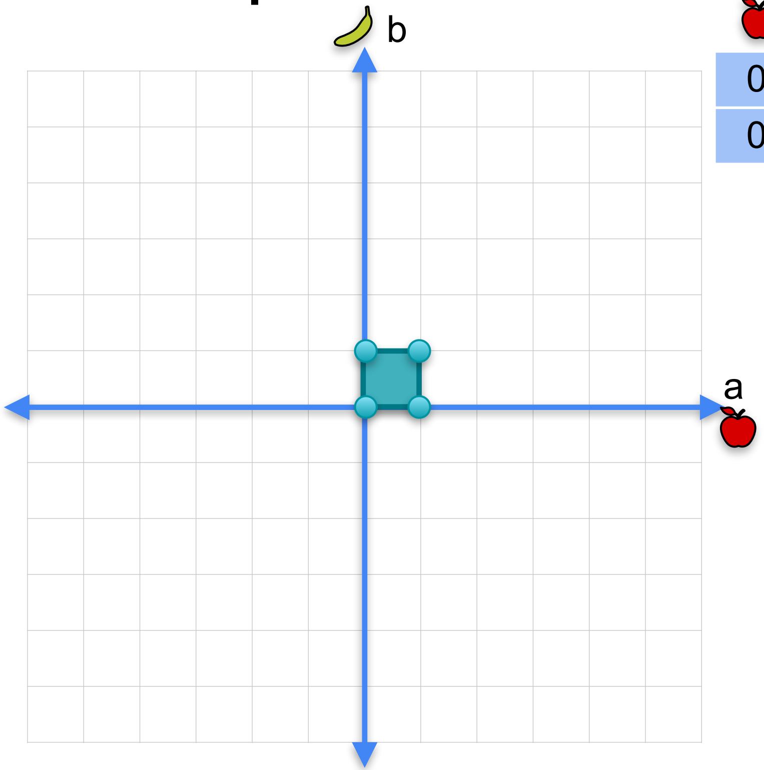
(1,0) → (0,0)

(0,1) → (0,0)

(1,1) → (0,0)



Row space



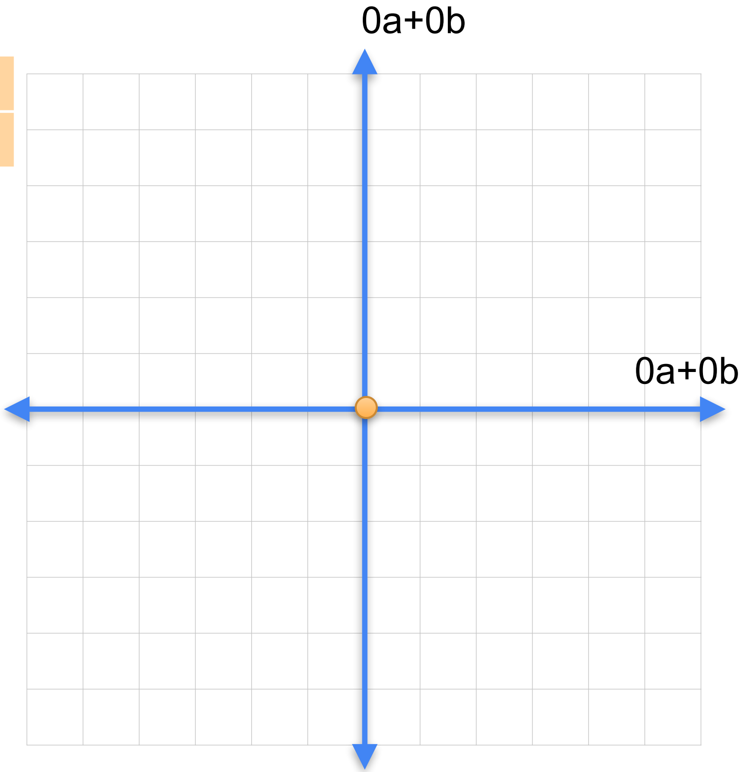
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (0,0)

(0,1) → (0,0)

(1,1) → (0,0)

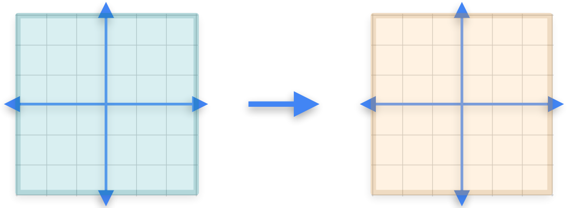


Row space

Non-singular

3	1
1	2

Rank = 2

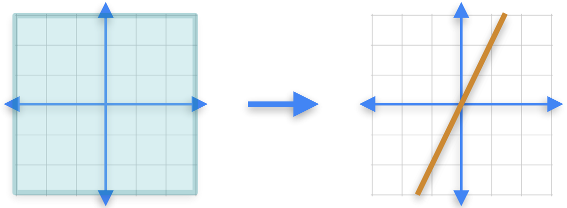


Dimension = 2

Singular

1	1
2	2

Rank = 1

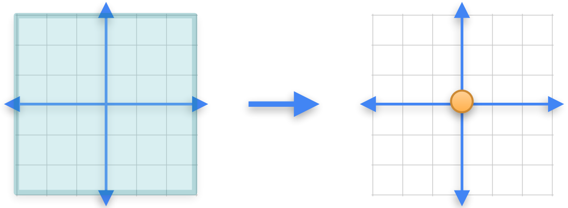


Dimension = 1

Singular

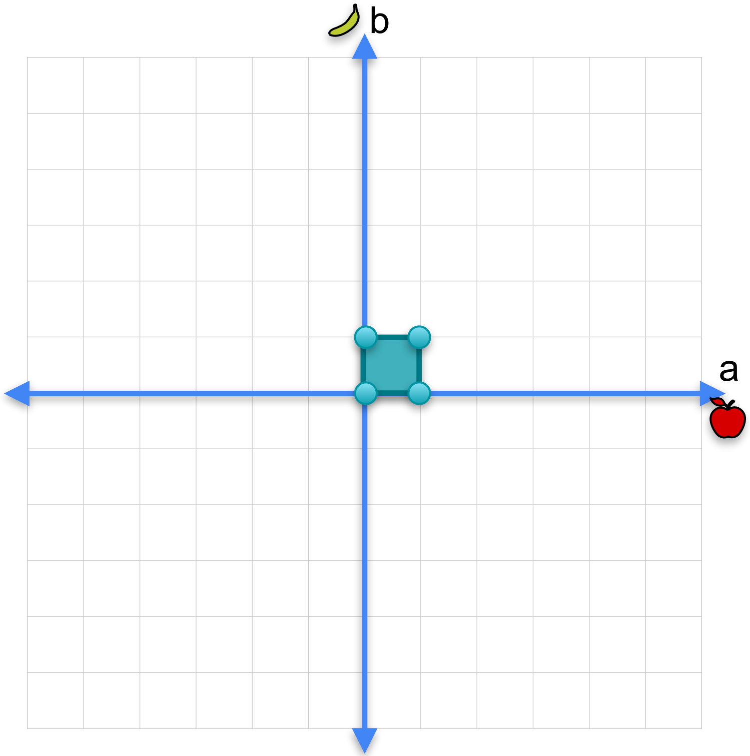
0	0
0	0

Rank = 0



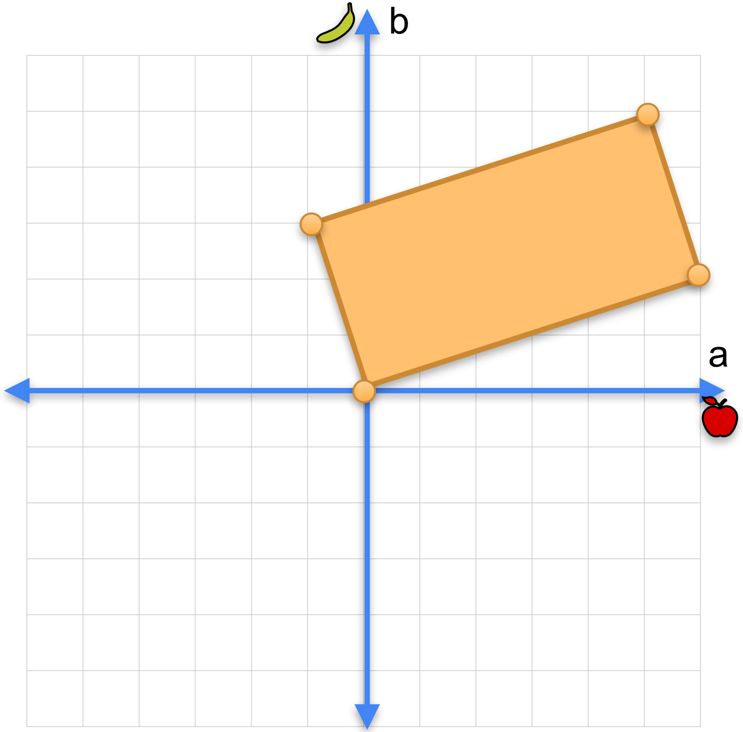
Dimension = 0

Orthogonal matrix

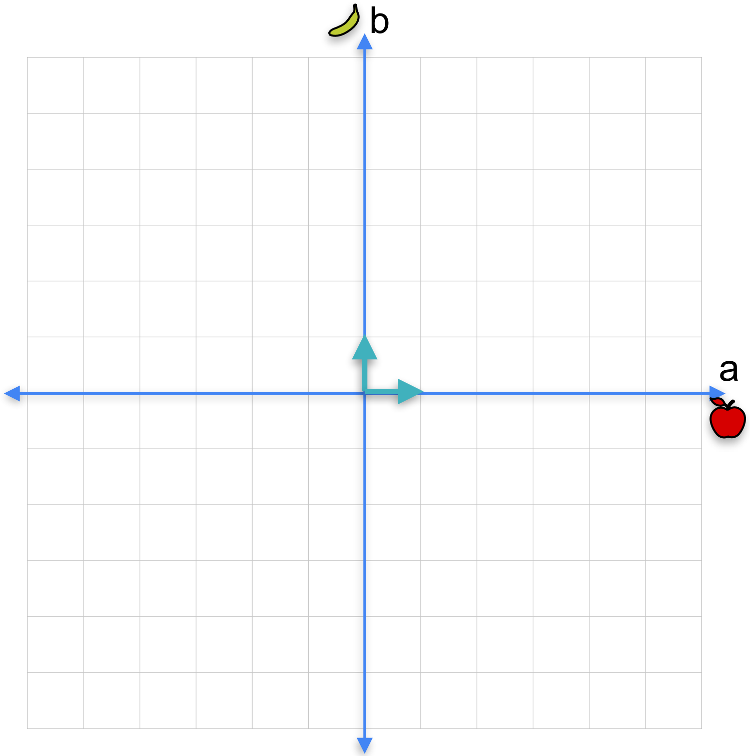


6	-1
2	3

(0,0) → (0,0)
(1,0) → (6,2)
(0,1) → (-1,3)
(1,1) → (5,5)

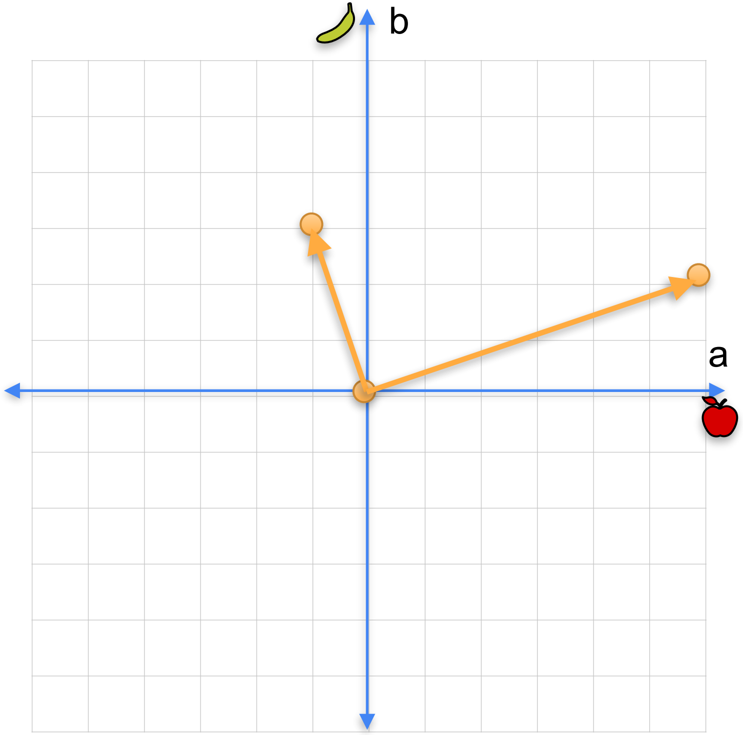


Orthogonal matrix



6	-1
2	3

(1,0) → (6,2)
(0,1) → (-1,3)



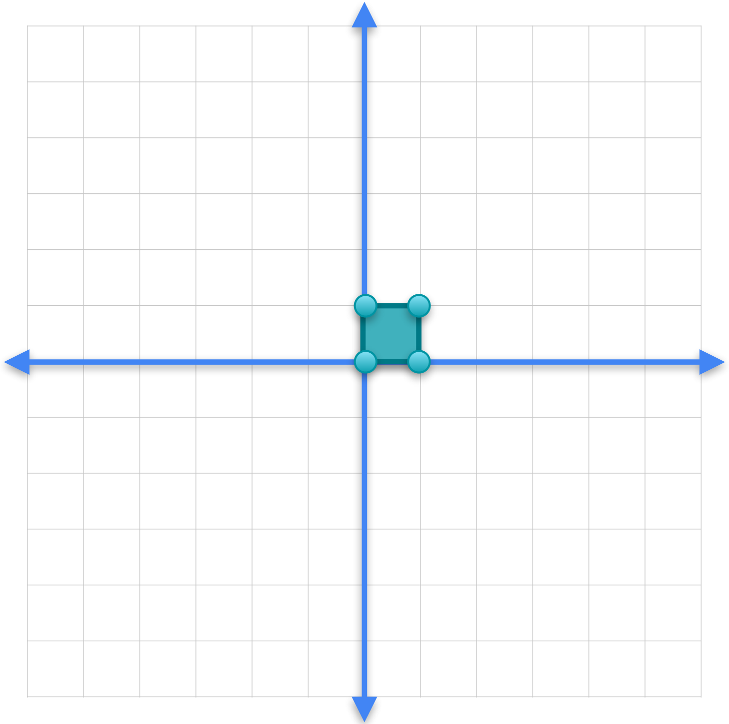
Orthogonal matrices have orthogonal columns

$$\begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \end{bmatrix} = 0$$

Orthogonal matrix



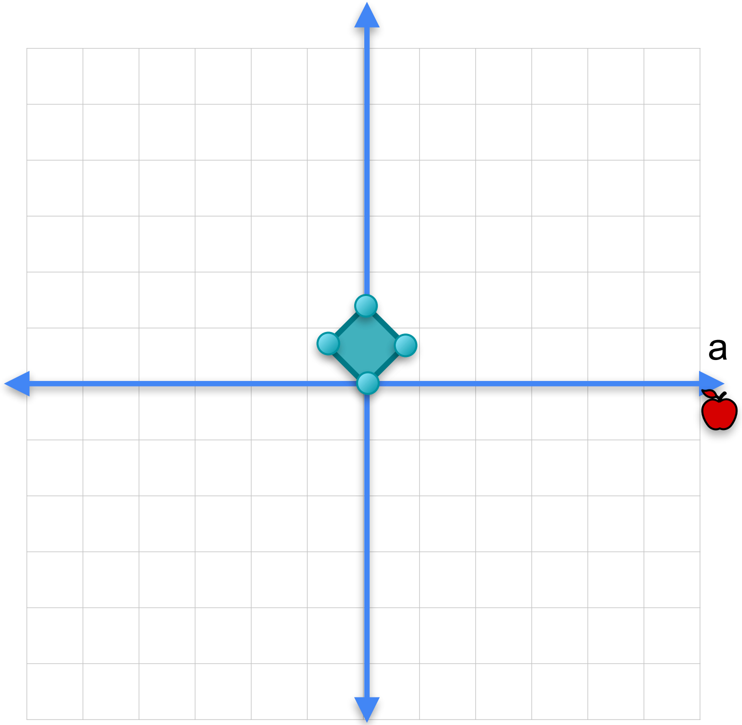
0.7071	0.7071
-0.7071	0.7071

(0,0) → (0,0)

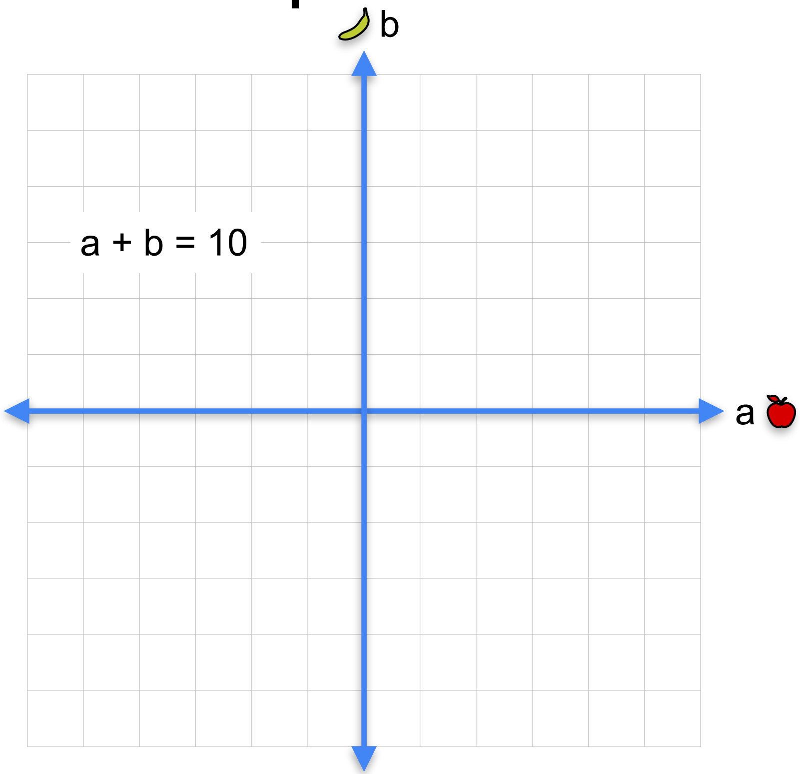
(1,0) → (0.7071, 0.7071)

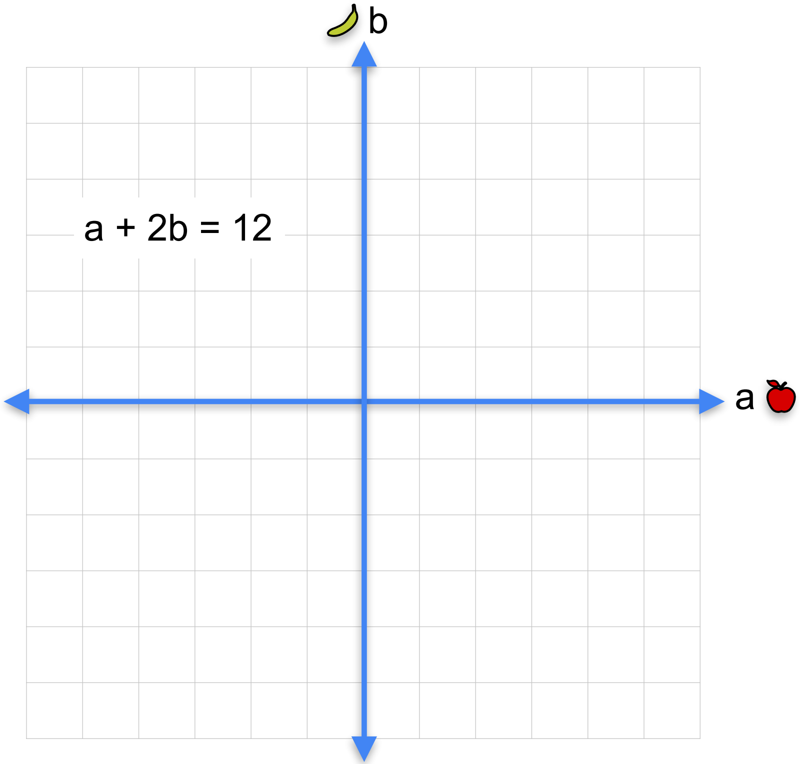
(0,1) → (-0.7071,0.7071)

(1,1) → (0,1.4142)

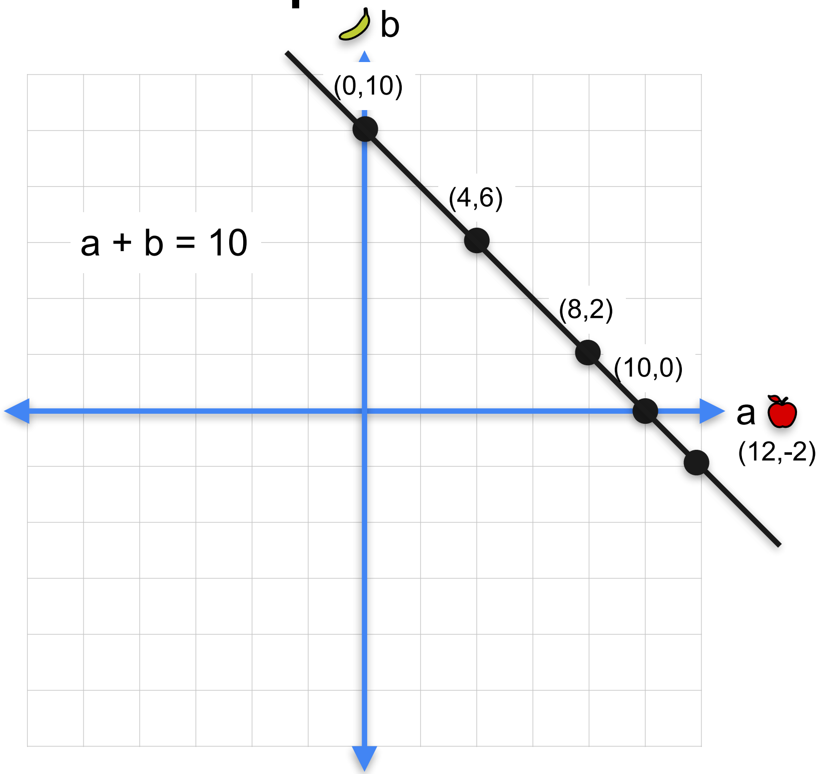


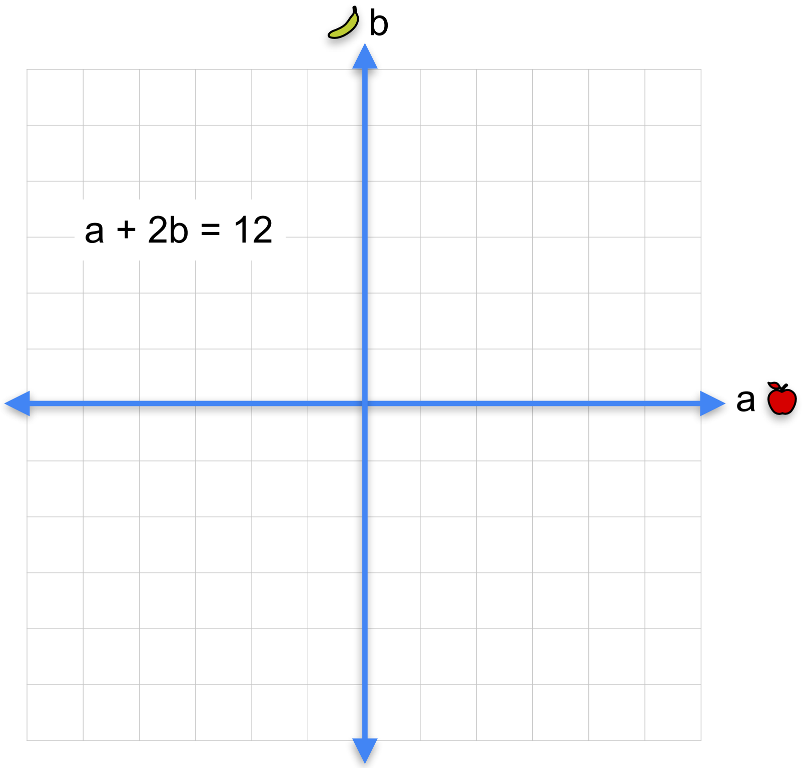
Linear equation in 2 variables -> Line



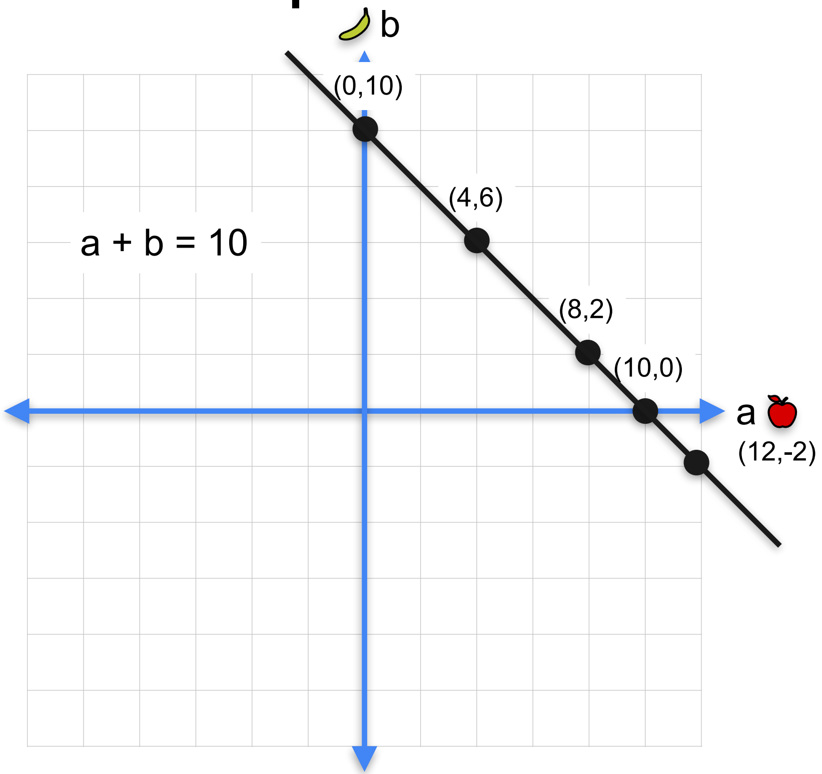


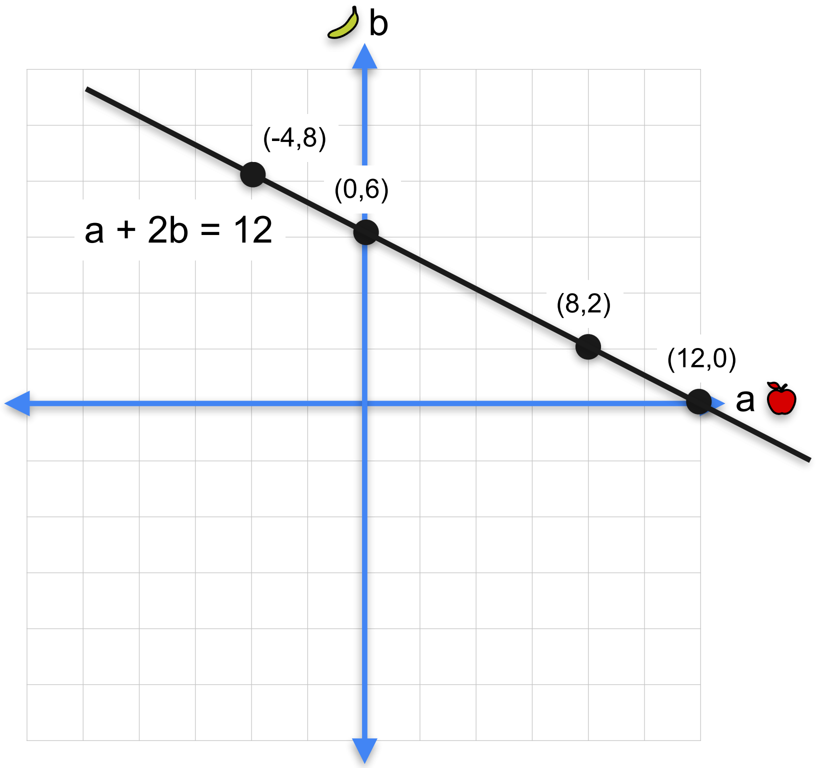
Linear equation in 2 variables -> Line





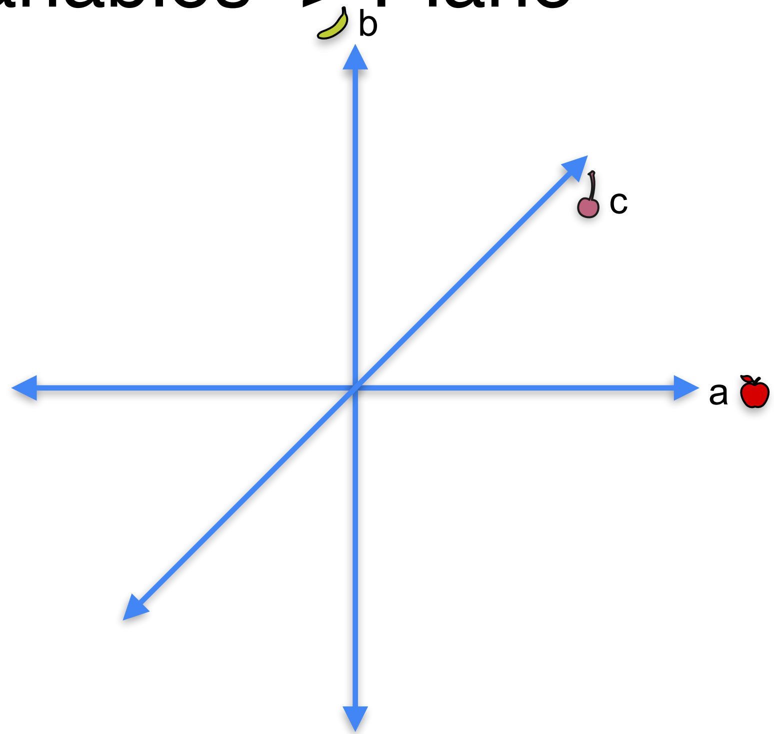
Linear equation in 2 variables -> Line





Linear equation in 3 variables -> Plane

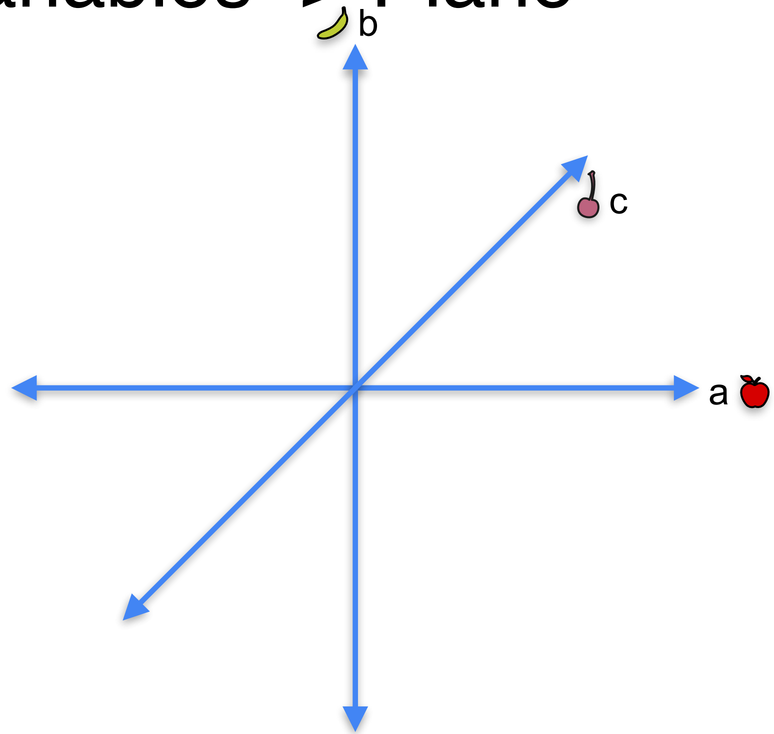
$a + b + c = 1$



Linear equation in 3 variables -> Plane

a + b + c = **1**

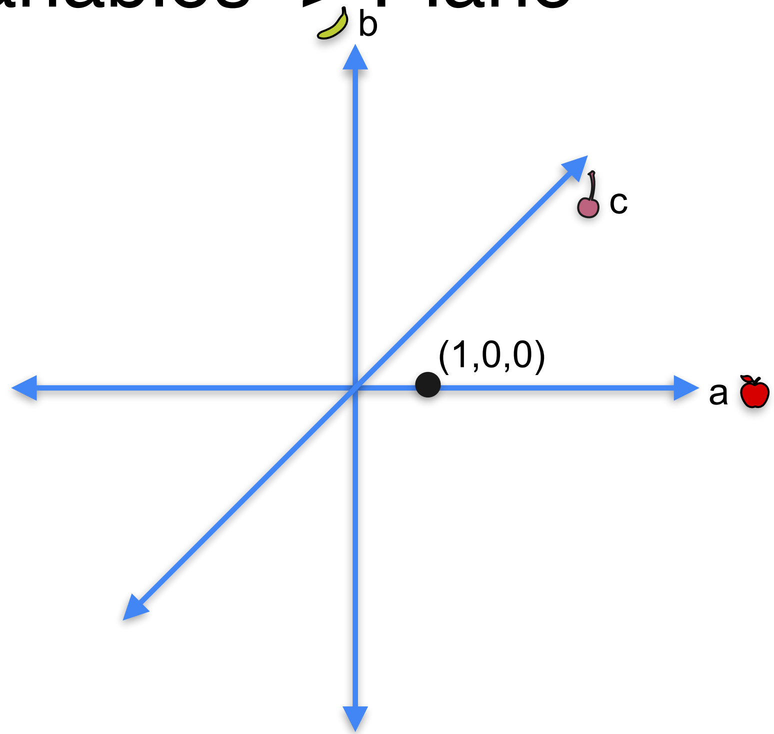
1 + 0 + 0 = **1**



Linear equation in 3 variables -> Plane

$a + b + c = \mathbf{1}$

$1 + 0 + 0 = \mathbf{1}$

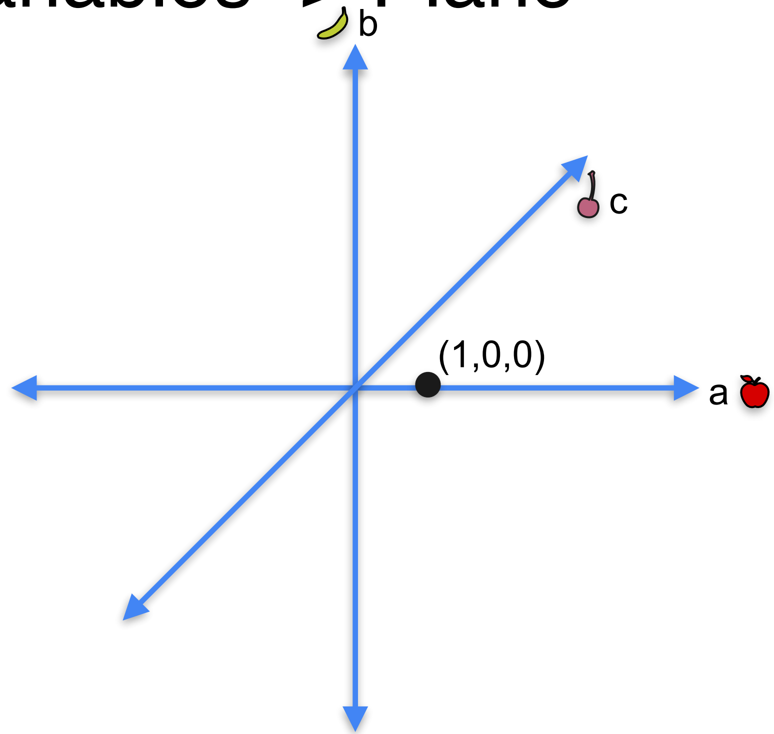


Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$

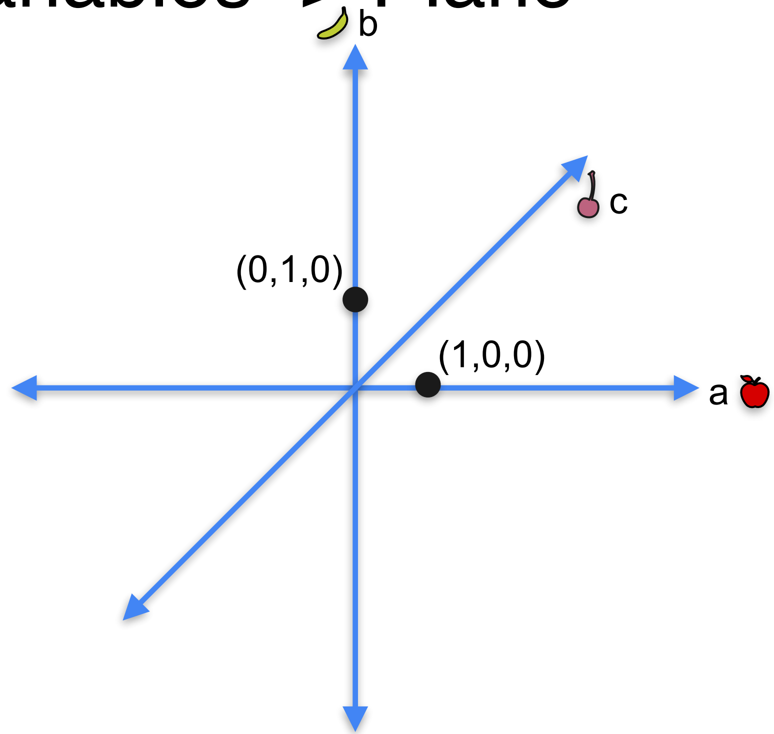


Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$



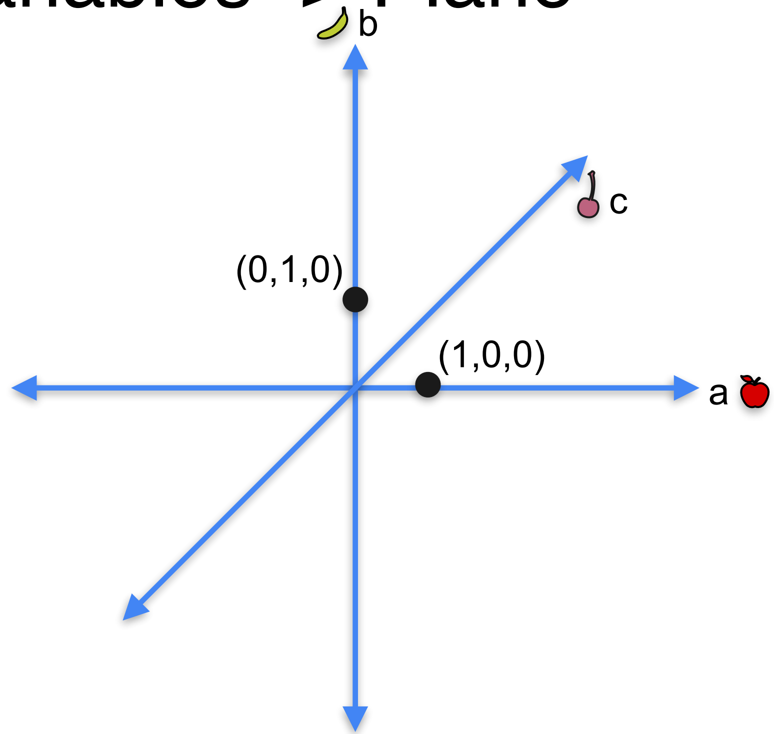
Linear equation in 3 variables -> Plane

a + b + c = 1

1 + 0 + 0 = 1

0 + 1 + 0 = 1

0 + 0 + 1 = 1



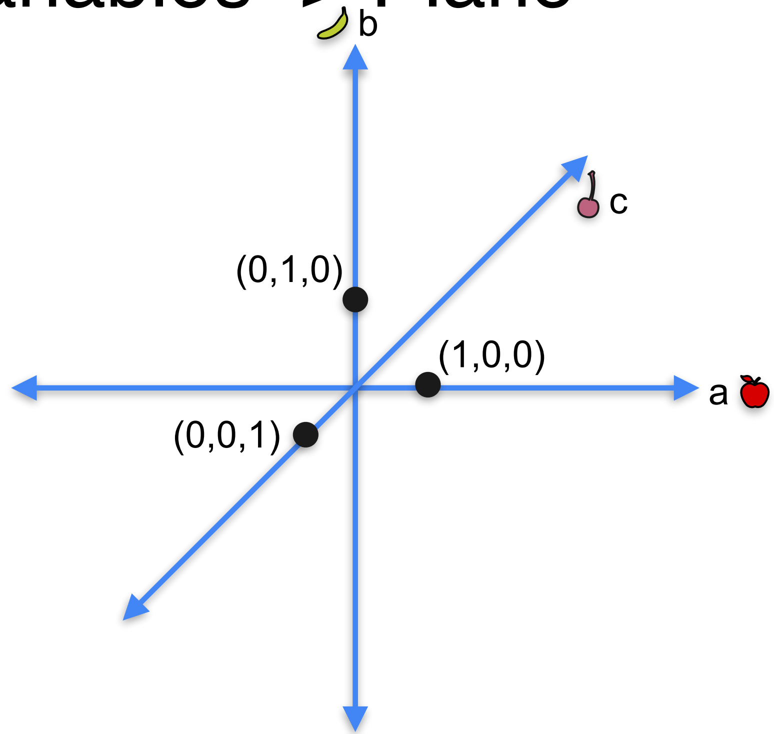
Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

$0 + 1 + 0 = 1$

$0 + 0 + 1 = 1$



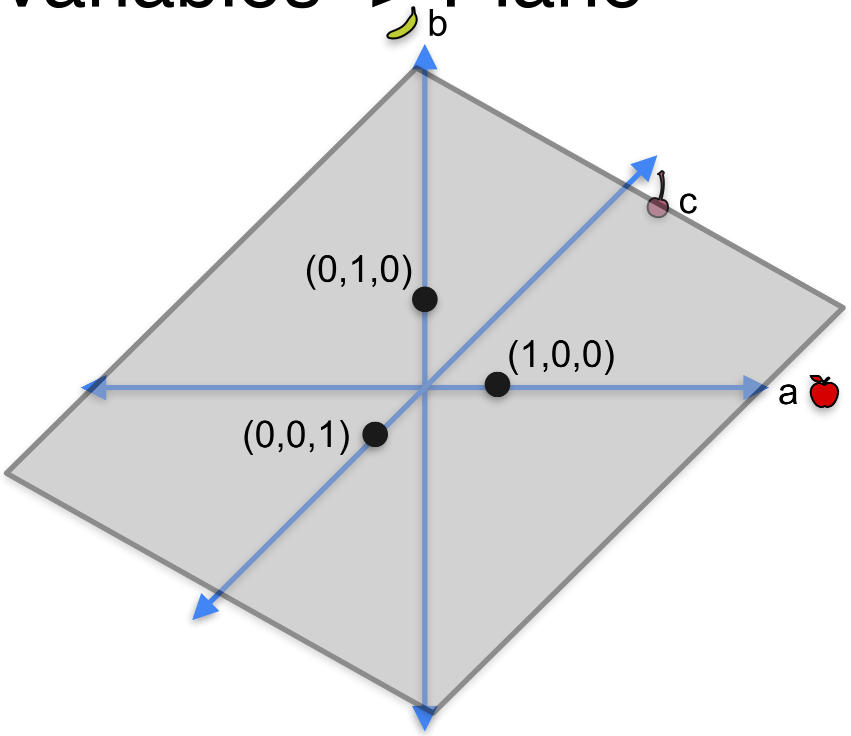
Linear equation in 3 variables -> Plane

$a + b + c = 1$

$1 + 0 + 0 = 1$

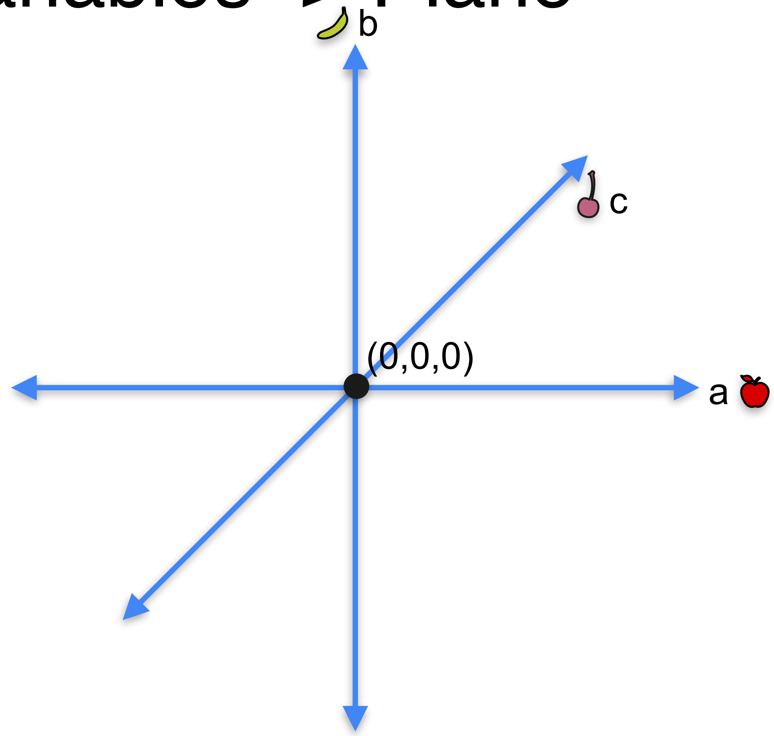
$0 + 1 + 0 = 1$

$0 + 0 + 1 = 1$



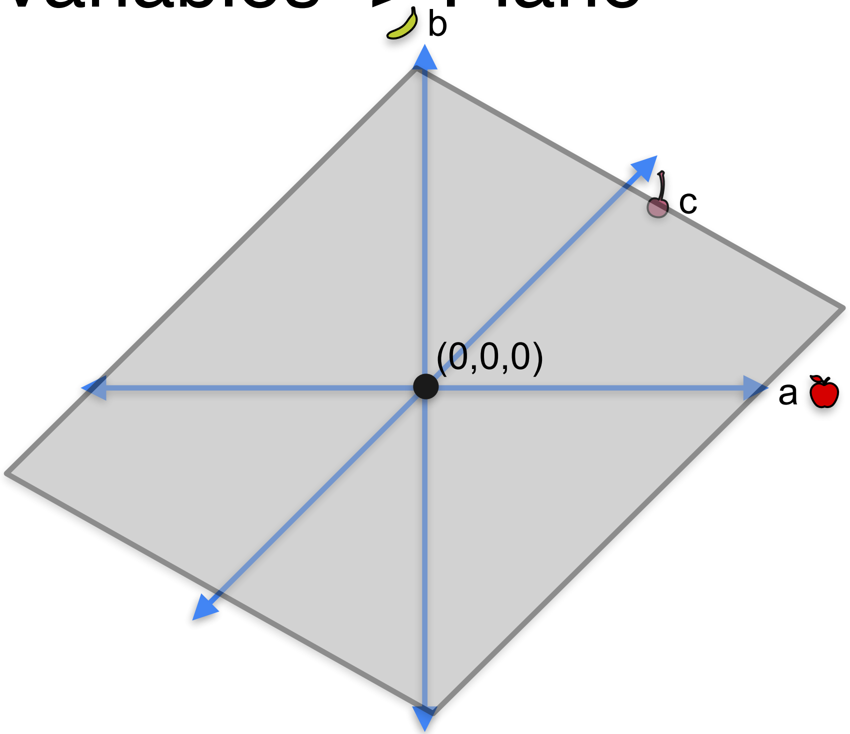
Linear equation in 3 variables -> Plane

$3a - 5b + 2c = \mathbf{0}$



Linear equation in 3 variables -> Plane

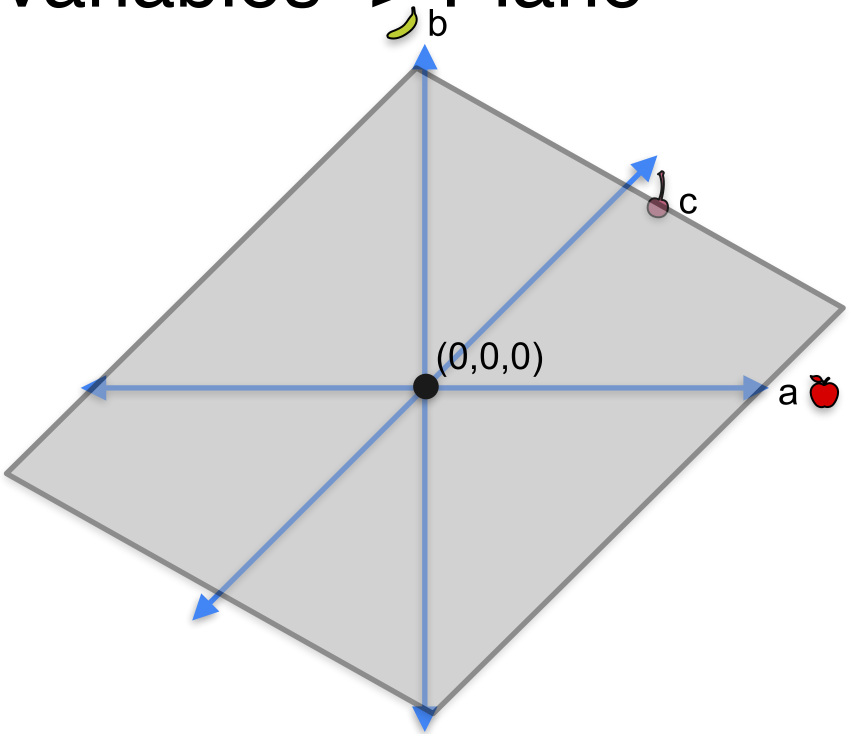
$3a - 5b + 2c = \mathbf{0}$



Linear equation in 3 variables -> Plane

3a - 5b + 2c = **0**

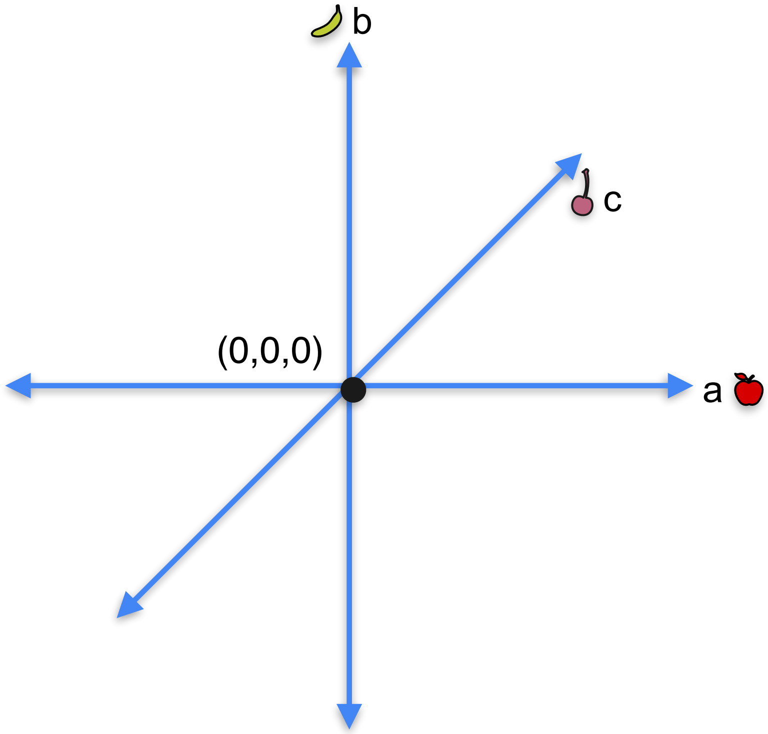
3(0) + 5(0) + 2(0) = **0**



System 1

System 1

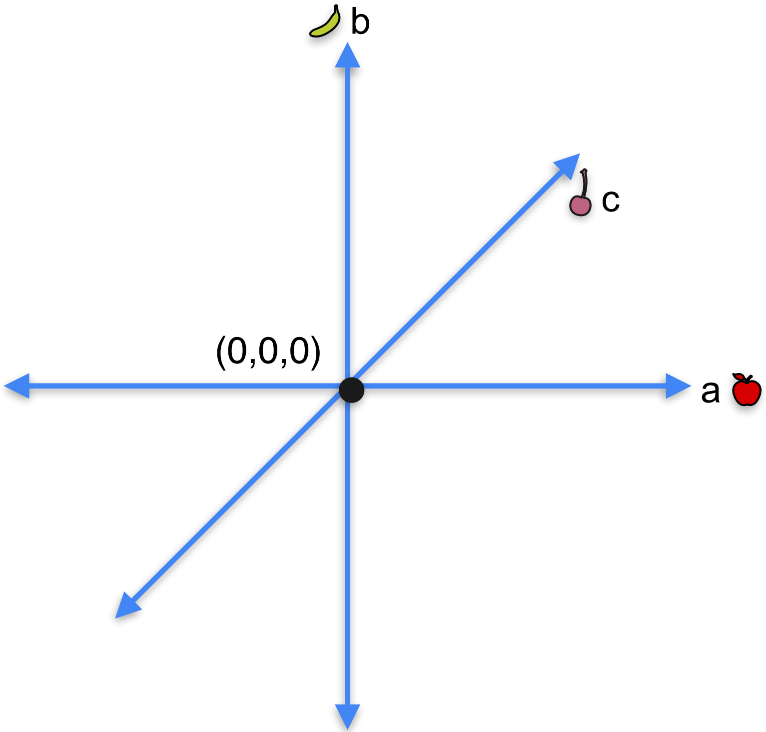
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

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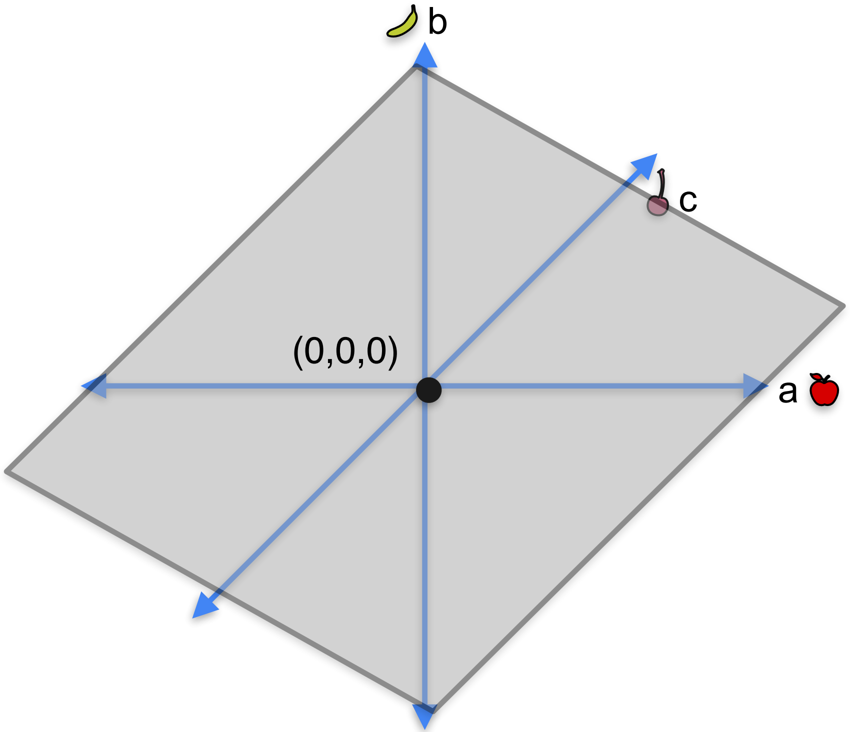

System 1

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
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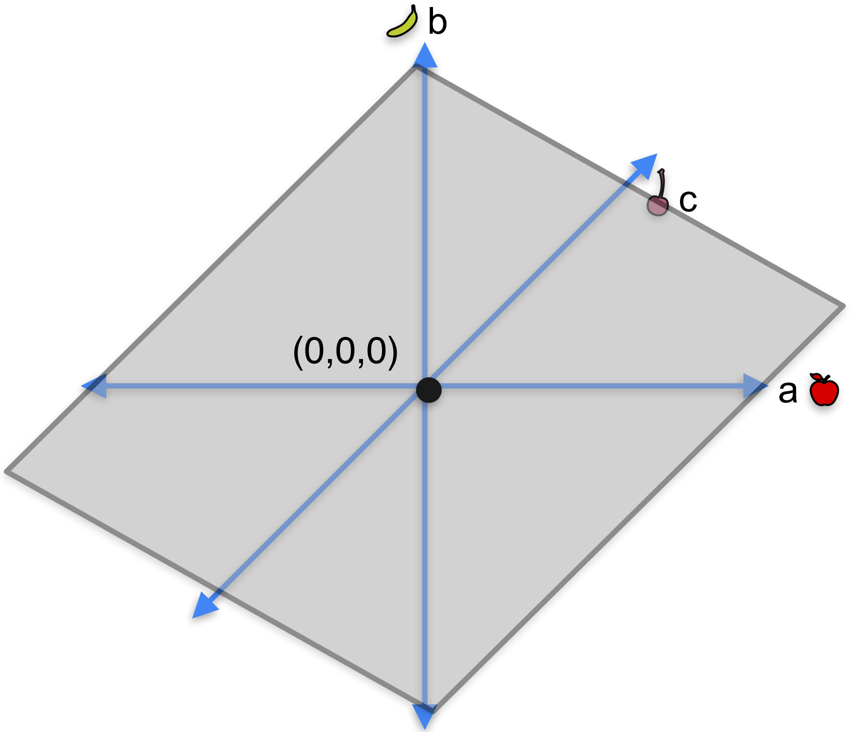
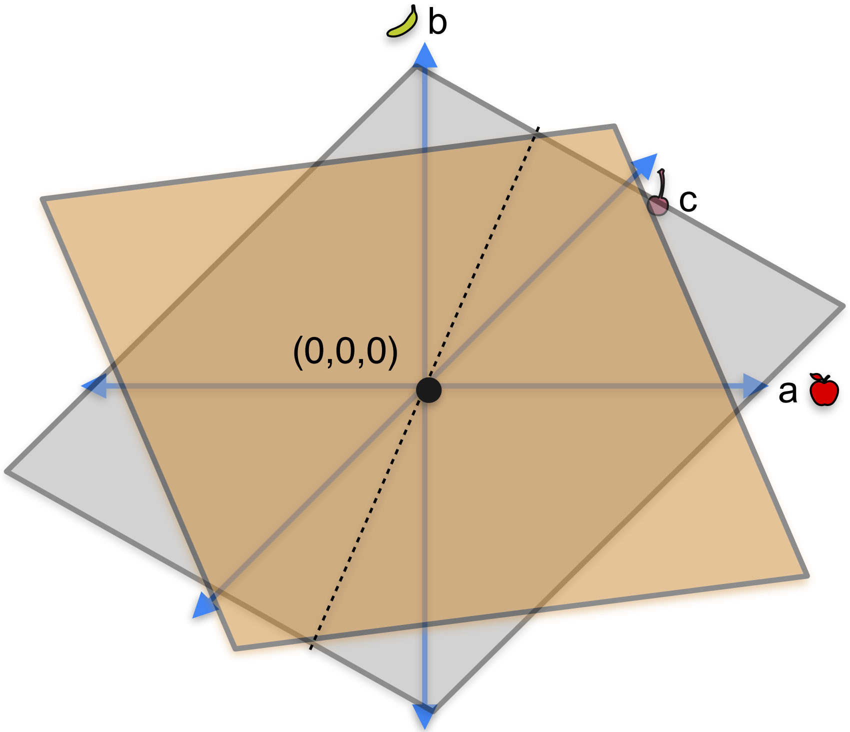


System 1

System 1

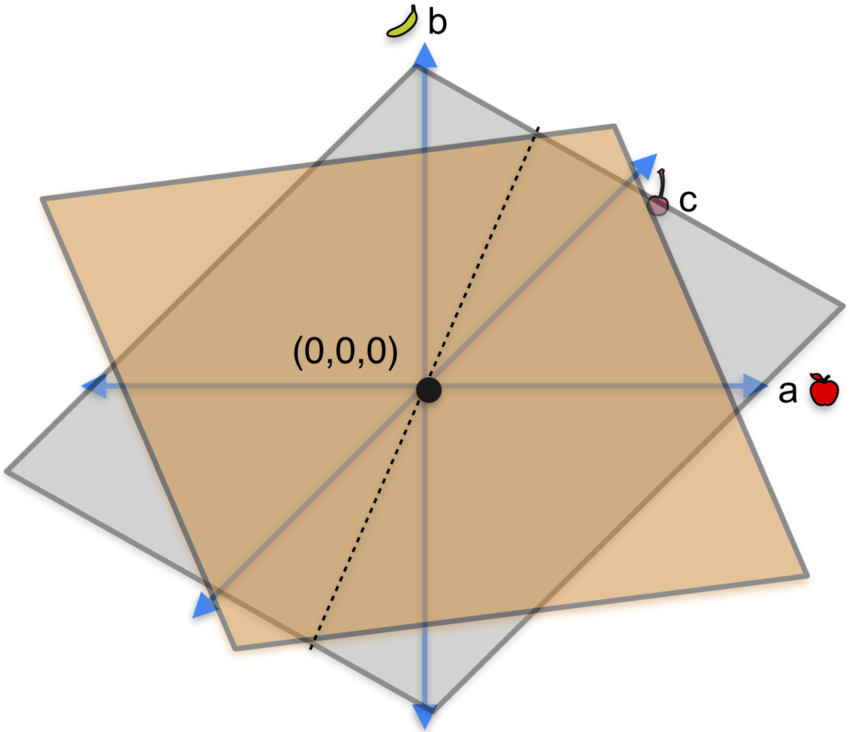
- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**



System 1

System 1

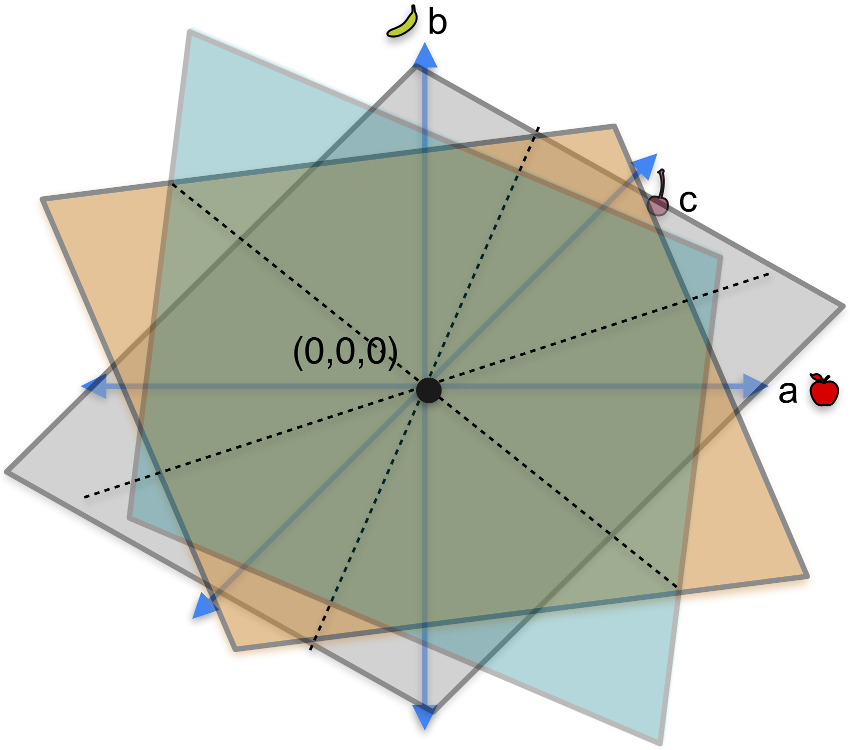
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**



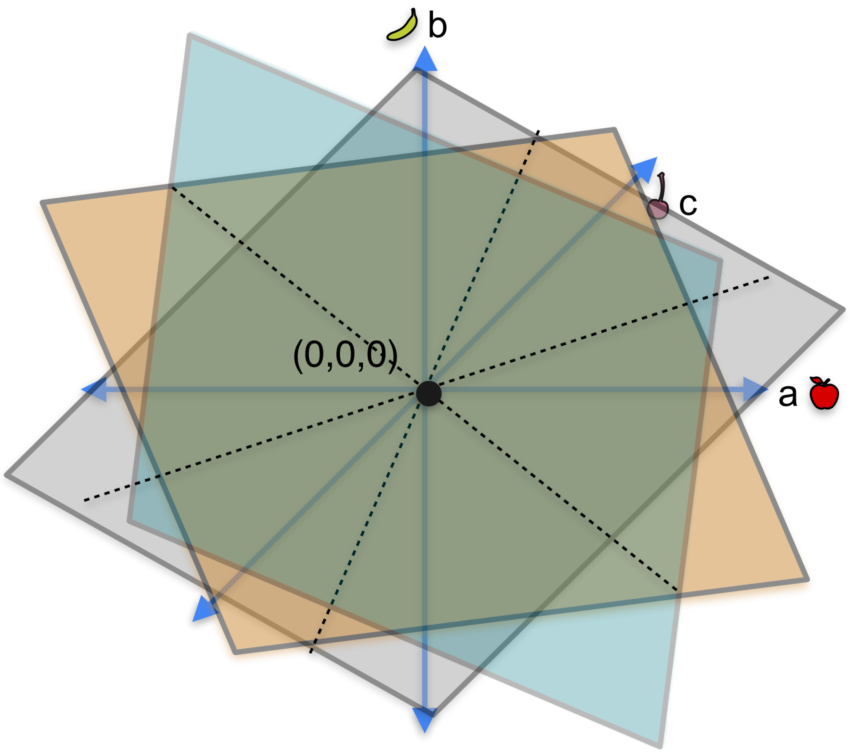
System 1

System 1

- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

Solution space

- $a = 0$
- $b = 0$
- $c = 0$



System 1

System 1

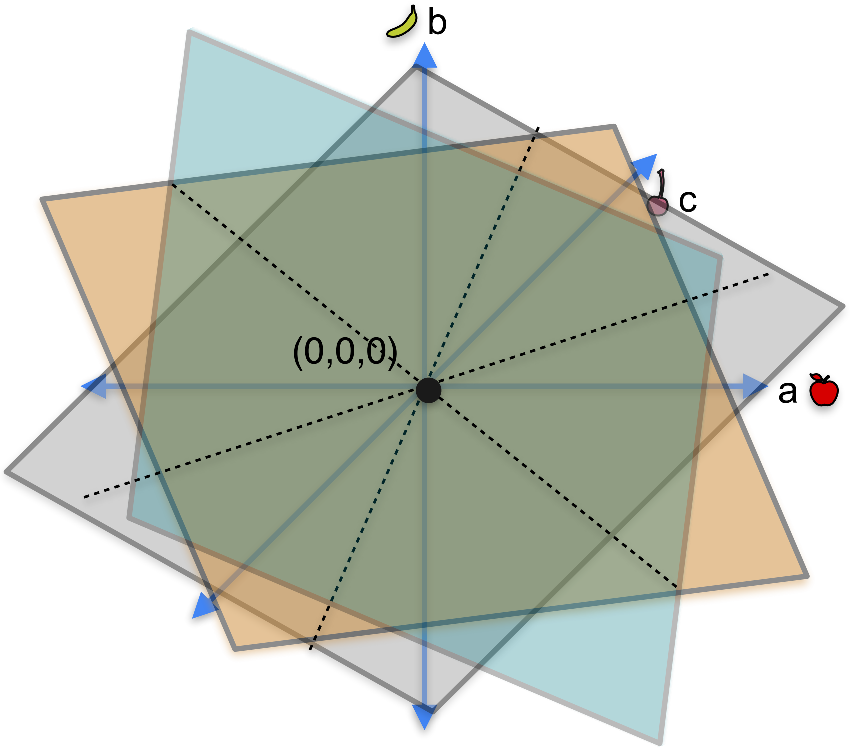
- a + b + c = **0**
- a + 2b + c = **0**
- a + b + 2c = **0**

Solution space

- $a = 0$
- $b = 0$
- $c = 0$

The point

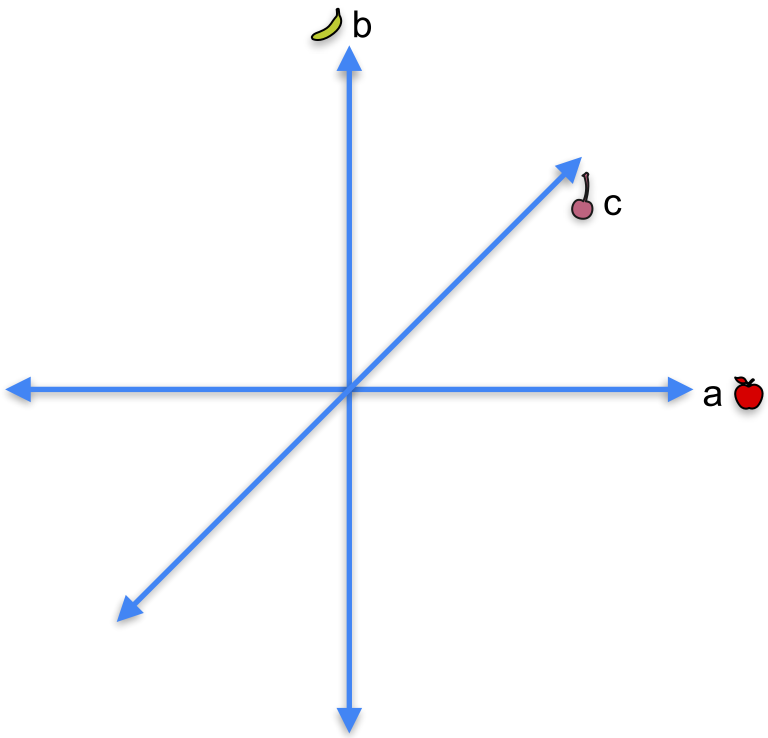
$(0,0,0)$



System 2

System 2

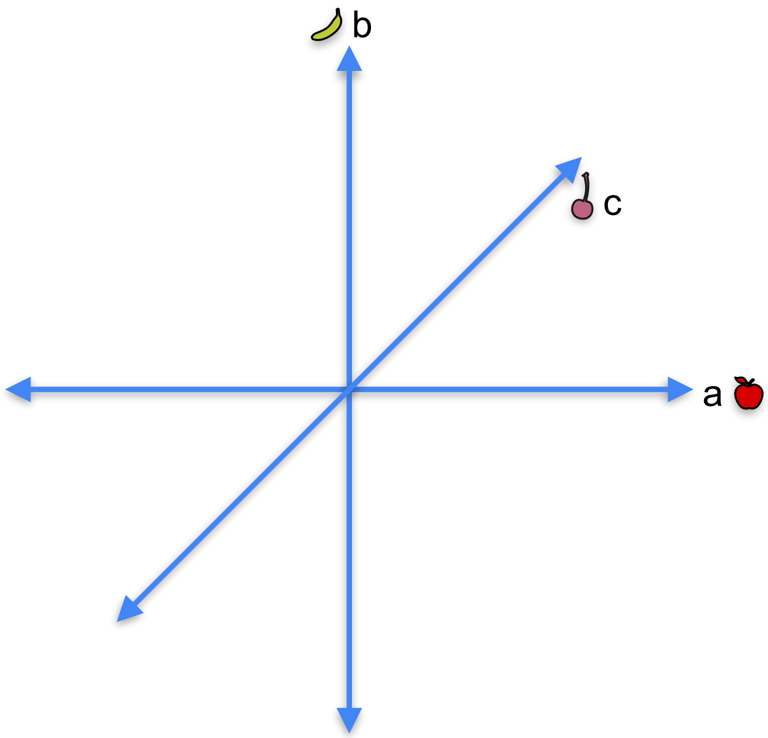
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



System 2

System 2

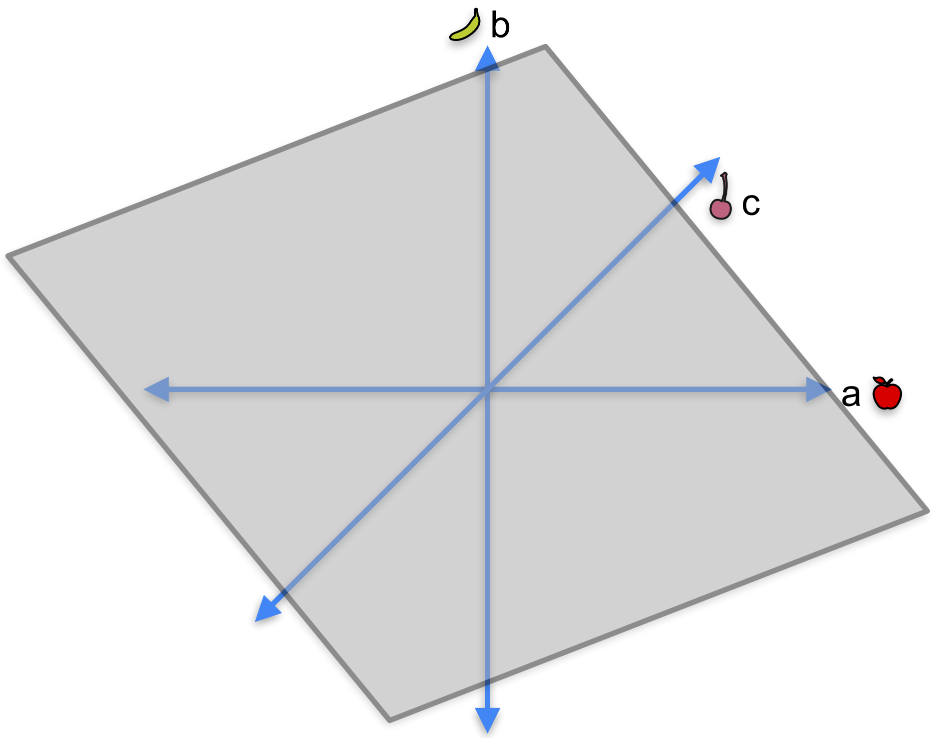
- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



System 2

System 2

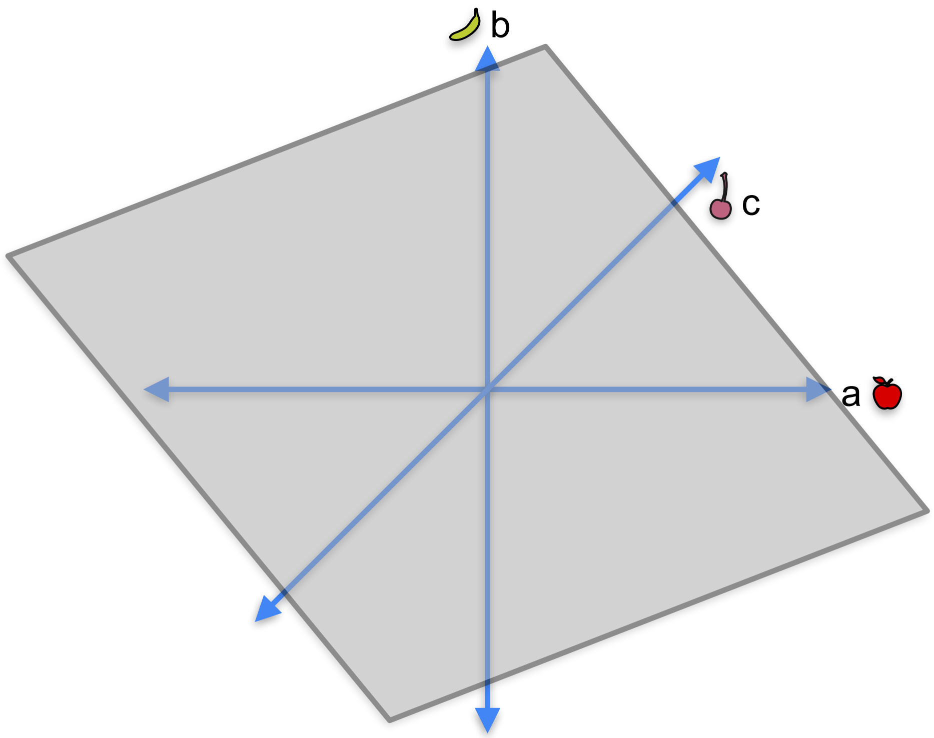
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



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System 2

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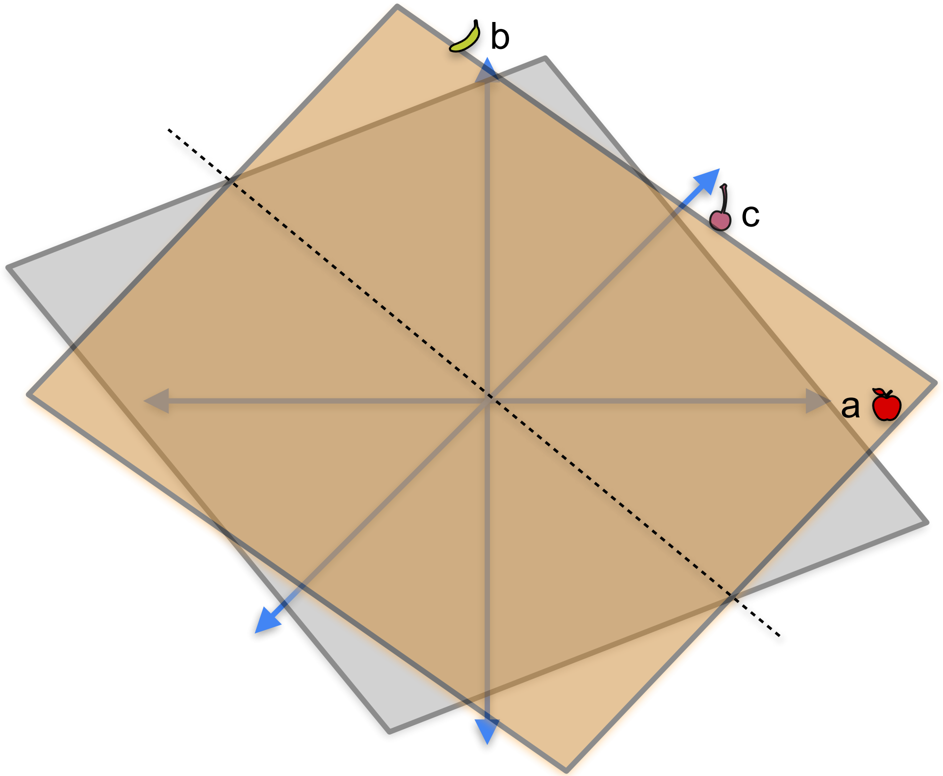

System 2

System 2

- $a + b + c = \mathbf{0}$
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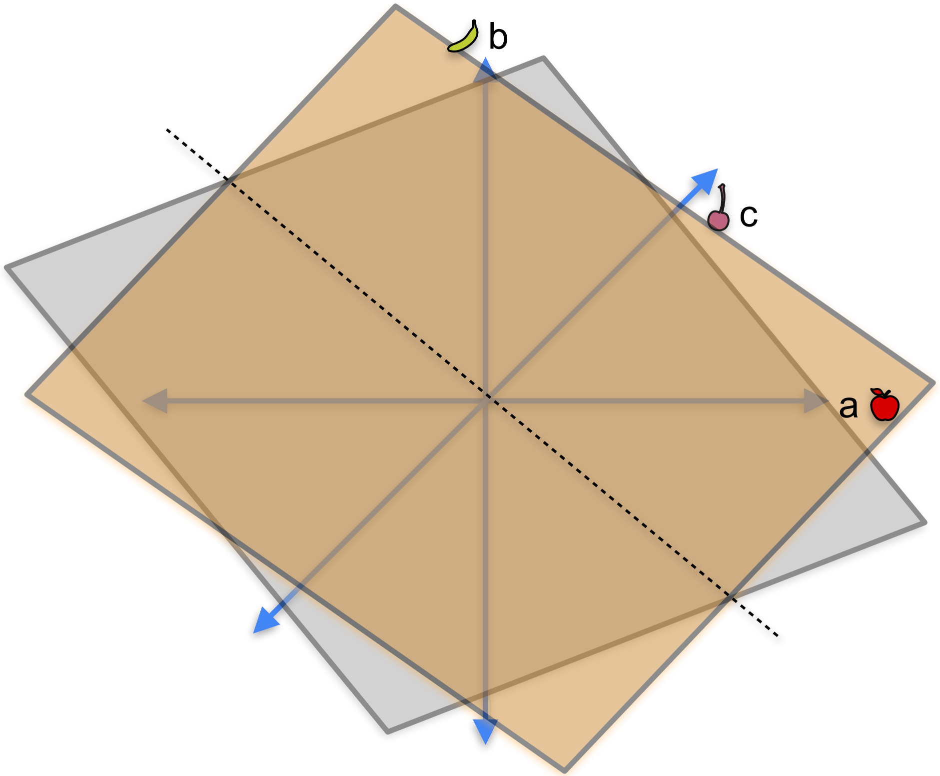
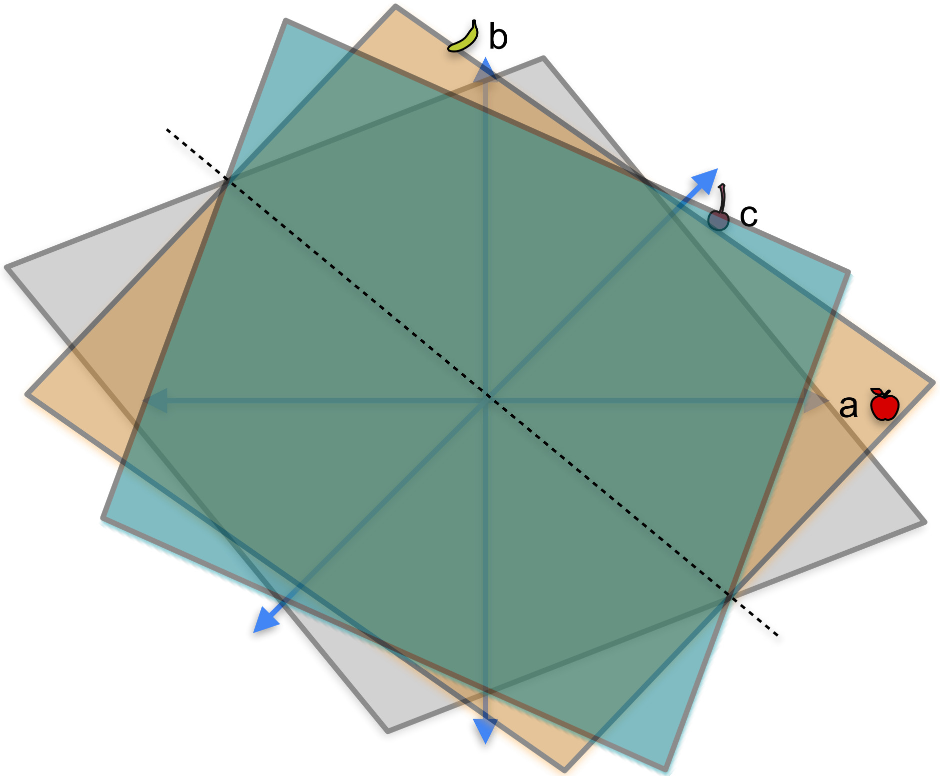
- $a + b + 3c = \mathbf{0}$



System 2

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



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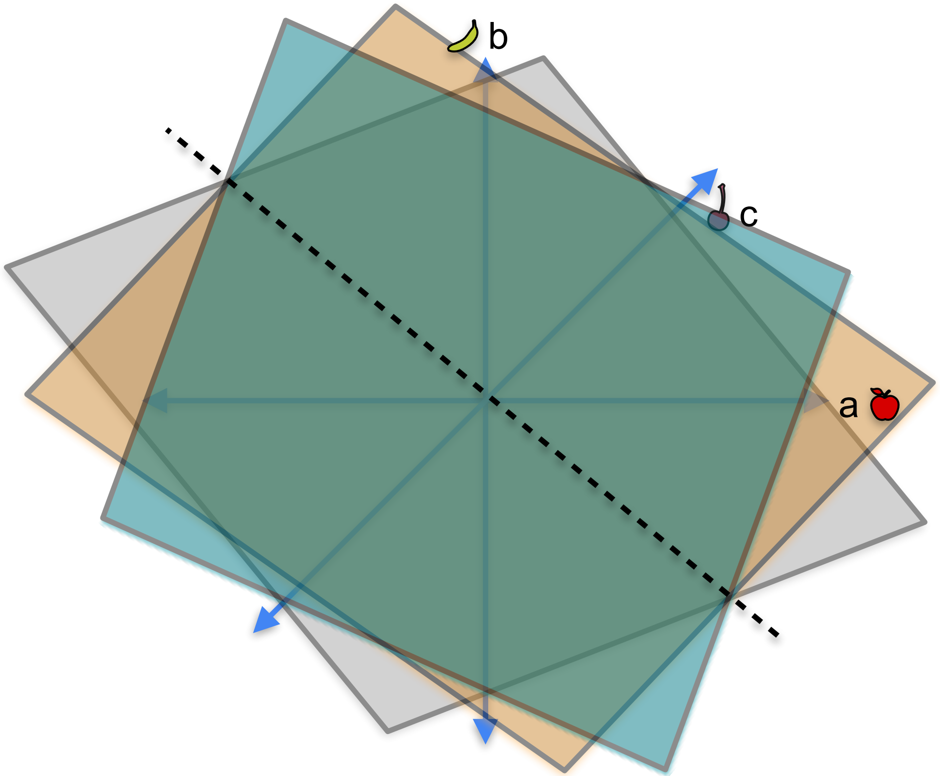
System 2

System 2

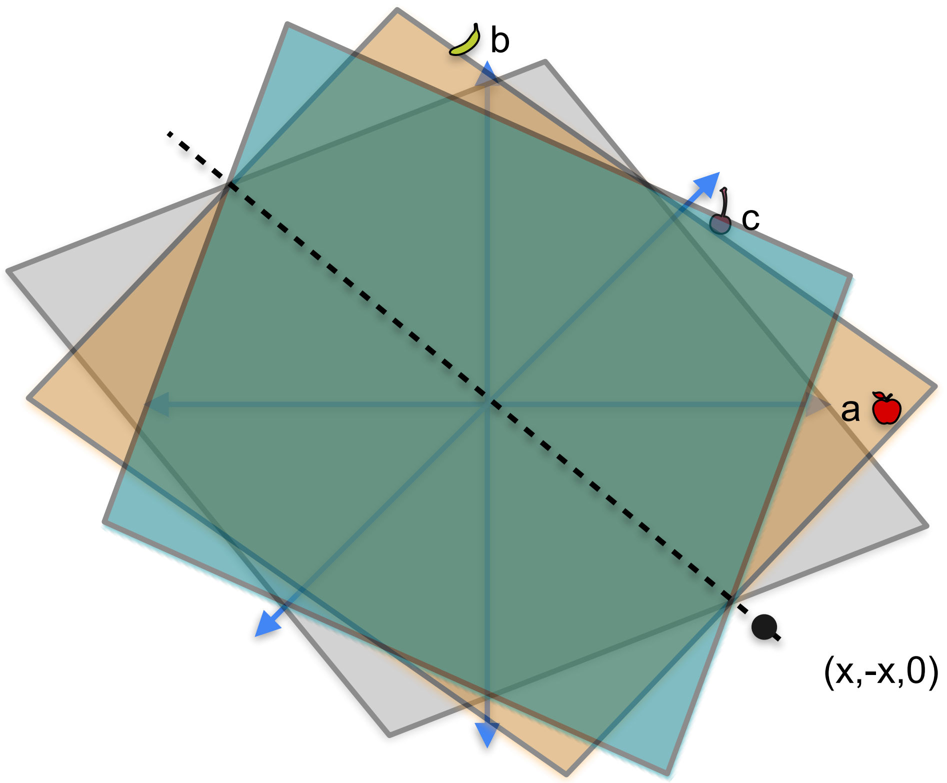
- a + b + c = 0
- a + b + 2c = 0
- a + b + 3c = 0



System 2

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**



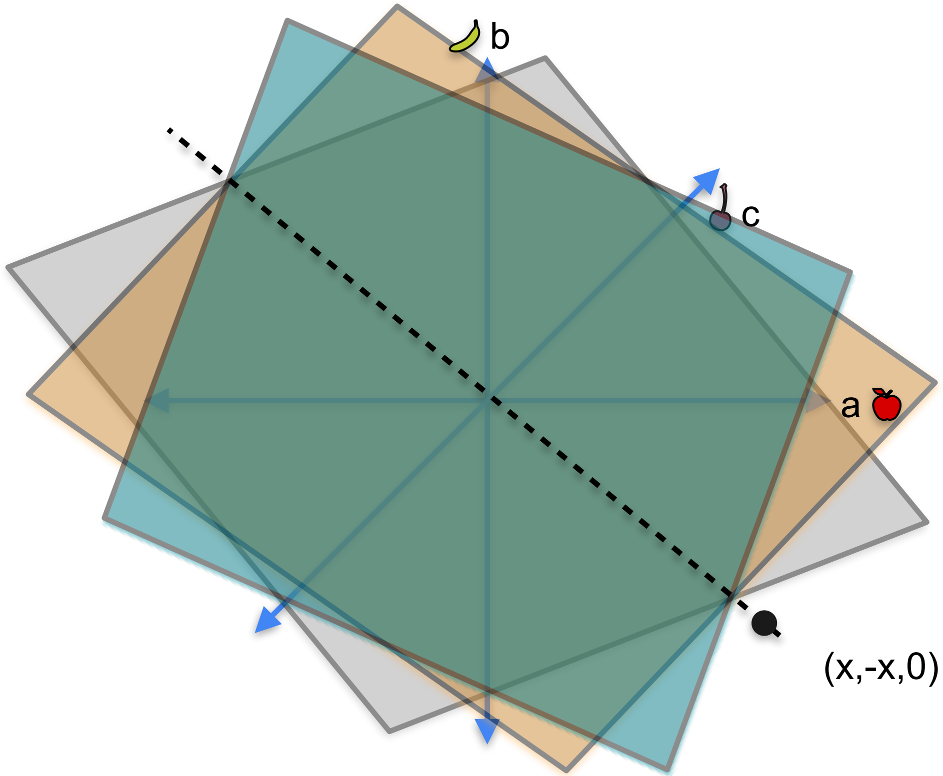
System 2

System 2

- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

Solution space

- $c = 0$
- $b = -a$



System 2

System 2

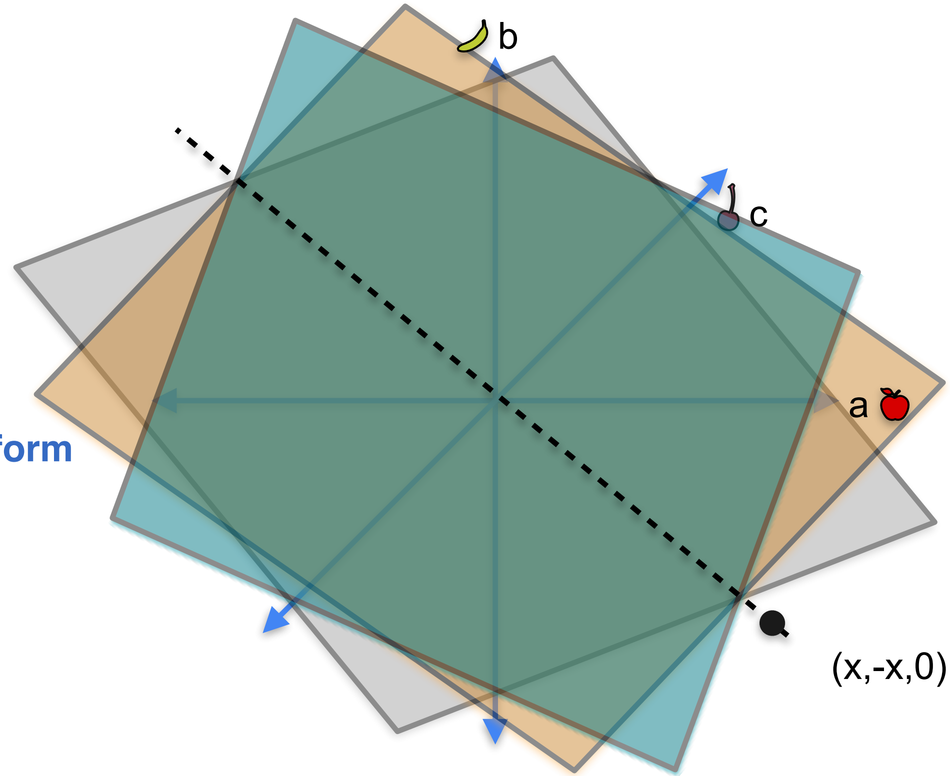
- a + b + c = **0**
- a + b + 2c = **0**
- a + b + 3c = **0**

Solution space

- $c = 0$
- $b = -a$

All points of the form

$(x, -x, 0)$



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System 2

- a + b + c = **0**
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Solution space

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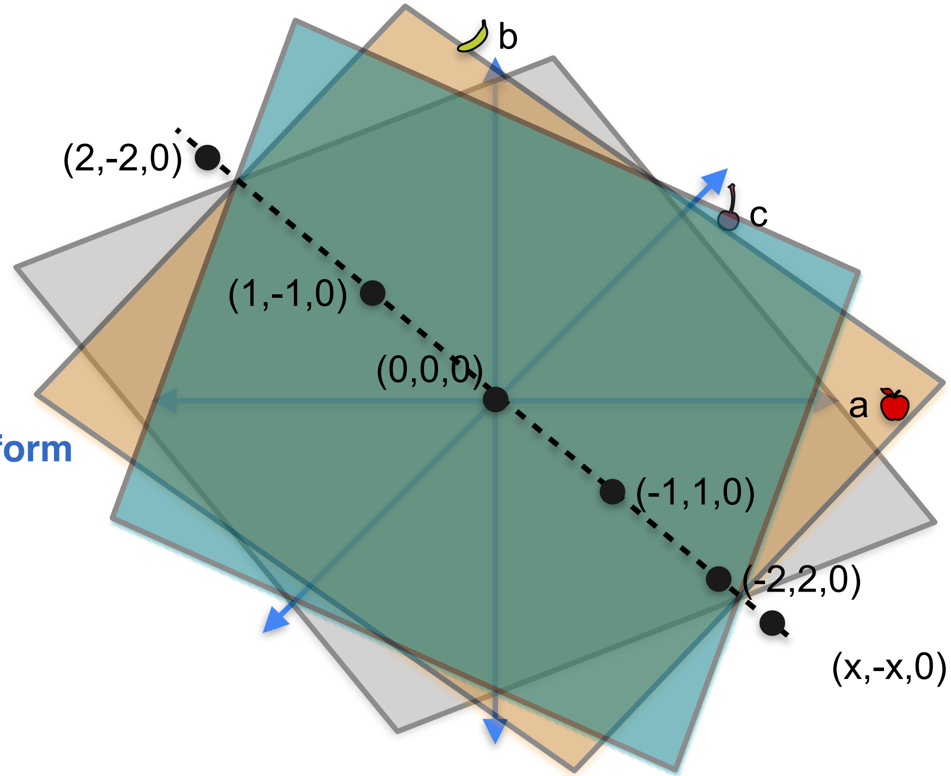
All points of the form

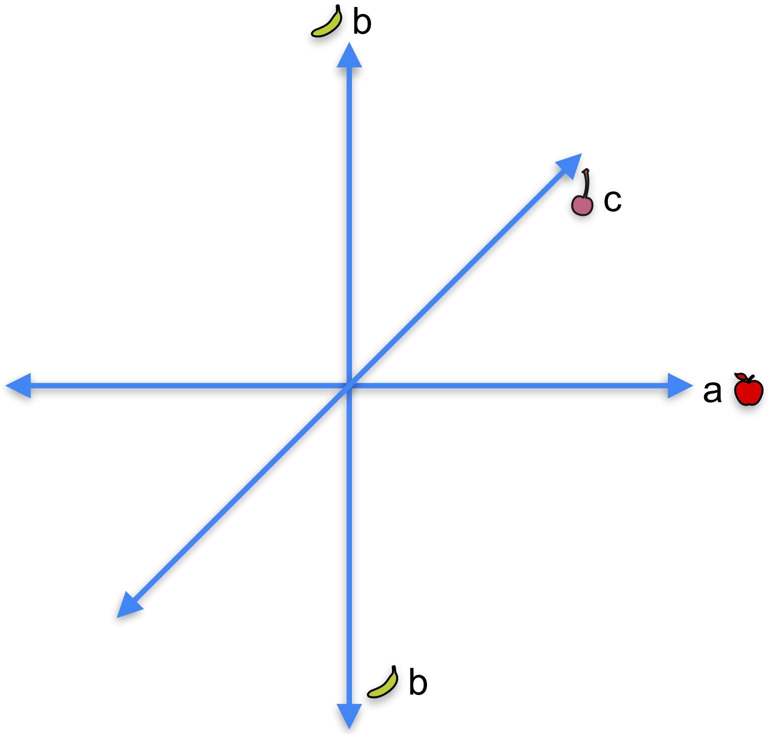
$(x, -x, 0)$



System 3

System 3

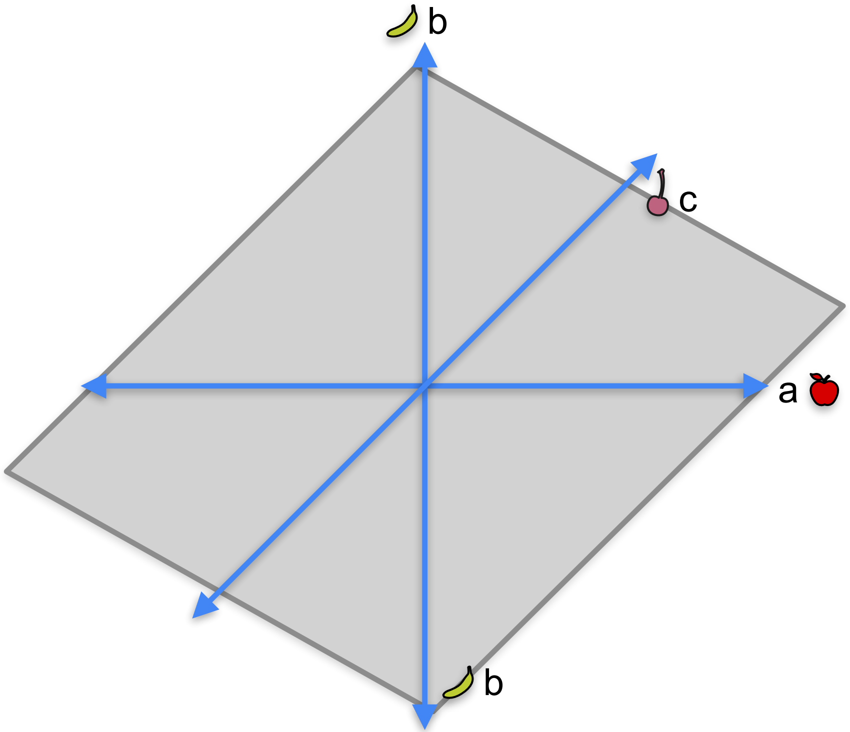
- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**



System 3

System 3

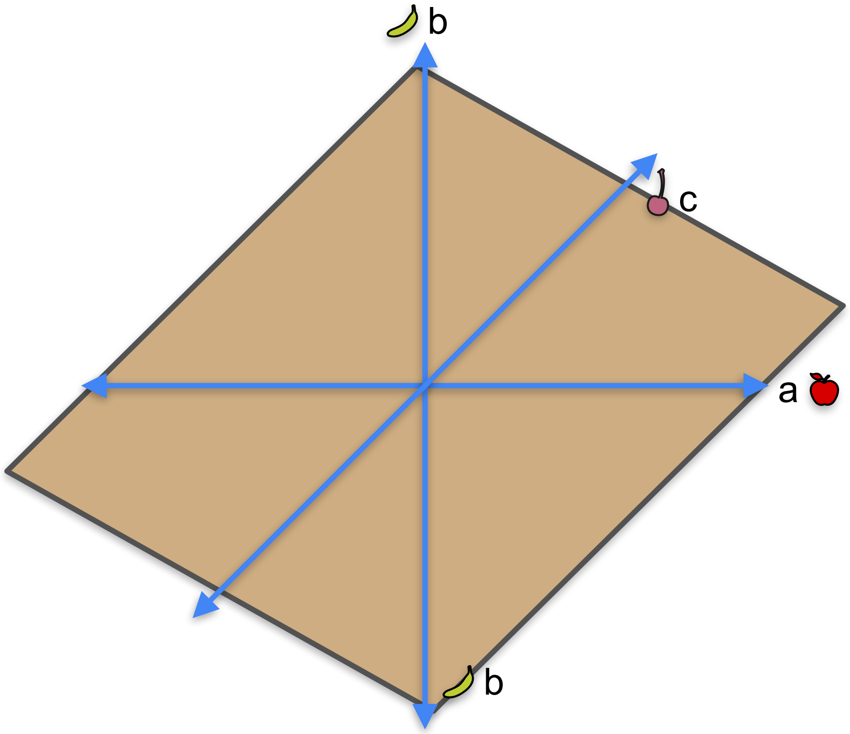
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

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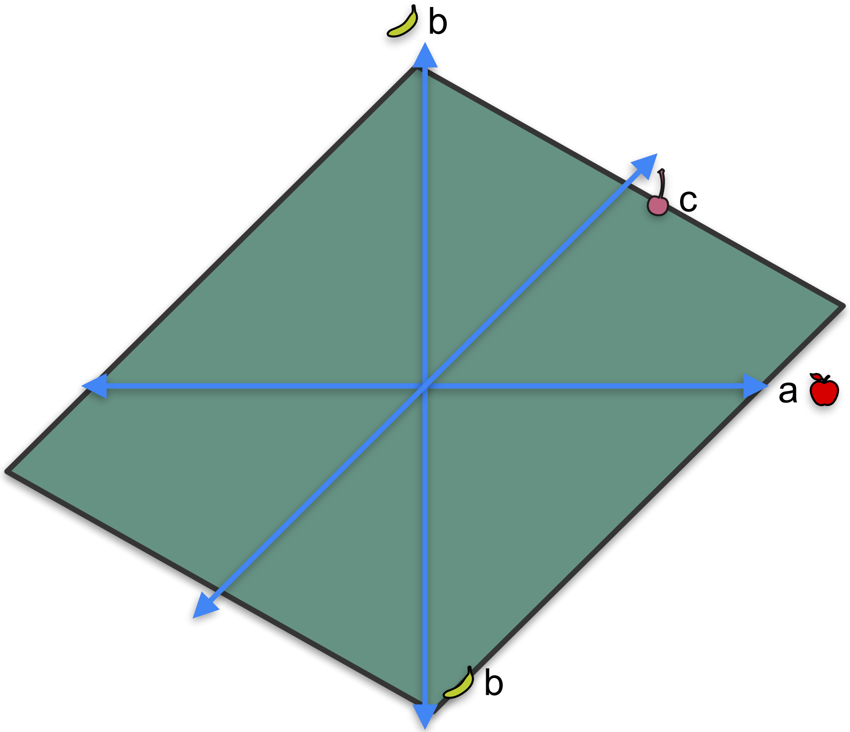

System 3

System 3

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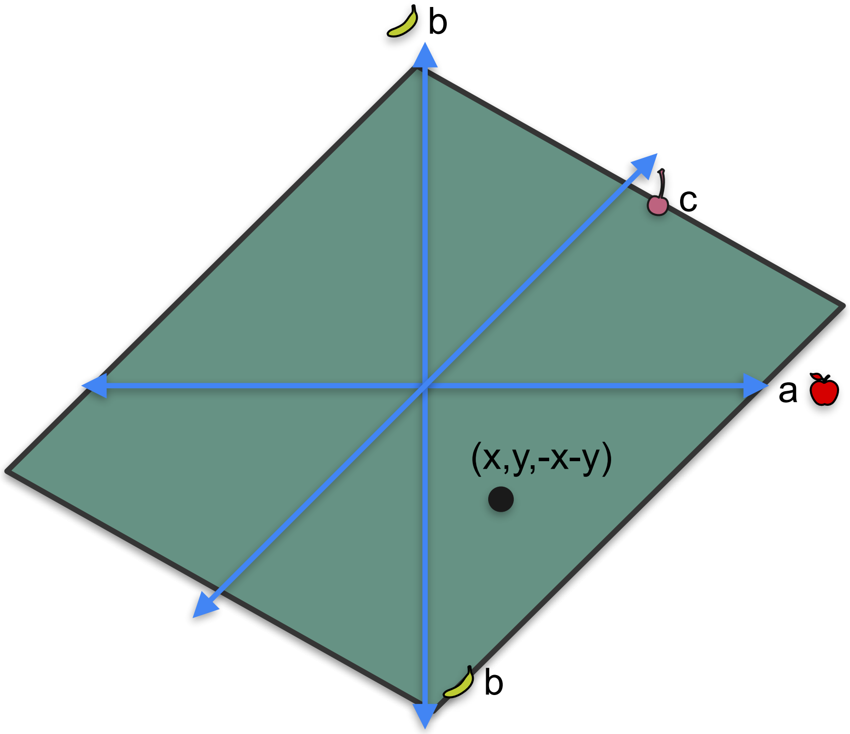


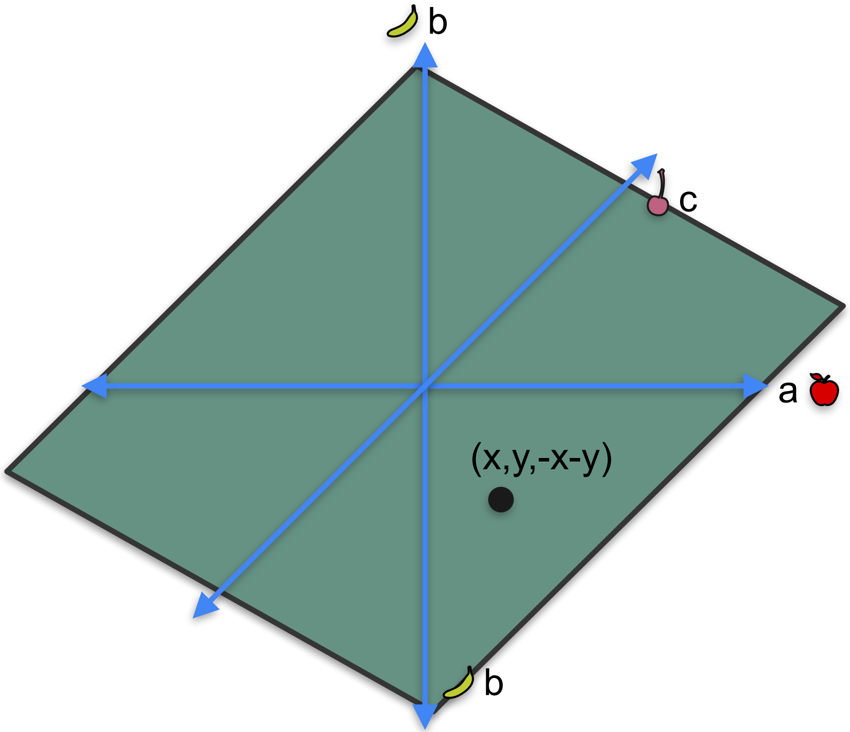
System 3

System 3

- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

Solution space

- $a + b + c = 0$



System 3

System 3

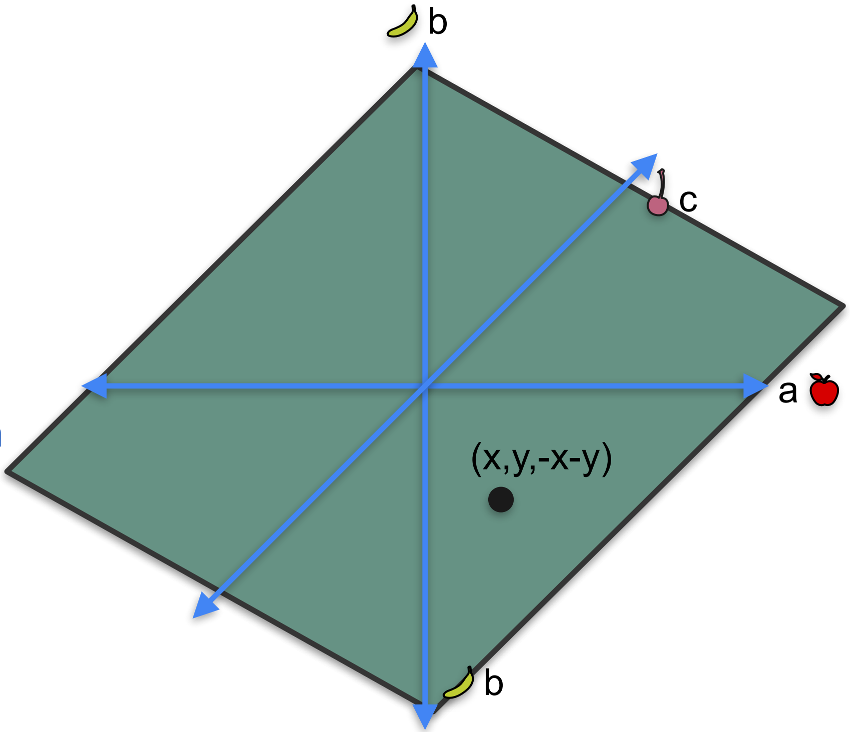
- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

Solution space

- $a + b + c = 0$

All points of the form

$(x, y, -x - y)$



System 3

System 3

- a + b + c = **0**
- 2a + 2b + 2c = **0**
- 3a + 3b + 3c = **0**

Solution space

- $a + b + c = 0$

All points of the form

$(x, y, -x - y)$