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Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations



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Vectors and Linear Transformations

Machine Learning motivation

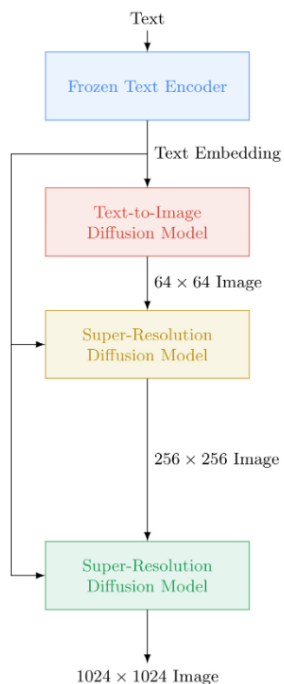
Neural Networks - AI generated images



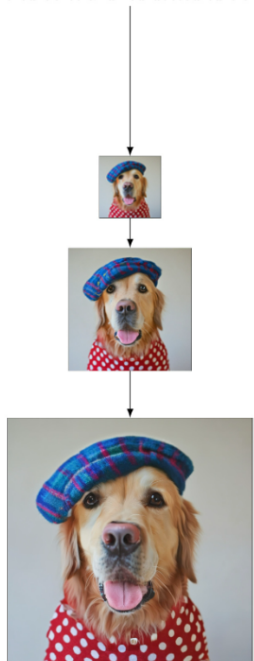
AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation



“A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck.”



The screenshot shows a user interface for a text-to-image generation model. The 'Input' field contains the text "wall clock - wall clock." and the 'Model' dropdown is set to "Model". The output area shows a generated image of a round wall clock with a black frame and white face. The output is labeled "Output" and has a duration of "3.0s".

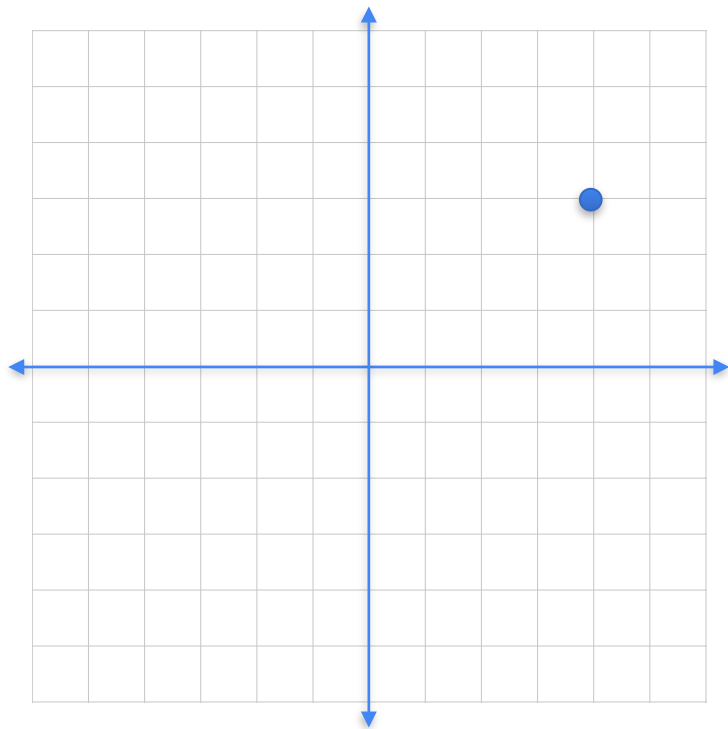


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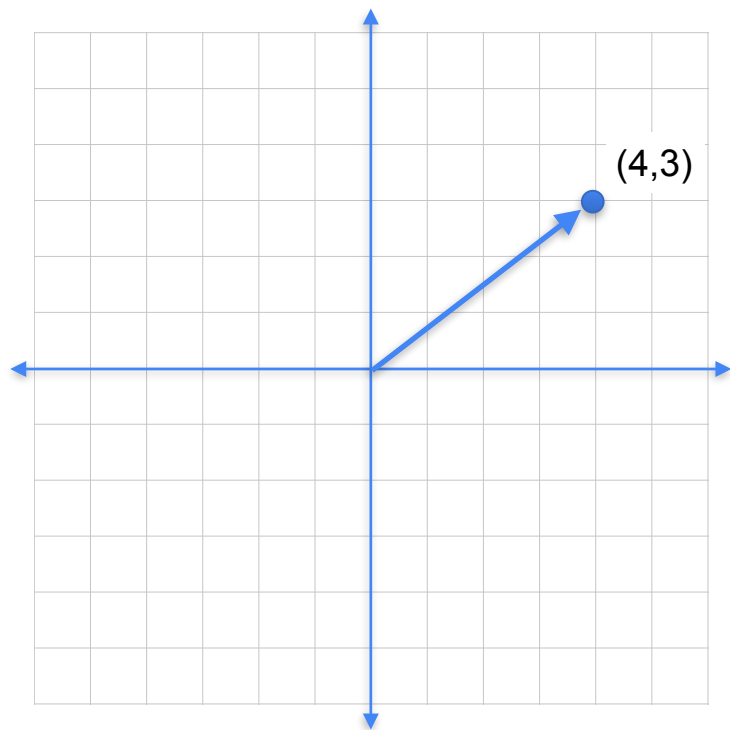
Vectors and Linear Transformations

Vectors and their properties

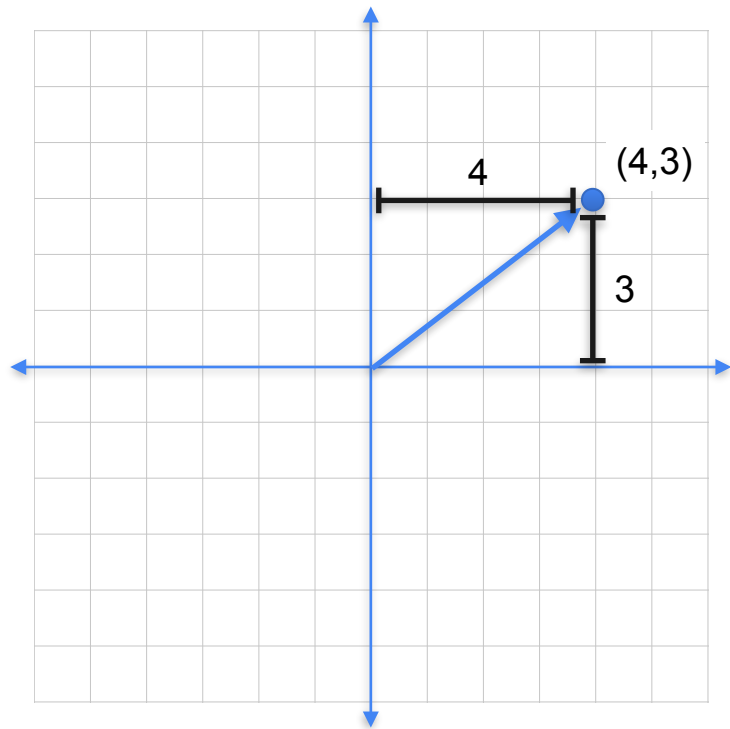
Vectors



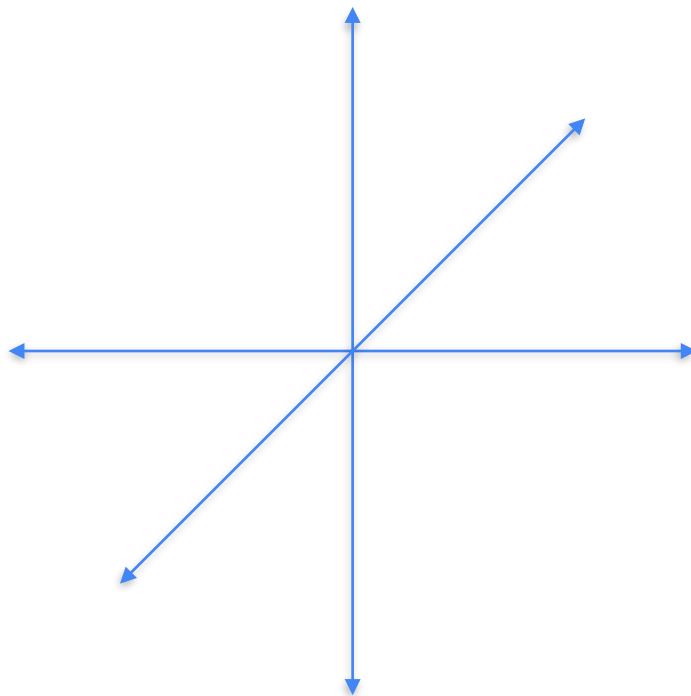
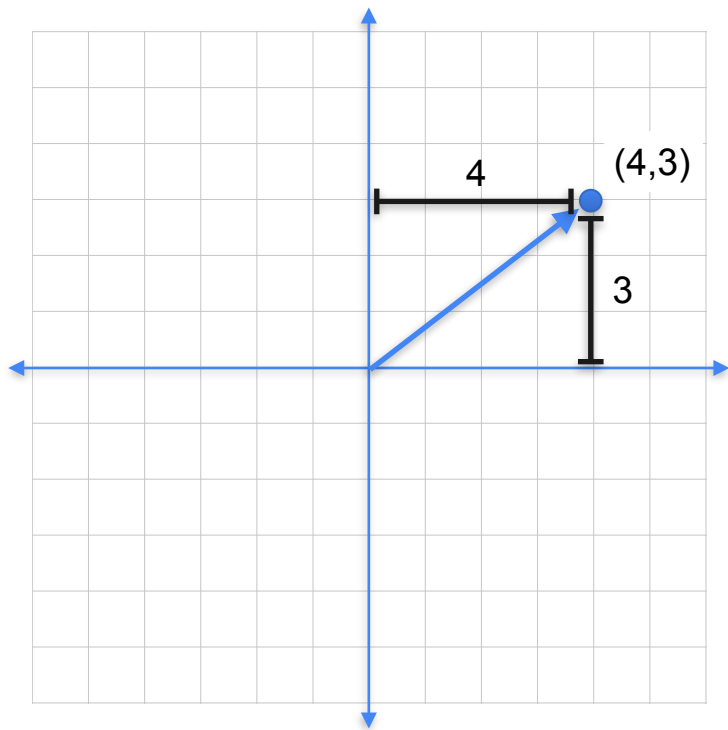
Vectors



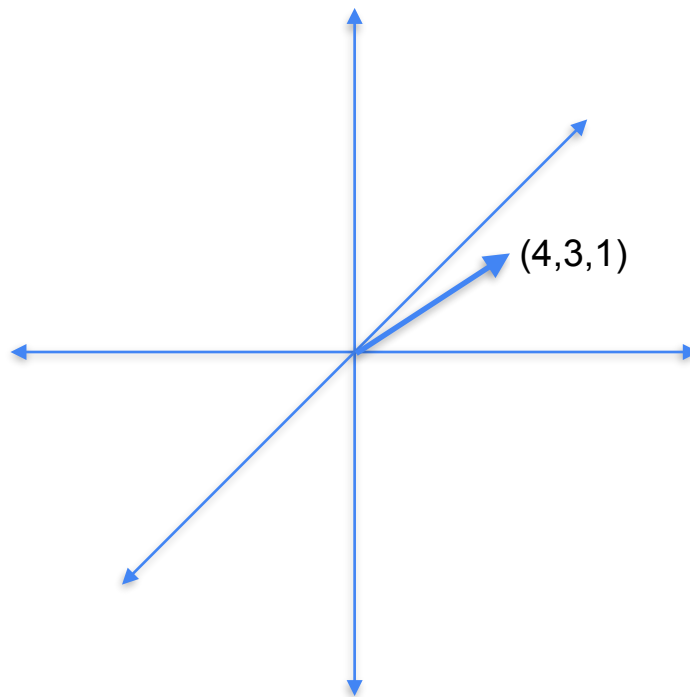
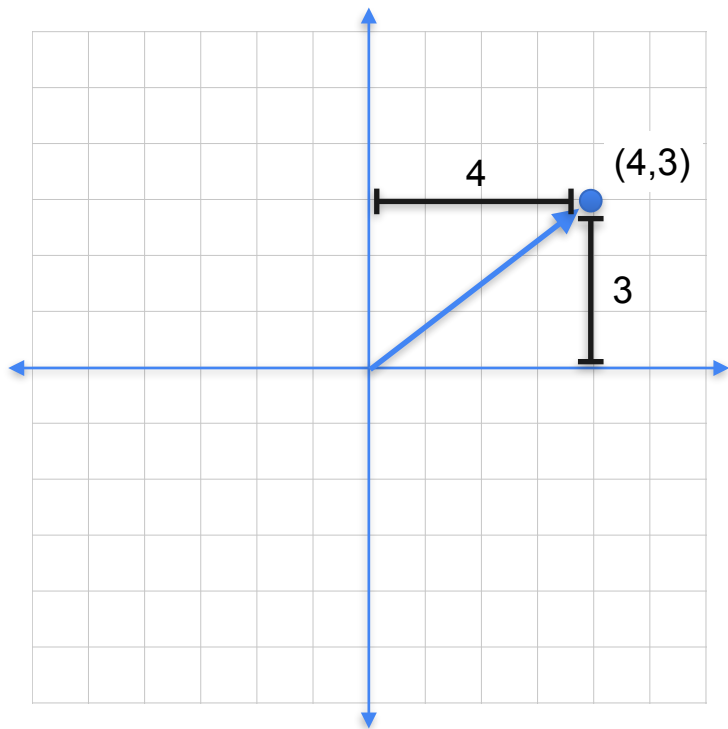
Vectors



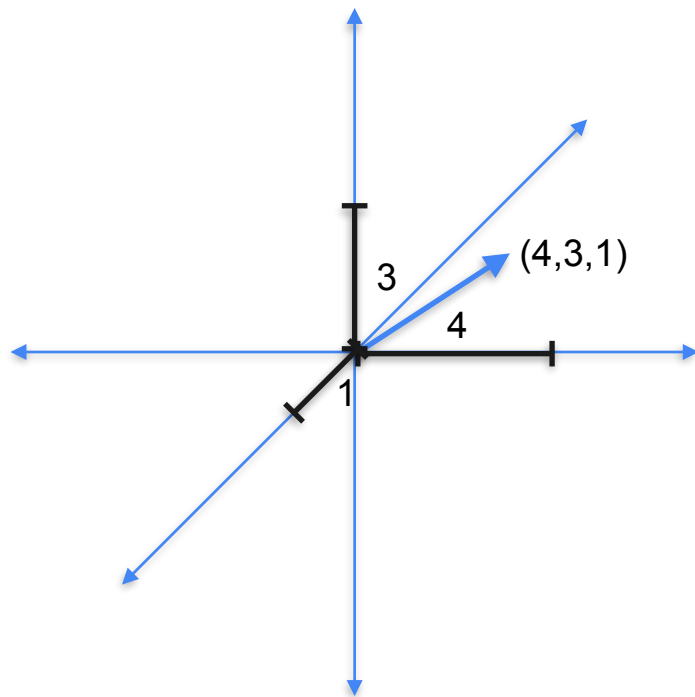
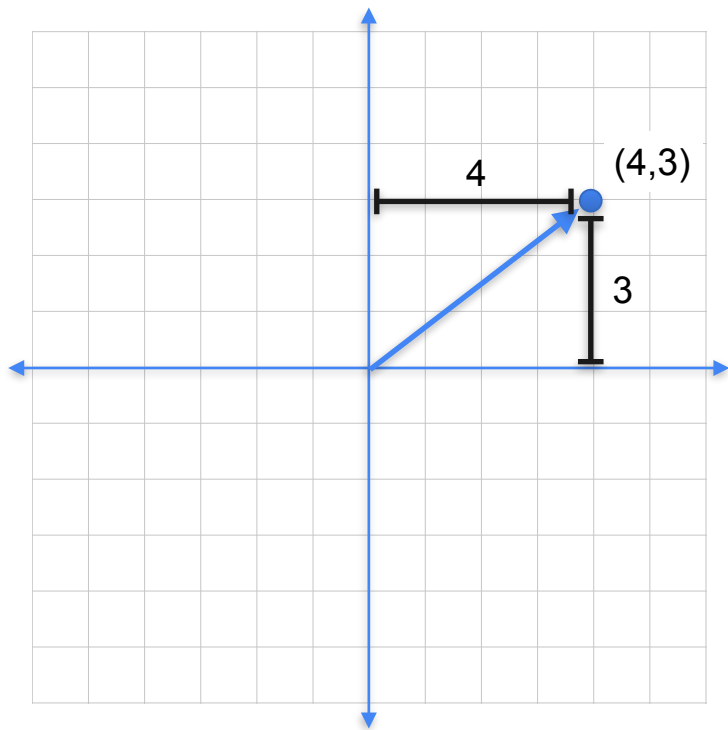
Vectors



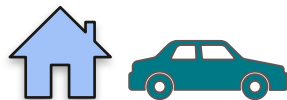
Vectors



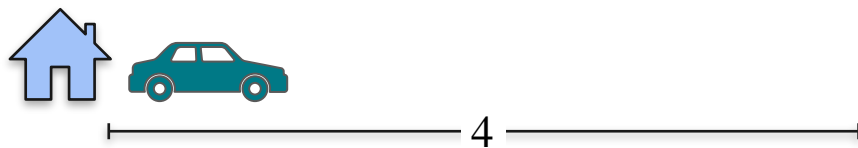
Vectors



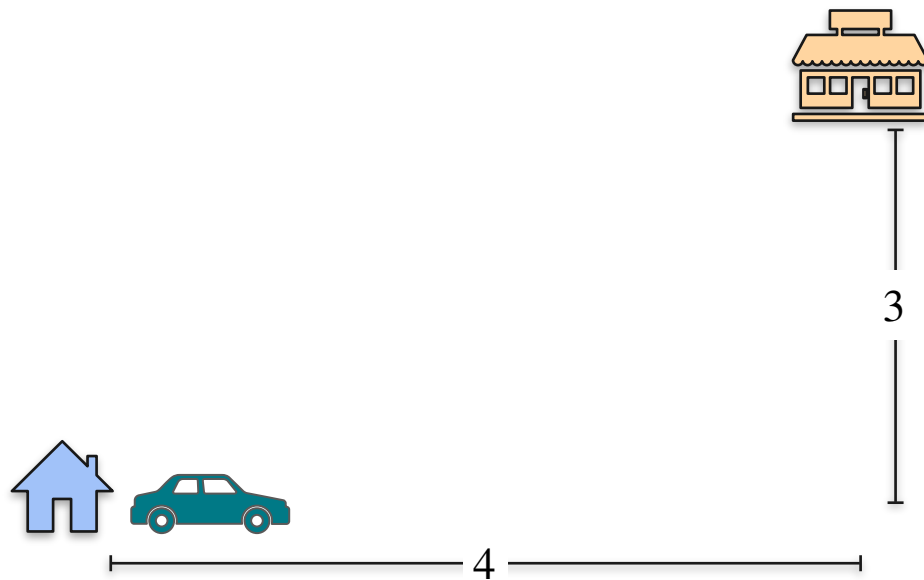
How to get from point A to point B?



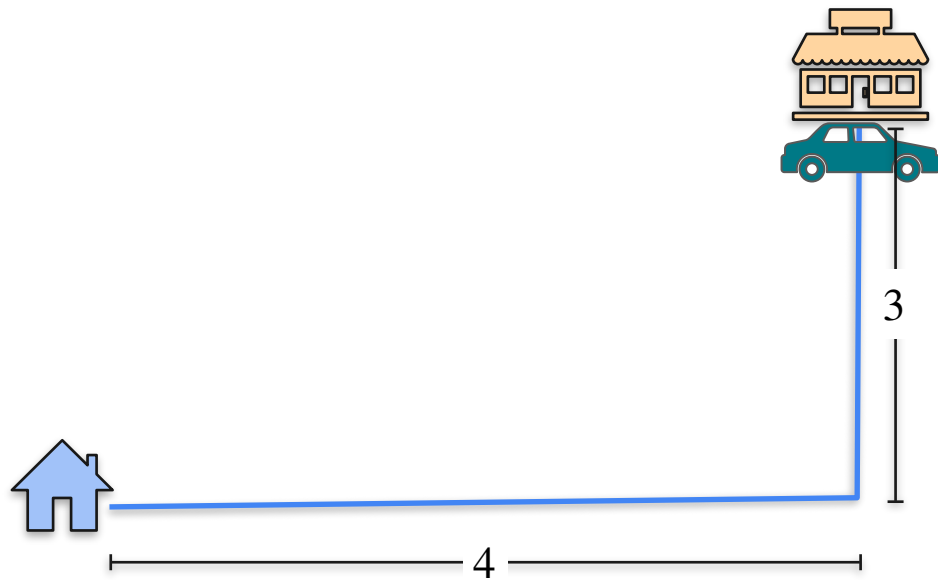
How to get from point A to point B?



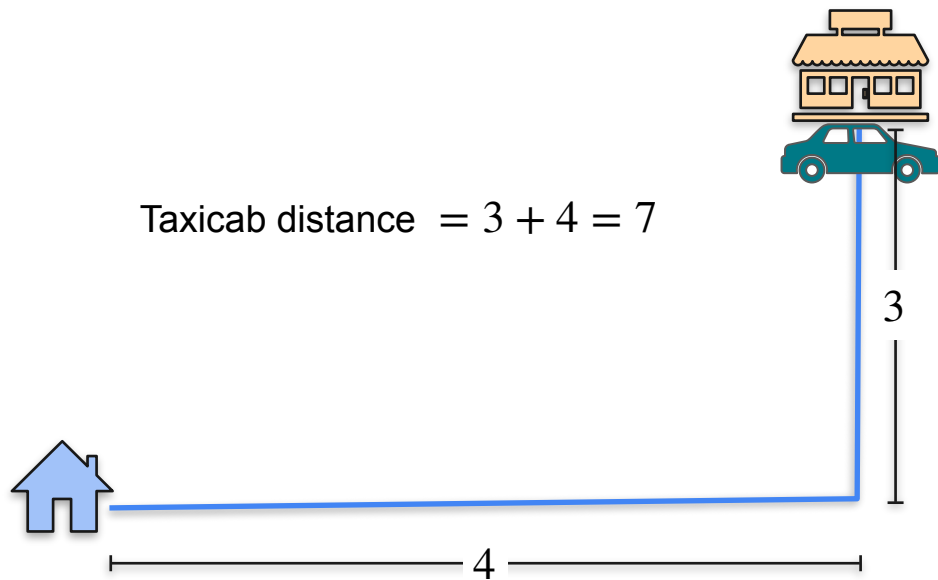
How to get from point A to point B?



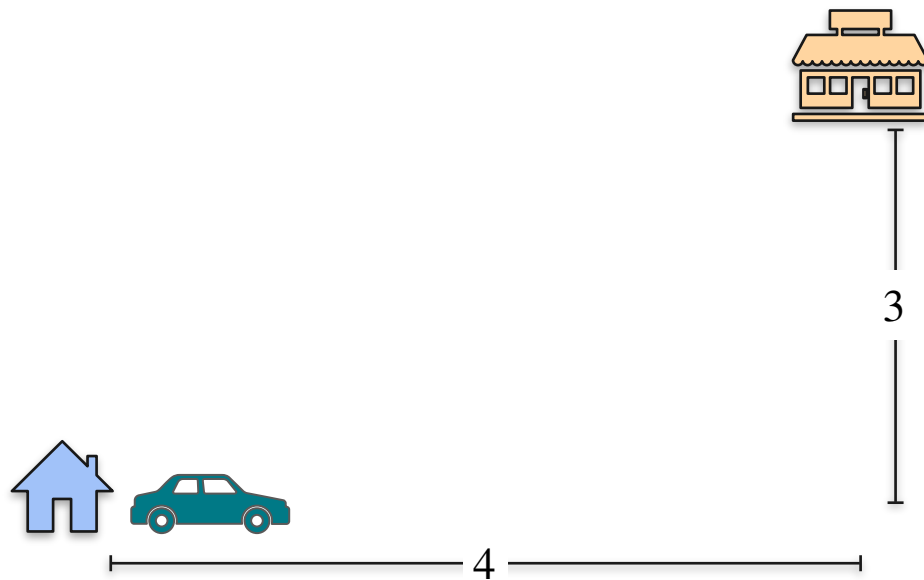
How to get from point A to point B?



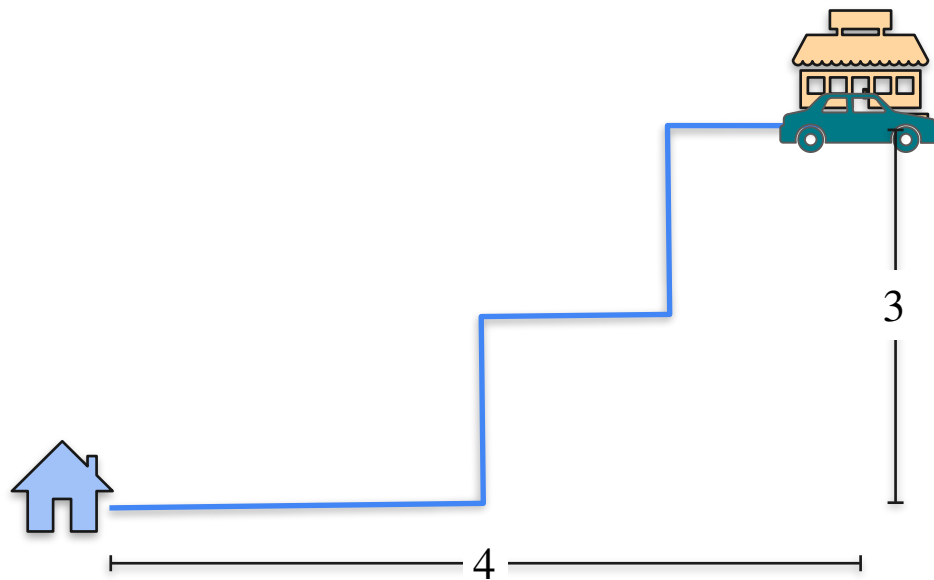
How to get from point A to point B?



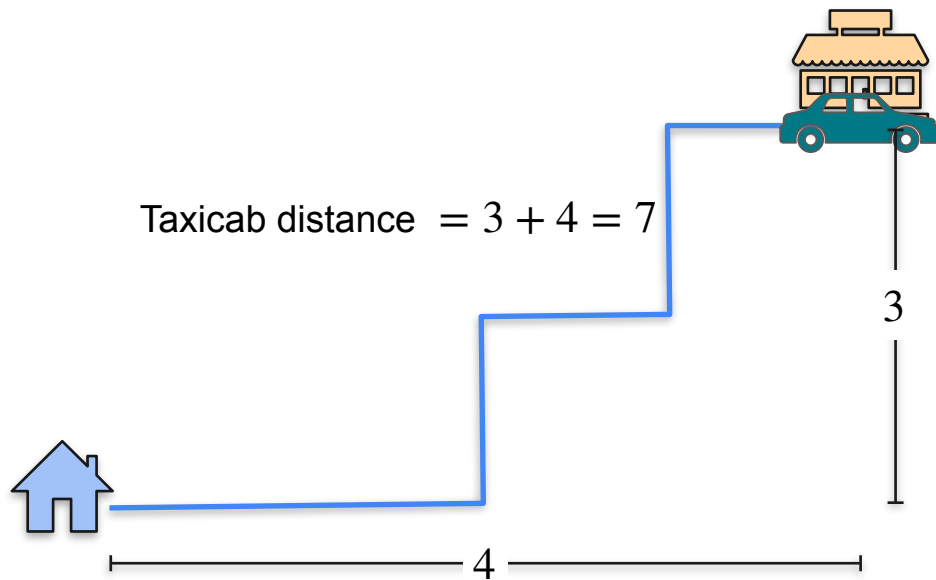
How to get from point A to point B?



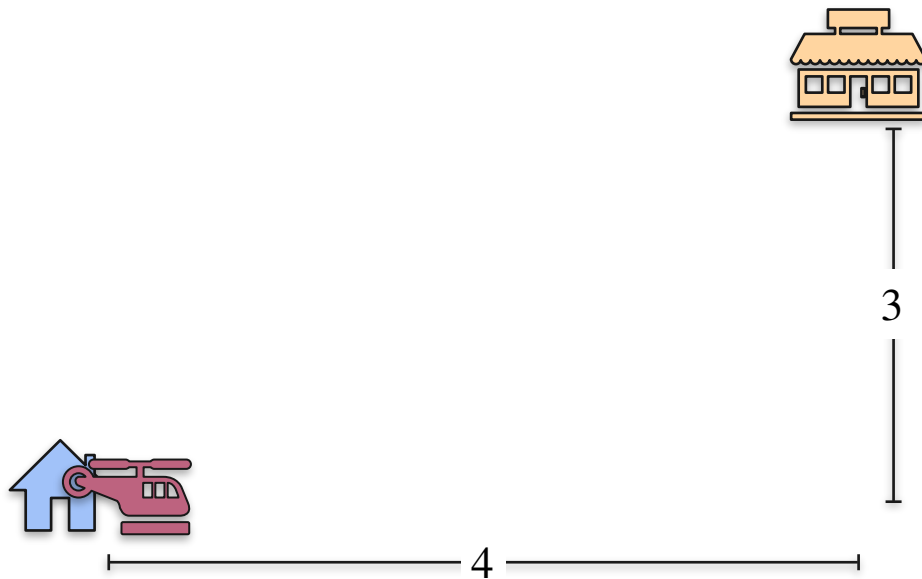
How to get from point A to point B?



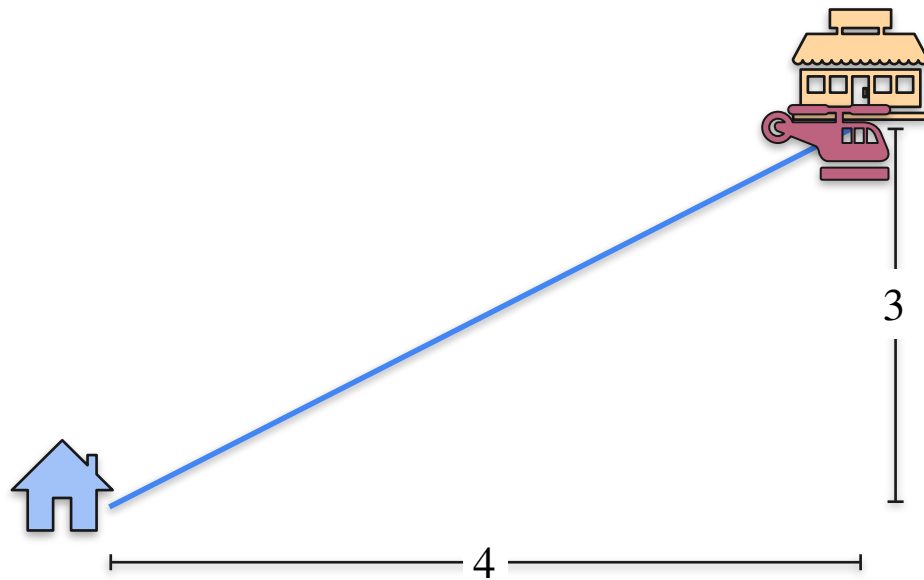
How to get from point A to point B?



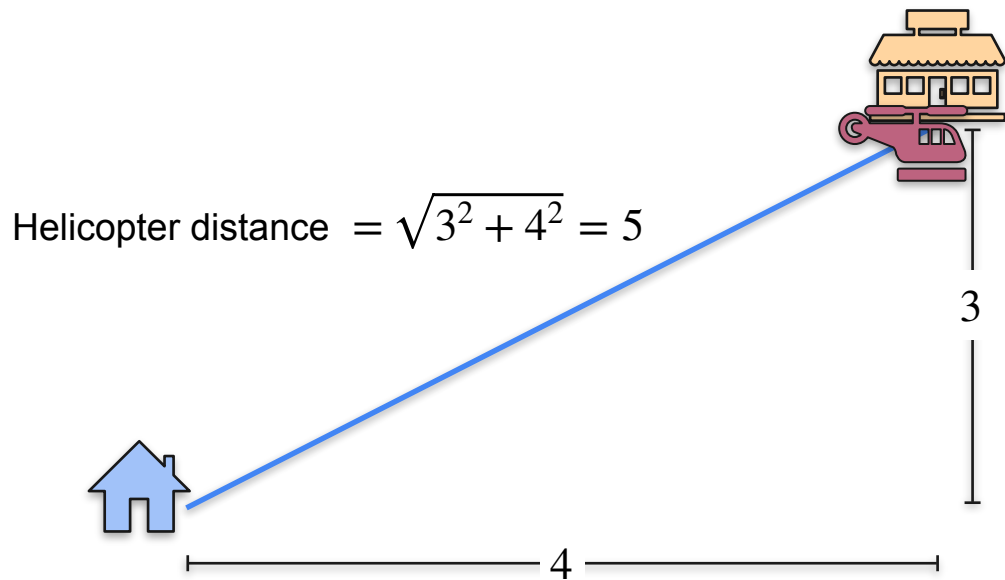
How to get from point A to point B?



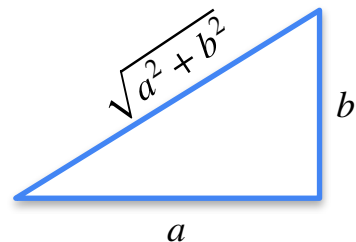
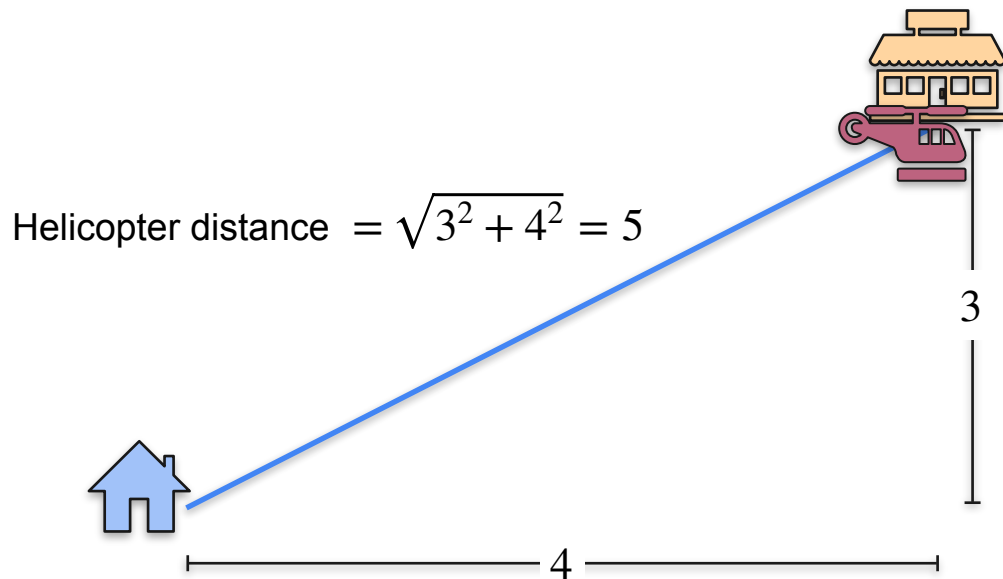
How to get from point A to point B?



How to get from point A to point B?

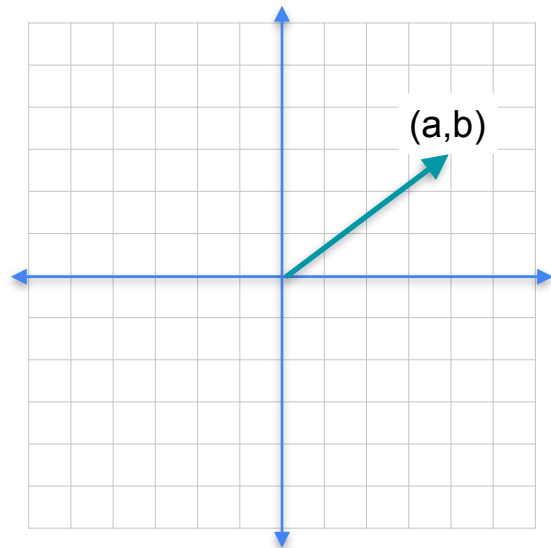


How to get from point A to point B?

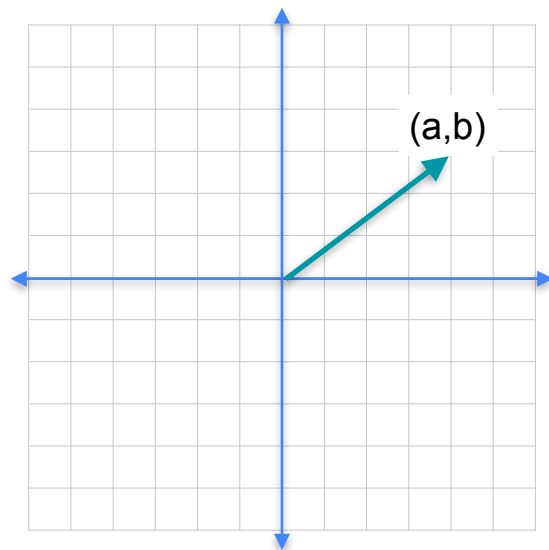


Pythagorean Theorem

Norms

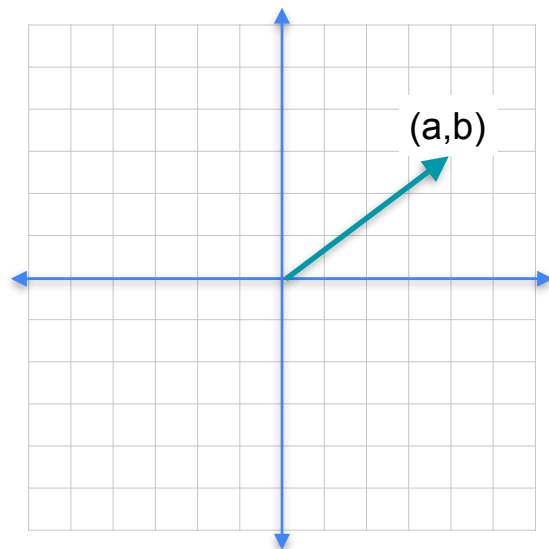


Norms



$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

Norms

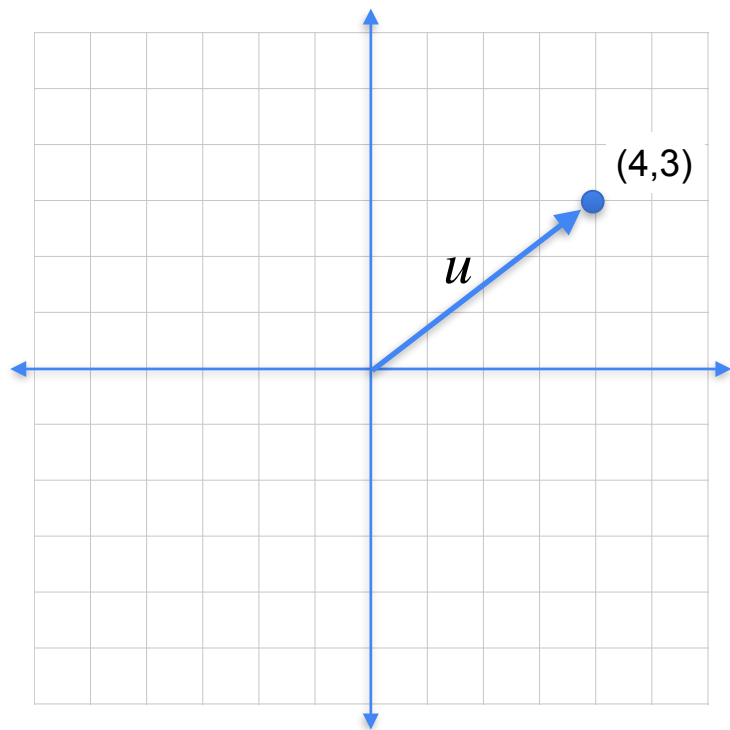


$$\text{L1-norm} = |(a, b)|_1 = |a| + |b|$$

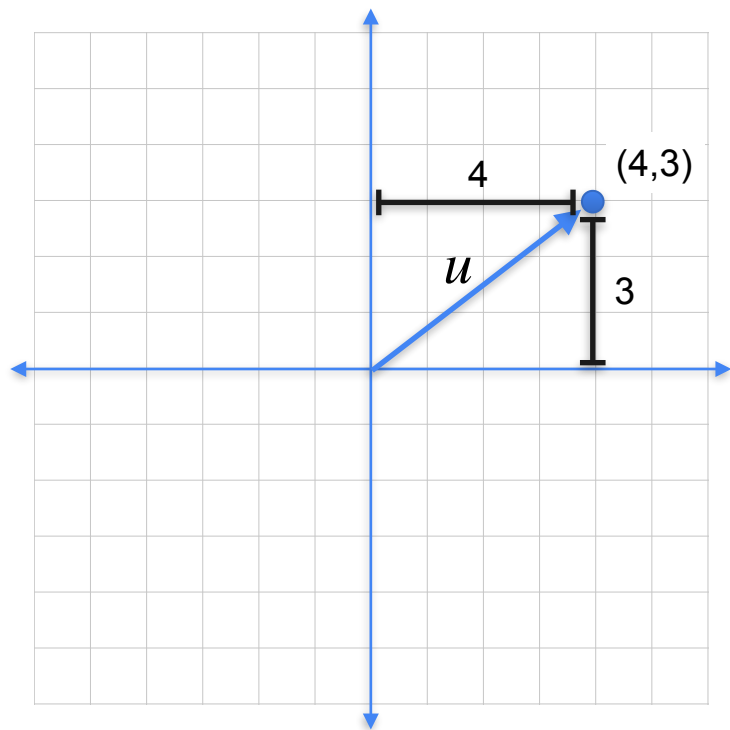


$$\text{L2-norm} = |(a, b)|_2 = \sqrt{a^2 + b^2}$$

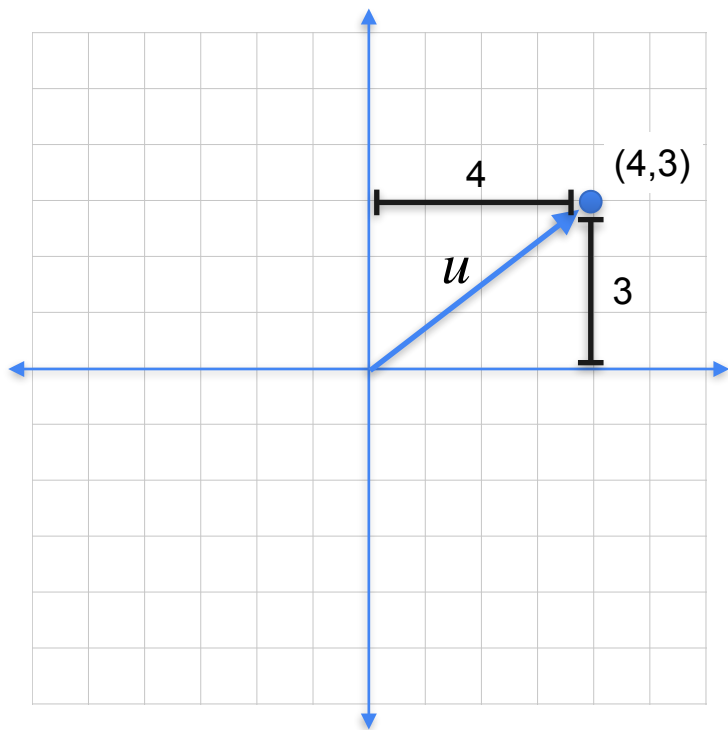
Norm of a vector



Norm of a vector

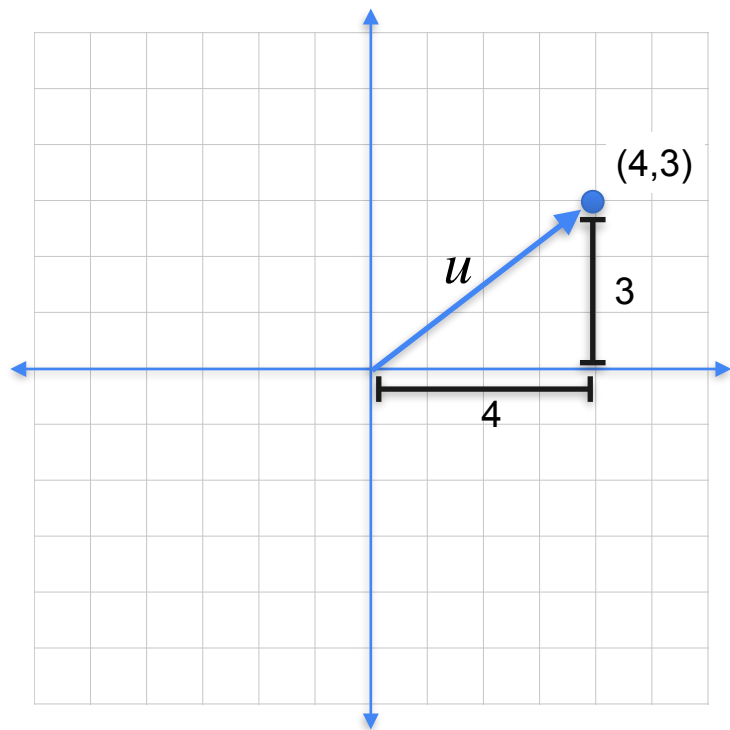


Norm of a vector

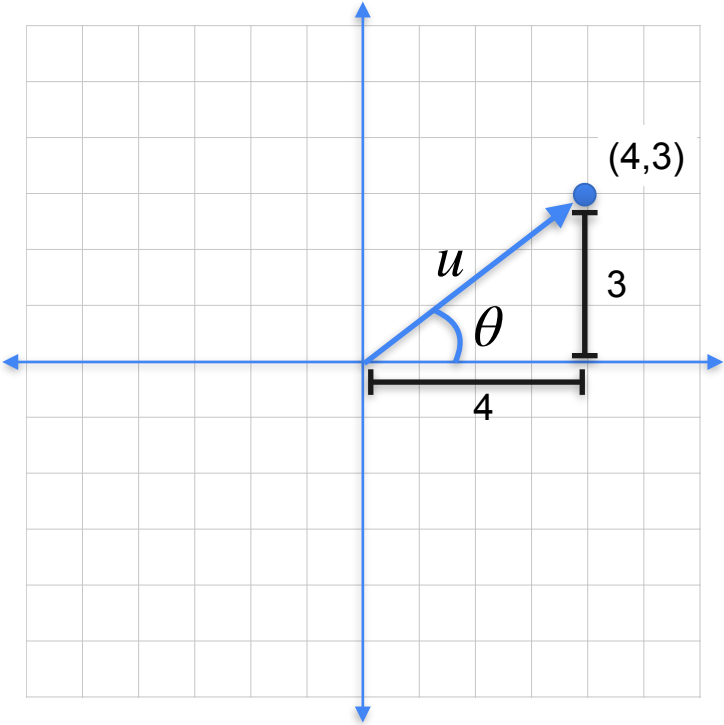


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

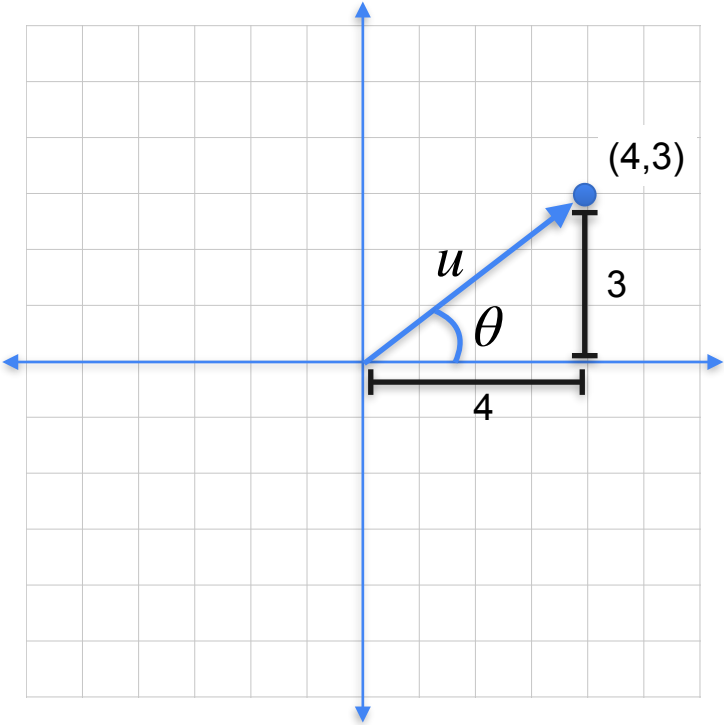
Direction of a vector



Direction of a vector

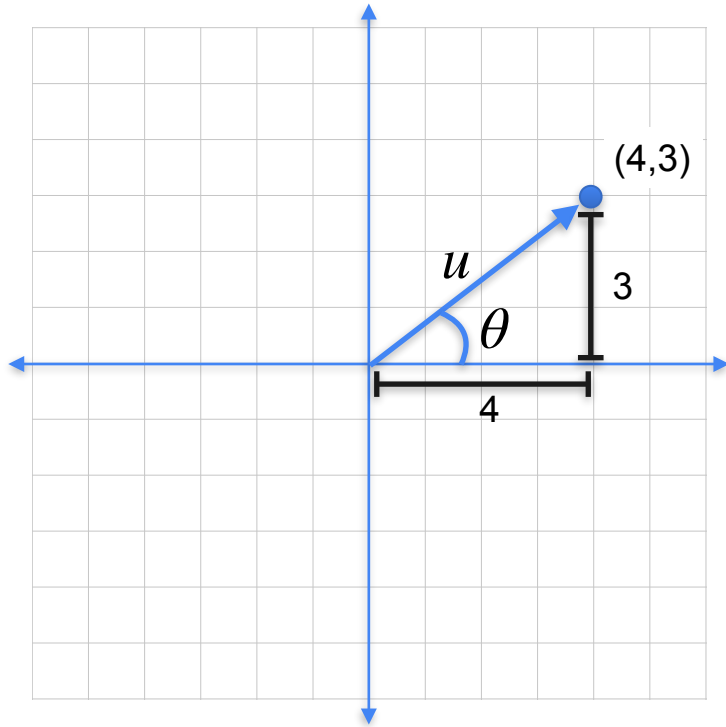


Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

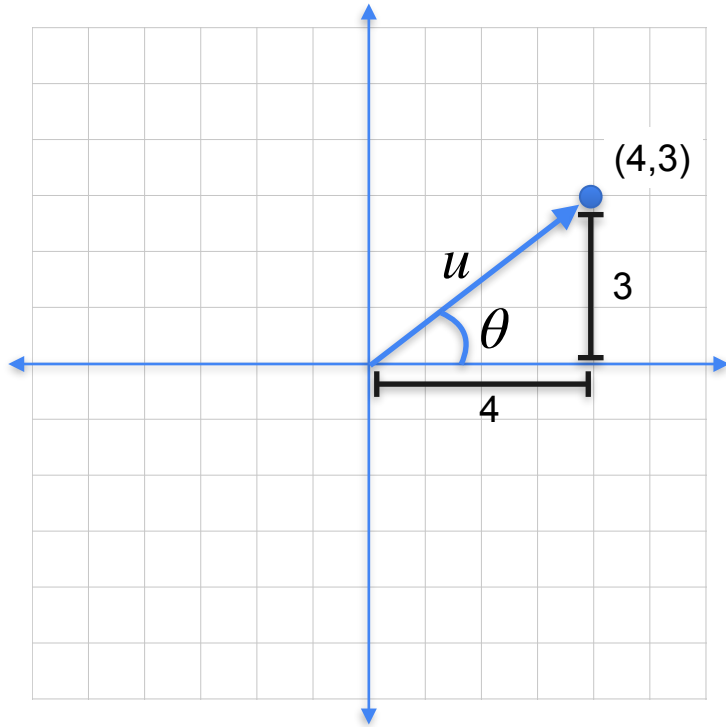
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

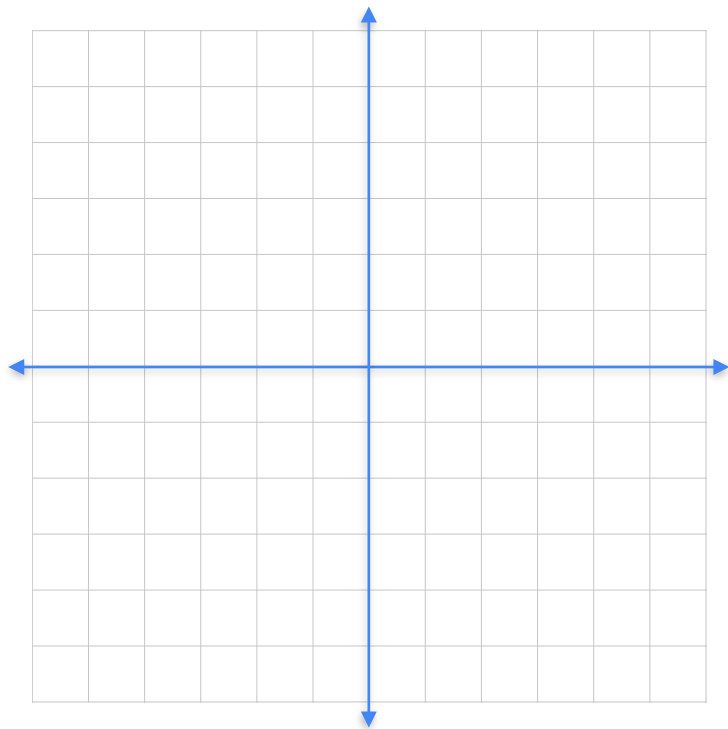


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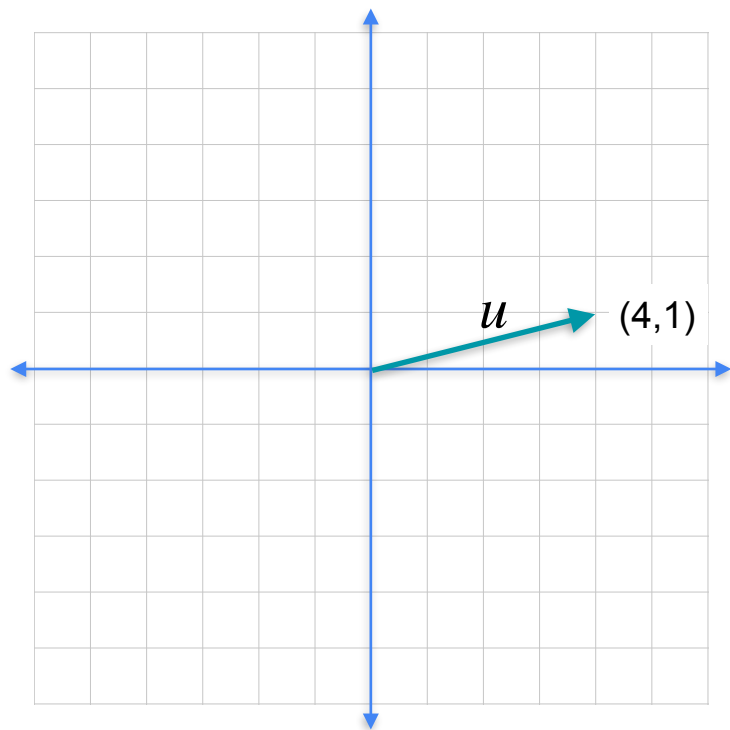
Vectors and Linear Transformations

Sum and difference of vectors

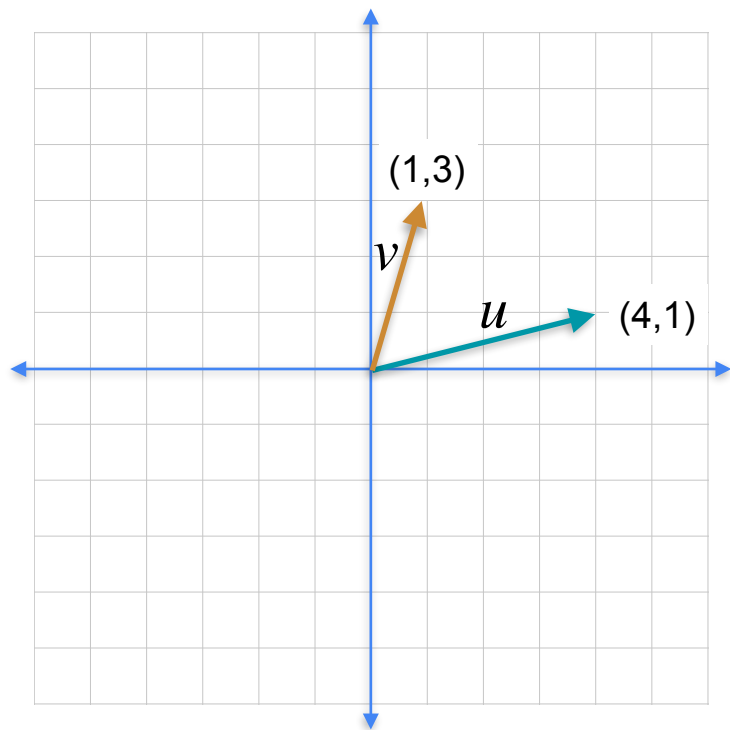
Sum of vectors



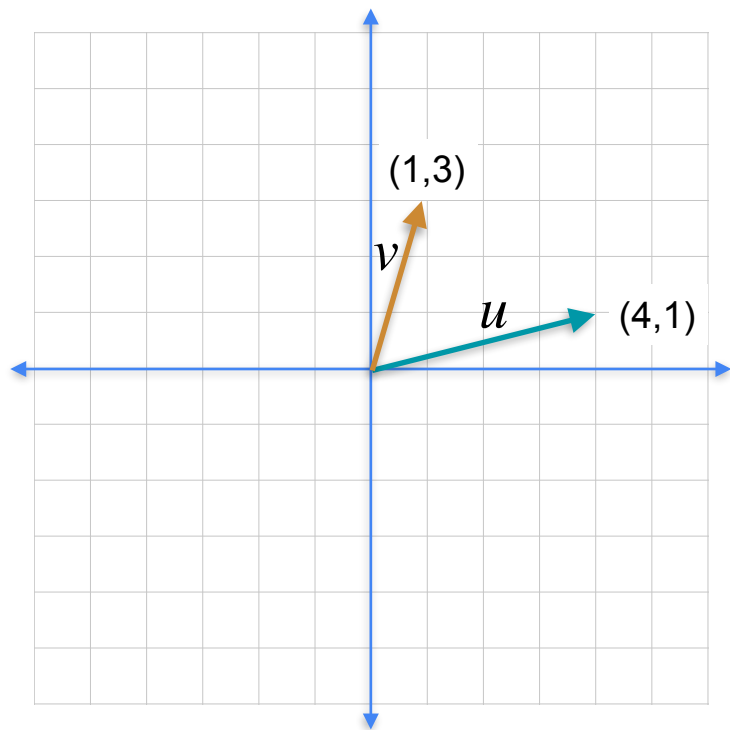
Sum of vectors



Sum of vectors

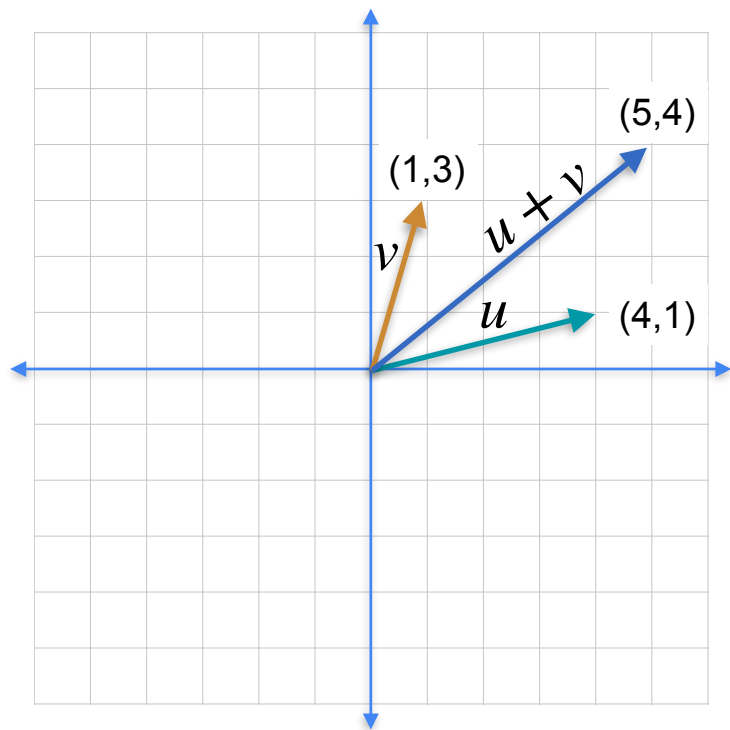


Sum of vectors



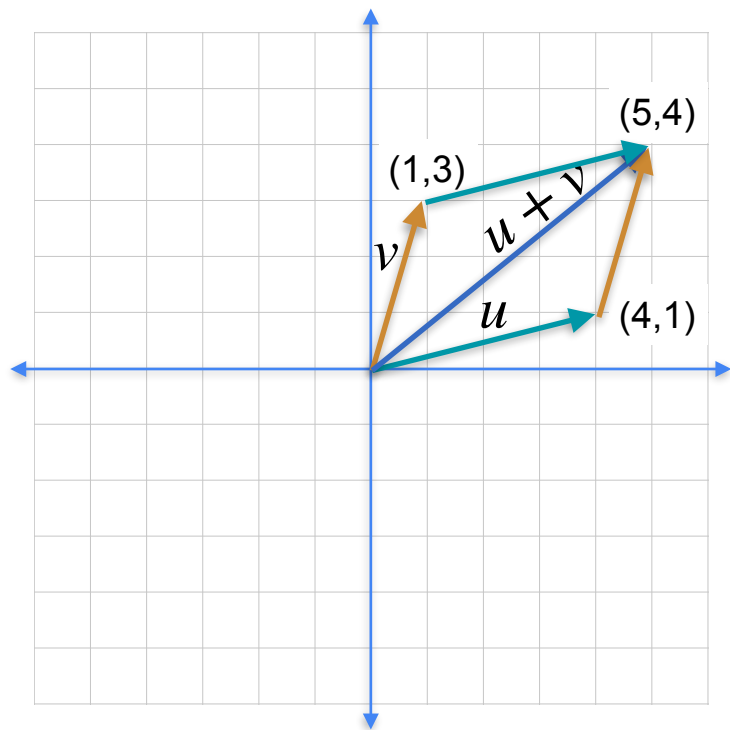
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors



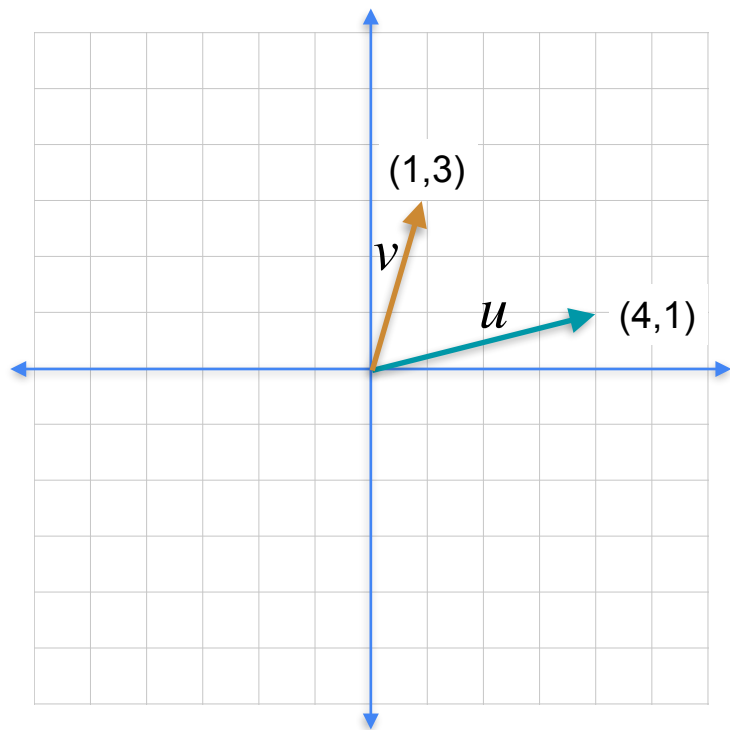
$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Sum of vectors

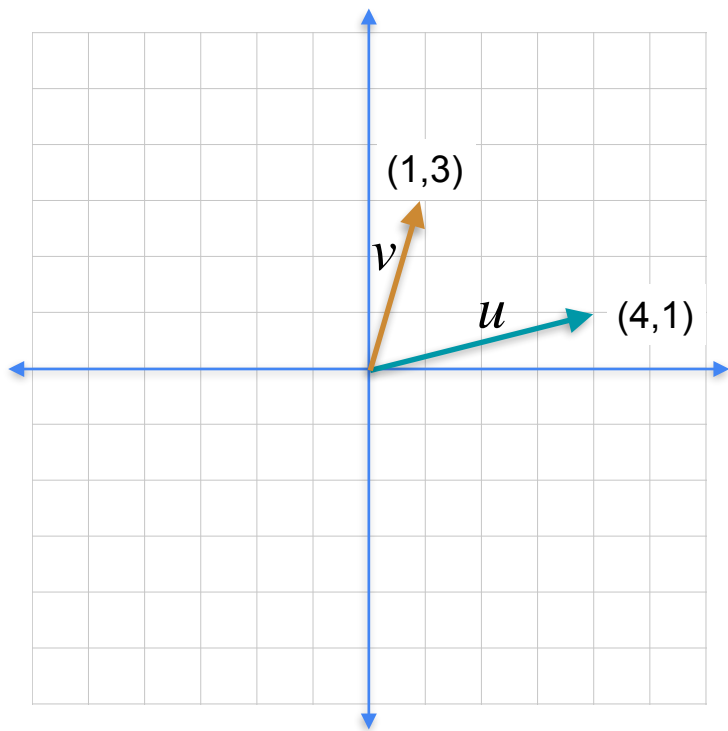


$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

Difference of vectors

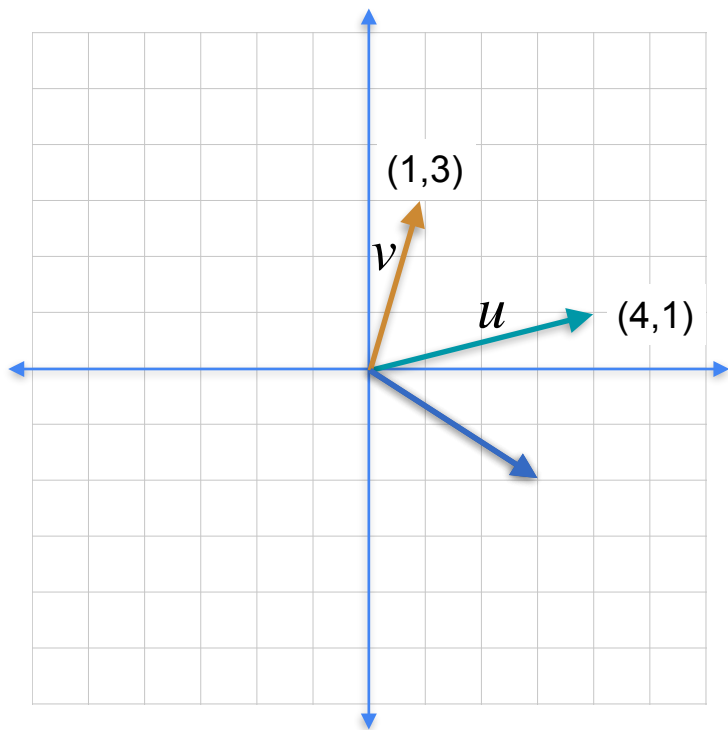


Difference of vectors



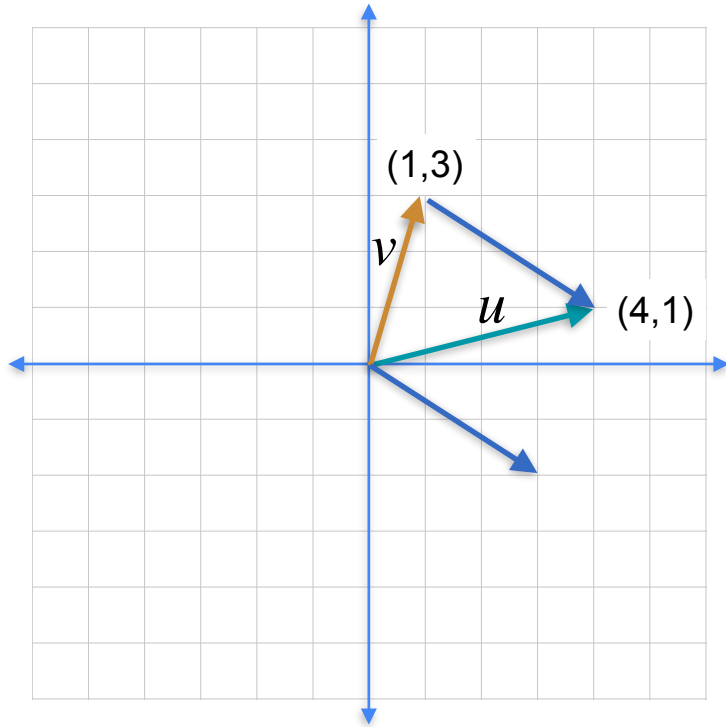
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



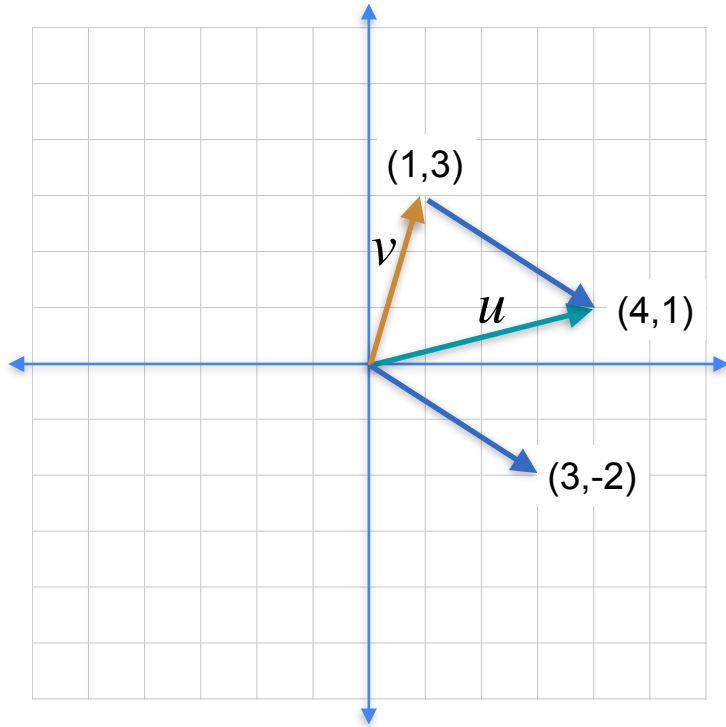
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

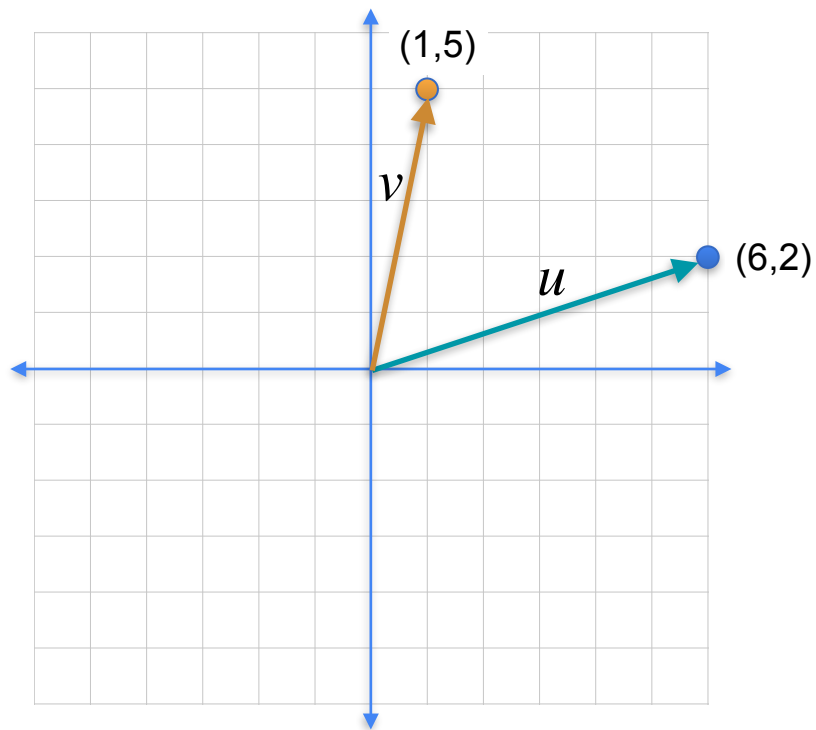


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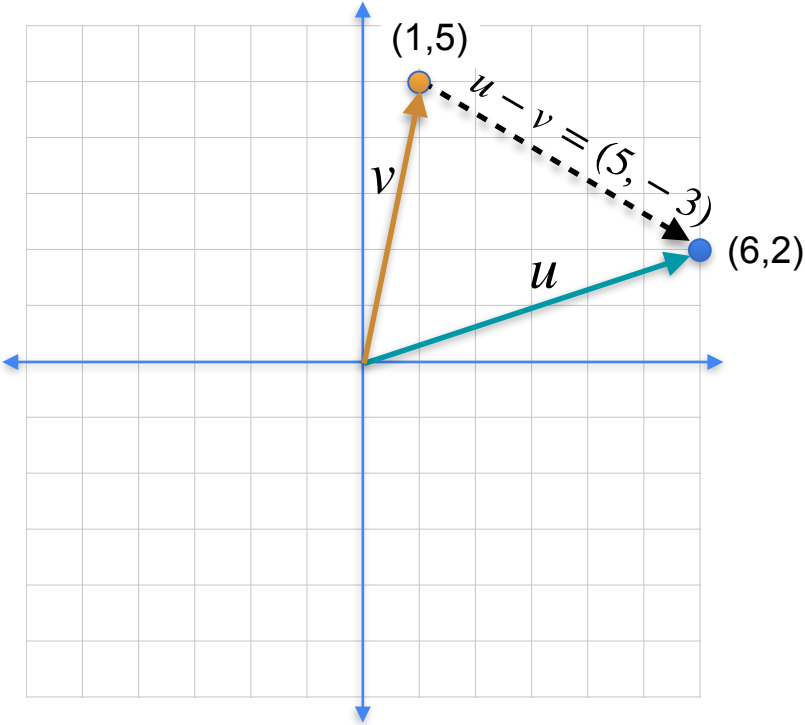
Vectors and Linear Transformations

Distance between vectors

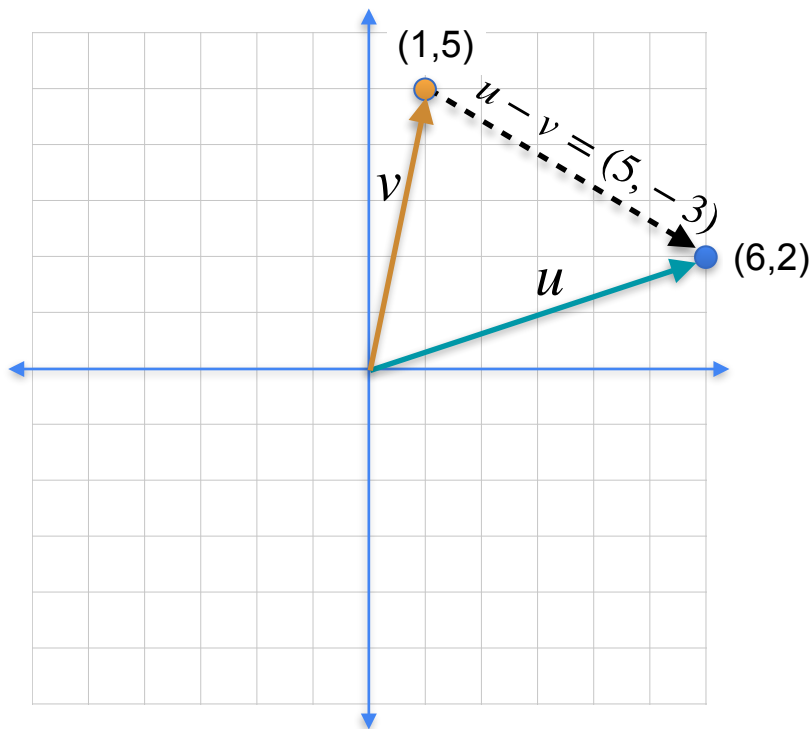
Distances



Distances



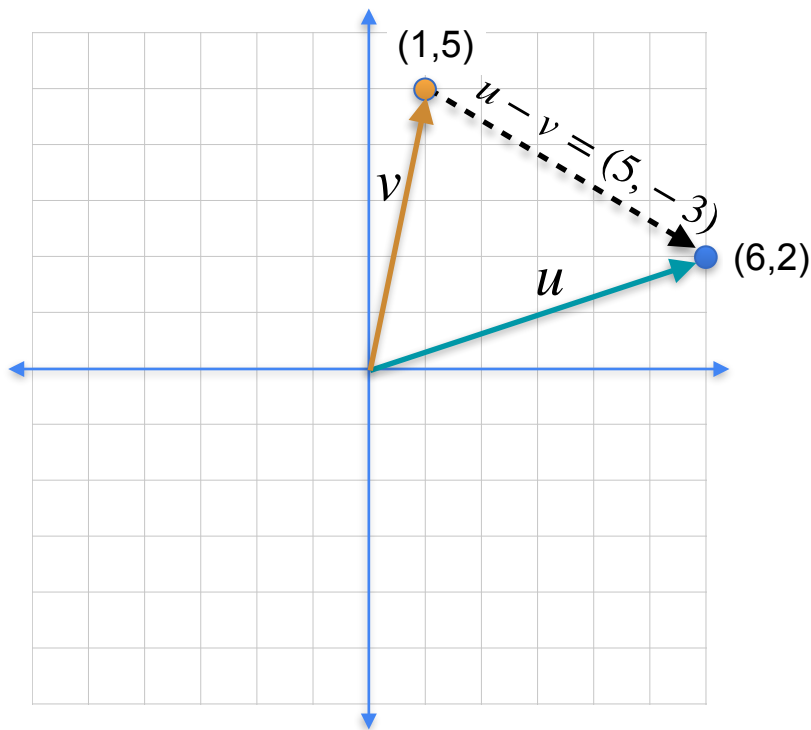
Distances



L1-distance

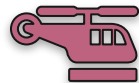
$$|u - v|_1 = |5| + |-3| = 8$$

Distances



L1-distance

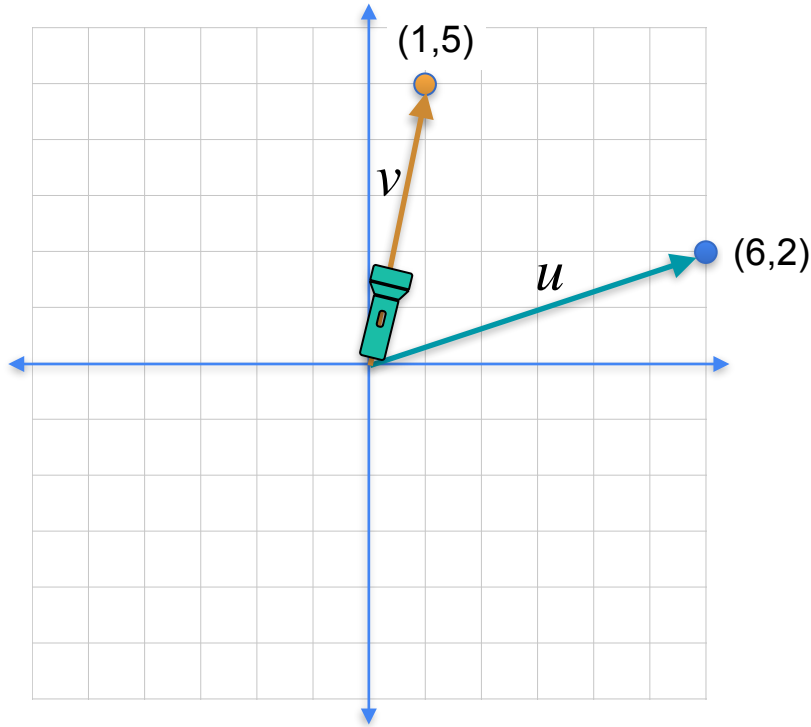
$$|u - v|_1 = |5| + |-3| = 8$$



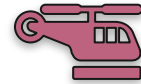
L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances

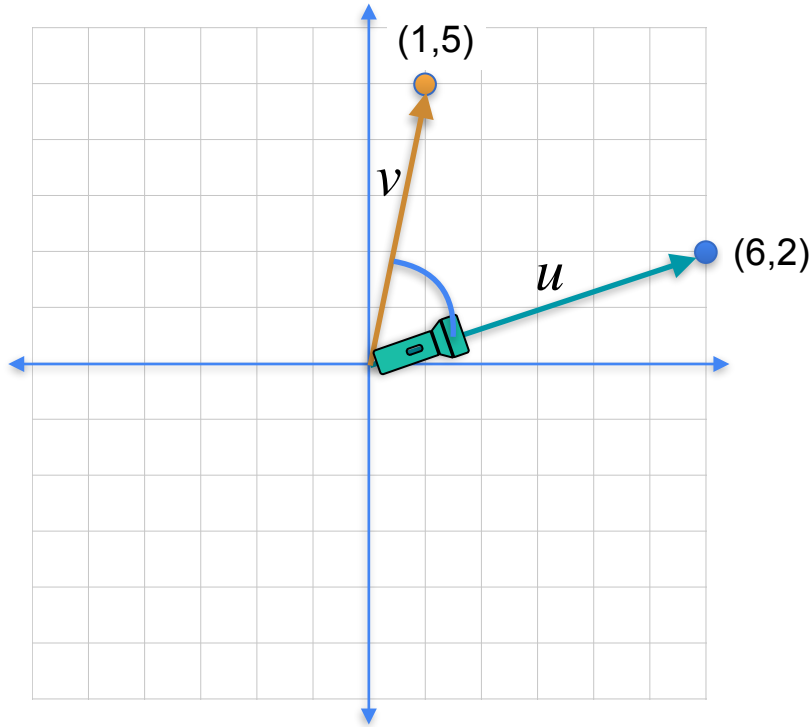


$$|u - v|_1 = |5| + |-3| = 8$$

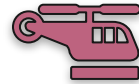


$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances

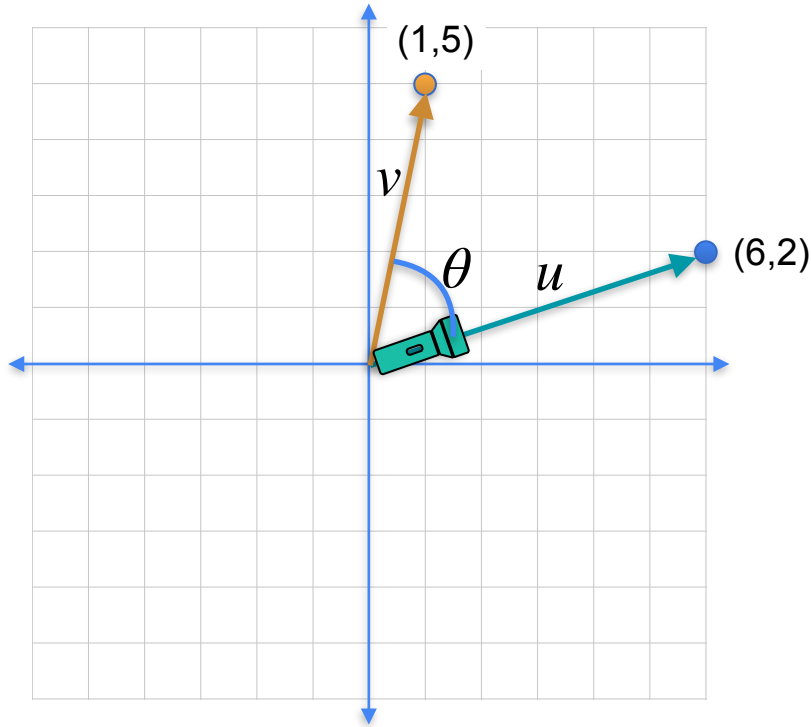



$$|u - v|_1 = |5| + |-3| = 8$$

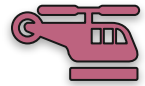


$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

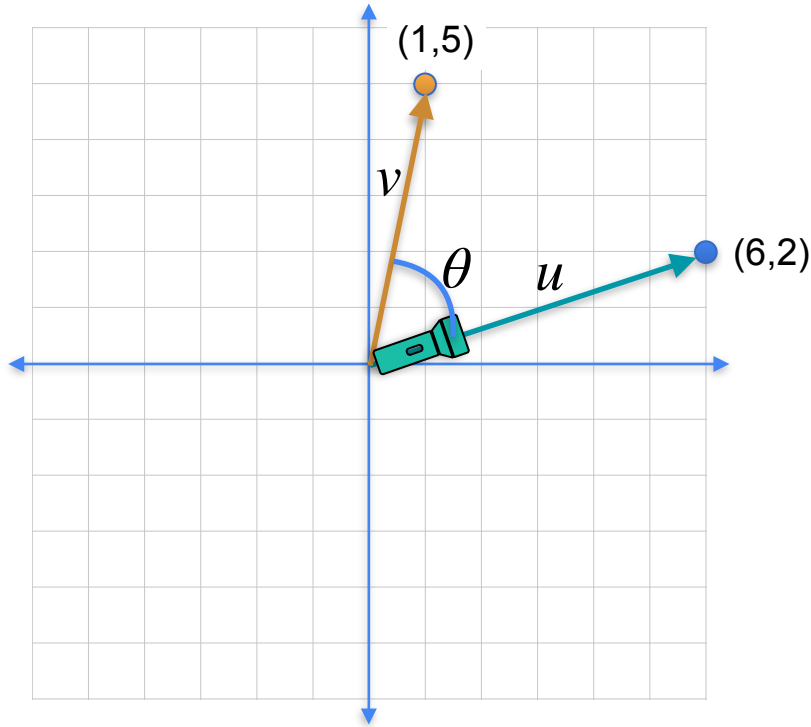
Distances





 L1-distance $|u - v|_1 = |5| + |-3| = 8$

 L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

Distances



 $|u - v|_1 = |5| + |-3| = 8$
L1-distance

 $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$
L2-distance

 $\cos(\theta)$
Cosine distance

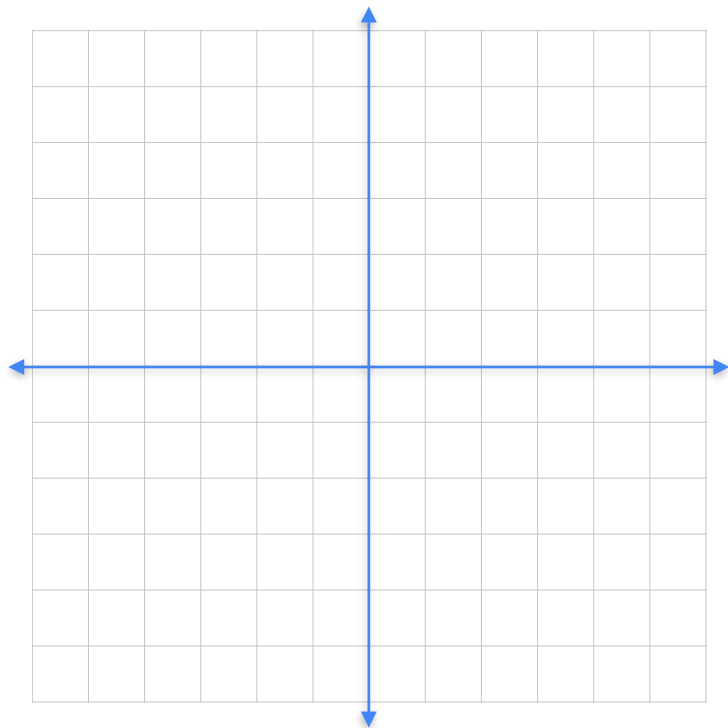


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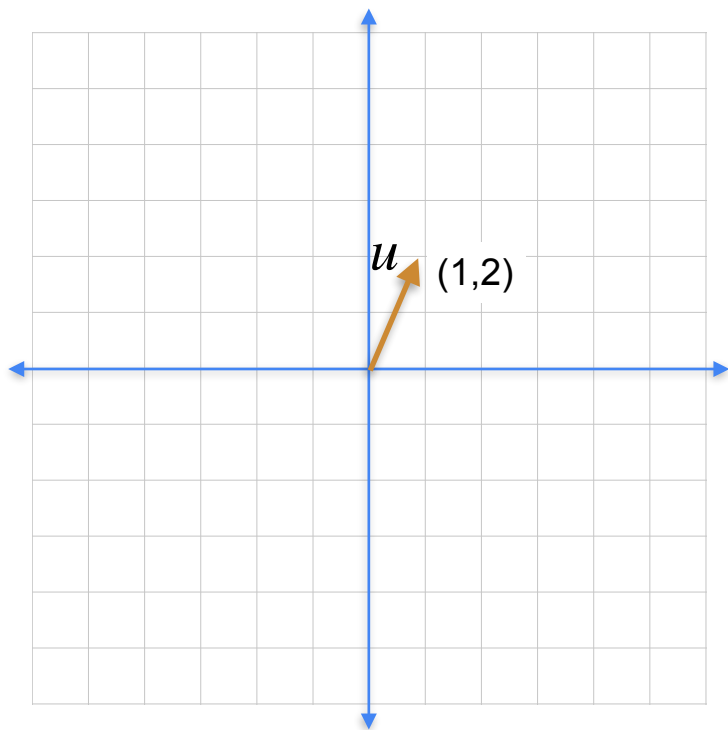
Vectors and Linear Transformations

Multiplying a vector by a scalar

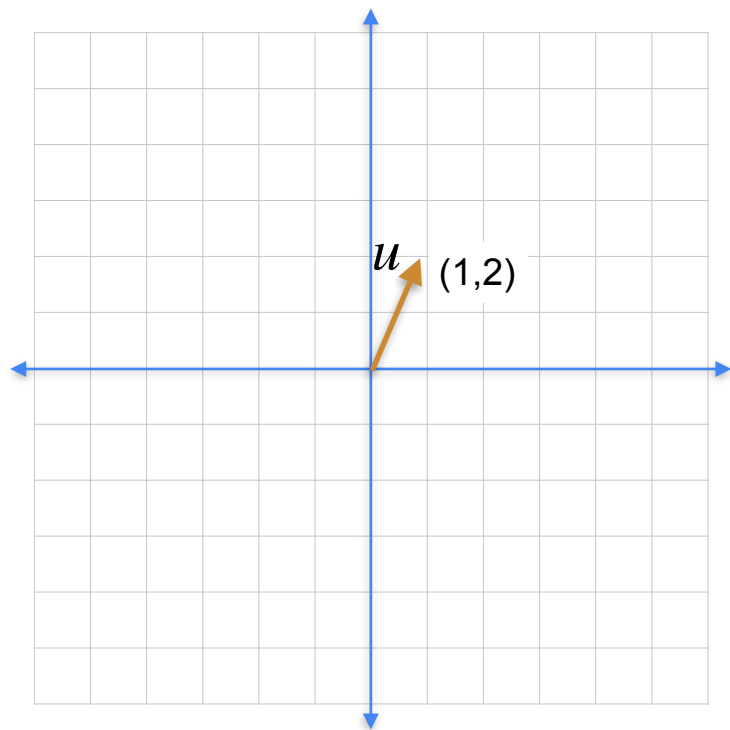
Multiplying a vector by a scalar



Multiplying a vector by a scalar

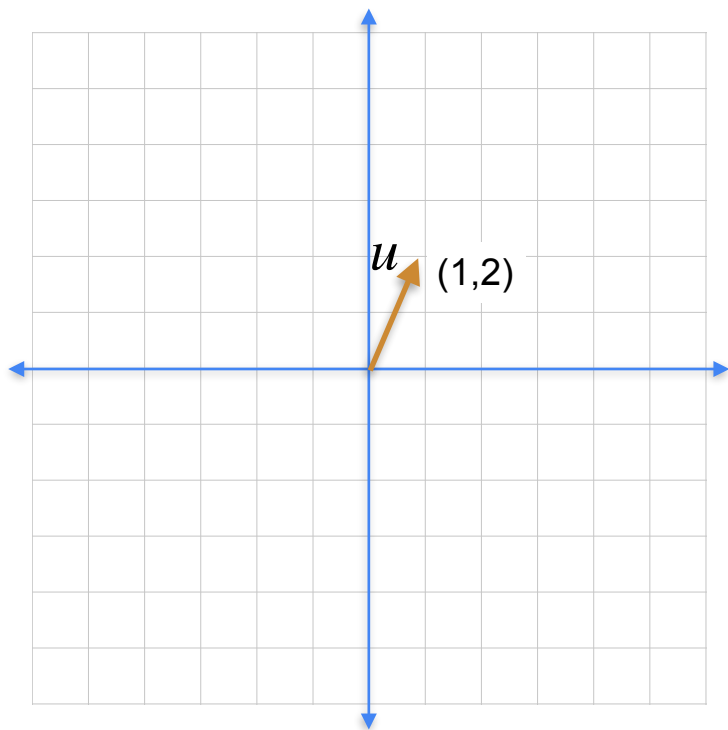


Multiplying a vector by a scalar



$$u = (1,2)$$

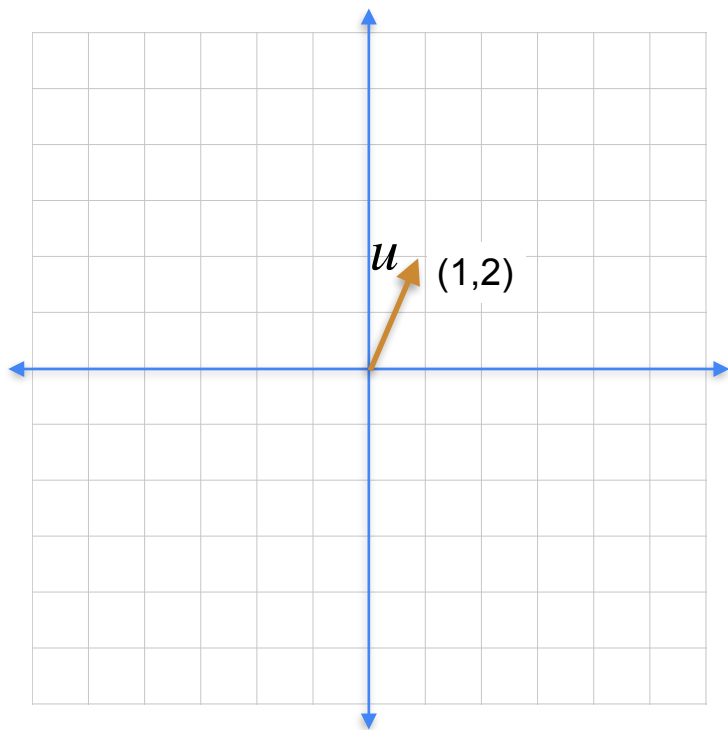
Multiplying a vector by a scalar



$$u = (1,2)$$

$$\lambda = 3$$

Multiplying a vector by a scalar

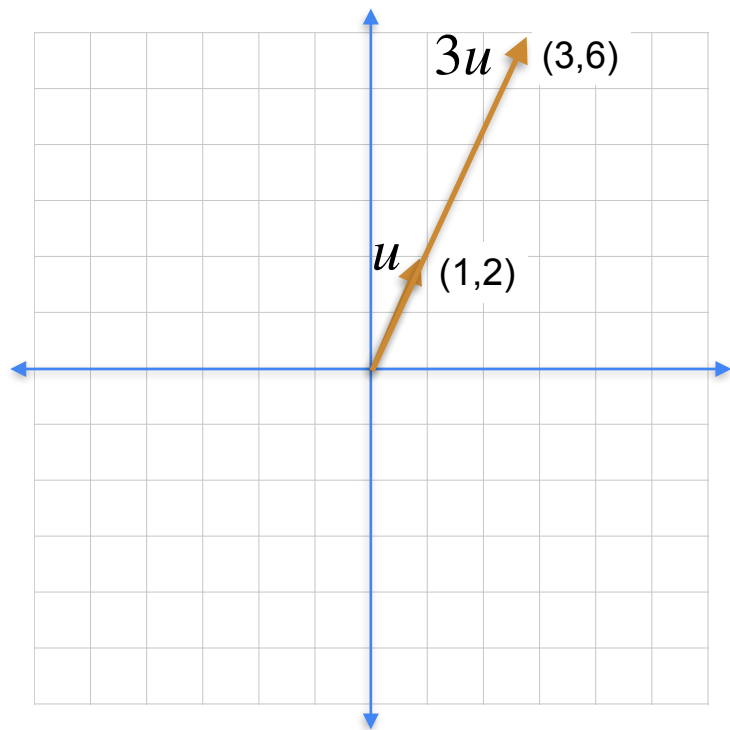


$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

Multiplying a vector by a scalar

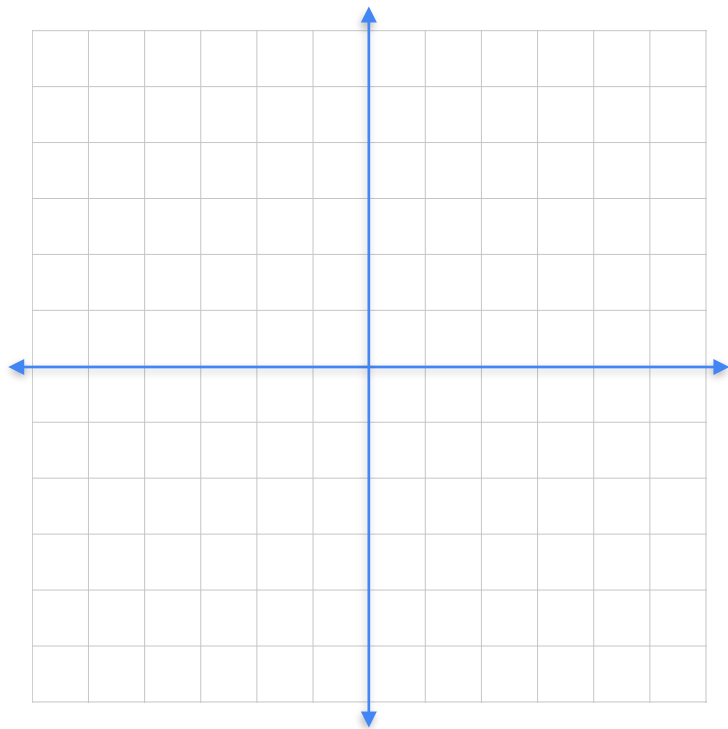


$$u = (1,2)$$

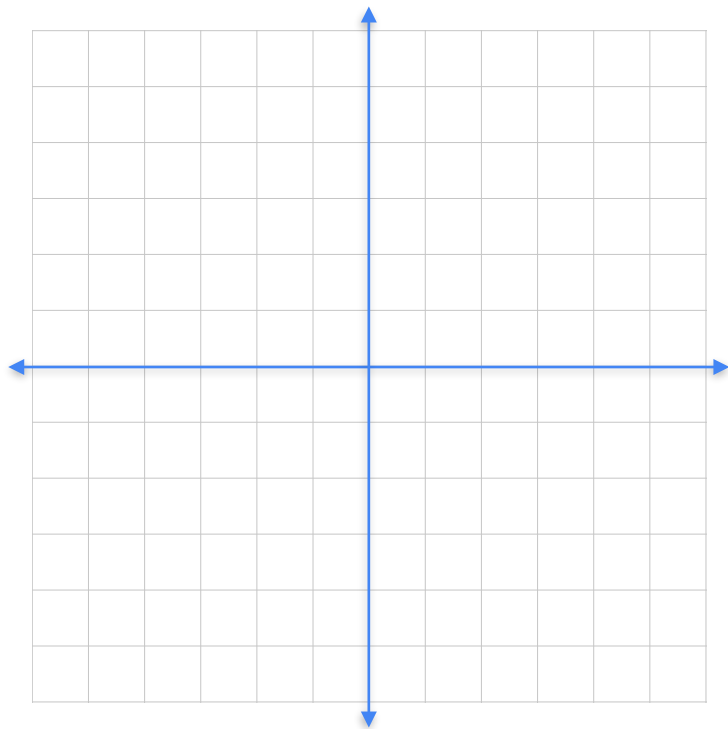
$$\lambda = 3$$

$$\lambda u = (3,6)$$

If the scalar is negative

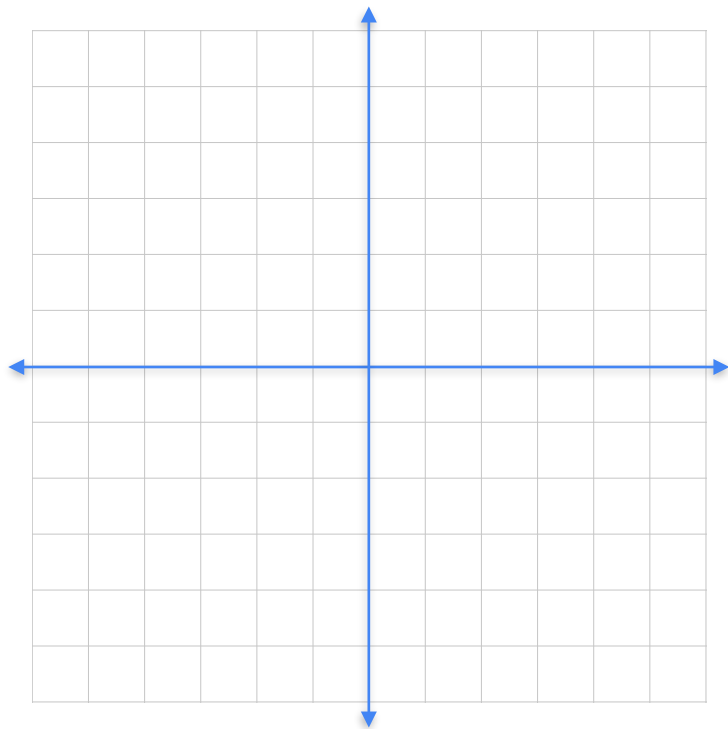


If the scalar is negative



$$u = (1,2)$$

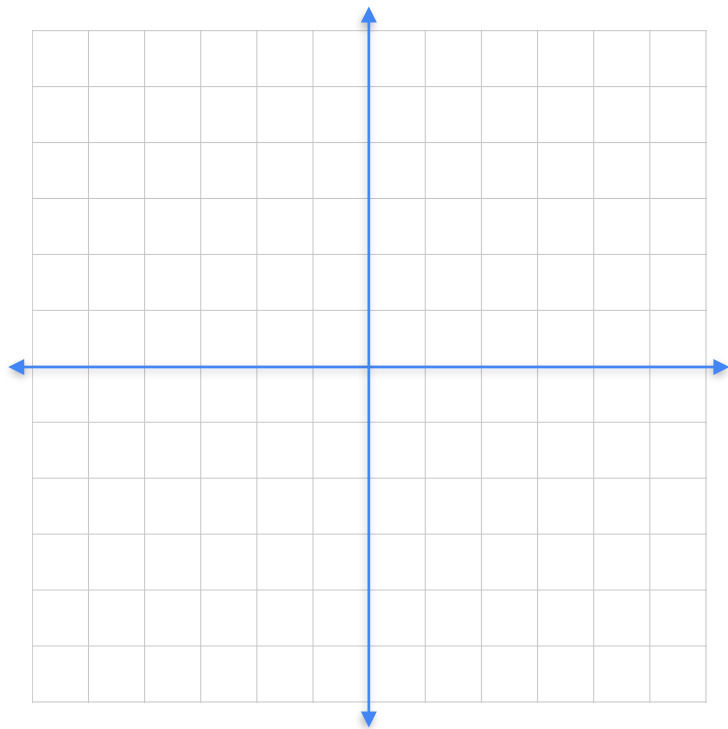
If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

If the scalar is negative

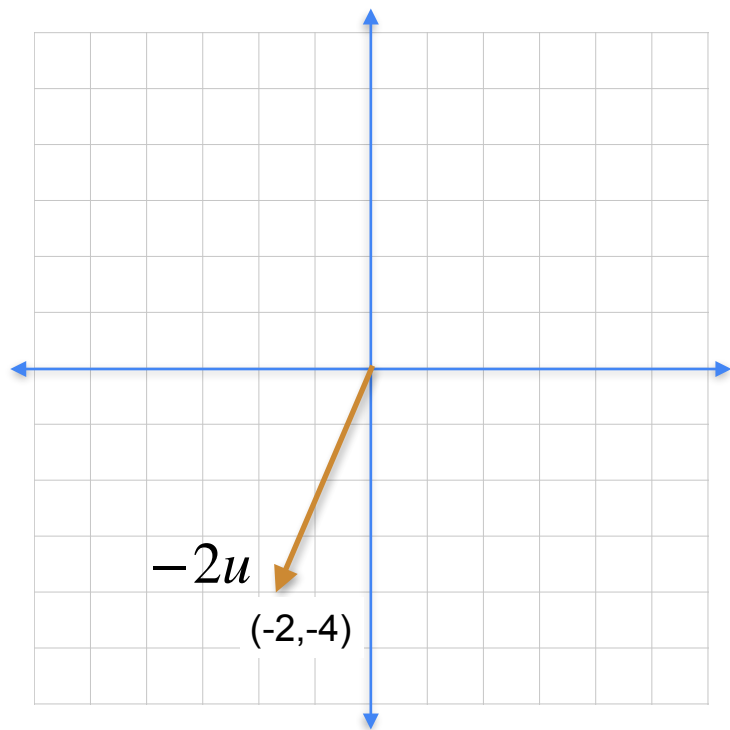


$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative

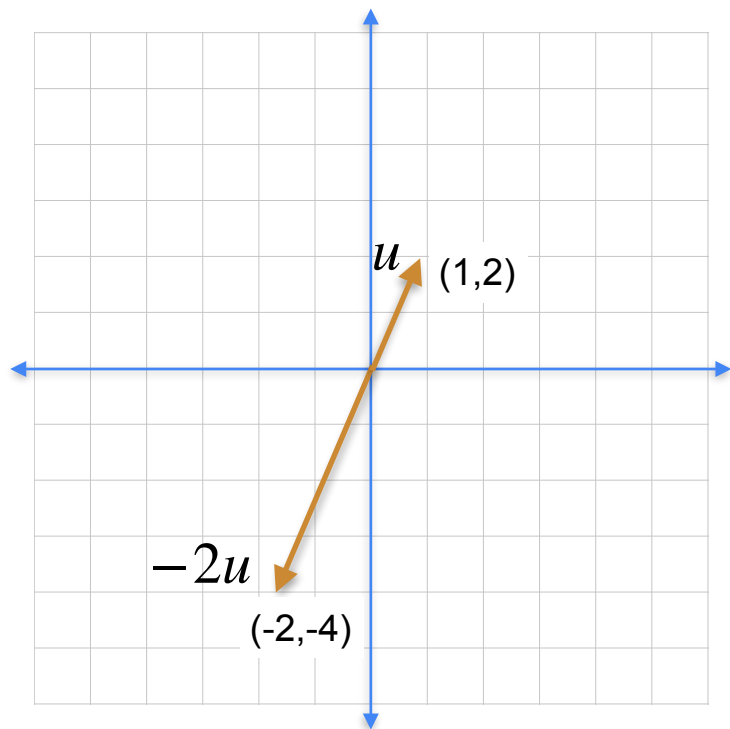


$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$



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Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

Prices

apples: \$3

bananas: \$5

cherries: \$2

A shortcut for linear operations

Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price




A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

bananas: \$5

cherries: \$2

Total price




A shortcut for linear operations

Quantities

2 apples

4 bananas

1 cherry

	2
	4
	1

Prices

apples: \$3

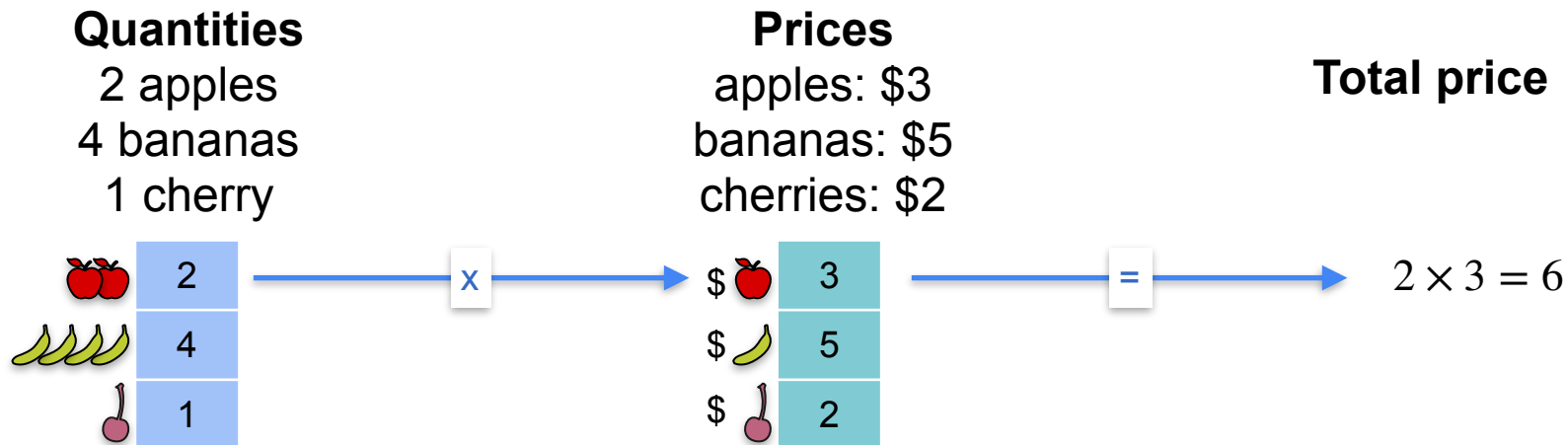
bananas: \$5

cherries: \$2

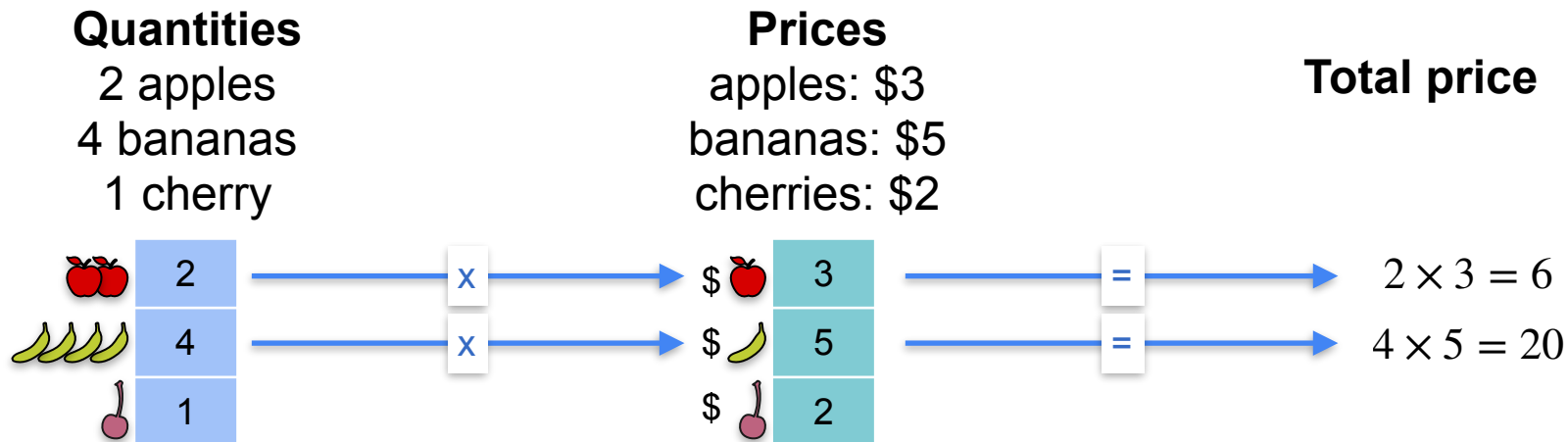
\$ 	3
\$ 	5
\$ 	2

Total price

A shortcut for linear operations



A shortcut for linear operations



A shortcut for linear operations

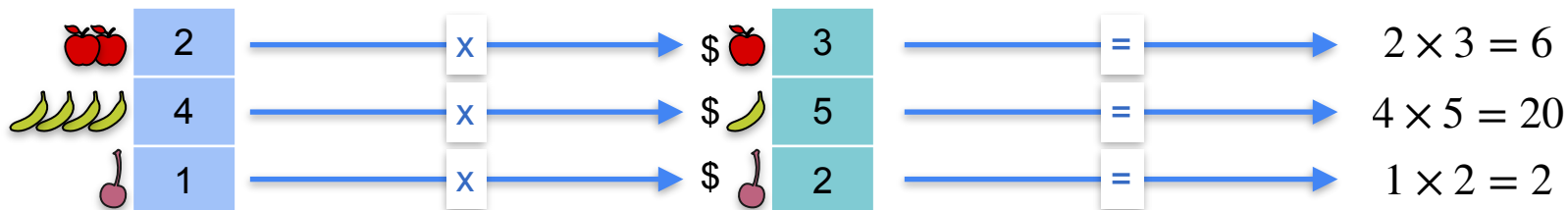
Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price



A shortcut for linear operations

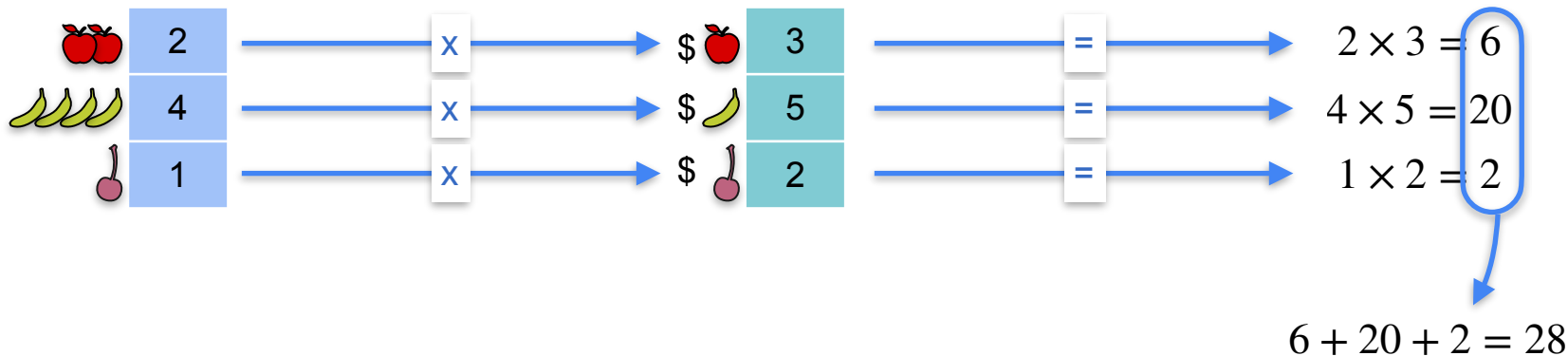
Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price



A shortcut for linear operations

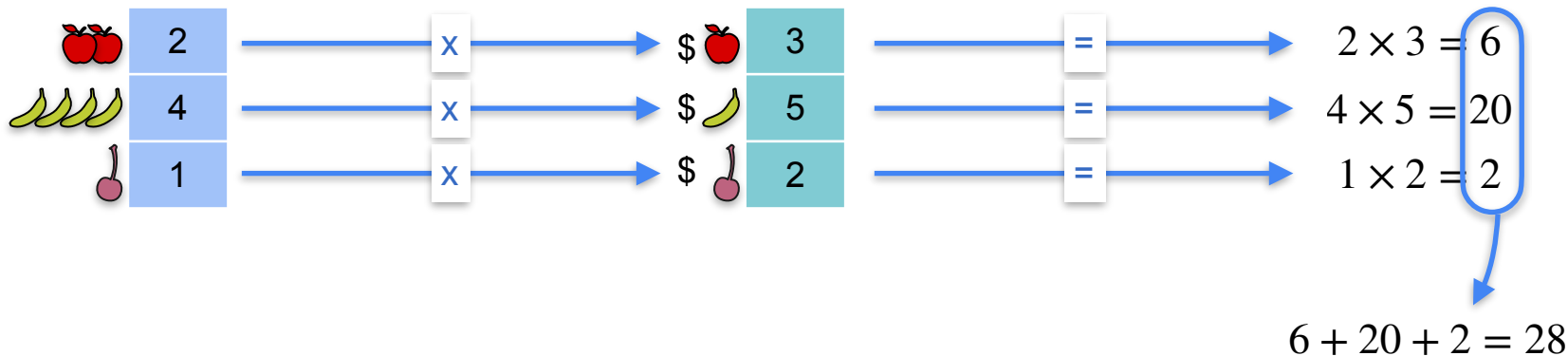
Quantities

2 apples
4 bananas
1 cherry

Prices

apples: \$3
bananas: \$5
cherries: \$2

Total price
\$28



The dot product

$$\begin{array}{|c|} \hline \text{🍏} \text{ } 2 \\ \hline \text{🍌} \text{ } 4 \\ \hline \text{🍒} \text{ } 1 \\ \hline \end{array} \cdot \begin{array}{|c|} \hline \$ \text{🍏} \text{ } 3 \\ \hline \$ \text{🍌} \text{ } 5 \\ \hline \$ \text{🍒} \text{ } 2 \\ \hline \end{array} = \$28$$

The dot product

The diagram shows two vectors being multiplied element-wise and summed. The first vector (blue) has components: 2 (apples), 4 (bananas), and 1 (cherry). The second vector (teal) has components: 3 (\$ apple), 5 (\$ banana), and 2 (\$ cherry). The result is \$28.

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

$$\begin{matrix} \text{🍏} & \text{🍌} & \text{🍒} \\ 2 & 4 & 1 \end{matrix} \cdot \begin{matrix} \$ \text{🍏} & 3 \\ \$ \text{🍌} & 5 \\ \$ \text{🍒} & 2 \end{matrix} = \$28$$

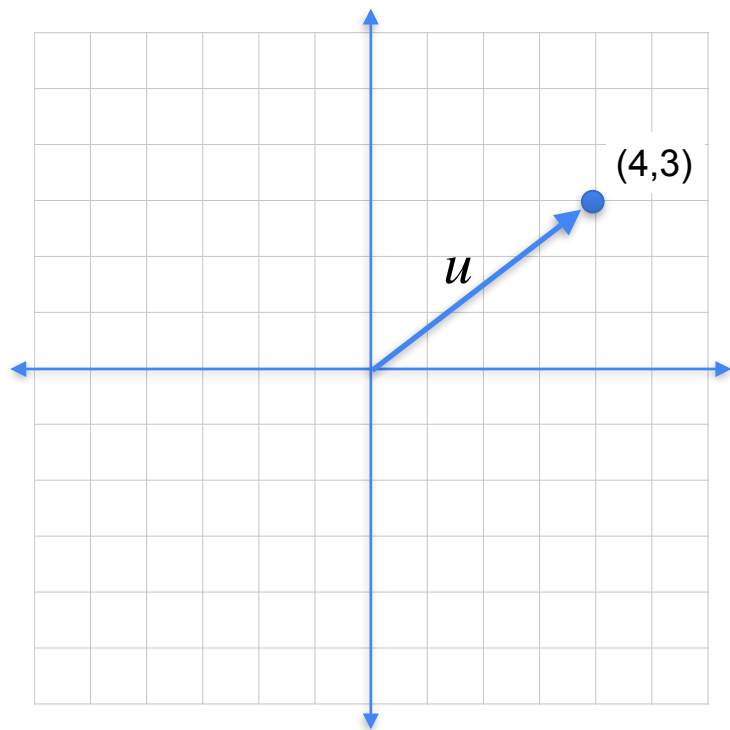
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

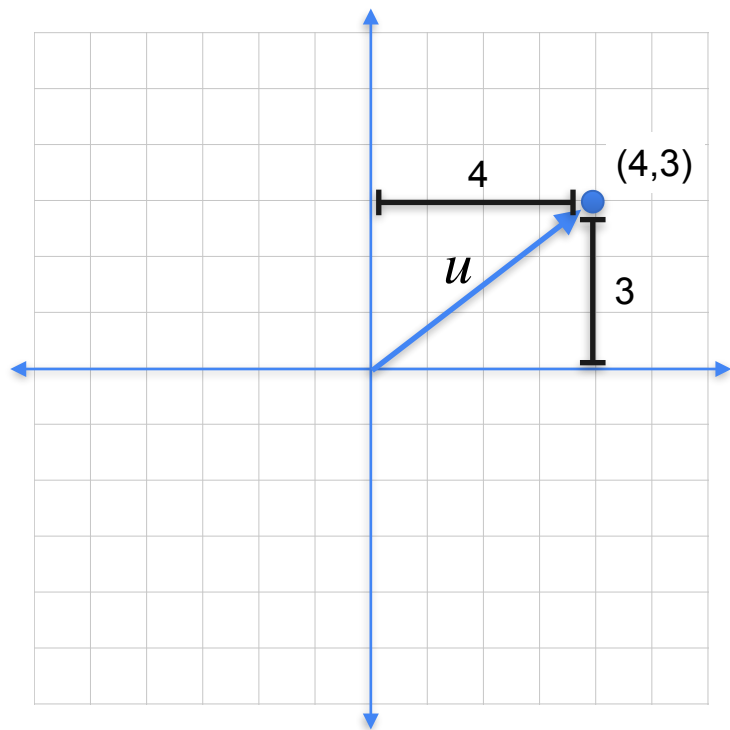
$$\begin{array}{|c|c|c|} \hline 2 & 4 & 1 \\ \hline \end{array} \cdot \begin{array}{|c|} \hline 3 \\ \hline 5 \\ \hline 2 \\ \hline \end{array} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

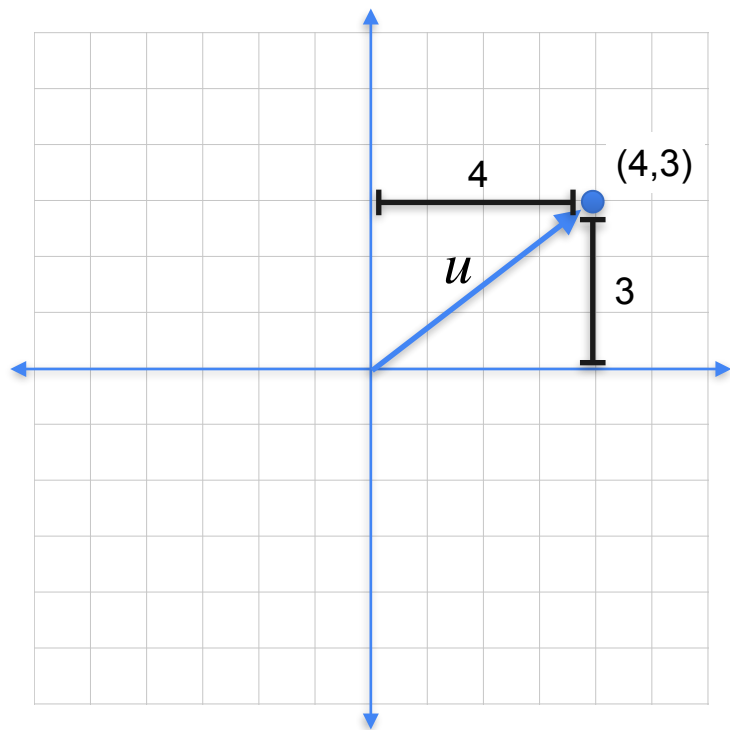
Norm of a vector using dot product



Norm of a vector using dot product

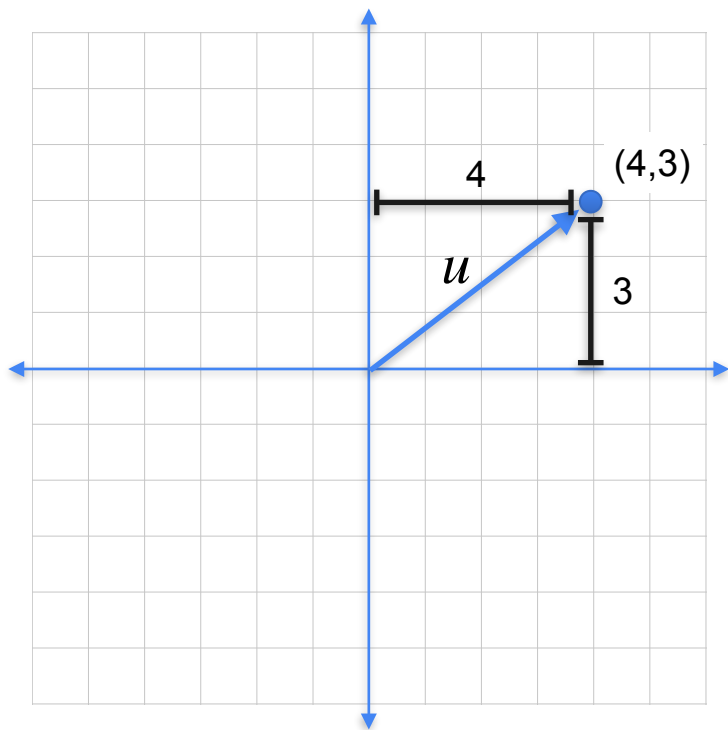


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

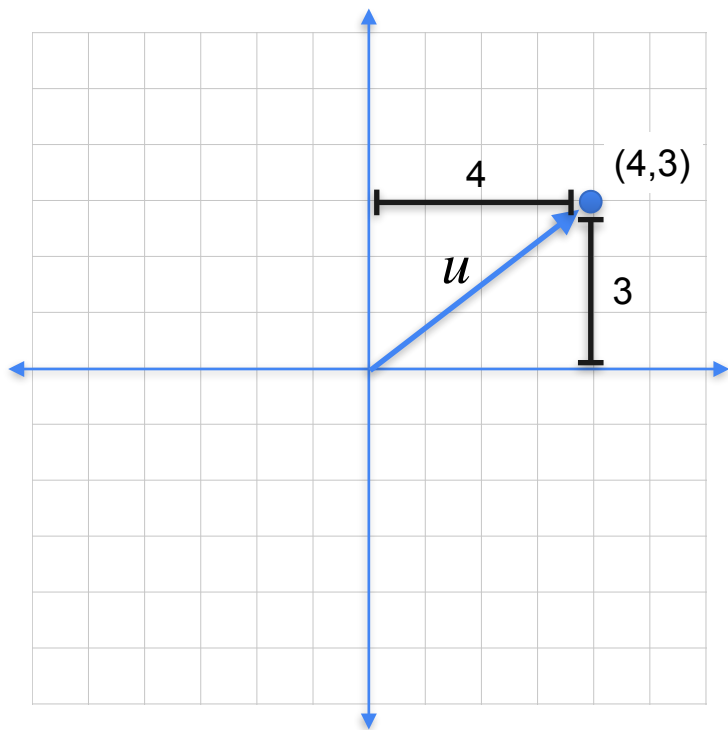
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{matrix} 4 & 3 & \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix} = 25$$

Norm of a vector using dot product

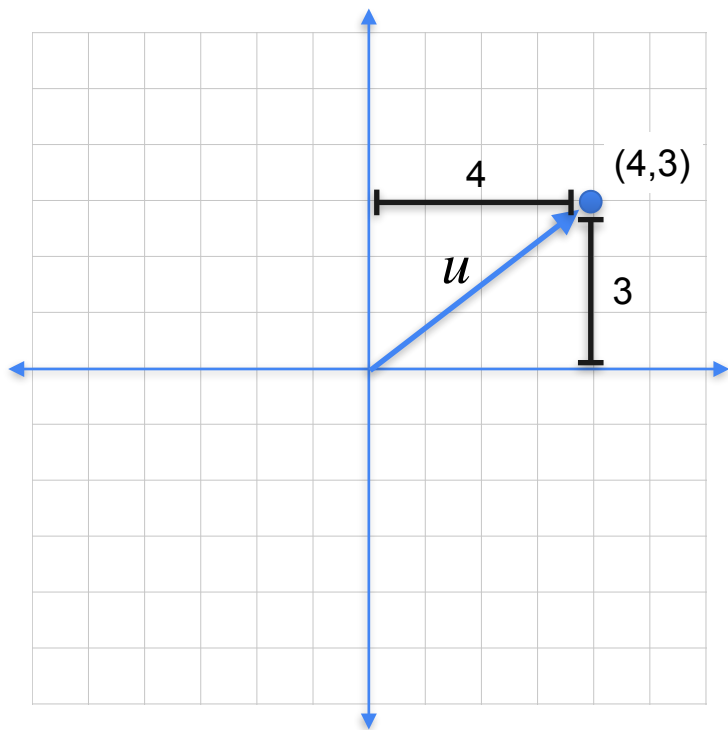


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{matrix} 4 & 3 \end{matrix} \cdot \begin{matrix} 4 \\ 3 \end{matrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$

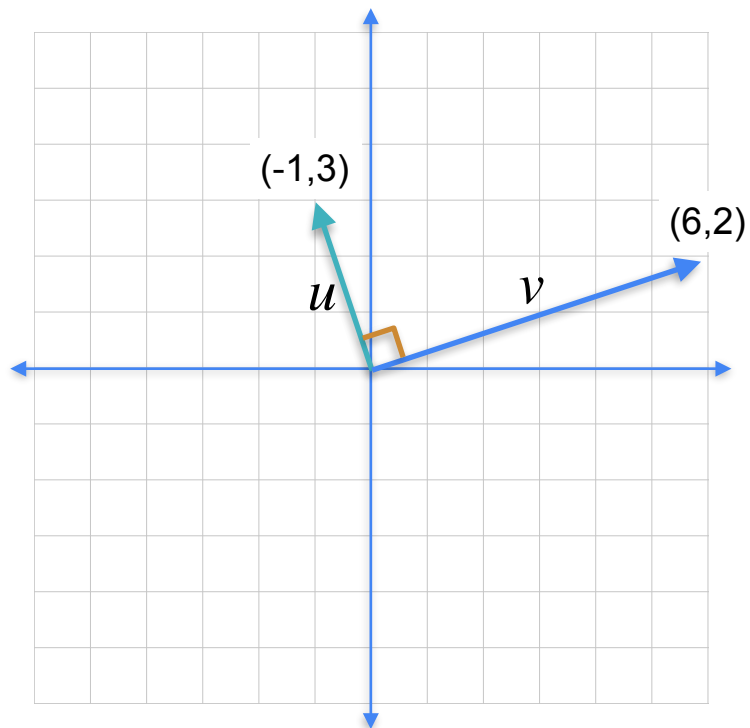


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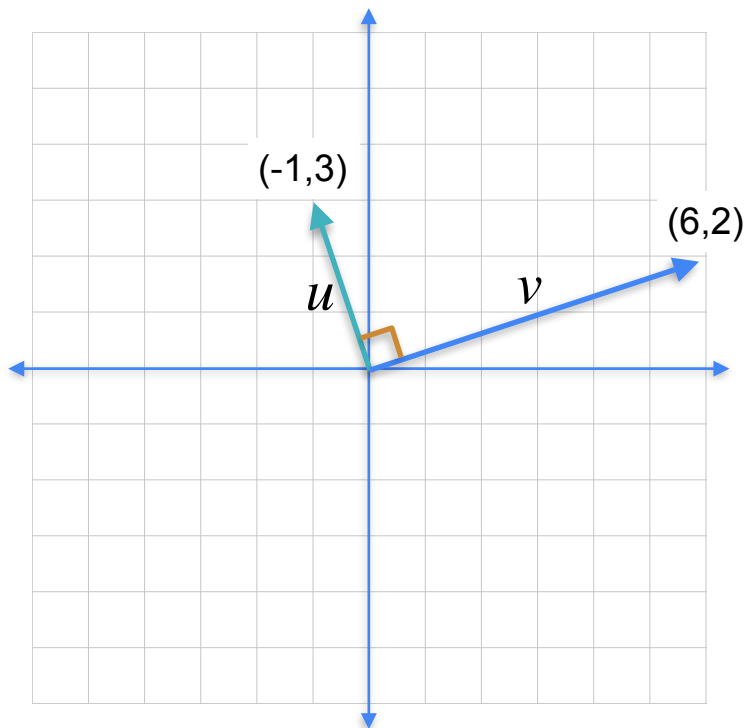
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0



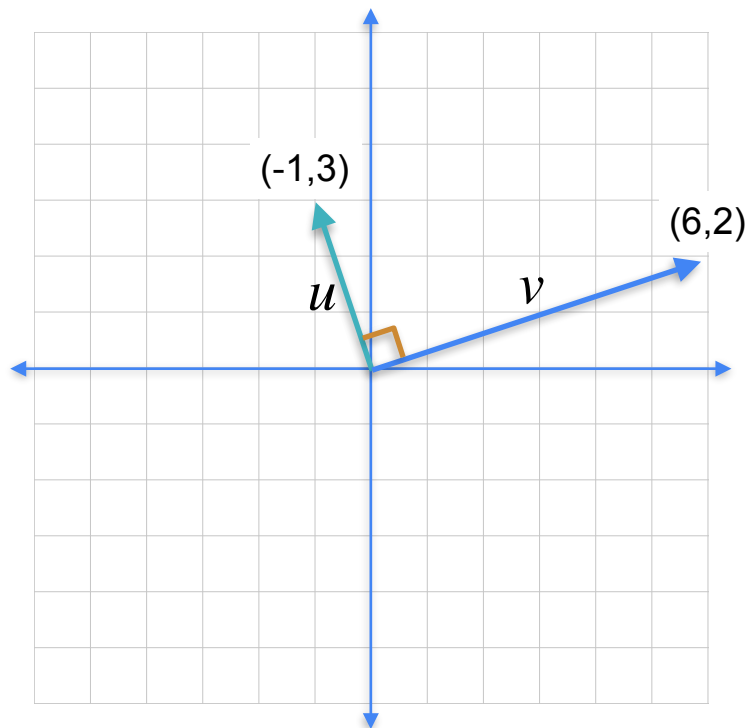
Orthogonal vectors have dot product 0



6

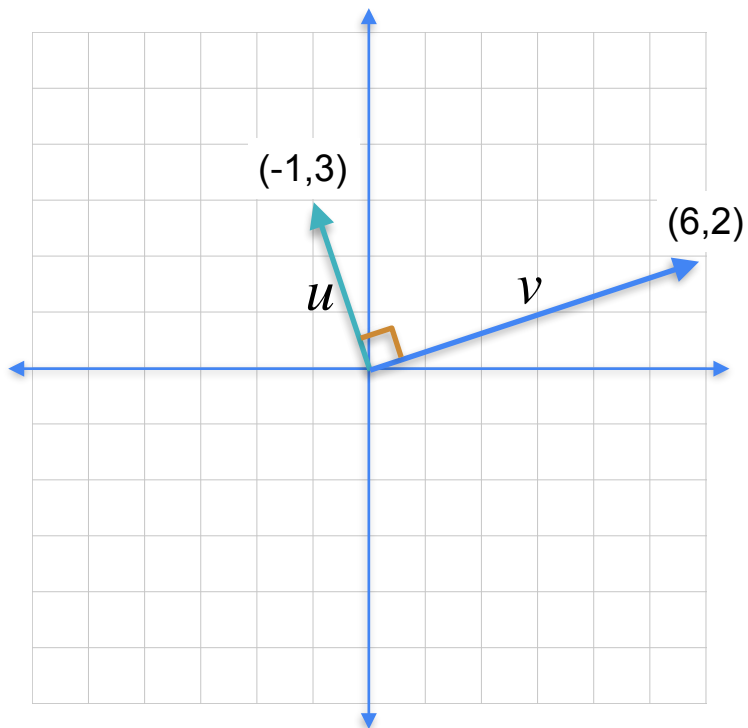
2

Orthogonal vectors have dot product 0



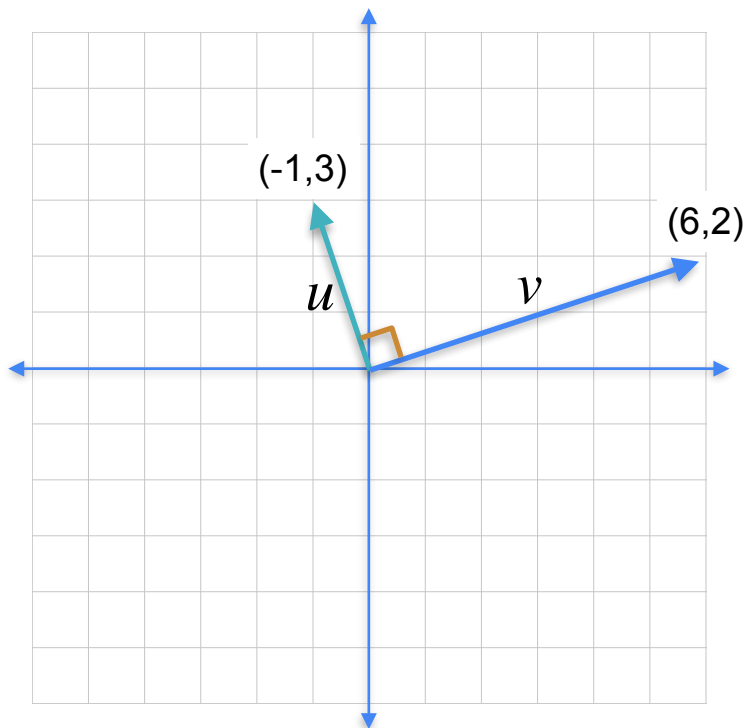
6	2	-1
		3

Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Orthogonal vectors have dot product 0

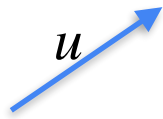


$$\begin{matrix} 6 & 2 & \begin{matrix} -1 \\ 3 \end{matrix} \end{matrix} = 0$$

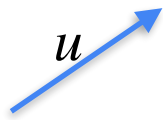
$$\langle u, v \rangle = 0$$

The dot product

The dot product

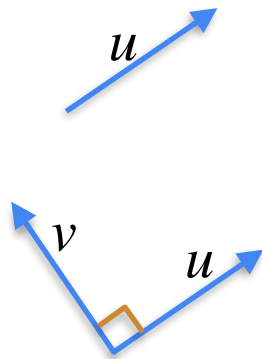


The dot product



$$\langle u, u \rangle = |u|^2$$

The dot product

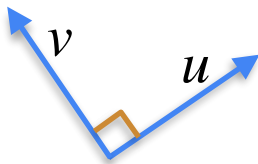


$$\langle u, u \rangle = |u|^2$$

The dot product

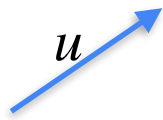


$$\langle u, u \rangle = |u|^2$$

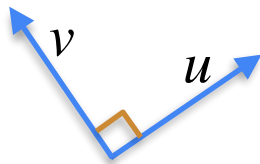


$$\langle u, v \rangle = 0$$

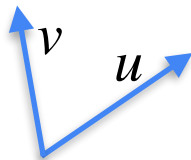
The dot product



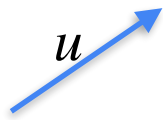
$$\langle u, u \rangle = |u|^2$$



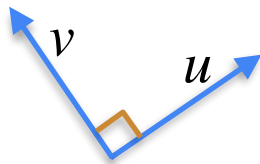
$$\langle u, v \rangle = 0$$



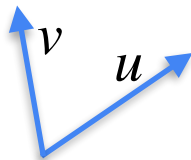
The dot product



$$\langle u, u \rangle = |u|^2$$



$$\langle u, v \rangle = 0$$



$$\langle u, v \rangle = ?$$

The dot product

The dot product

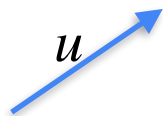


The dot product



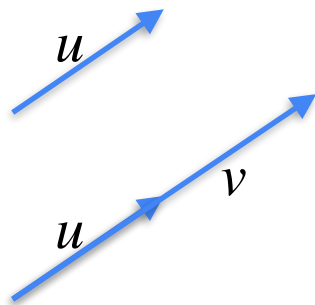
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



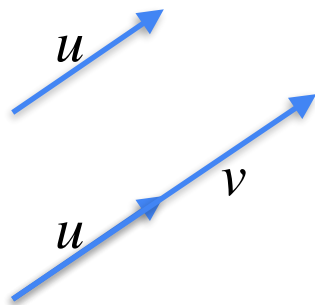
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

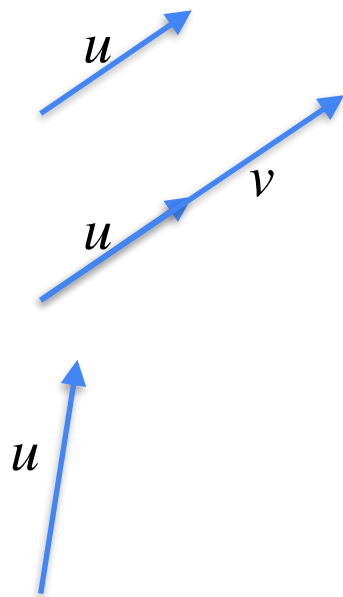
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product



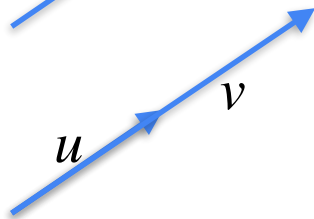
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

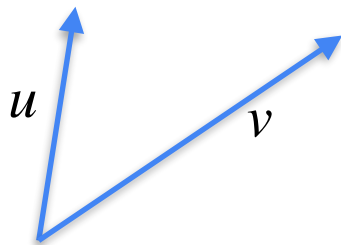
The dot product



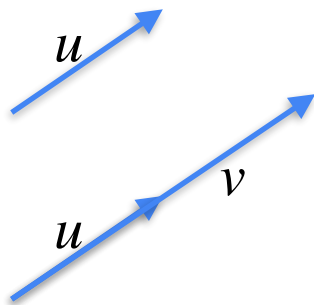
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



$$\langle u, v \rangle = |u| \cdot |v|$$

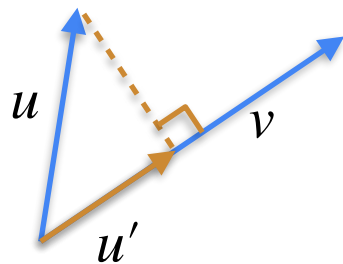


The dot product

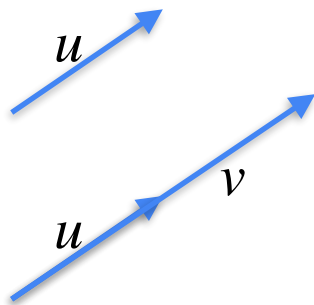


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

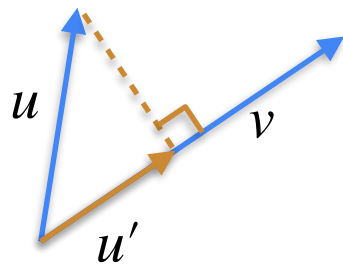


The dot product



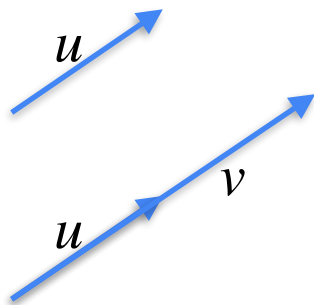
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$



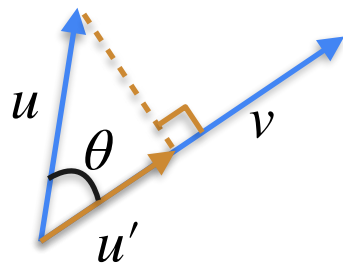
$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



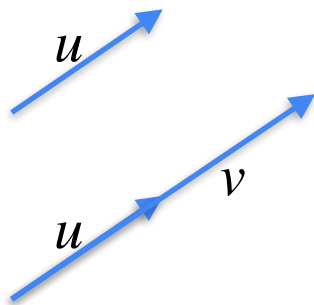
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$



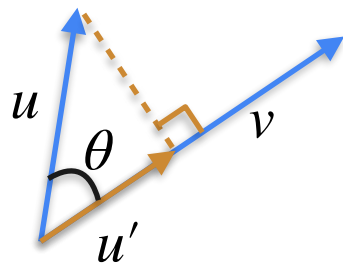
$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



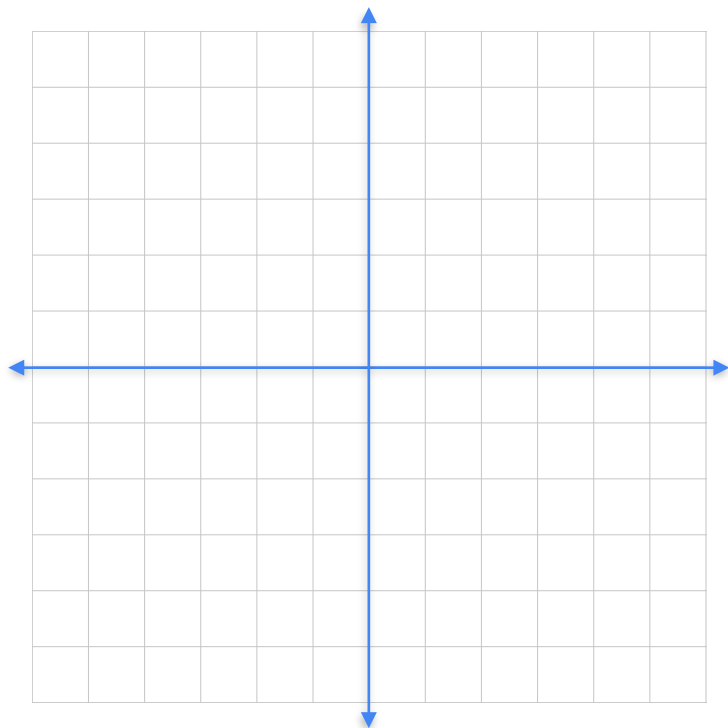
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

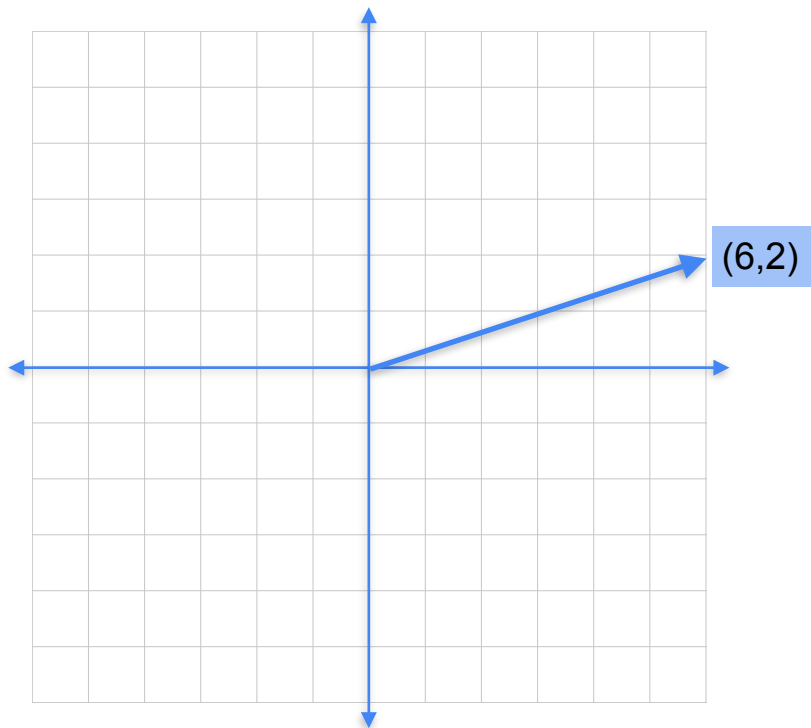


$$\begin{aligned}\langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta)\end{aligned}$$

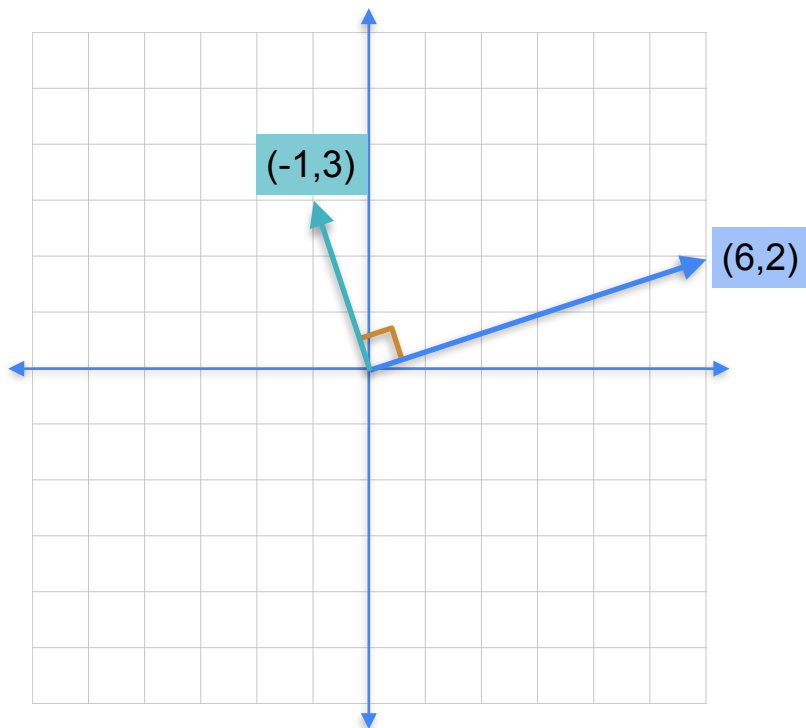
Geometric dot product



Geometric dot product

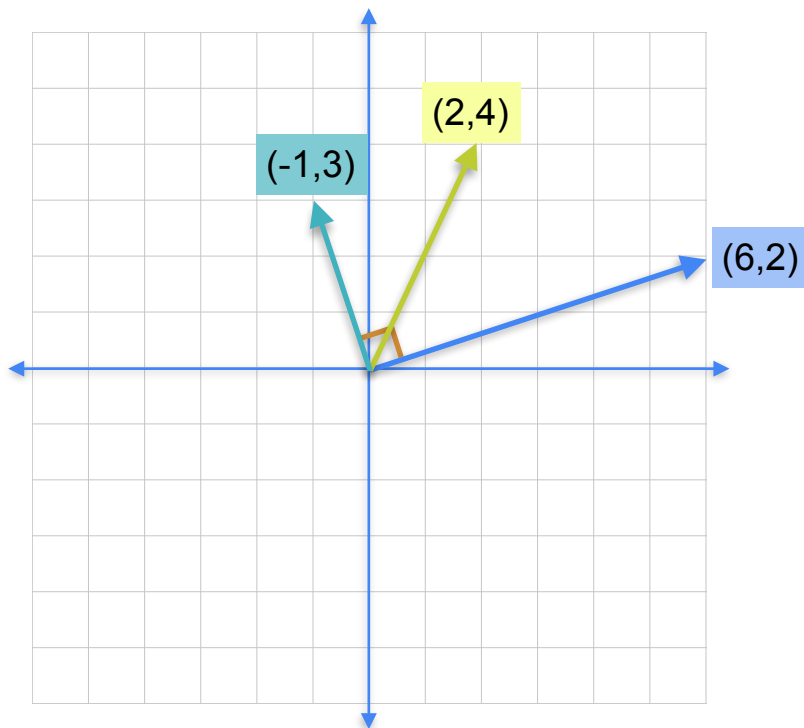


Geometric dot product



$$\begin{matrix} 6 & 2 \end{matrix} \cdot \begin{matrix} -1 \\ 3 \end{matrix} = 0$$

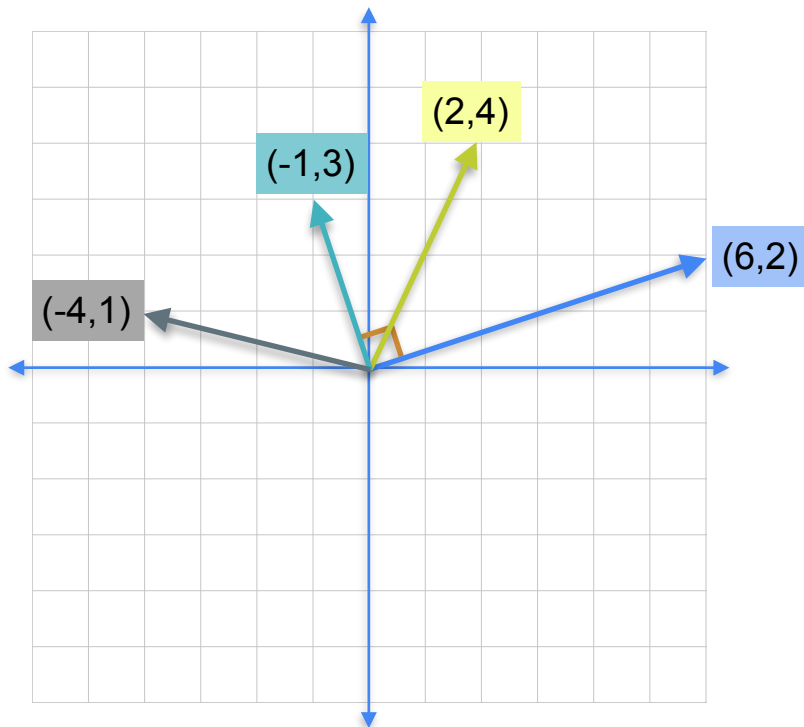
Geometric dot product



$$\begin{array}{|c|} \hline 6 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 6 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline \end{array} \begin{array}{|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

Geometric dot product

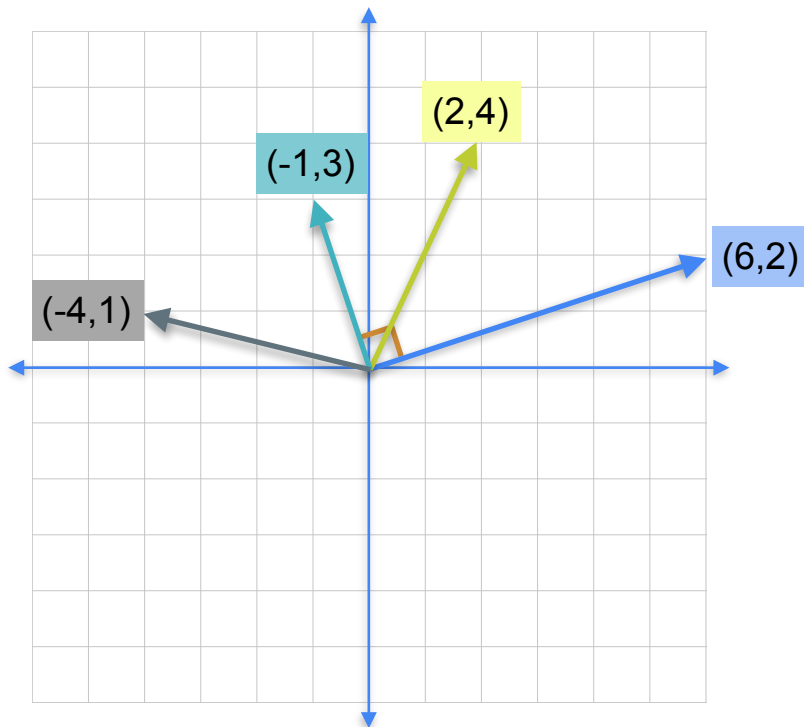


$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22$$

Geometric dot product

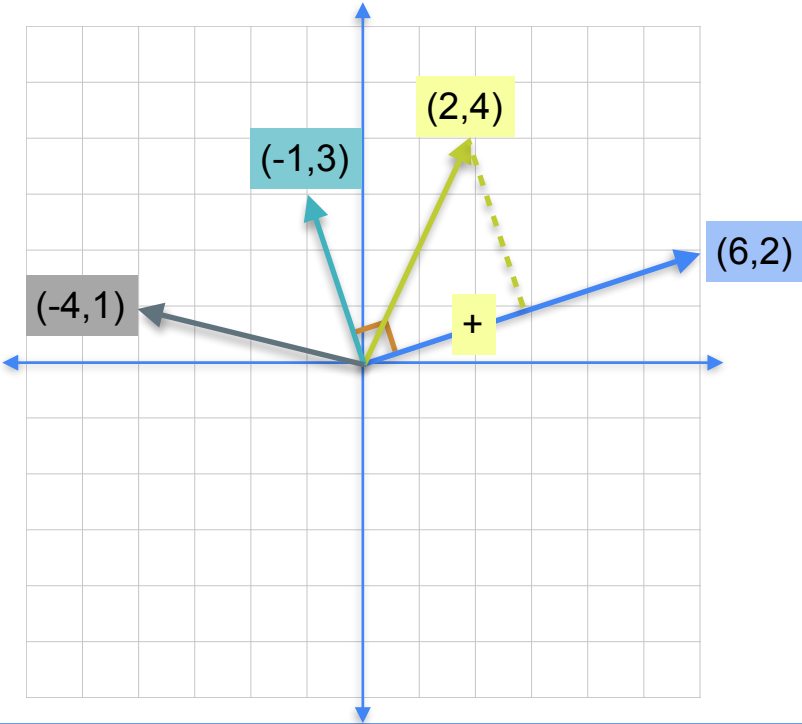


$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22$$

Geometric dot product

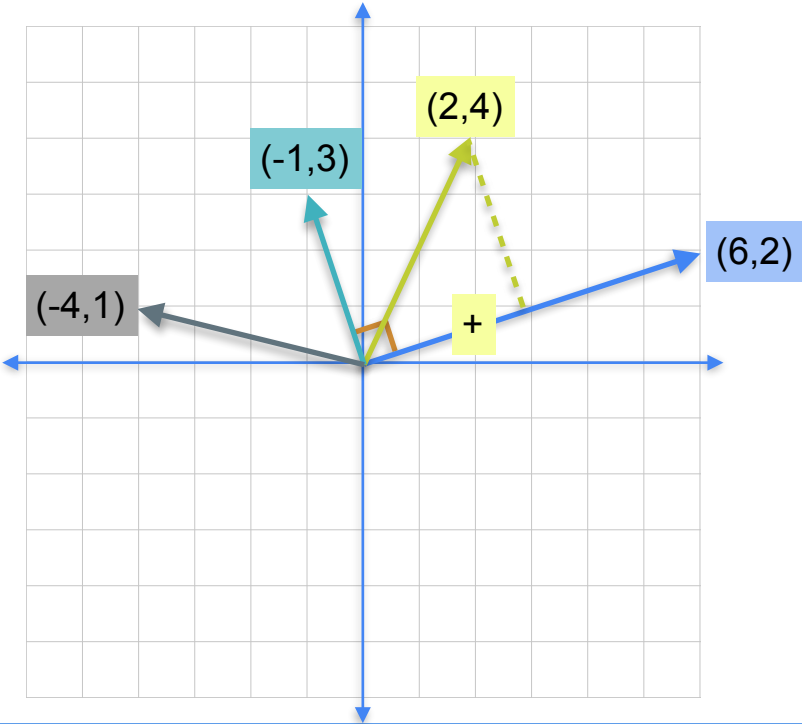


$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22$$

Geometric dot product

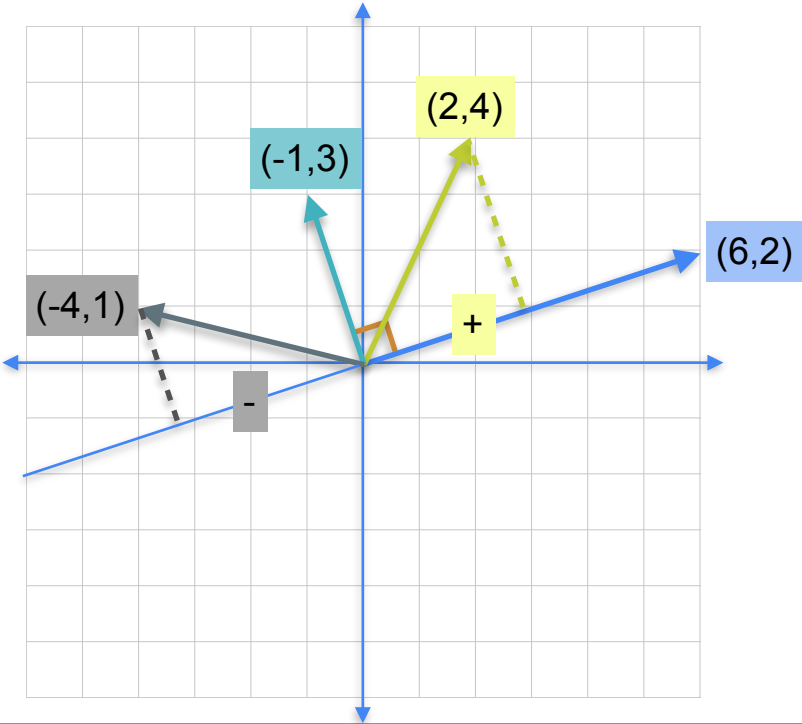


$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22 \quad \text{Negative}$$

Geometric dot product

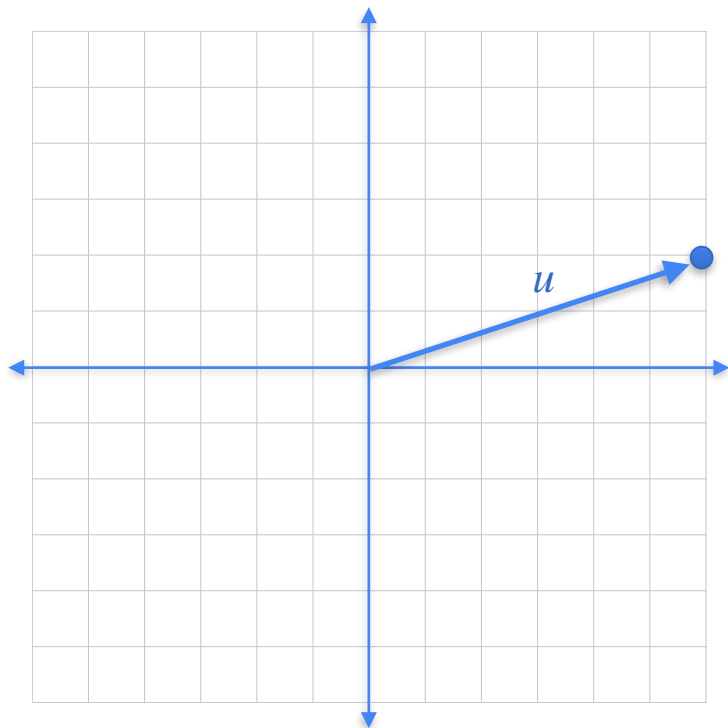


$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

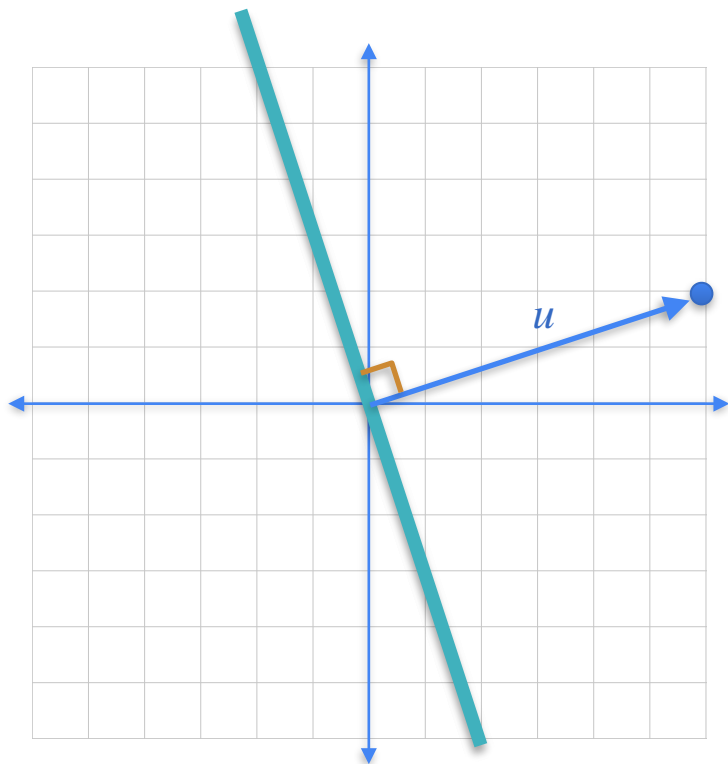
$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22 \quad \text{Negative}$$

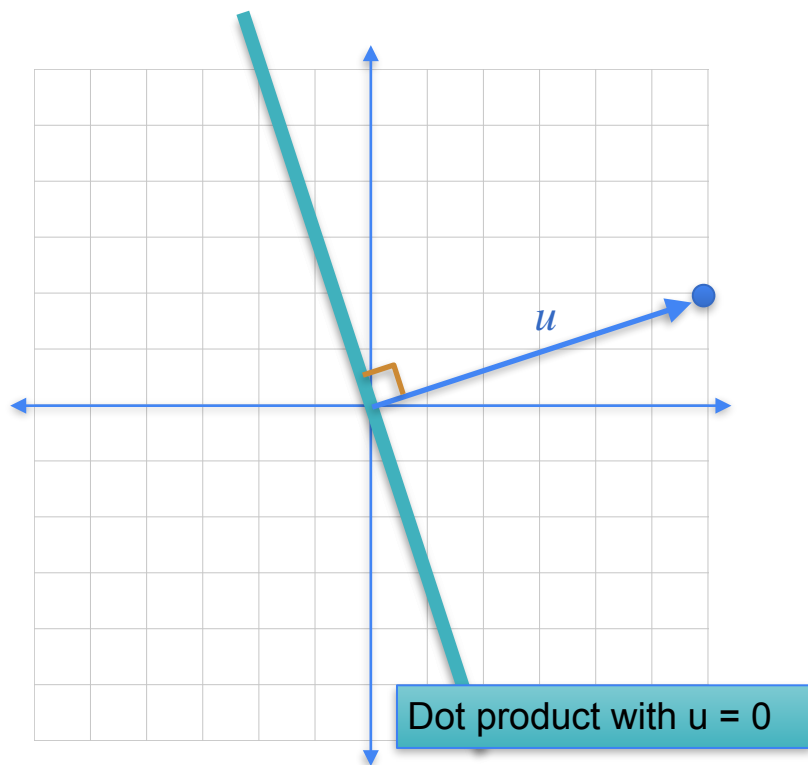
Geometric dot product



Geometric dot product

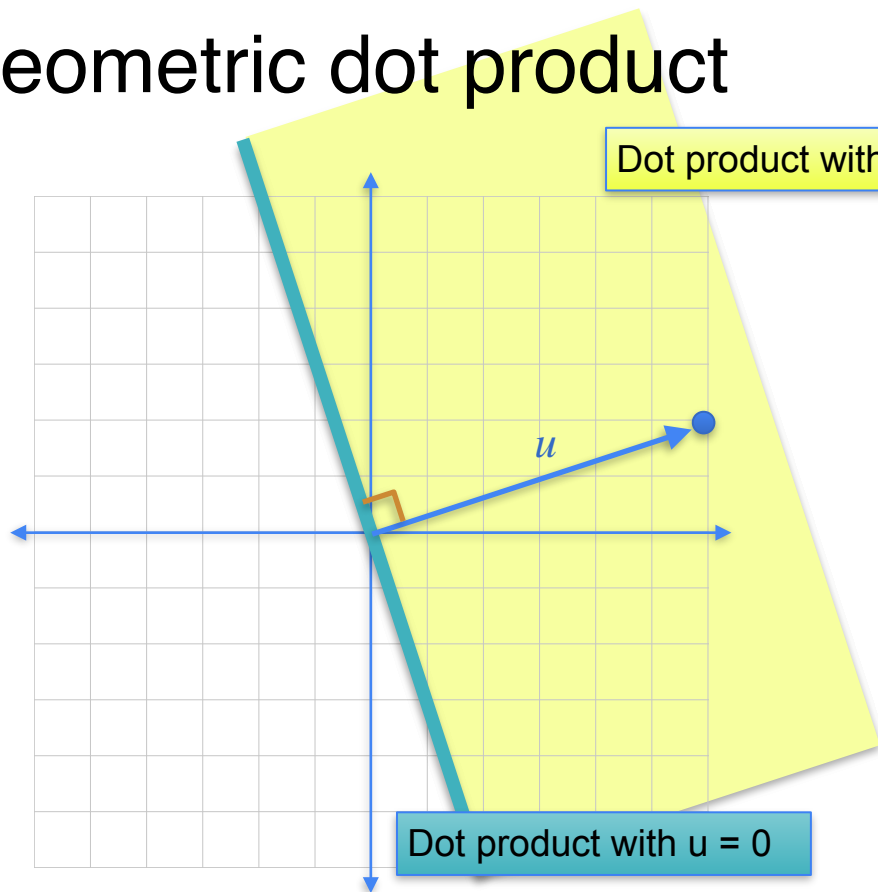


Geometric dot product



$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product

Dot product with $u > 0$

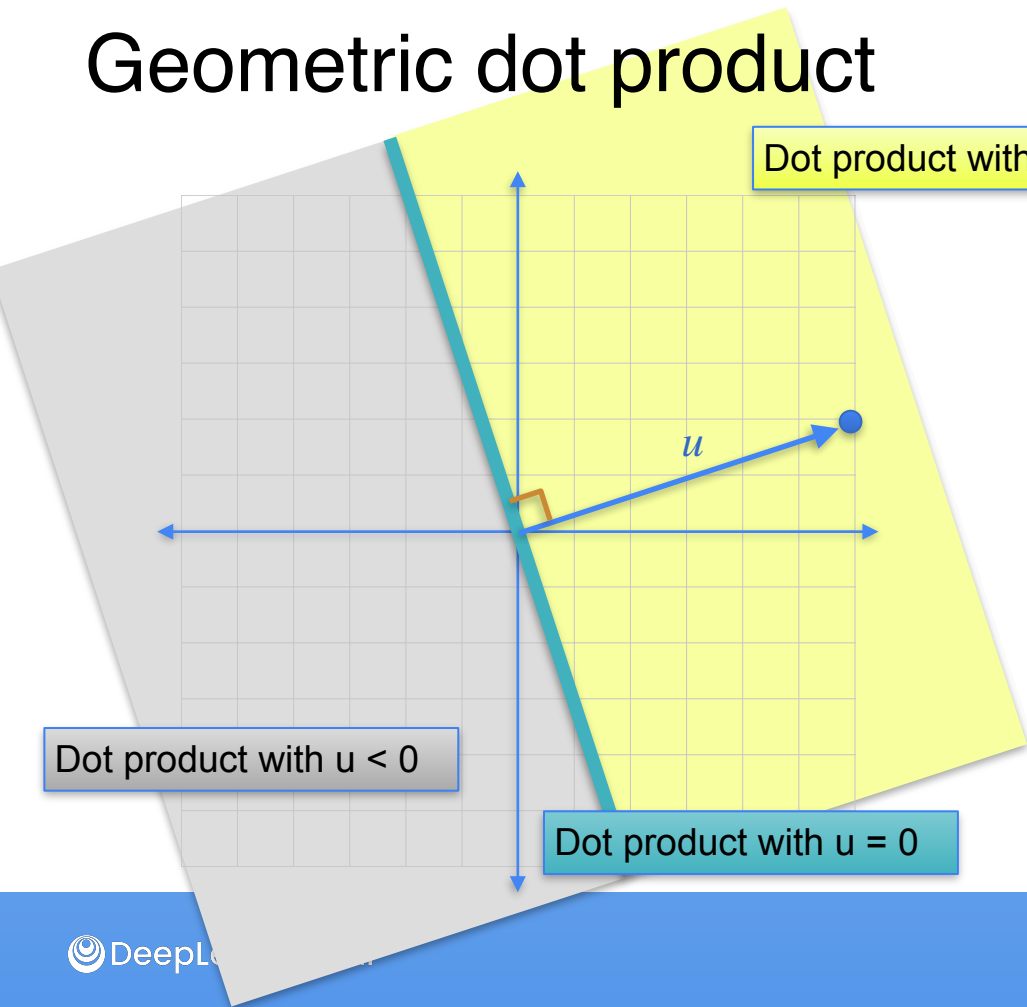
$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

Dot product with $u < 0$

Dot product with $u = 0$





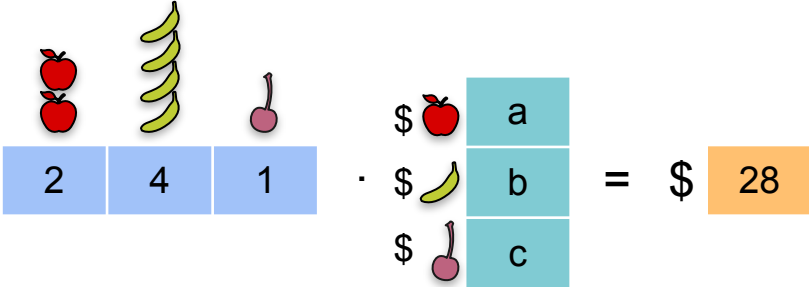
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Vectors and Linear Transformations

**Multiplying a matrix by a
vector**

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



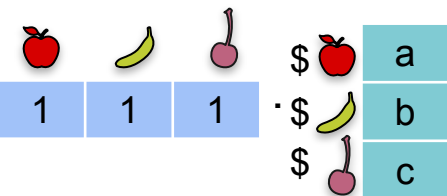
1	1	1
---	---	---

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$










$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{ apple} & a \\ \$ \text{ banana} & b \\ \$ \text{ cherry} & c \end{bmatrix} = \$ 10$


Equations as dot product

$$a + b + c = 10$$

			\$			a	
1	1	1	·	\$			b
			\$			c	

= \$ 10

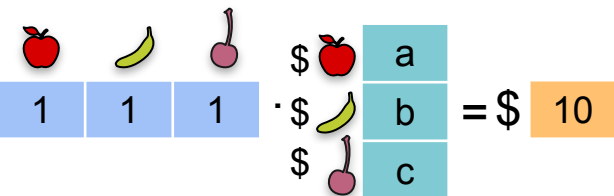
$$a + 2b + c = 15$$

		
1	2	1

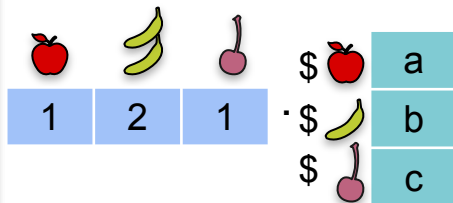
$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{ 🍏 } a \\ \$ \text{ 🍌 } b \\ \$ \text{ 🍒 } c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{ 🍏 } a \\ \$ \text{ 🍌 } b \\ \$ \text{ 🍒 } c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{ 🍏 } a \\ \$ \text{ 🍌 } b \\ \$ \text{ 🍒 } c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

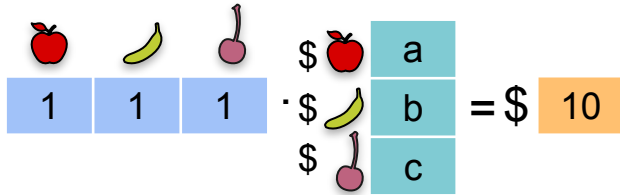
$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{ 🍏 } a \\ \$ \text{ 🍌 } b \\ \$ \text{ 🍒 } c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

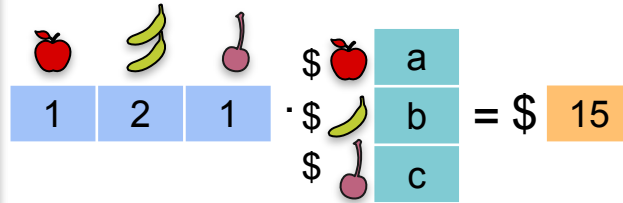
$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \$ \text{ 🍏 } a \\ \$ \text{ 🍌 } b \\ \$ \text{ 🍒 } c \end{bmatrix}$

Equations as dot product

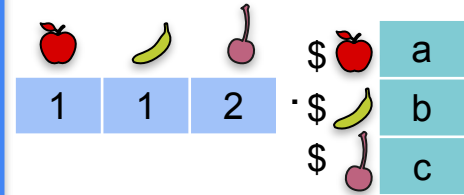
$$a + b + c = 10$$



$$a + 2b + c = 15$$



$$a + b + 2c = 12$$



Equations as dot product

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 12$

Equations as dot product

$$a + b + c = 10$$

Diagram illustrating the equation $a + b + c = 10$ as a dot product. The coefficients are represented by a row vector $[1, 1, 1]$ (with icons of an apple, banana, and cherry above the numbers). The variables are represented by a column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (with icons of an apple, banana, and cherry to the left of the variables). The result is $= \$ 10$.

$$a + 2b + c = 15$$

Diagram illustrating the equation $a + 2b + c = 15$ as a dot product. The coefficients are represented by a row vector $[1, 2, 1]$ (with icons of an apple, two bananas, and a cherry above the numbers). The variables are represented by a column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (with icons of an apple, banana, and cherry to the left of the variables). The result is $= \$ 15$.

$$a + b + 2c = 12$$







Diagram illustrating the equation $a + b + 2c = 12$ as a dot product. The coefficients are represented by a row vector $[1, 1, 2]$ (with icons of an apple, a banana, and two cherries above the numbers). The variables are represented by a column vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (with icons of an apple, banana, and cherry to the left of the variables). The result is $= \$ 12$.







Equations as dot product







$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

			·	\$ 	a	= \$ 10
1	1	1		\$ 	b	
				\$ 	c	

			·	\$ 	a	= \$ 15
1	2	1		\$ 	b	
				\$ 	c	

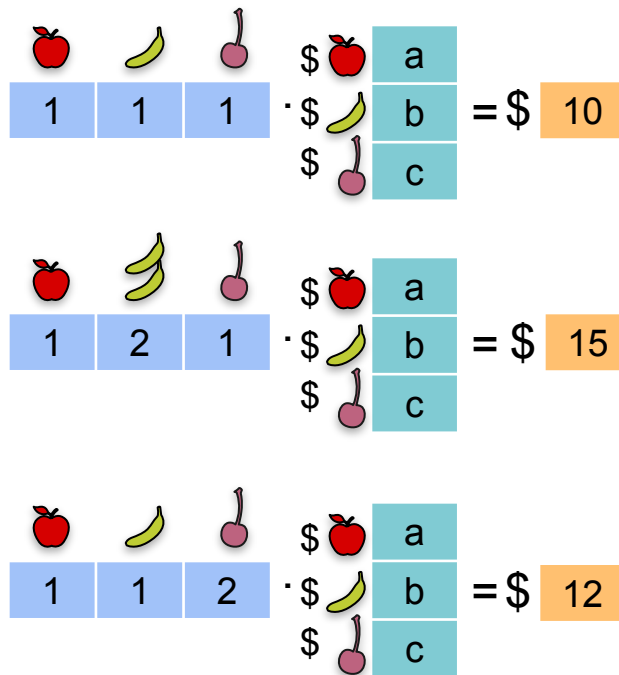
			·	\$ 	a	= \$ 12
1	1	2		\$ 	b	
				\$ 	c	

Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



Equations as dot product







System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

					
1	1	1	\$ 	a	10
1	2	1	· \$ 	b	15
1	1	2	\$ 	c	12

Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

1	1	1	a	=	10
1	2	1	b	=	15
1	1	2	c	=	12





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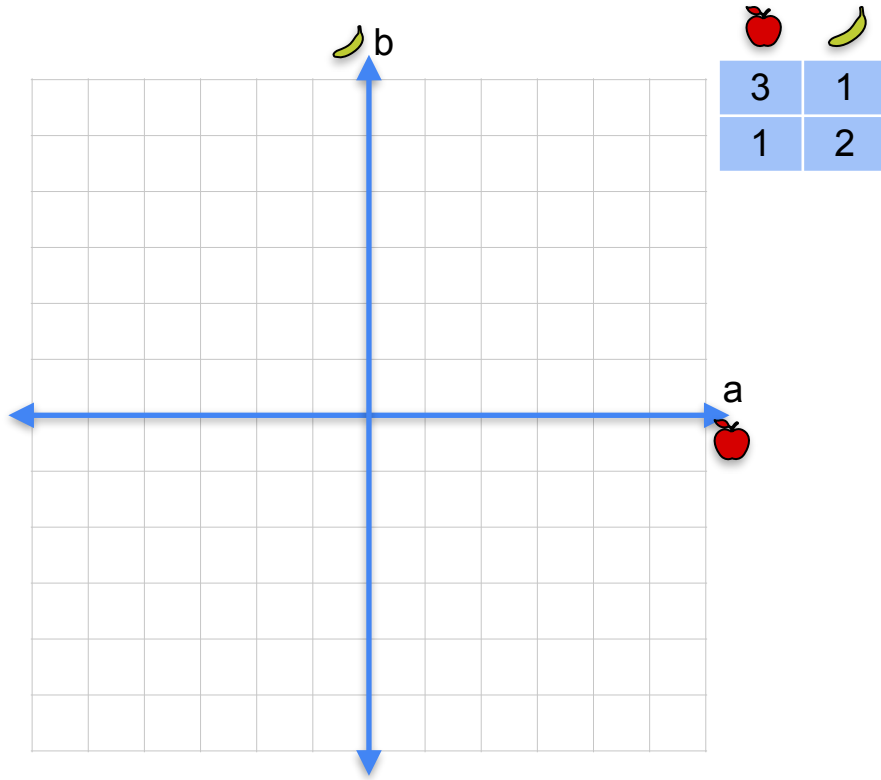
Vectors and Linear Transformations

**Matrices as linear
transformations**

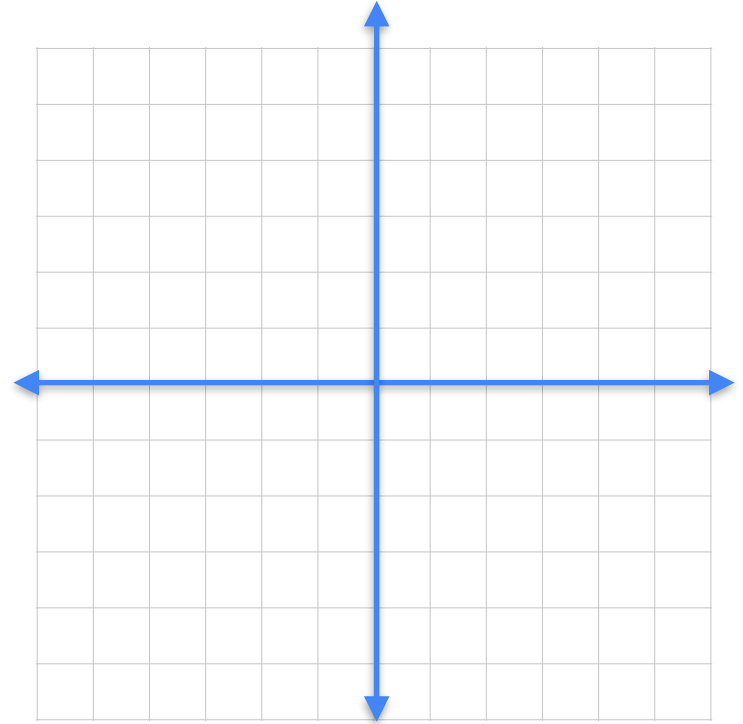
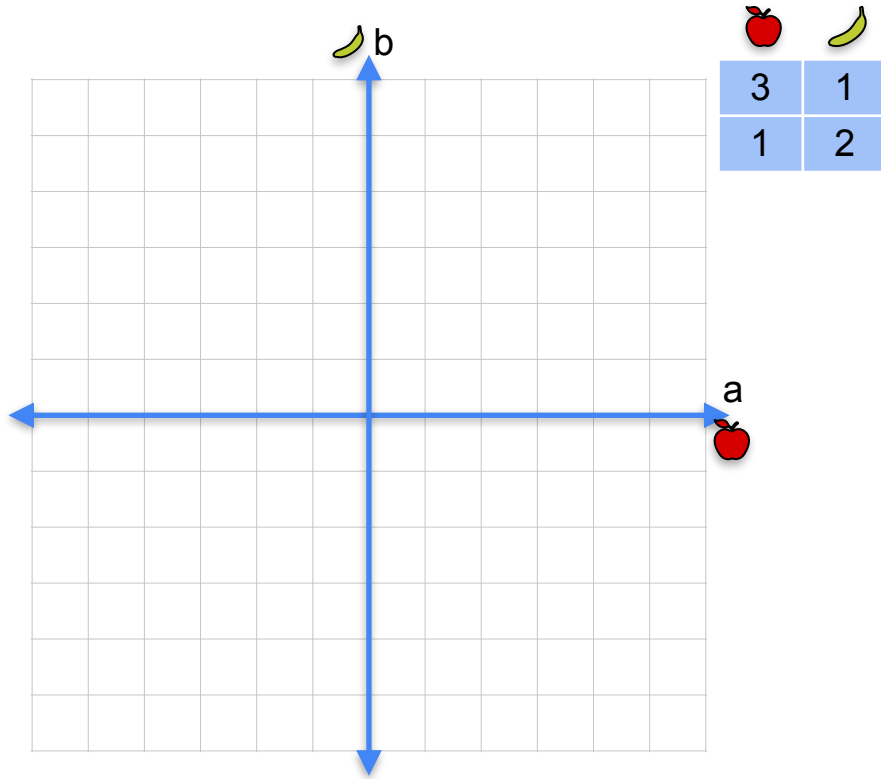
Matrices as linear transformations

	
3	1
1	2

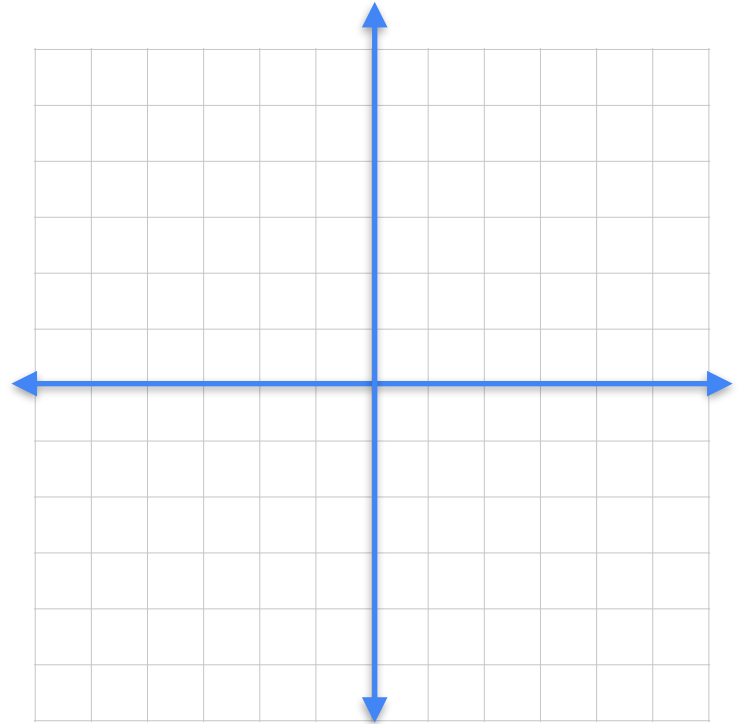
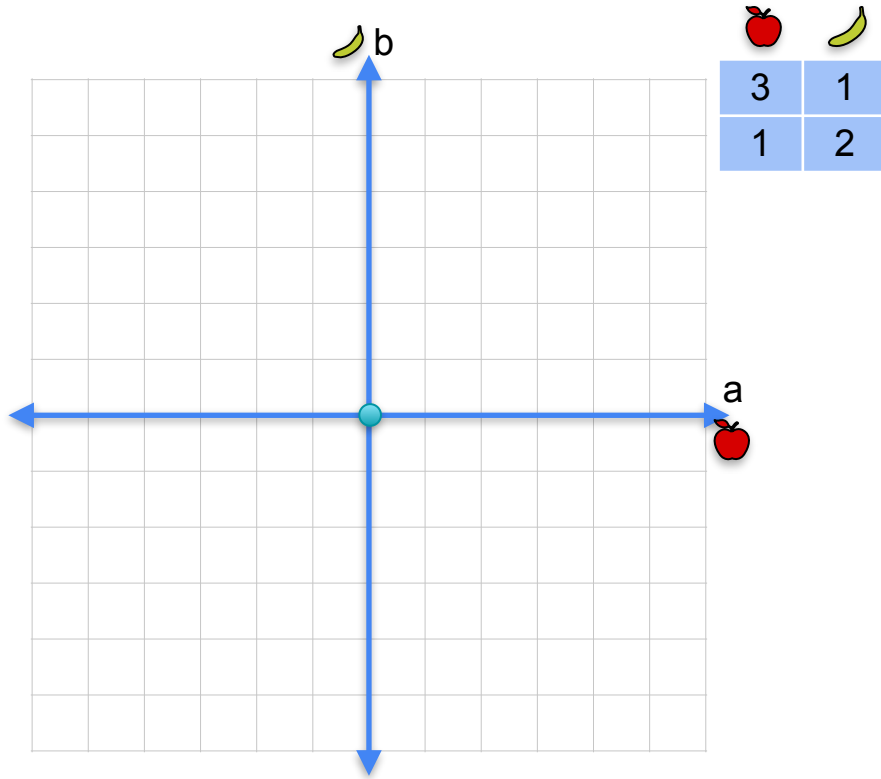
Matrices as linear transformations



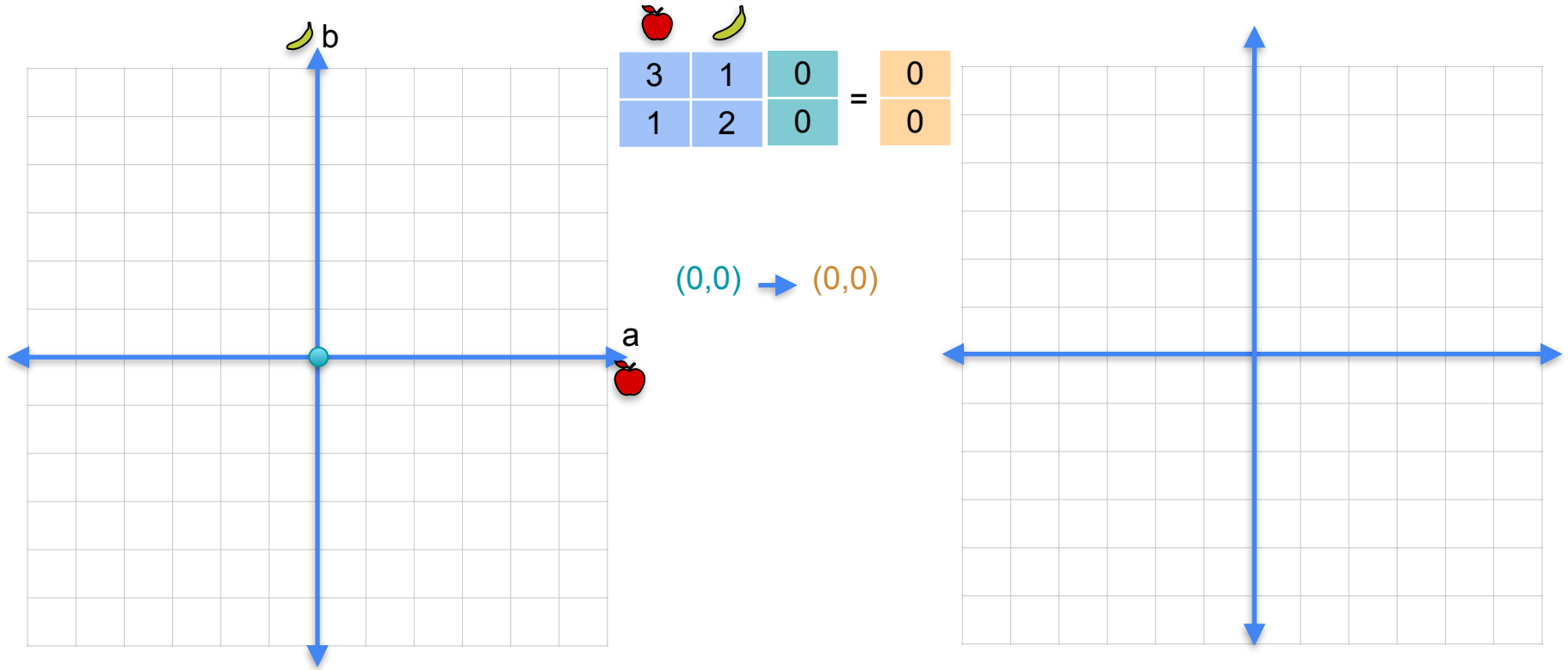
Matrices as linear transformations



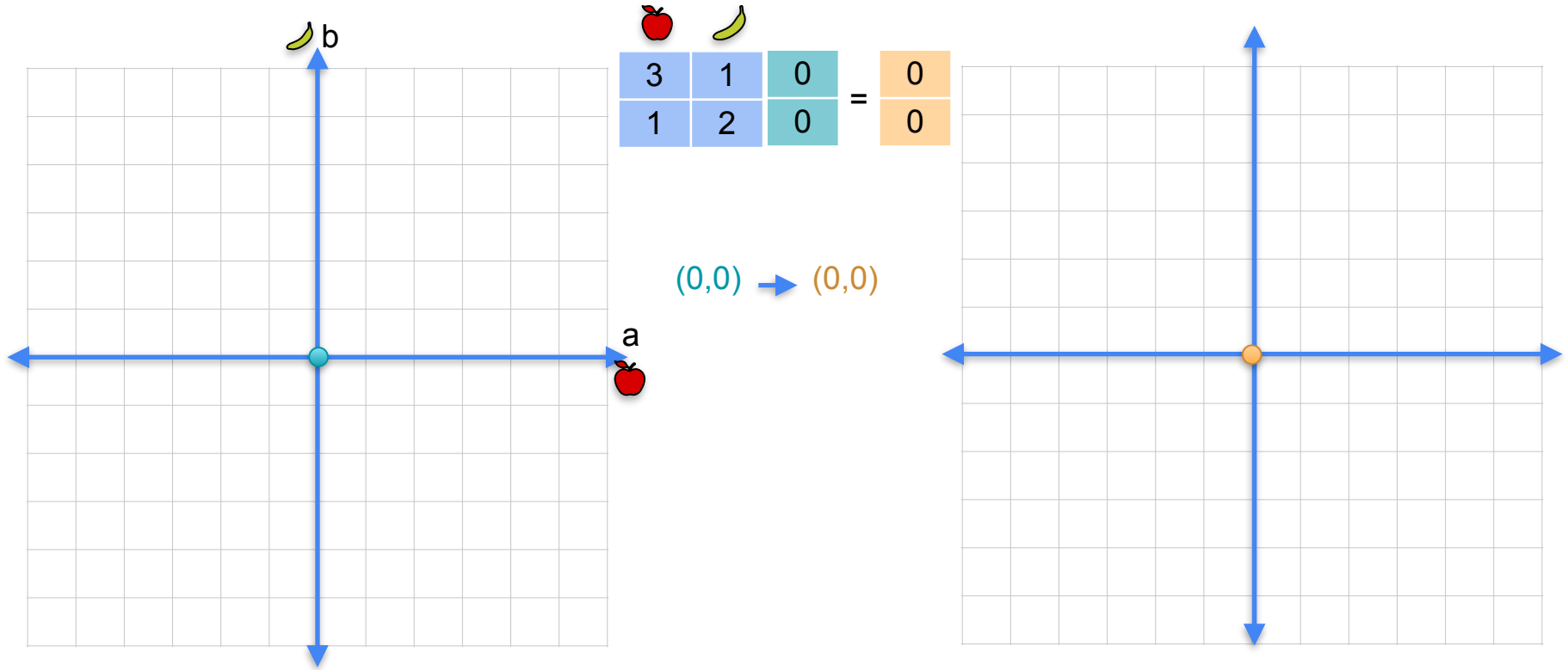
Matrices as linear transformations



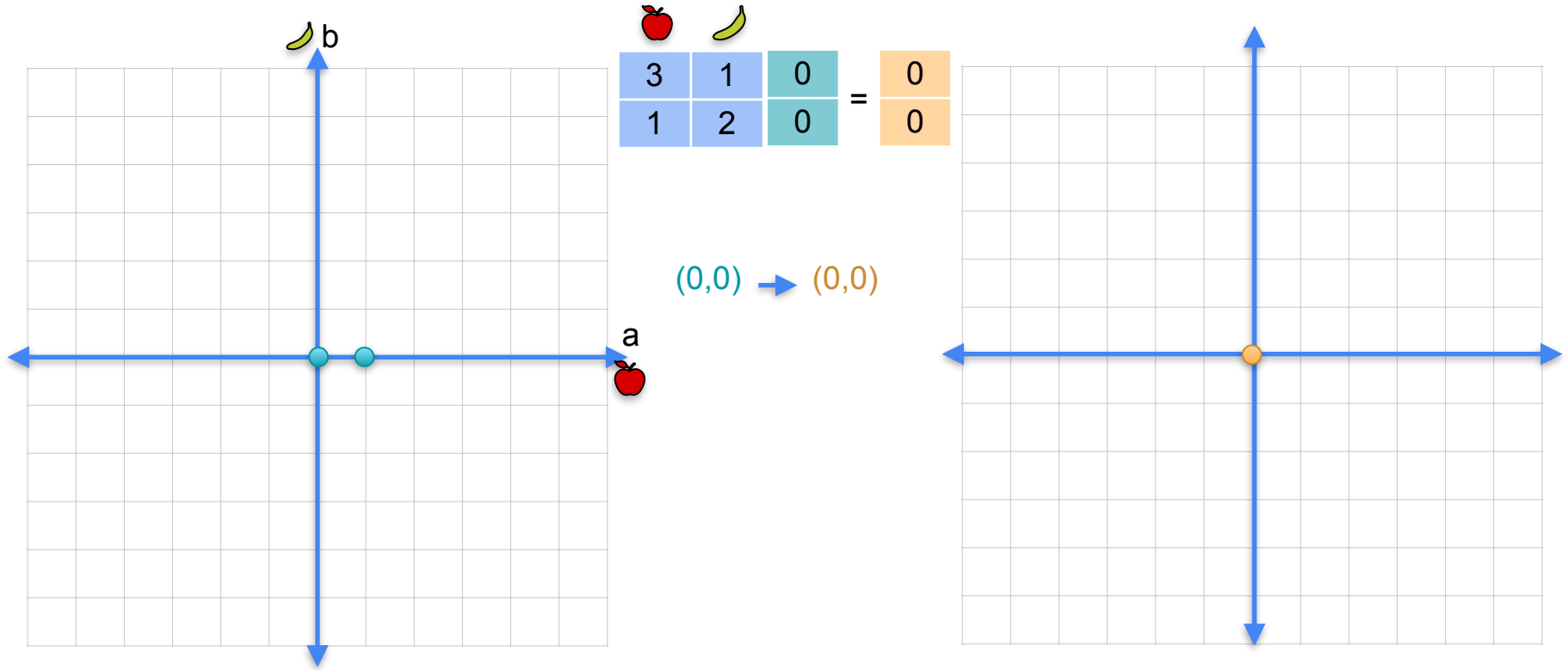
Matrices as linear transformations



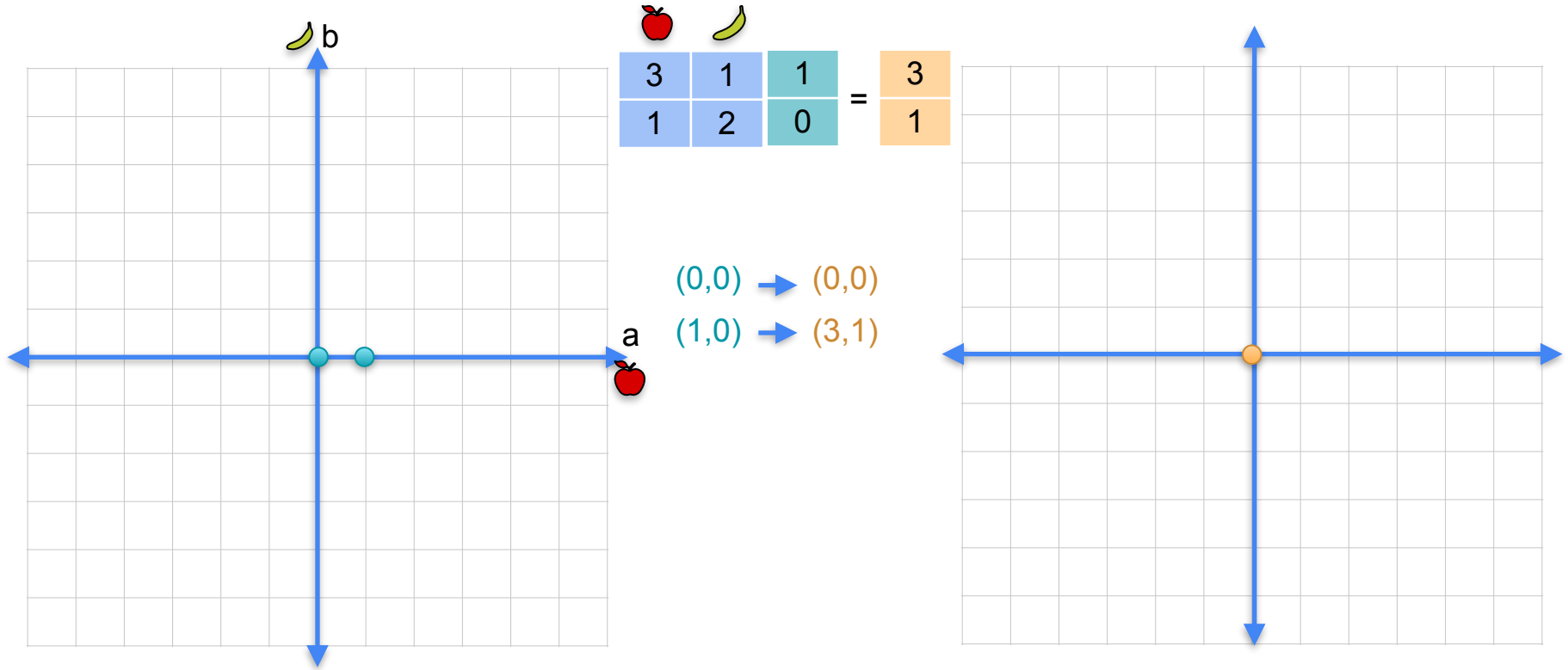
Matrices as linear transformations



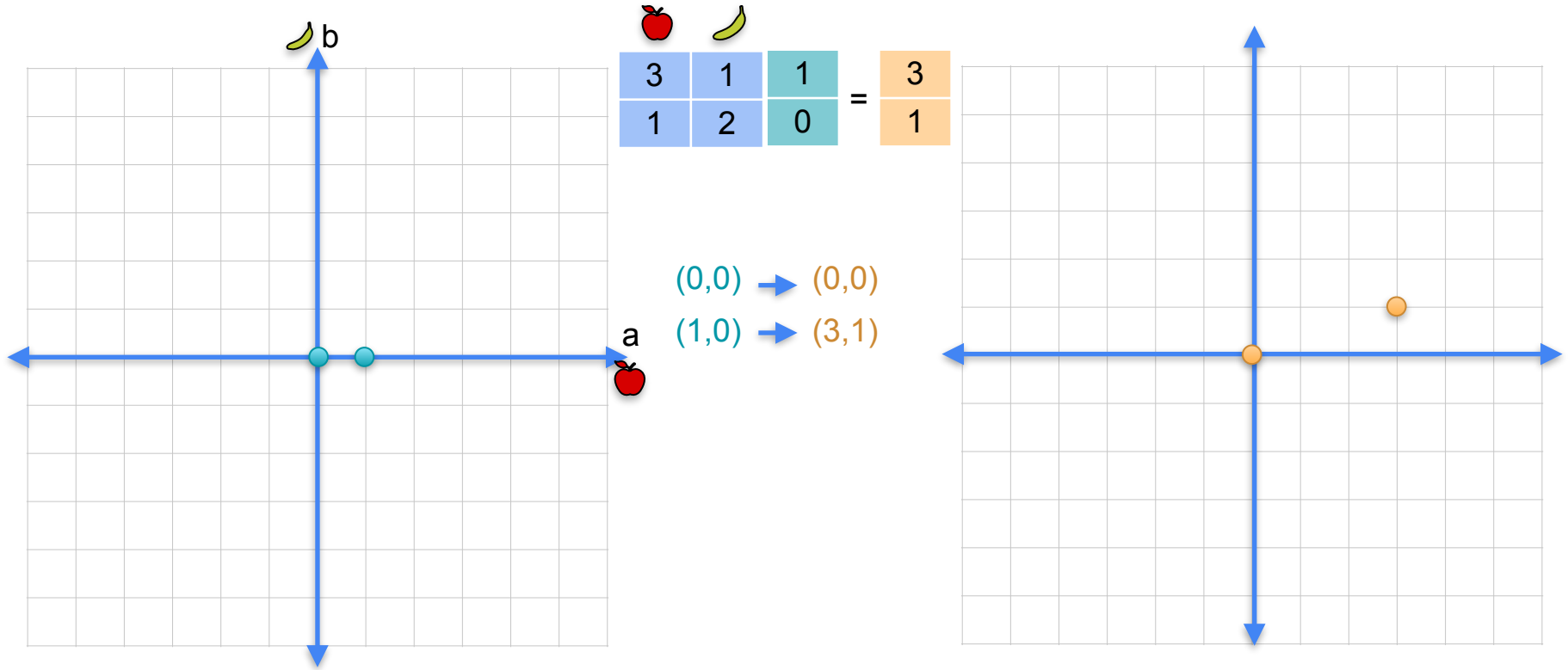
Matrices as linear transformations



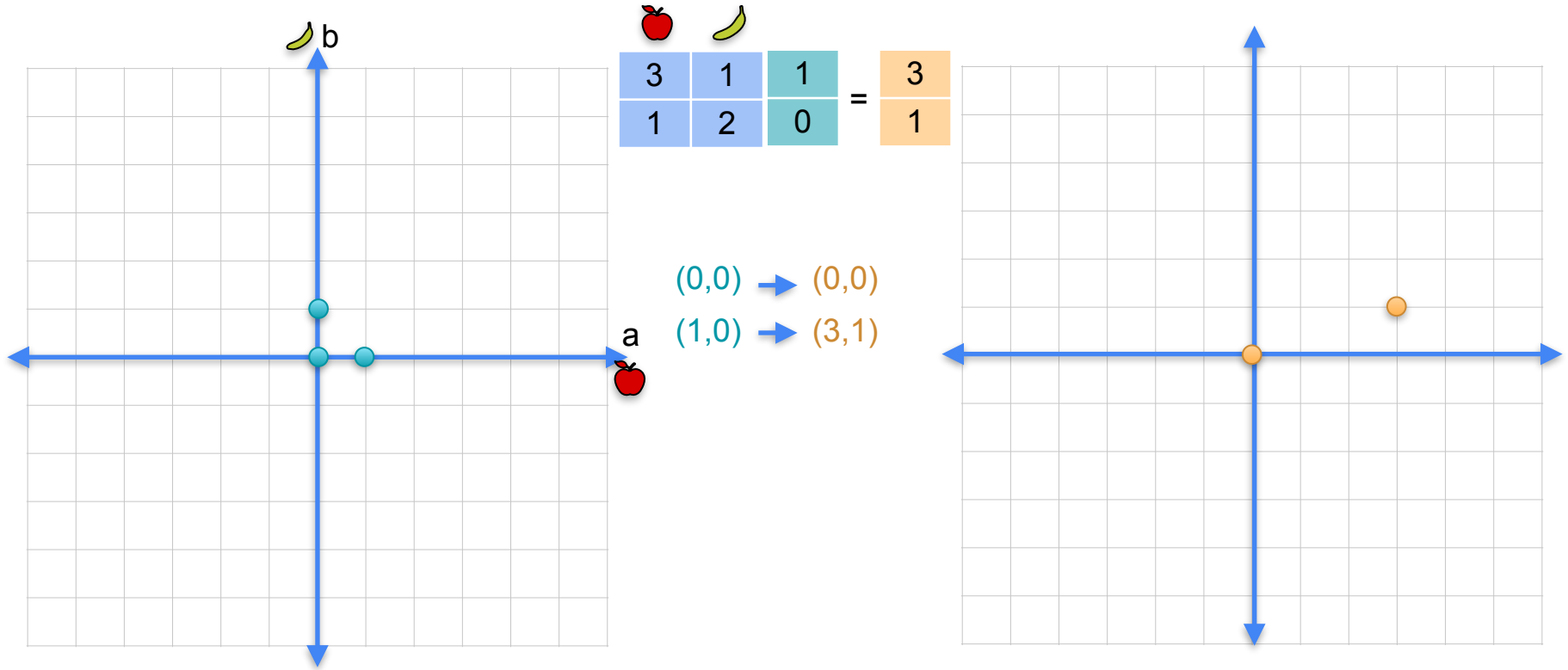
Matrices as linear transformations



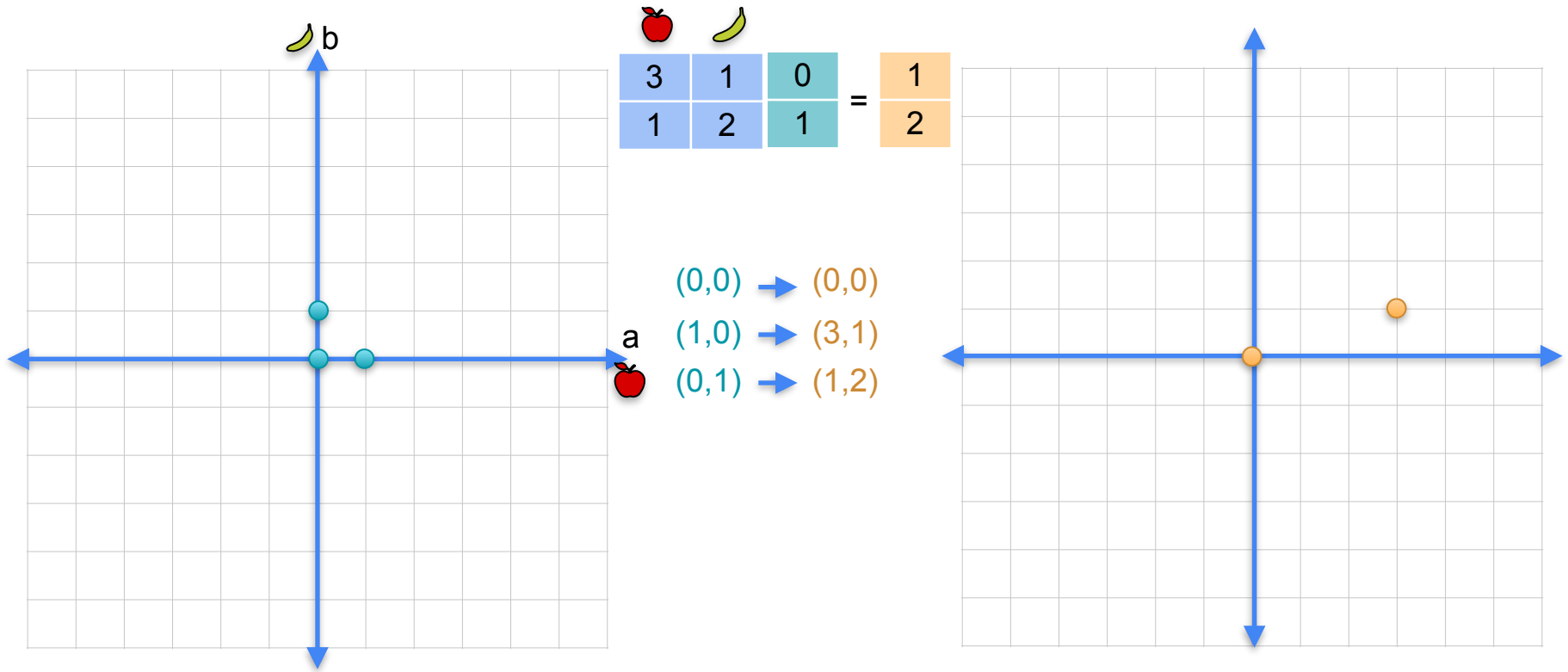
Matrices as linear transformations



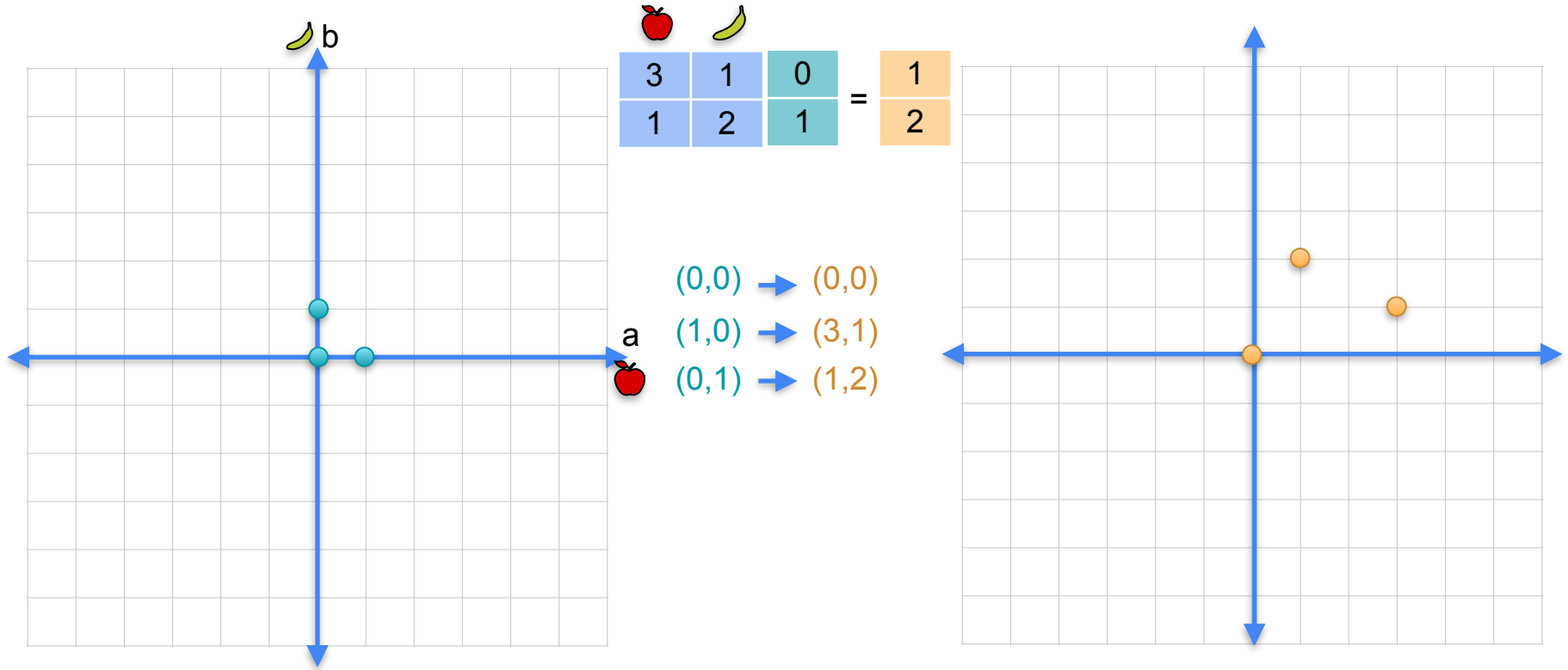
Matrices as linear transformations



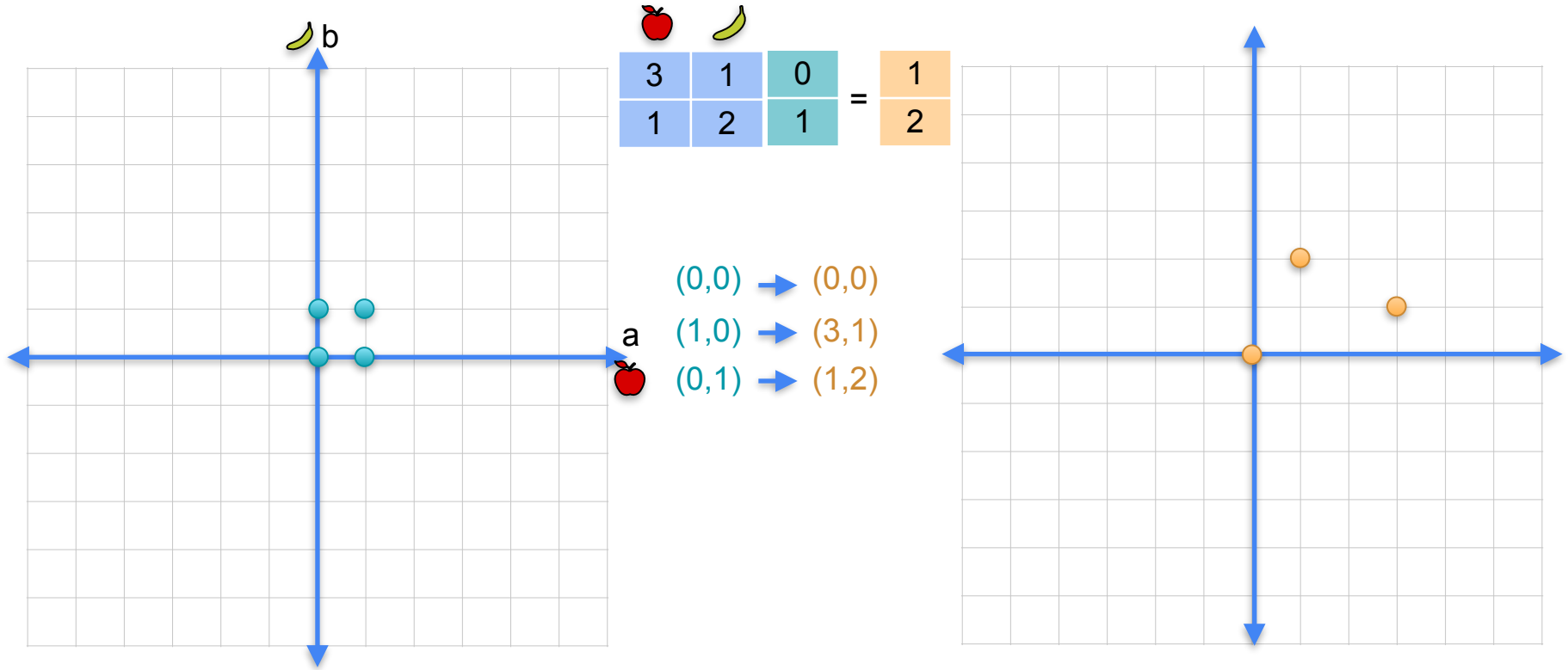
Matrices as linear transformations



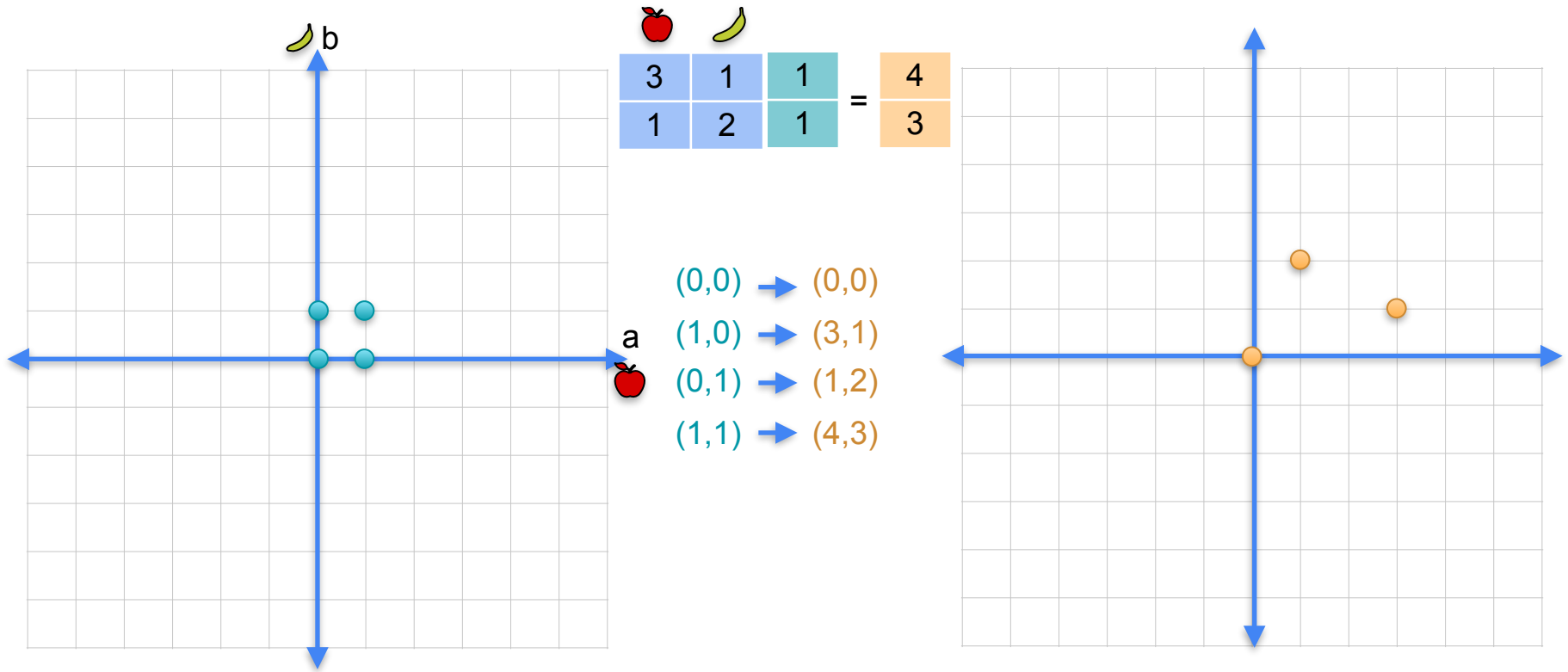
Matrices as linear transformations



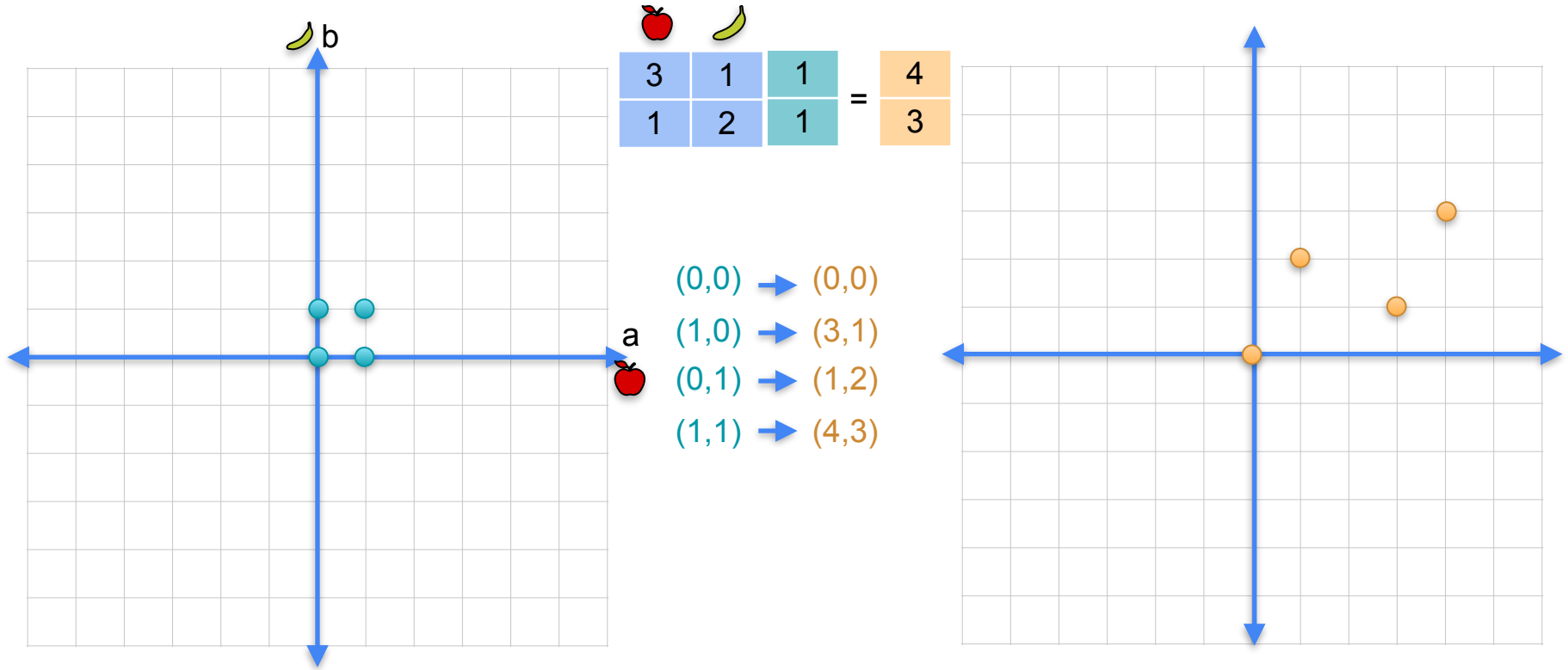
Matrices as linear transformations



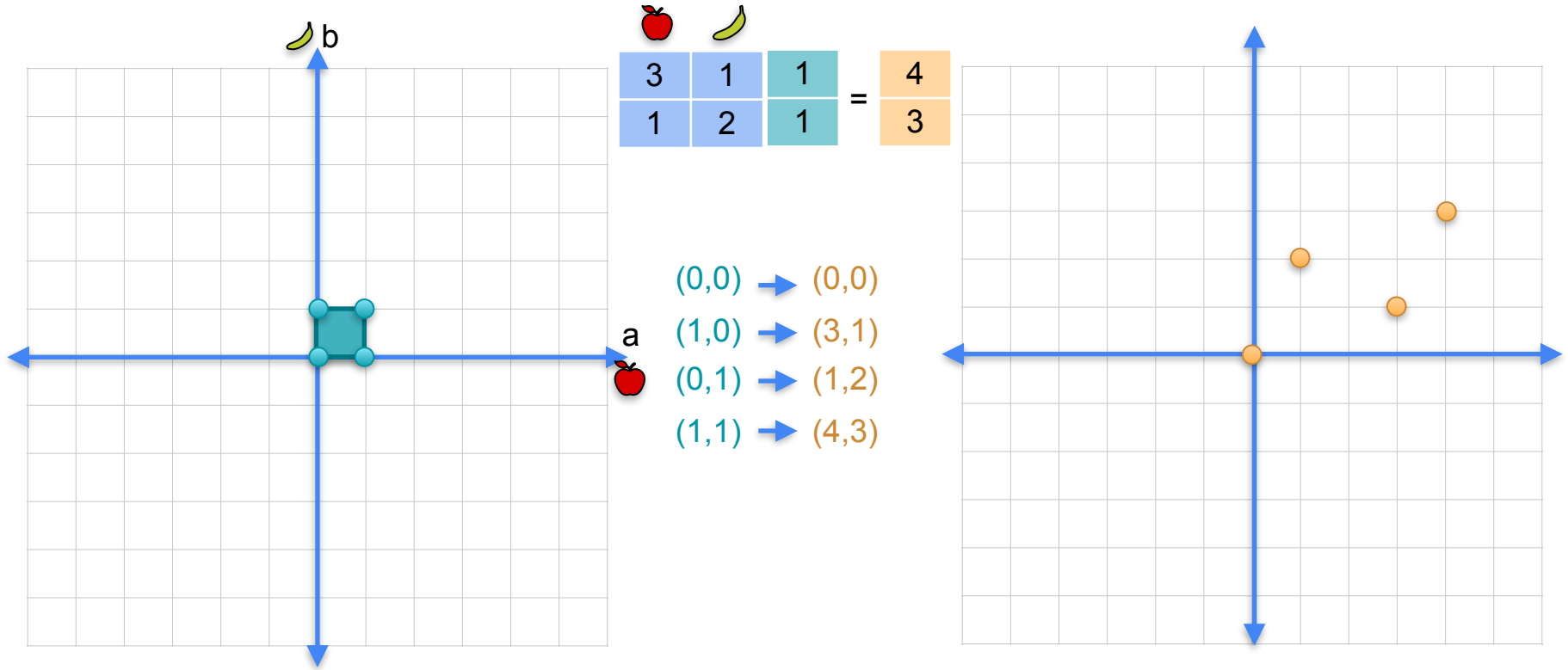
Matrices as linear transformations



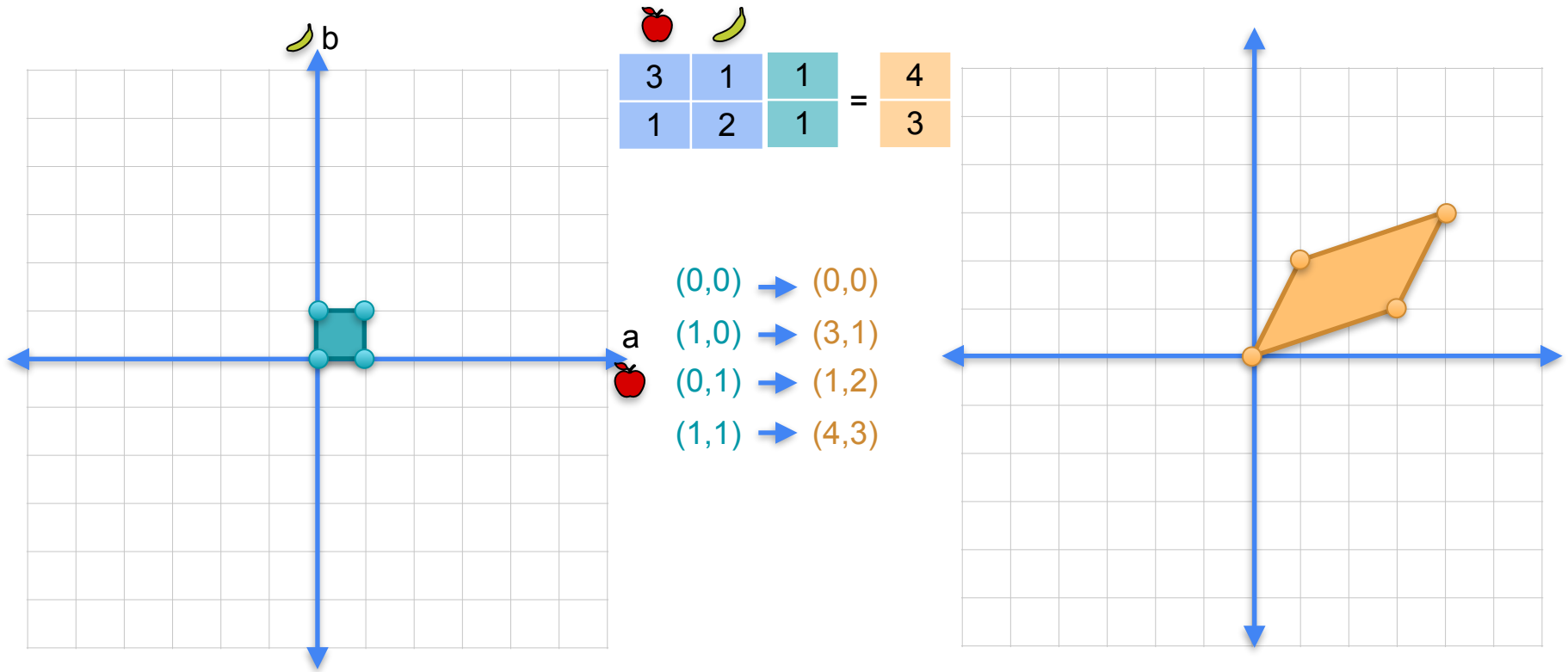
Matrices as linear transformations



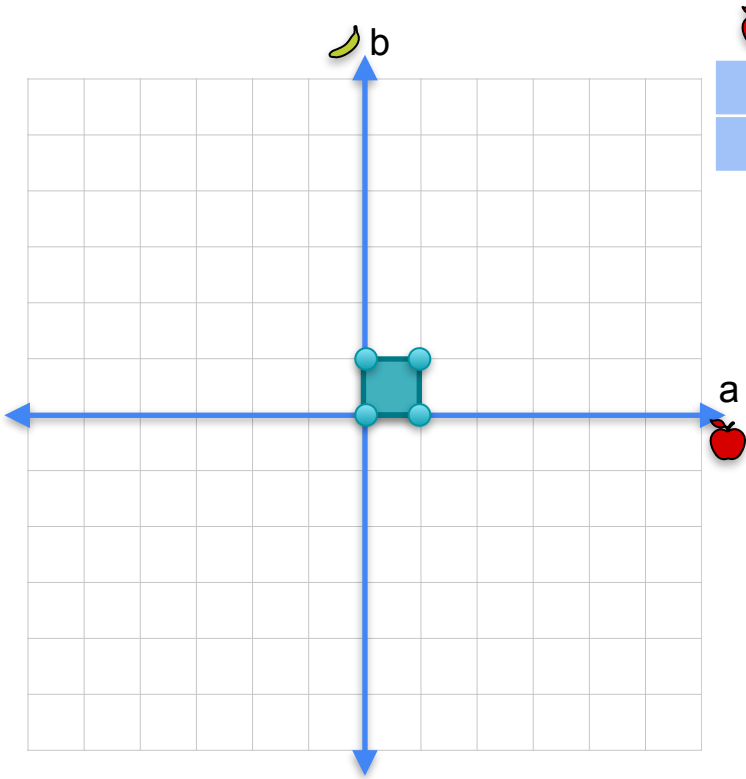
Matrices as linear transformations



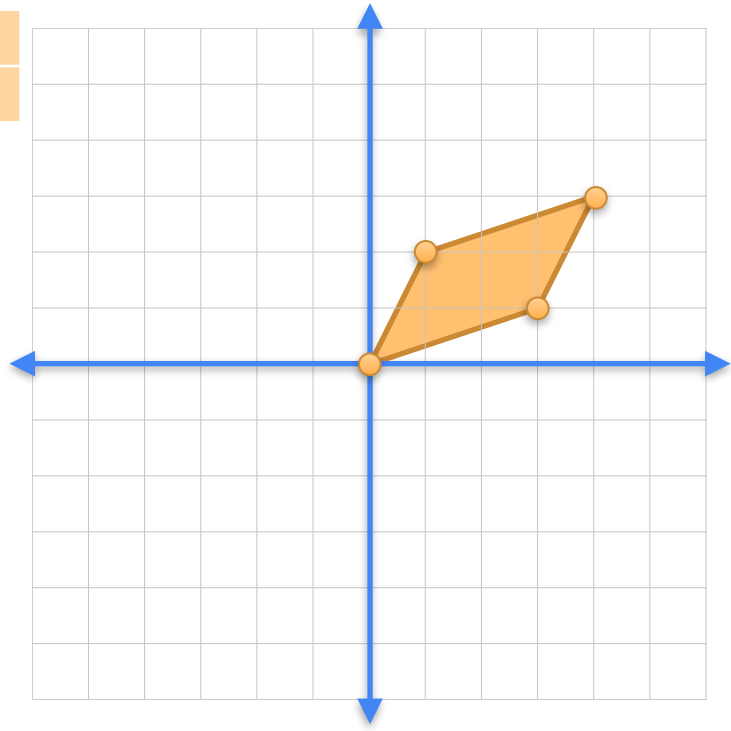
Matrices as linear transformations



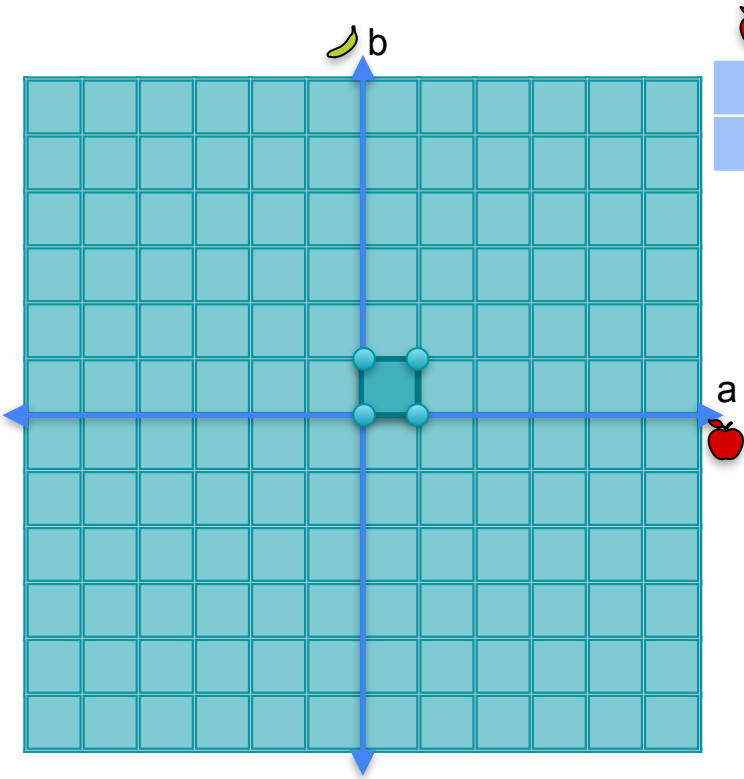
Matrices as linear transformations



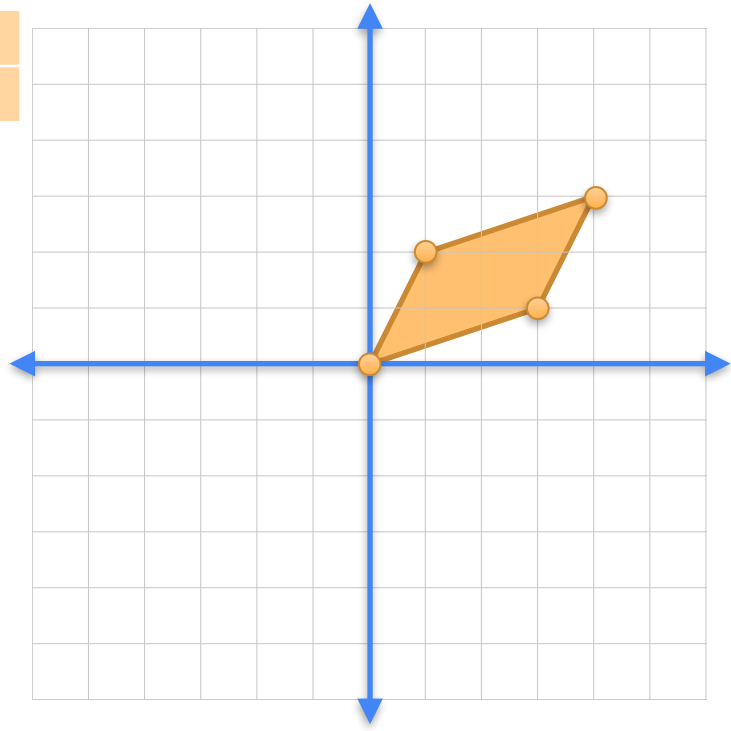
3	1	-2	=	-3
1	2	3		4



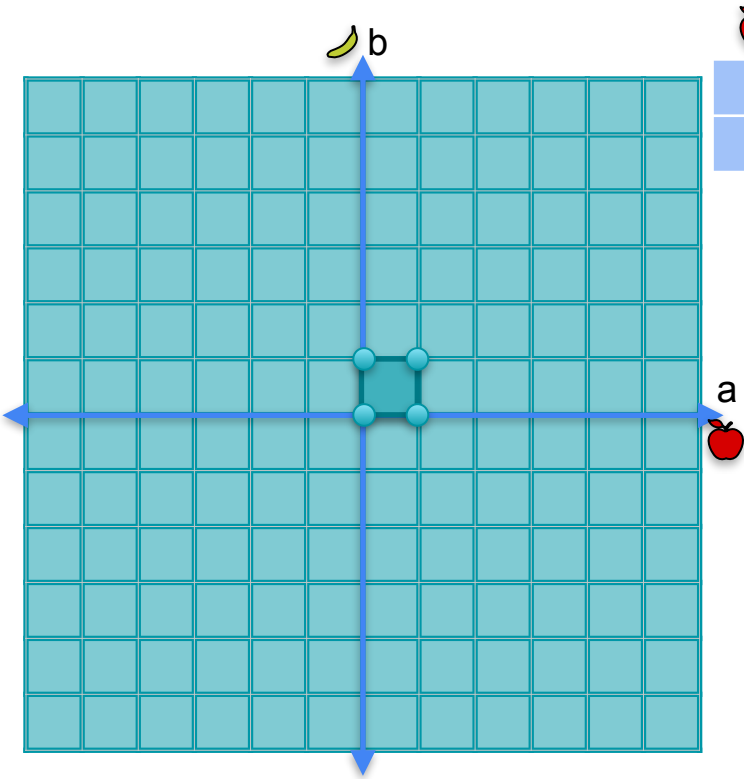
Matrices as linear transformations





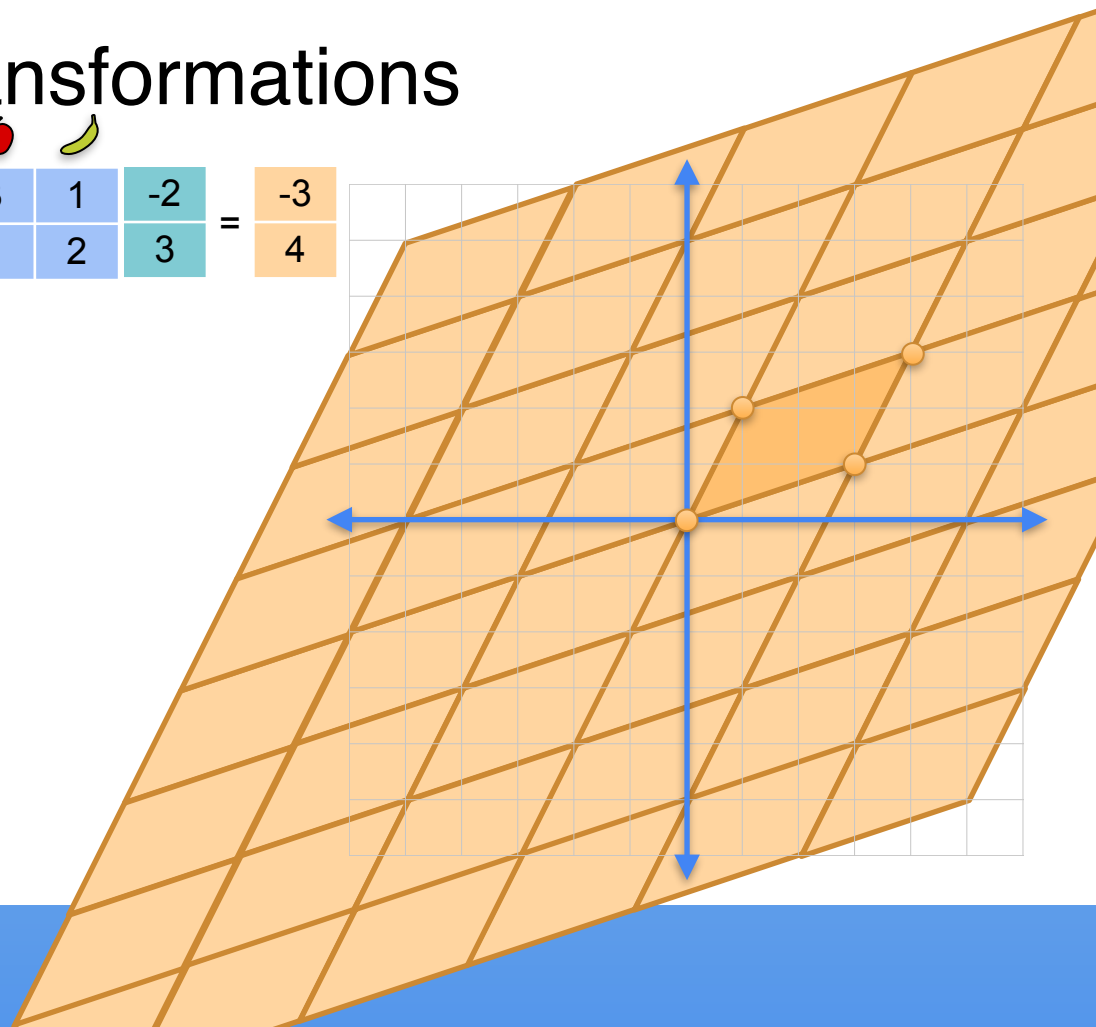
3	1	-2	=	-3
1	2	3		4



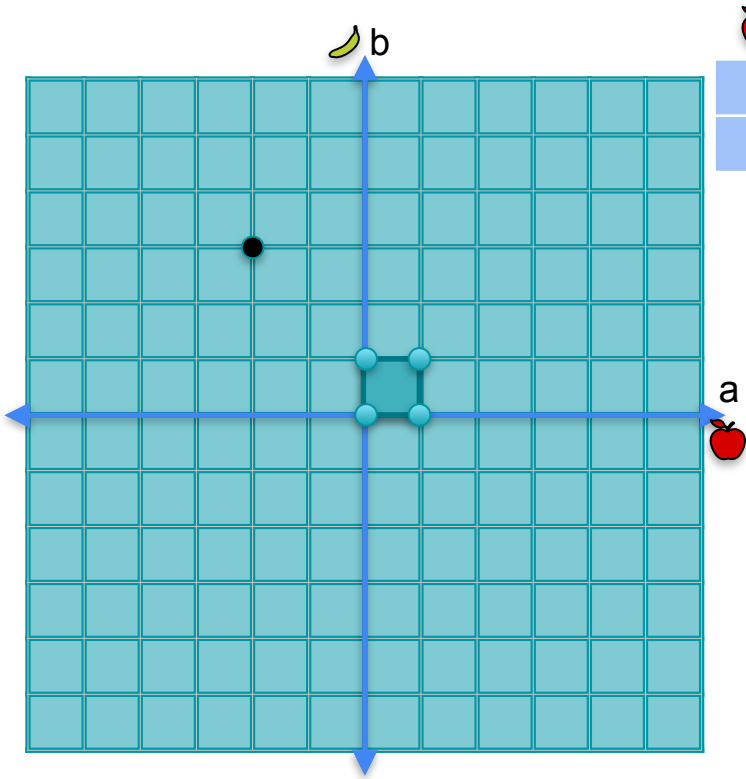
Matrices as linear transformations





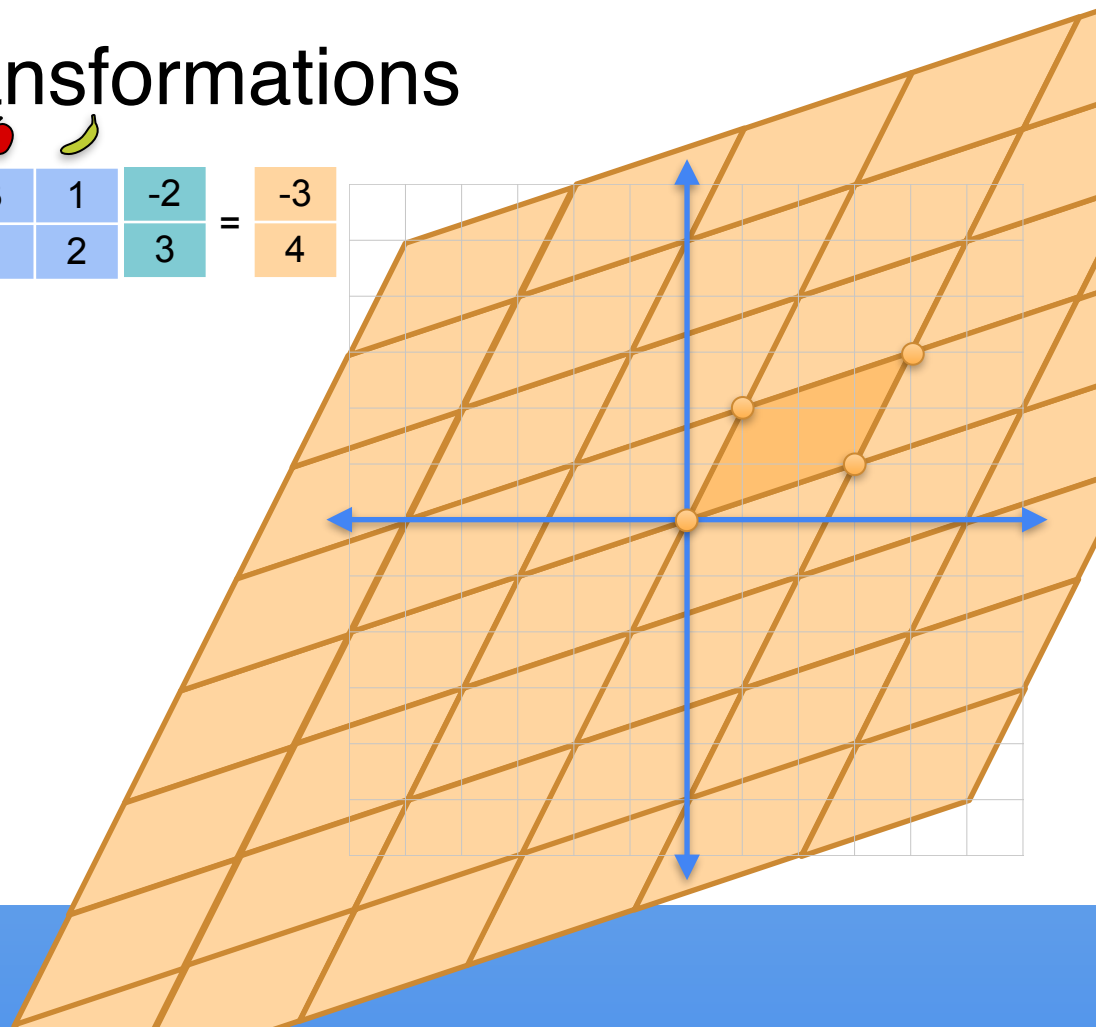
				
3	1	-2	=	-3
1	2	3		4



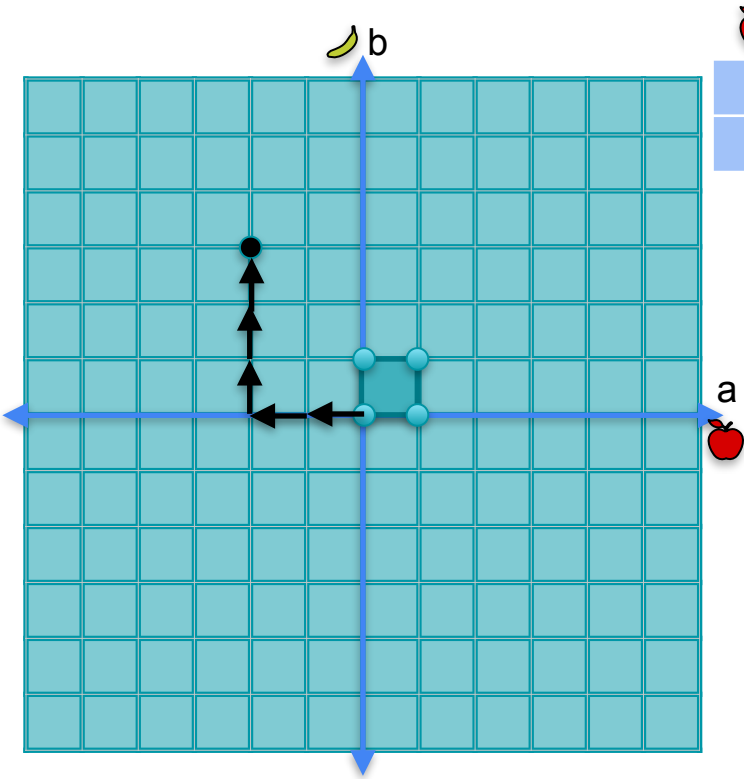
Matrices as linear transformations





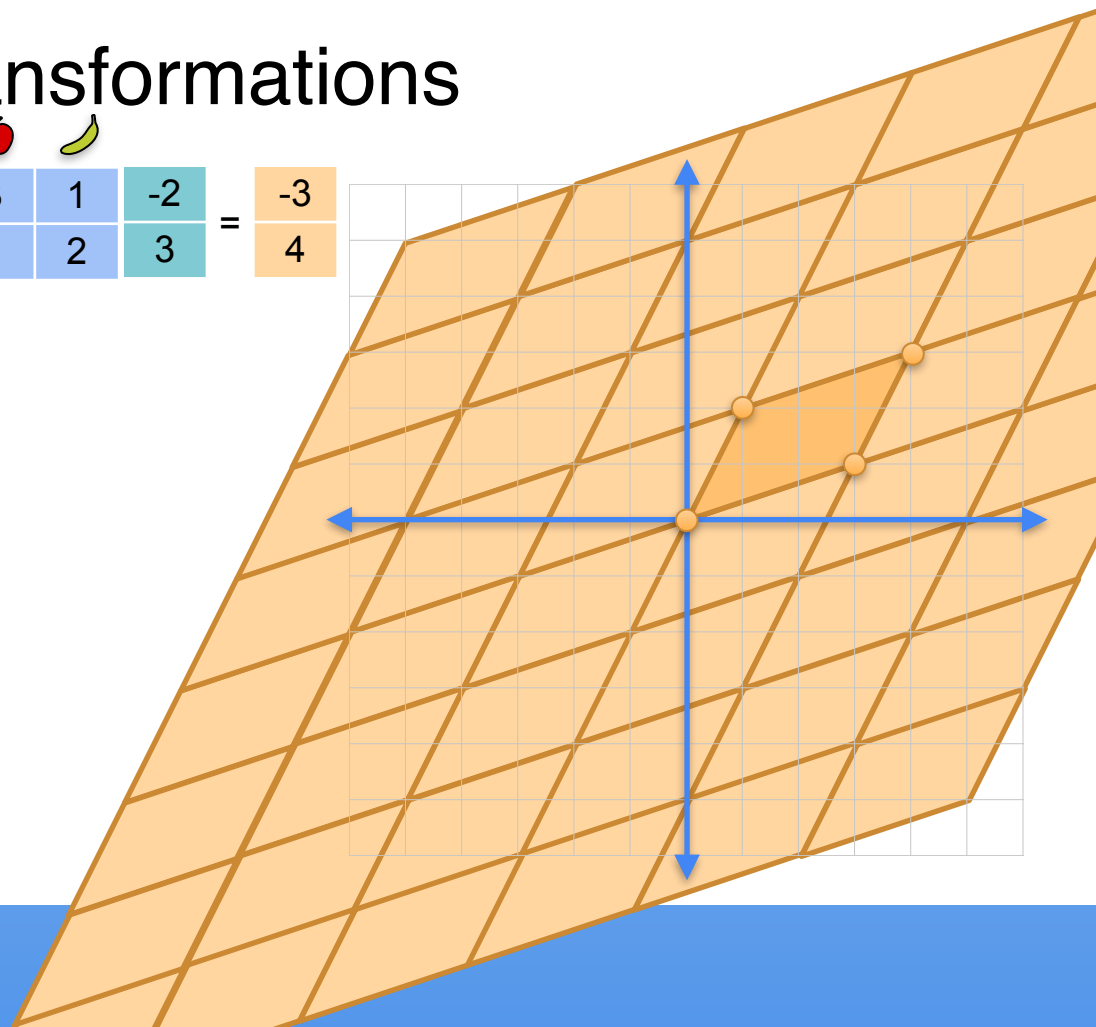
				
3	1	-2	=	-3
1	2	3		4



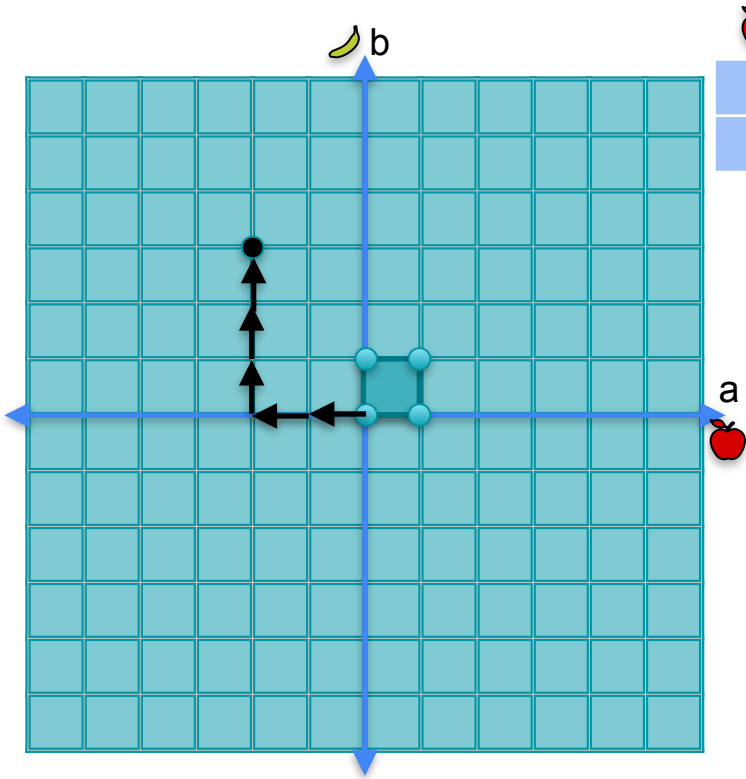
Matrices as linear transformations





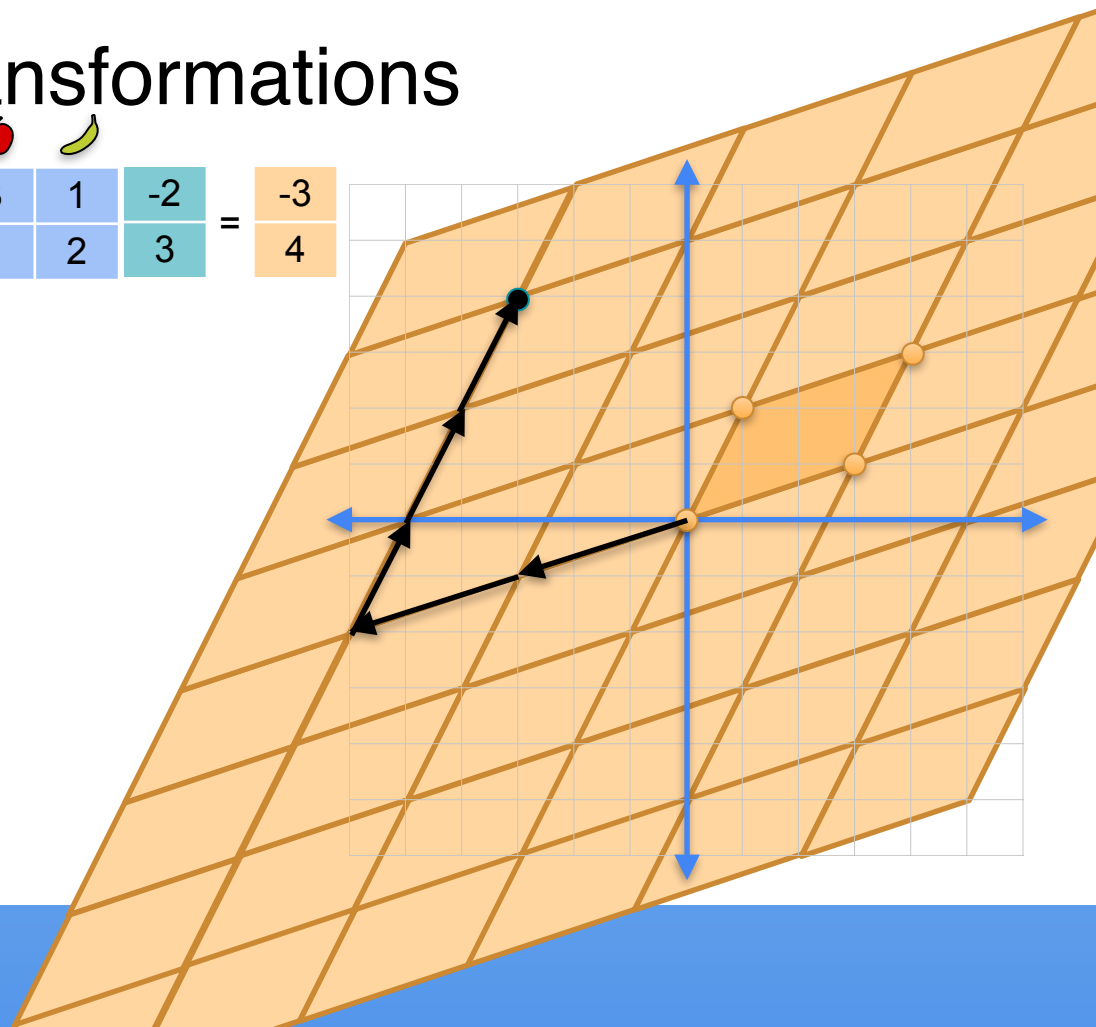
				
3	1	-2	=	-3
1	2	3		4



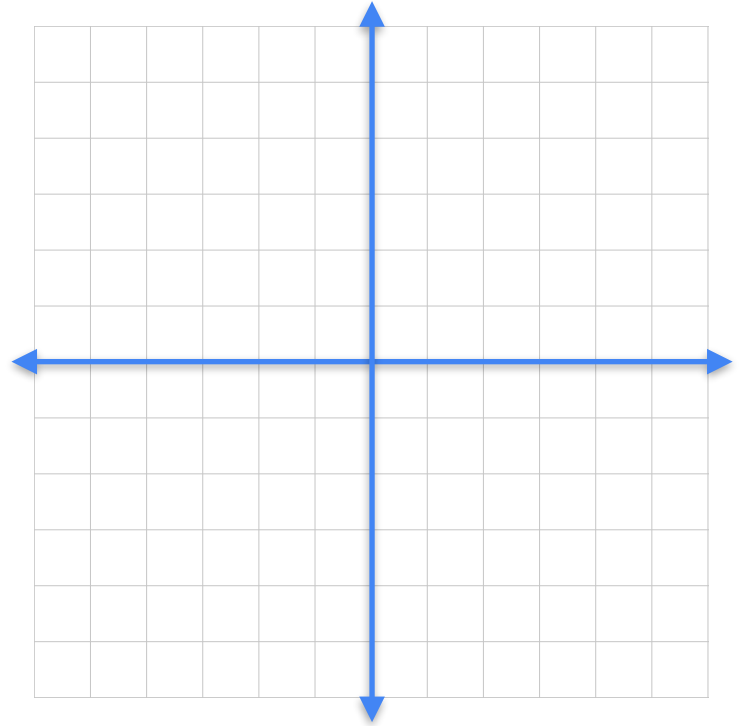
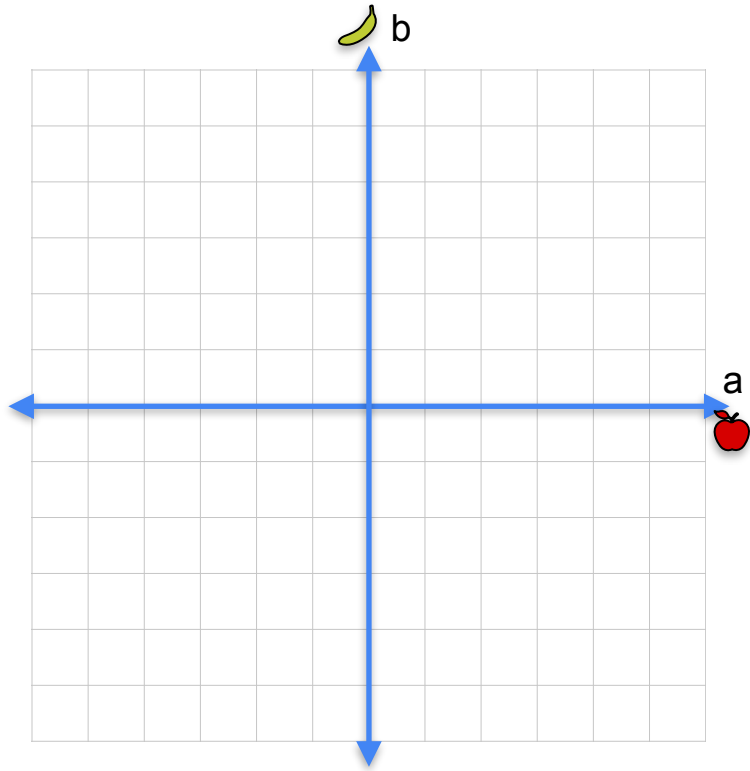
Matrices as linear transformations



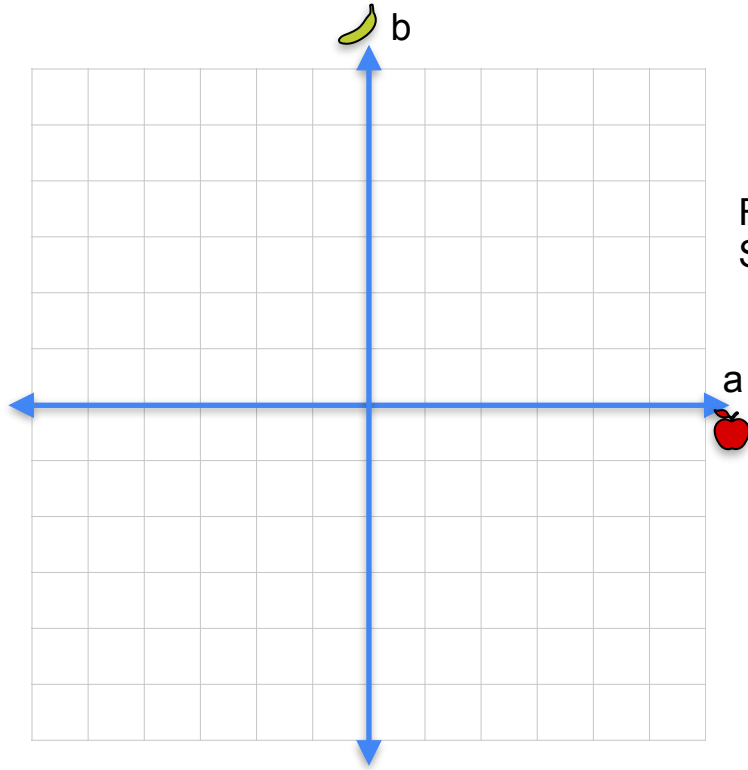
				
3	1	-2	=	-3
1	2	3		4



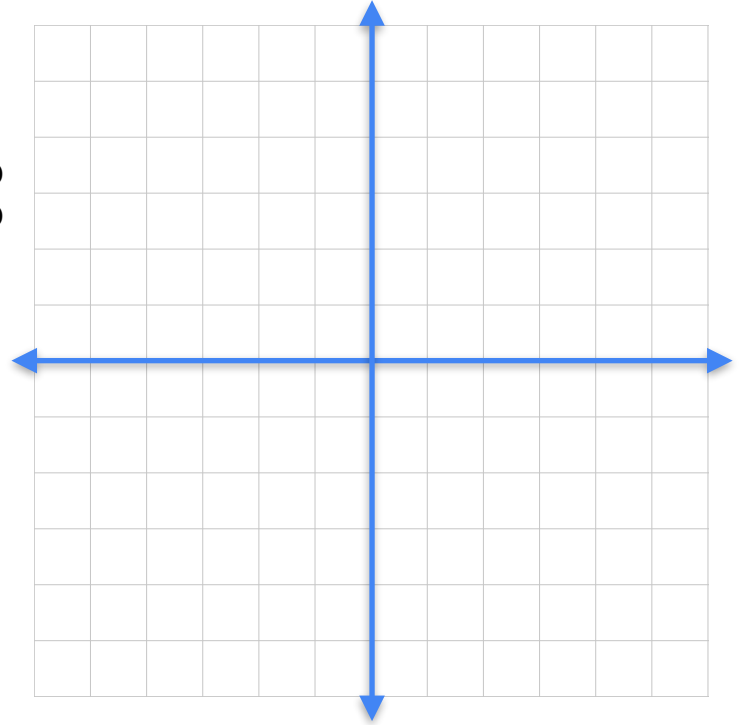
Systems of equations as linear transformations



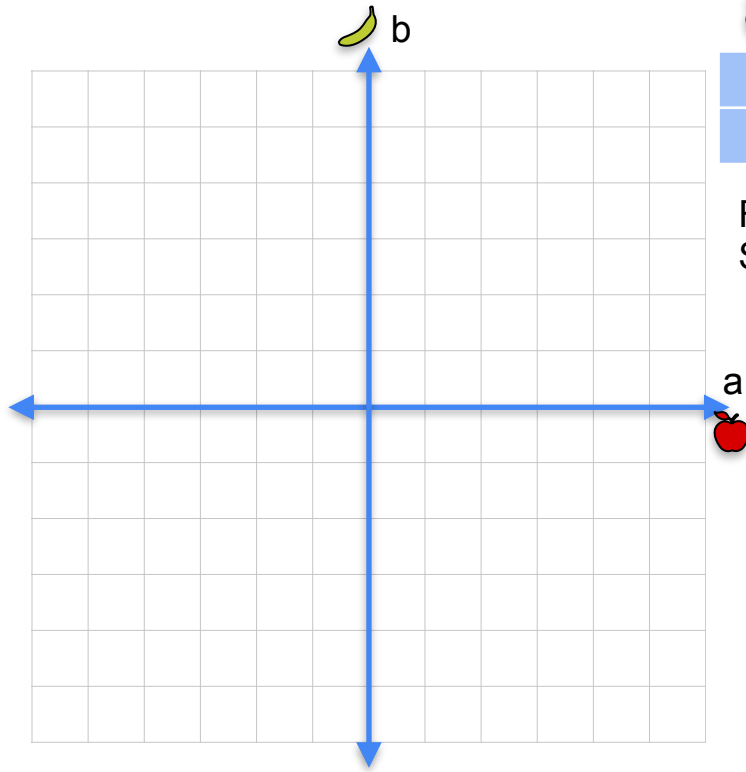
Systems of equations as linear transformations





First day: $3a + b$
Second day: $a + 2b$

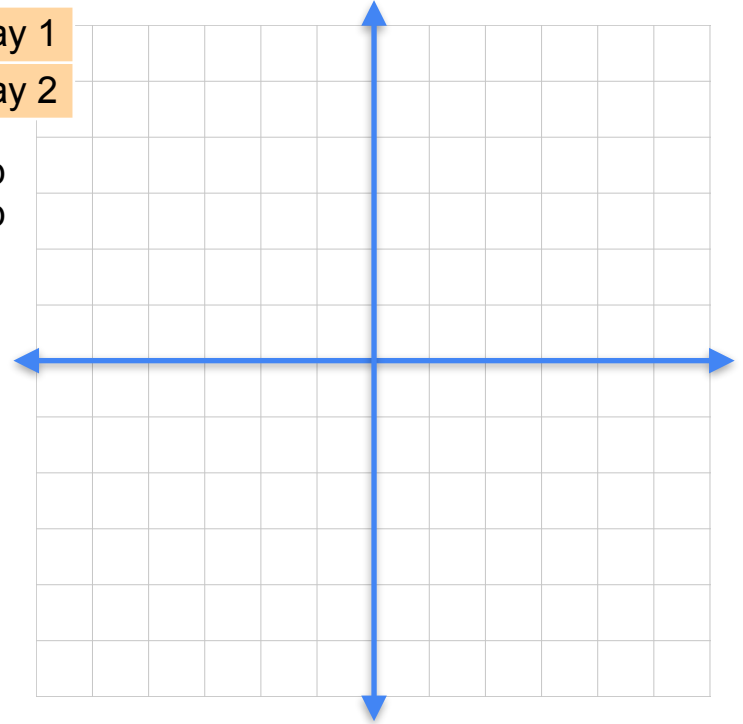


Systems of equations as linear transformations

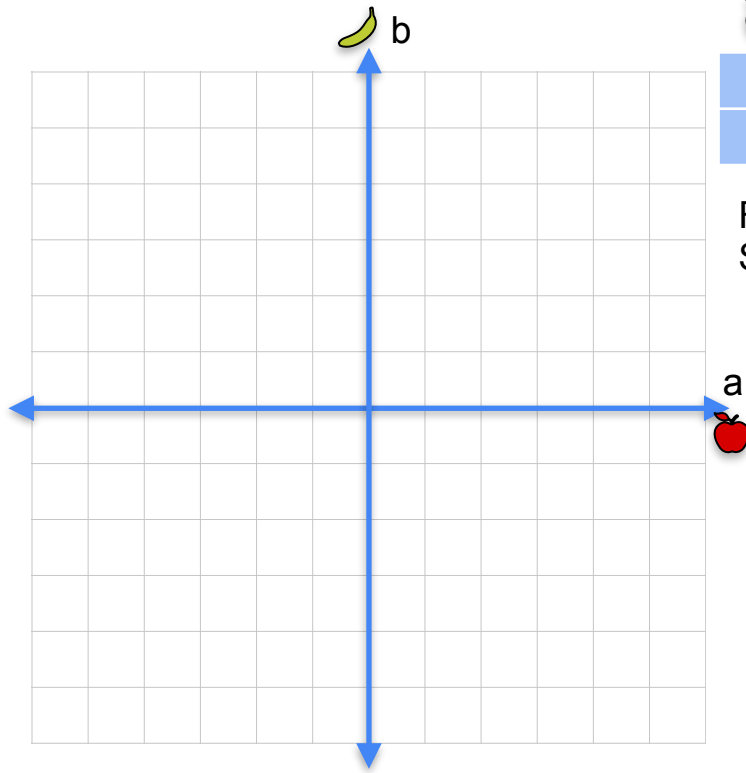


			
3	1	a	= Day 1
1	2	b	= Day 2

First day: $3a + b$
Second day: $a + 2b$

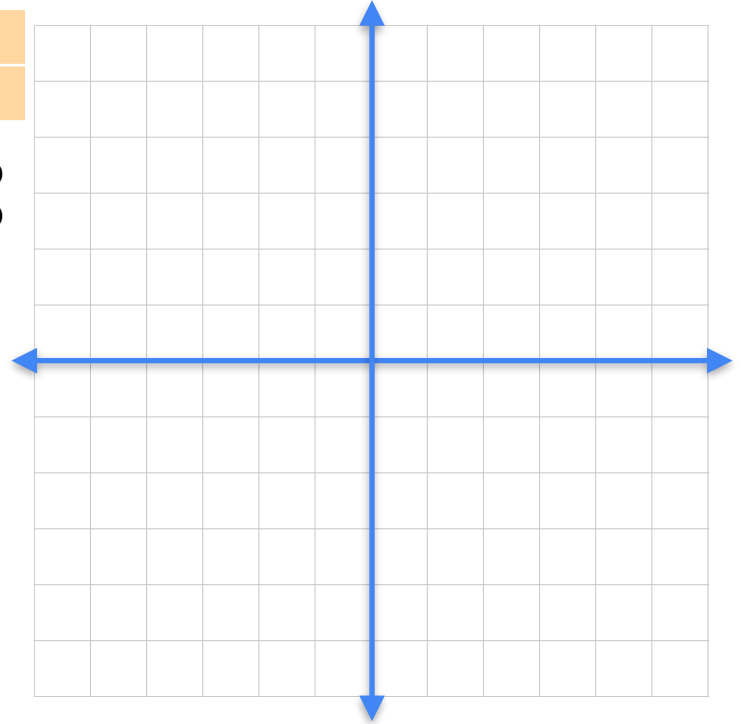


Systems of equations as linear transformations



3	1	1	=	4
1	2	1	=	3

First day: $3a + b$
Second day: $a + 2b$



Systems of equations as linear transformations

 b

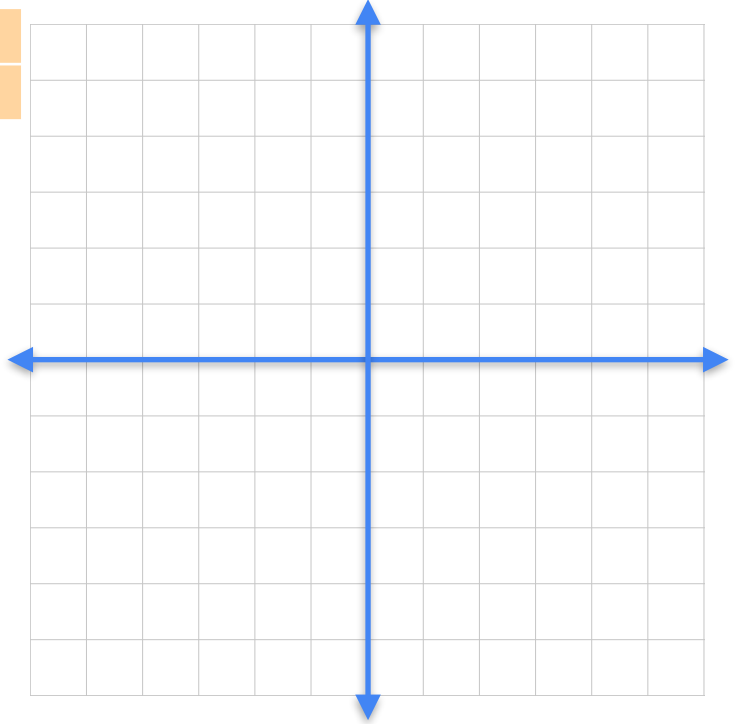
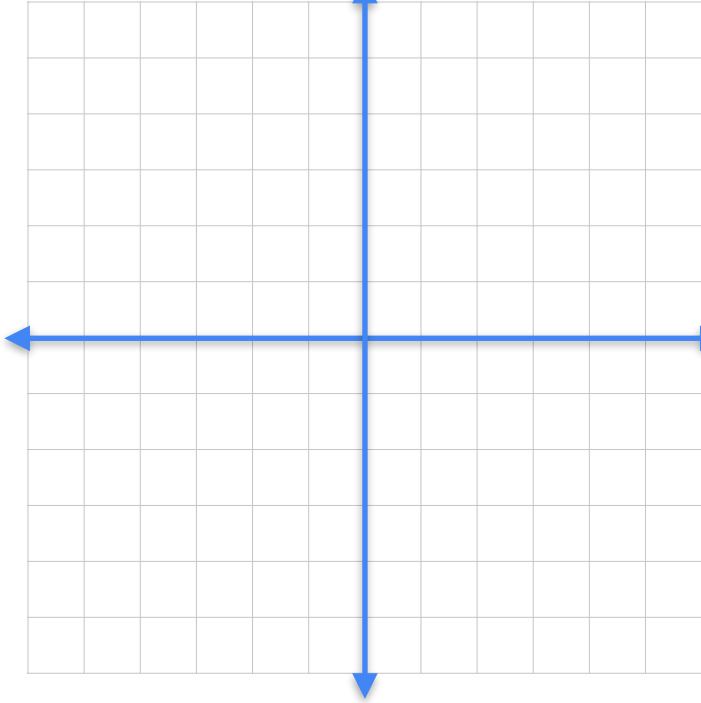
 

3	1	1	=	4
1	2	1	=	3

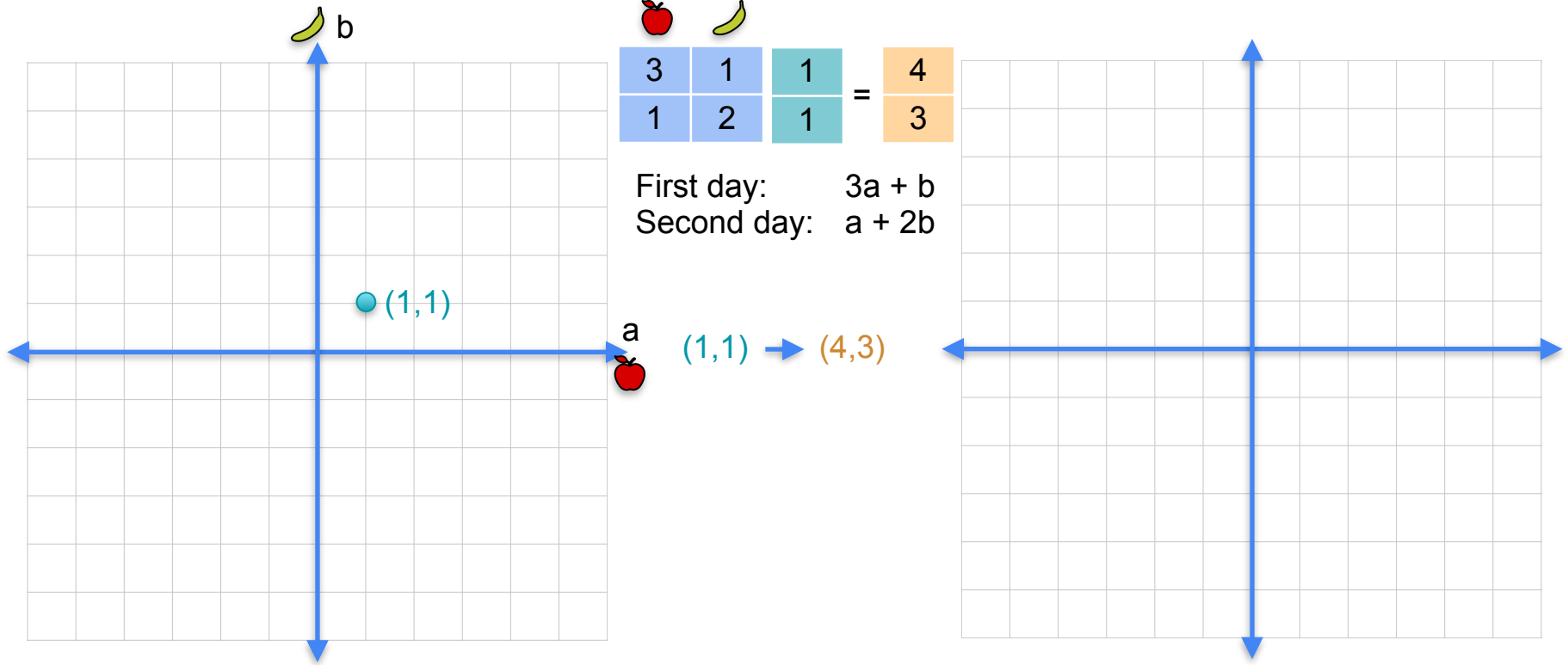
First day: $3a + b$
Second day: $a + 2b$

a

$(1,1) \rightarrow (4,3)$

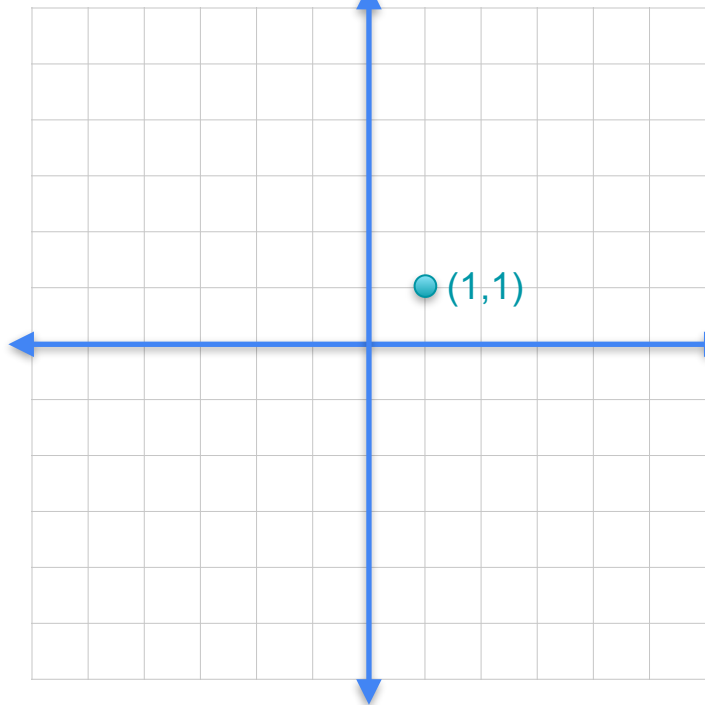




Systems of equations as linear transformations



Systems of equations as linear transformations

 b

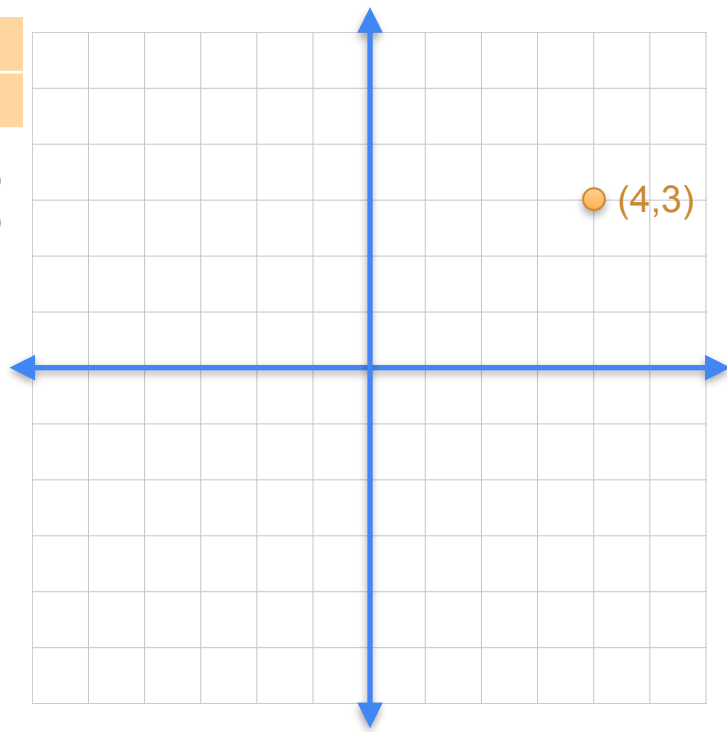


			
3	1	1	= 4
1	2	1	= 3

First day: $3a + b$
Second day: $a + 2b$

$(1, 1) \rightarrow (4, 3)$

 a



Systems of equations as linear transformations

 b

3	1	1	=	4
1	2	1	=	3

First day: $3a + b$

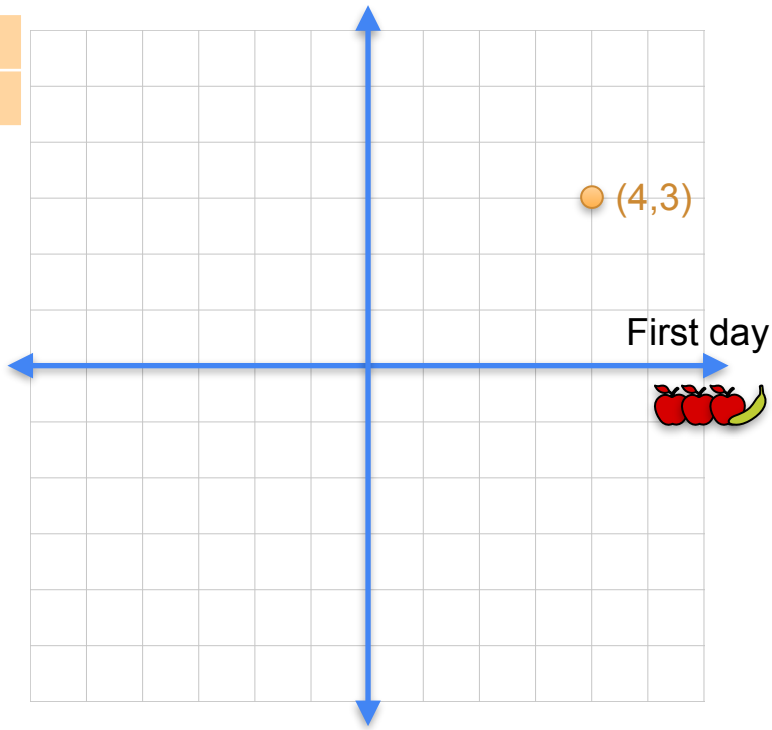
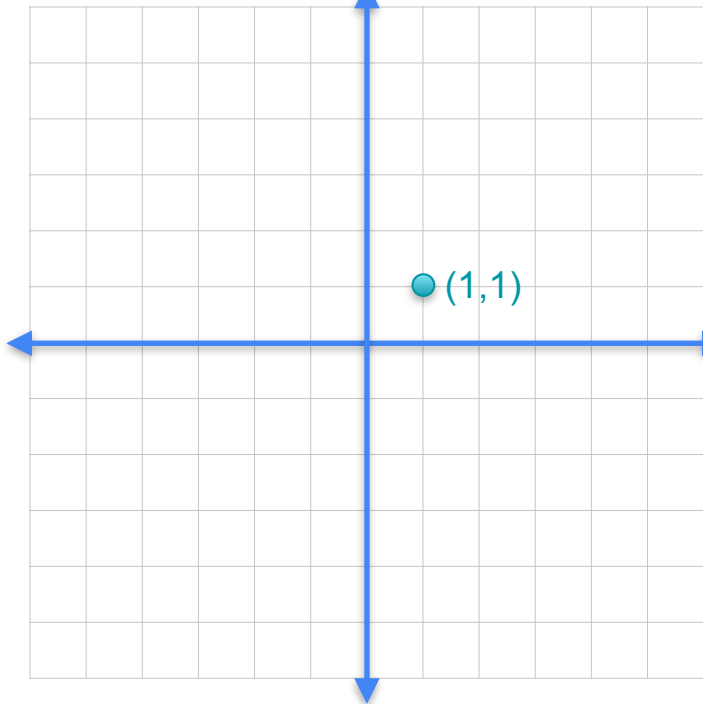
Second day: $a + 2b$

$(1,1) \rightarrow (4,3)$

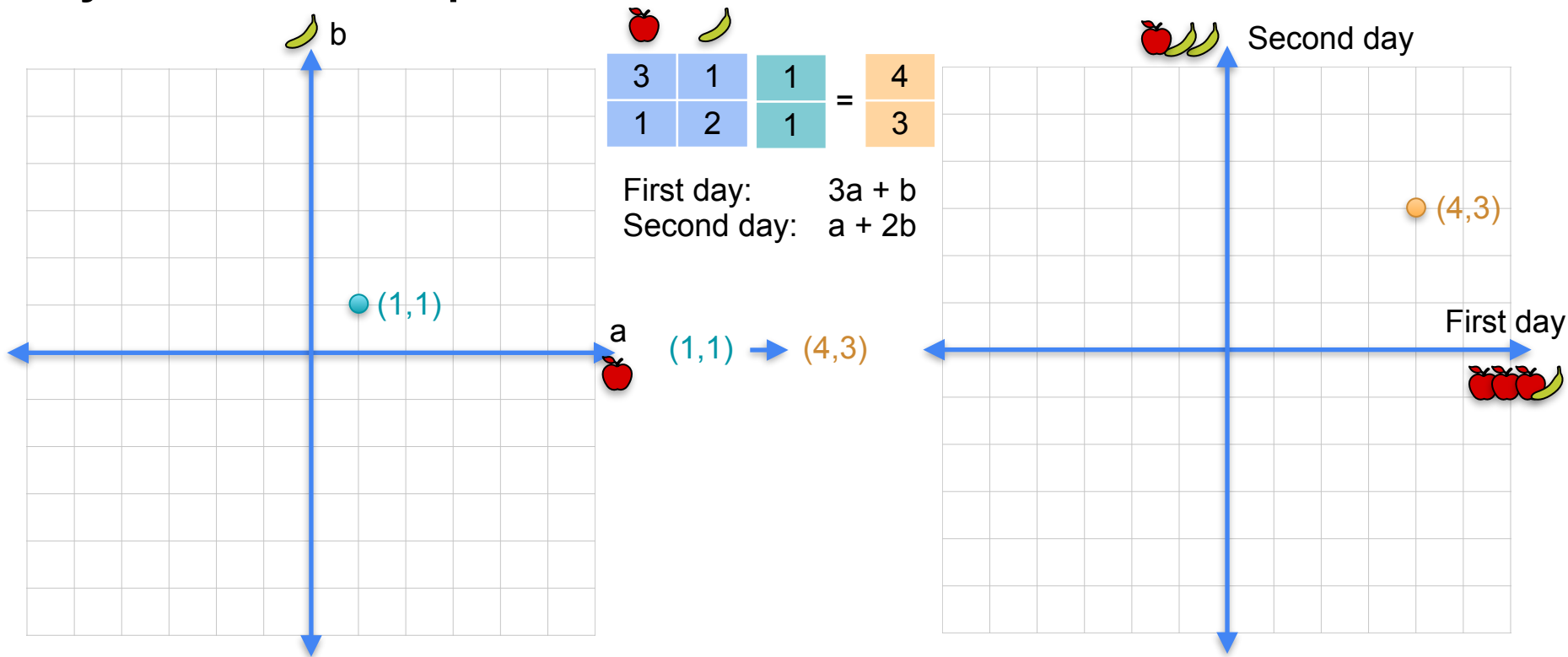
a 

First day



Systems of equations as linear transformations



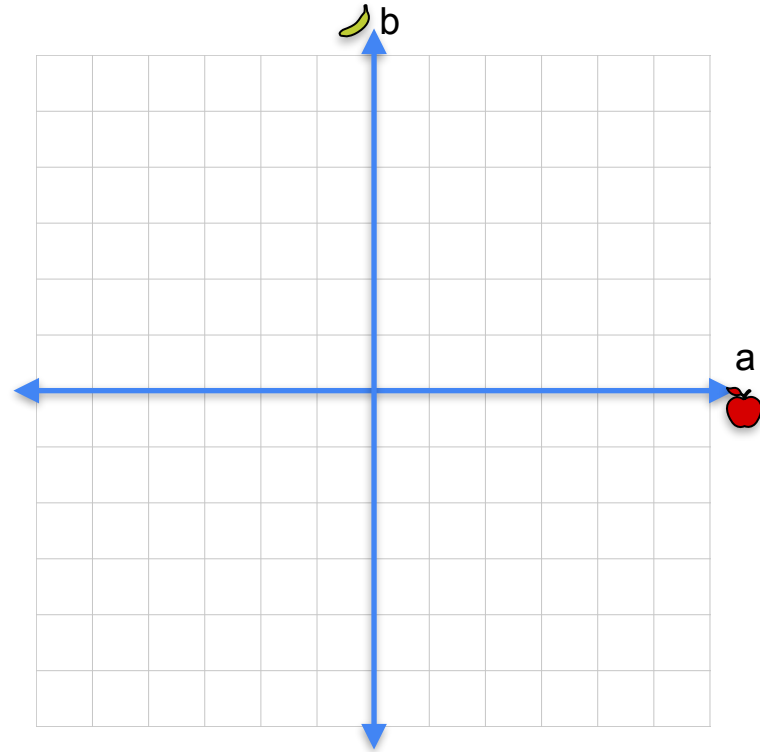
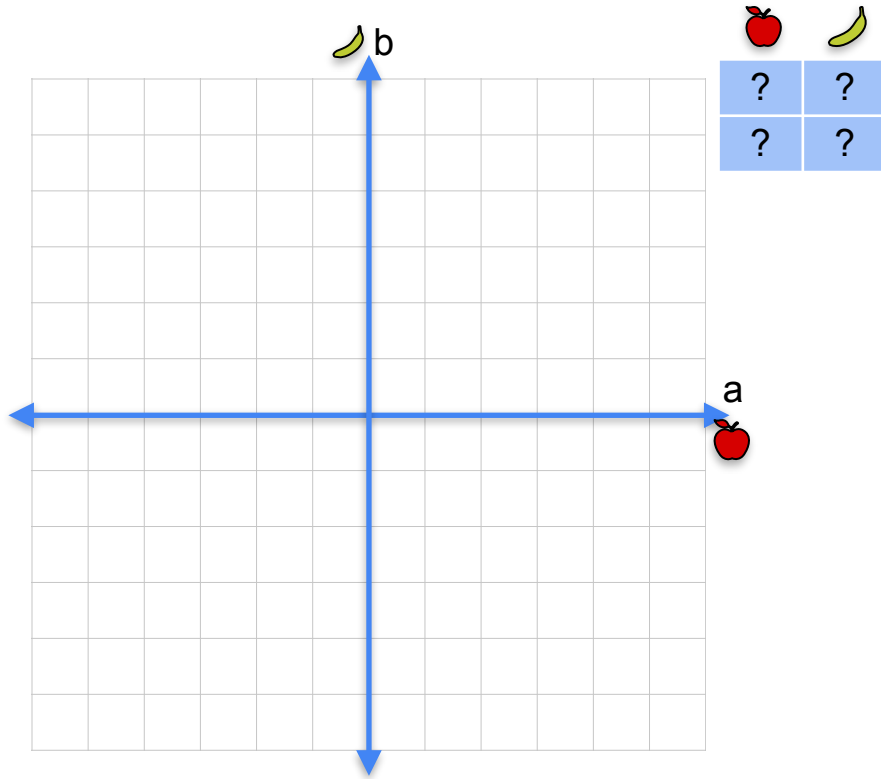


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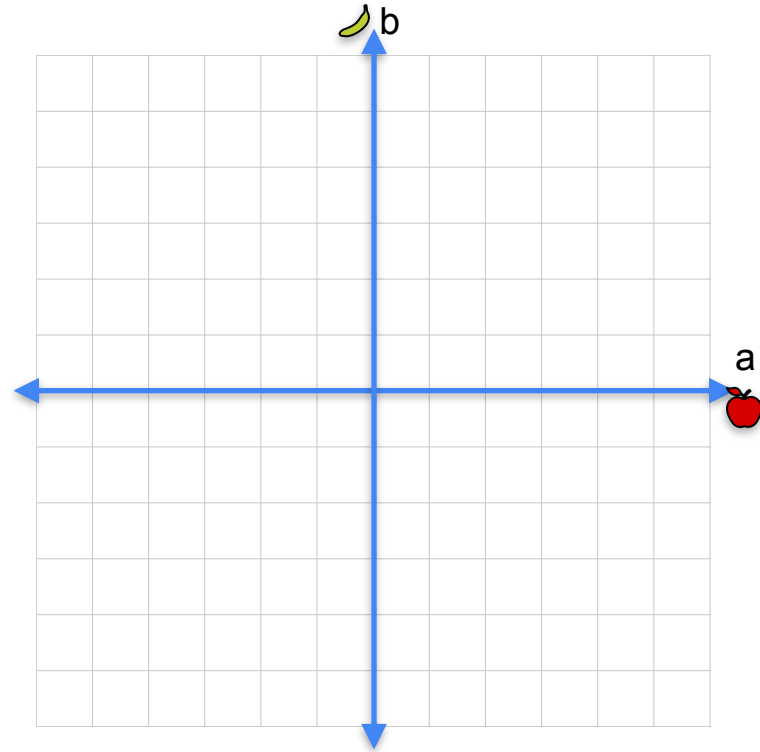
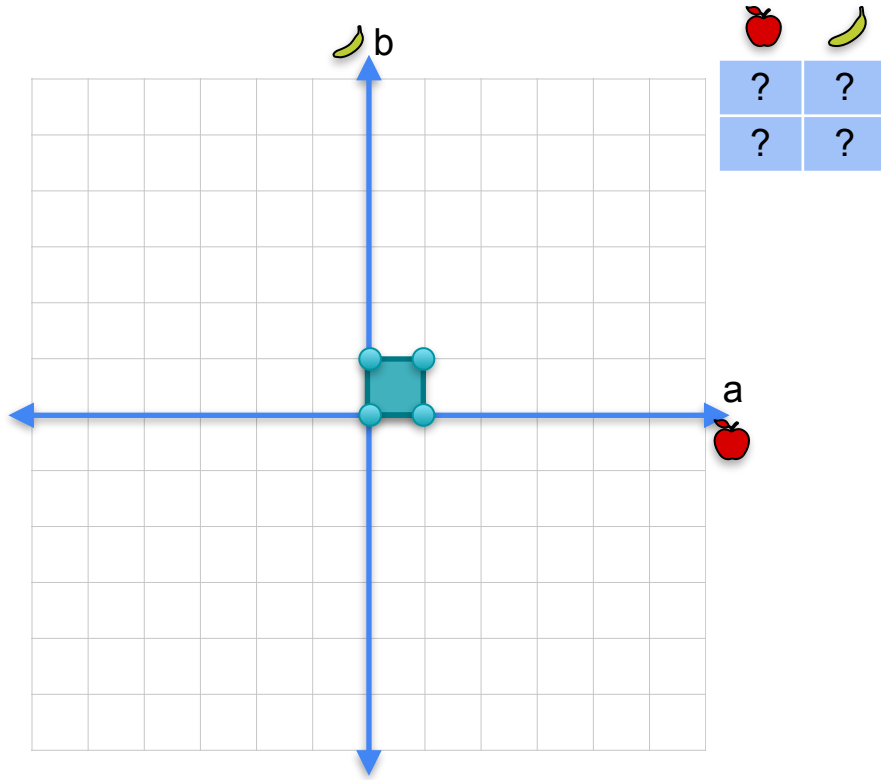
Vectors and Linear Transformations

**Linear transformations as
matrices**

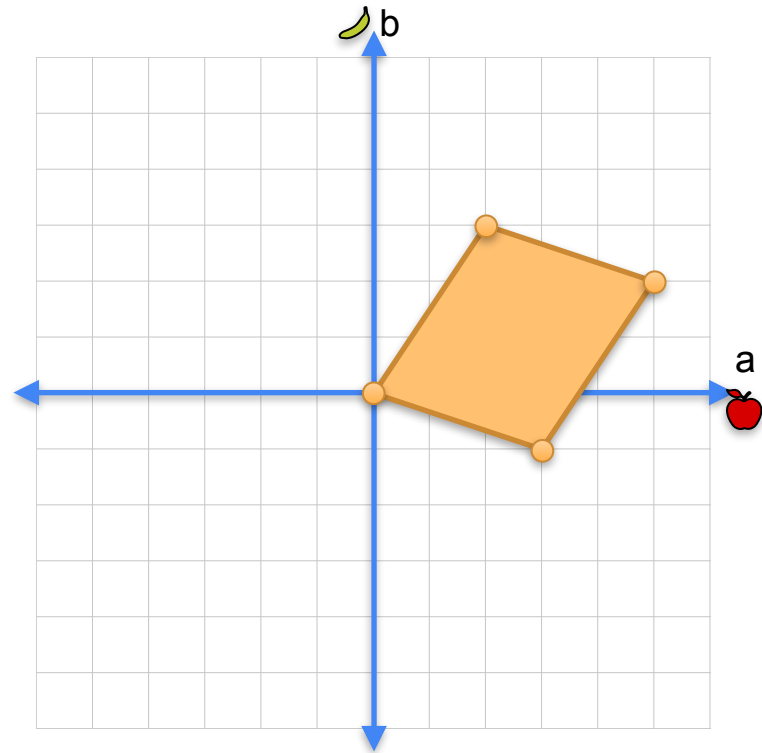
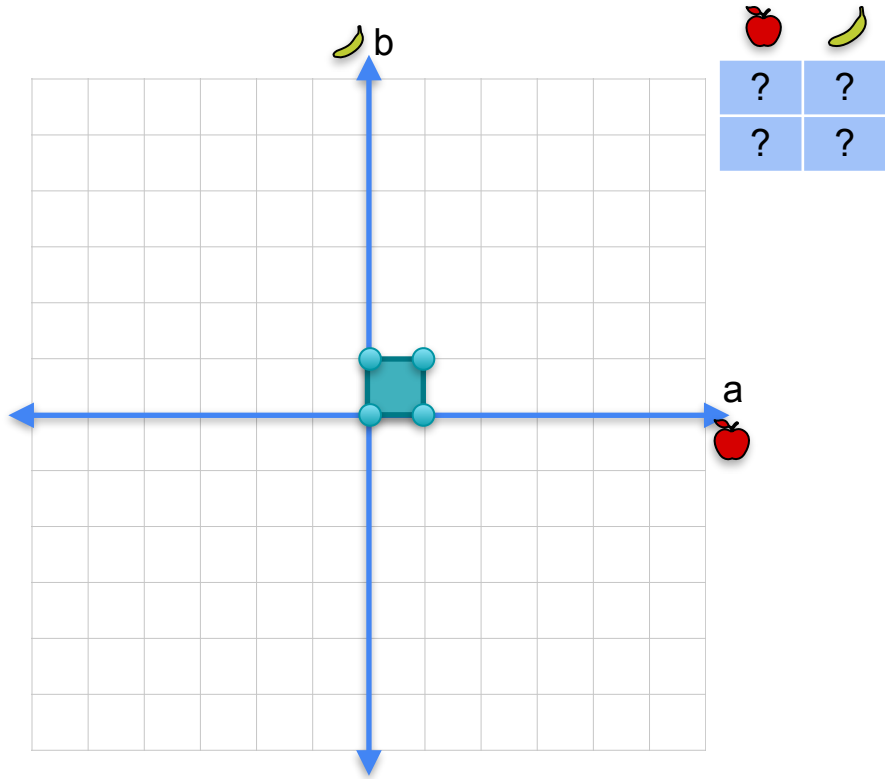
Linear transformations as matrices



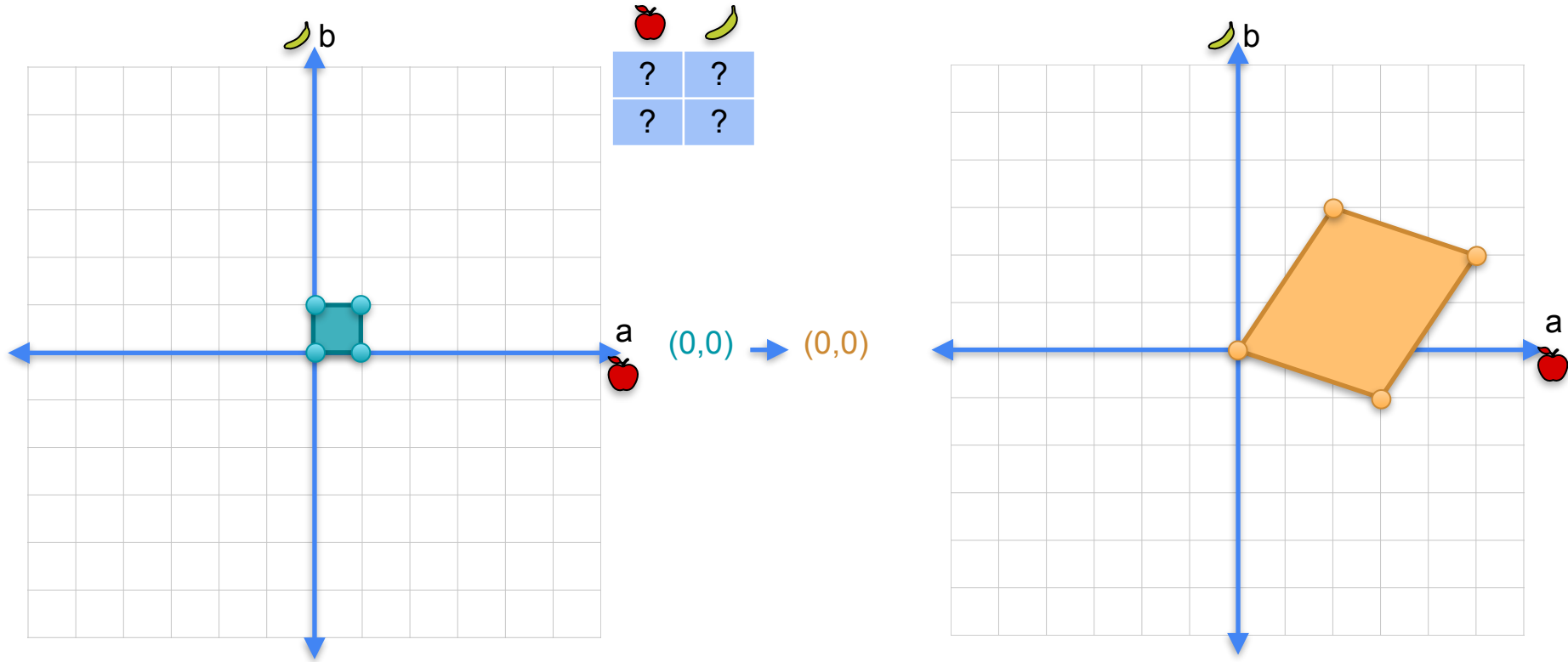
Linear transformations as matrices



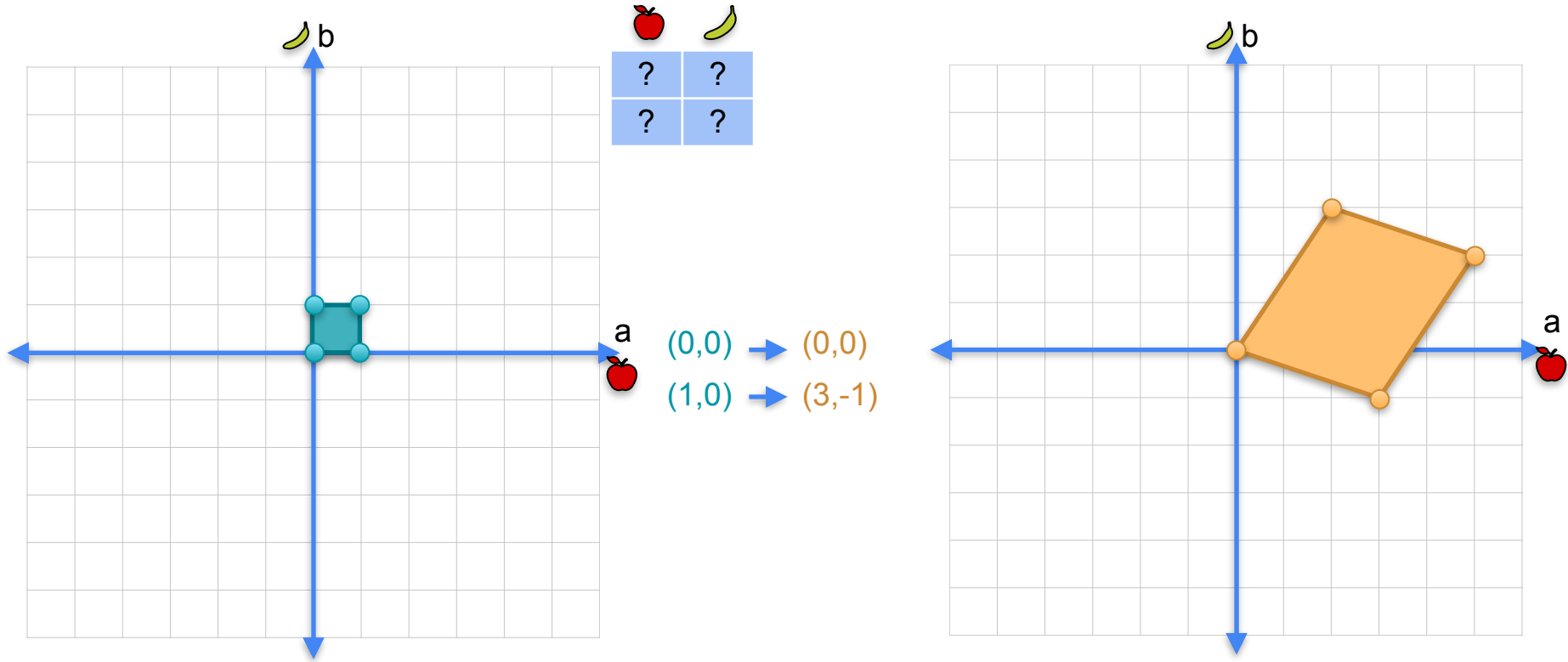
Linear transformations as matrices



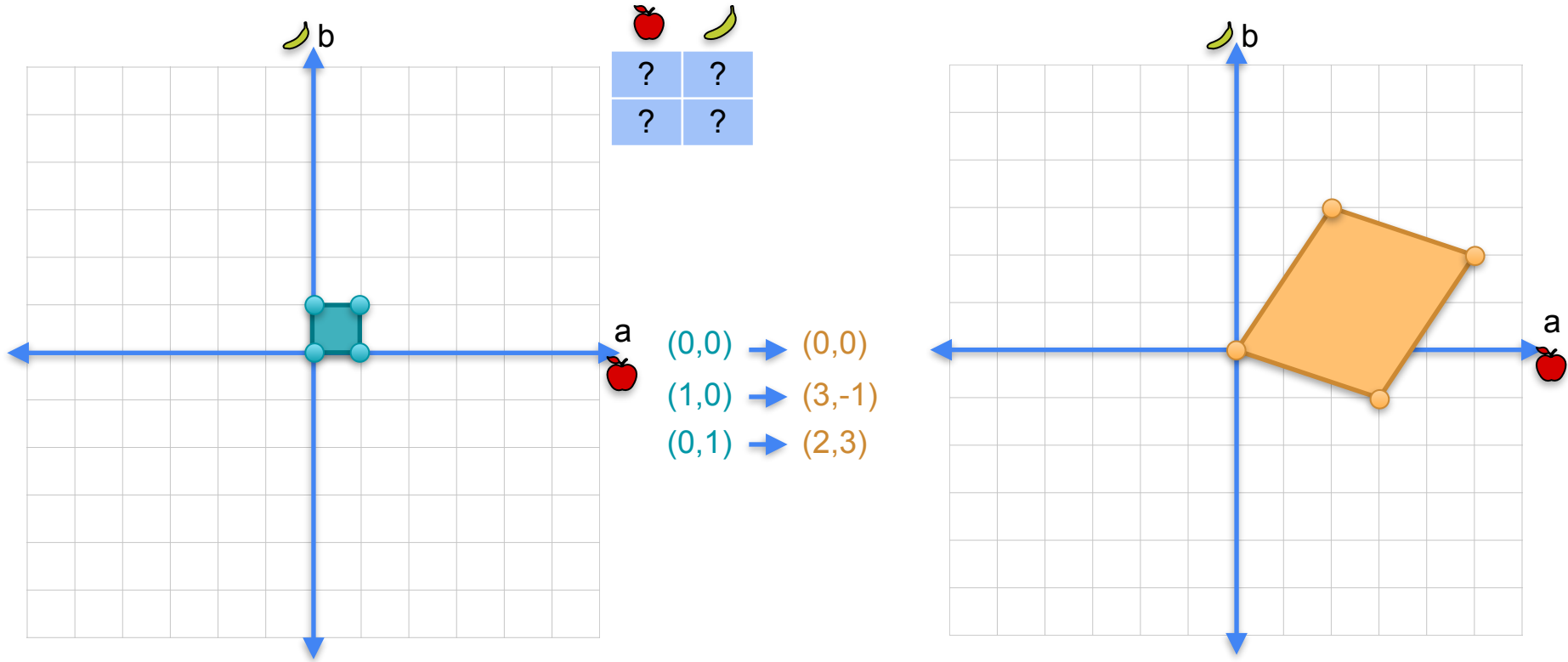
Linear transformations as matrices



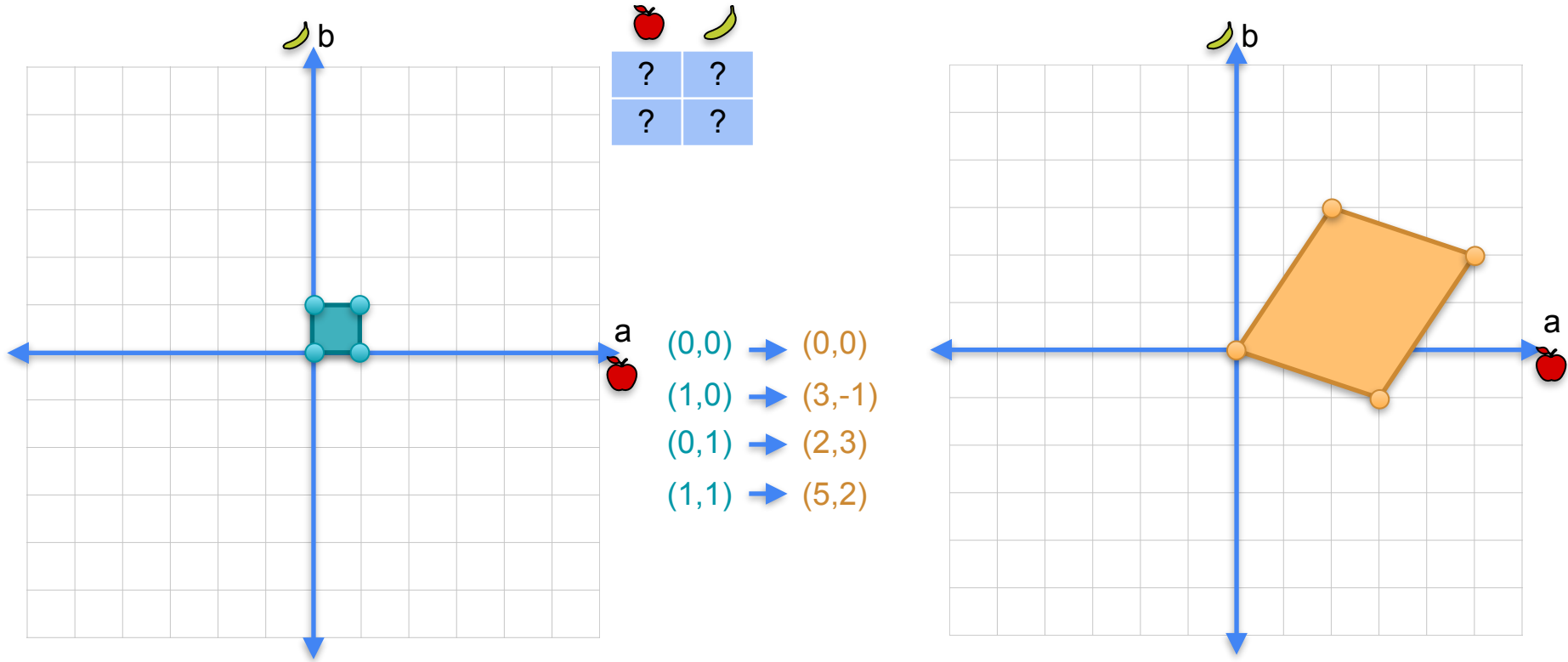
Linear transformations as matrices



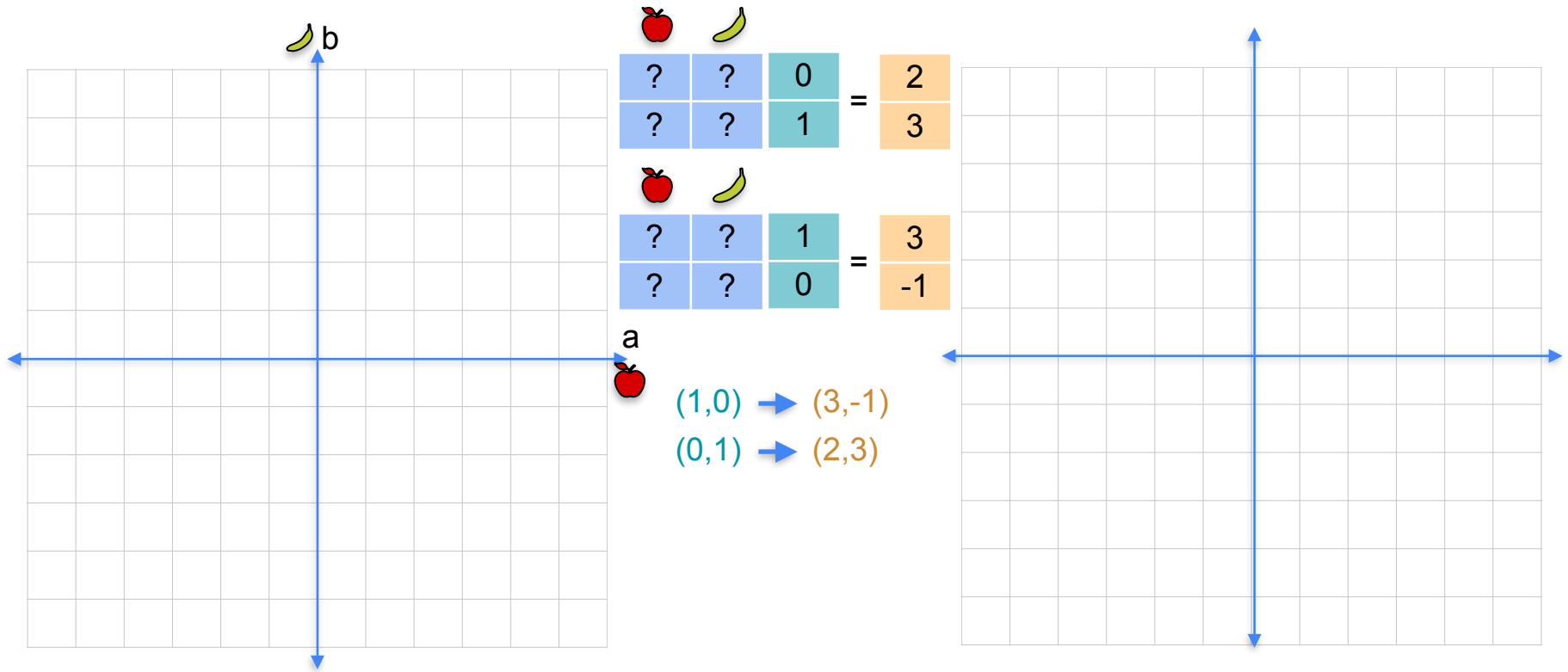
Linear transformations as matrices



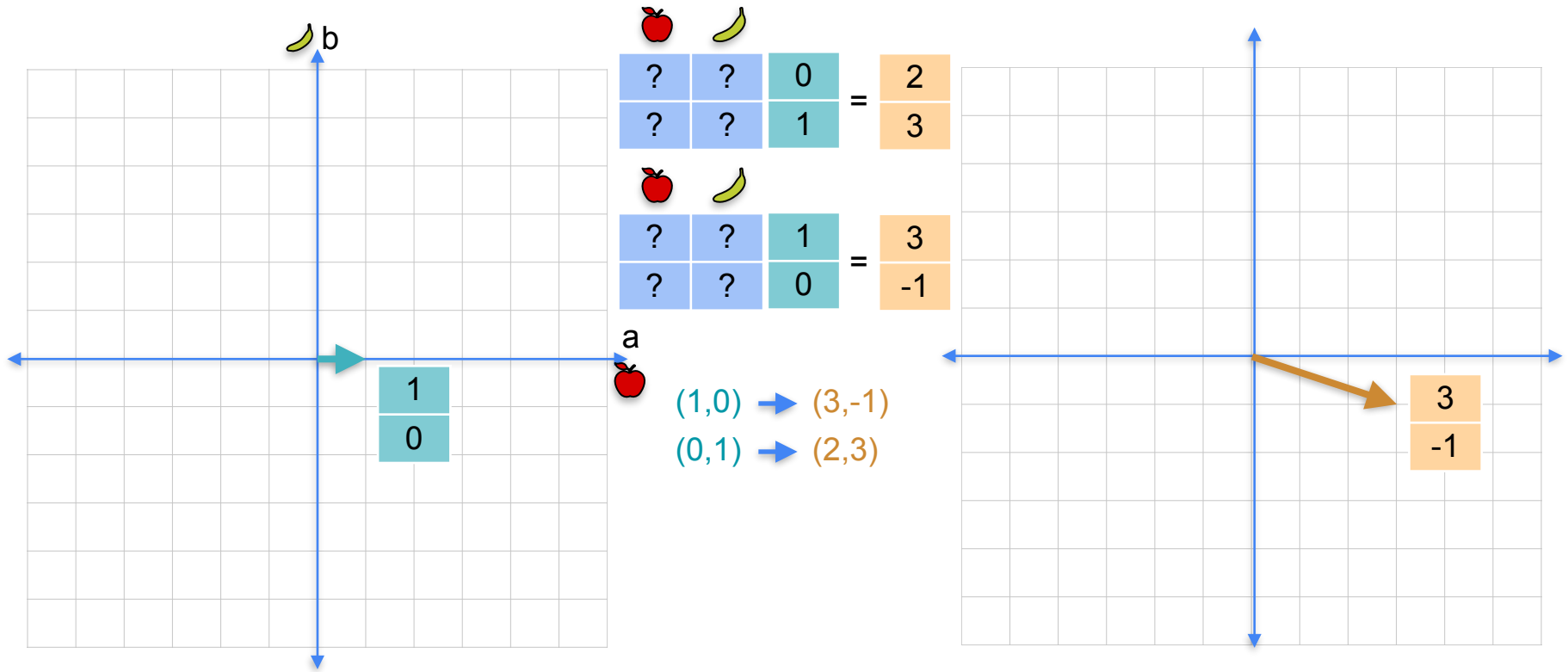
Linear transformations as matrices



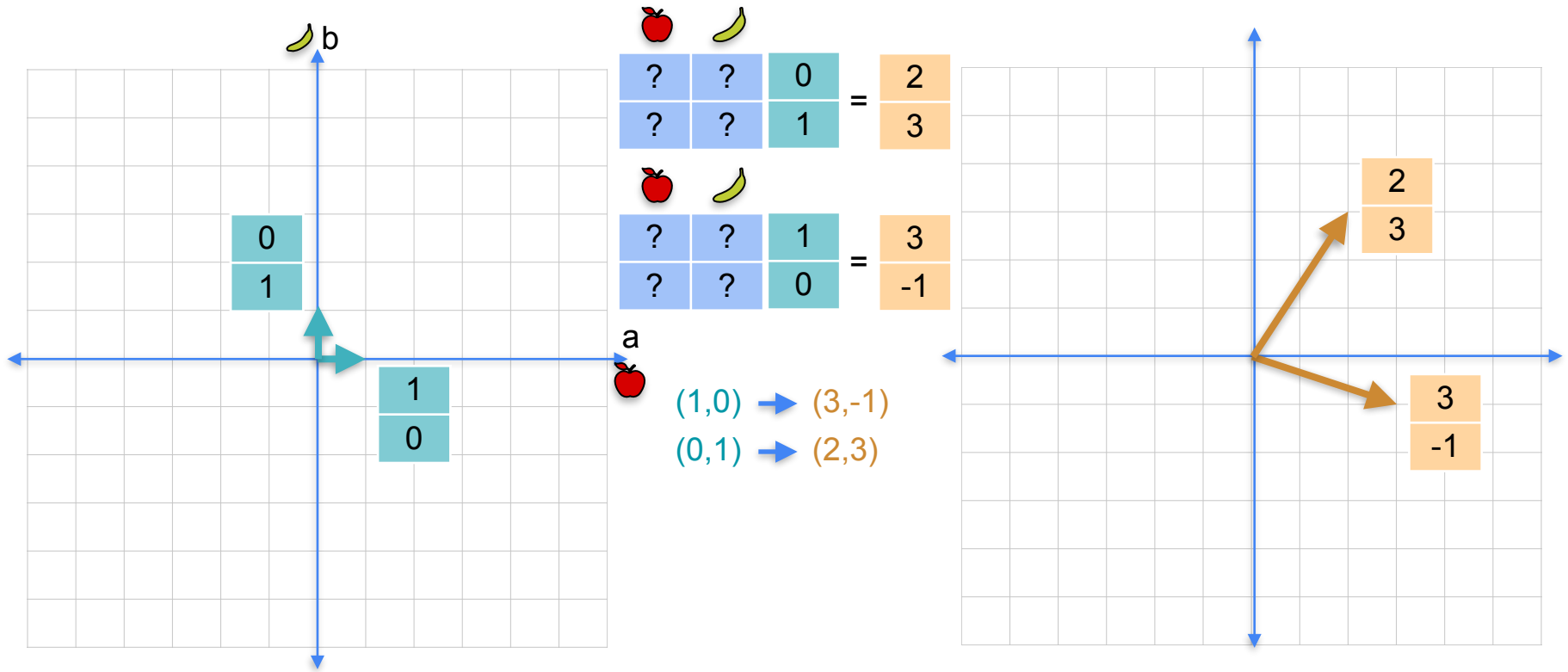
Linear transformations as matrices



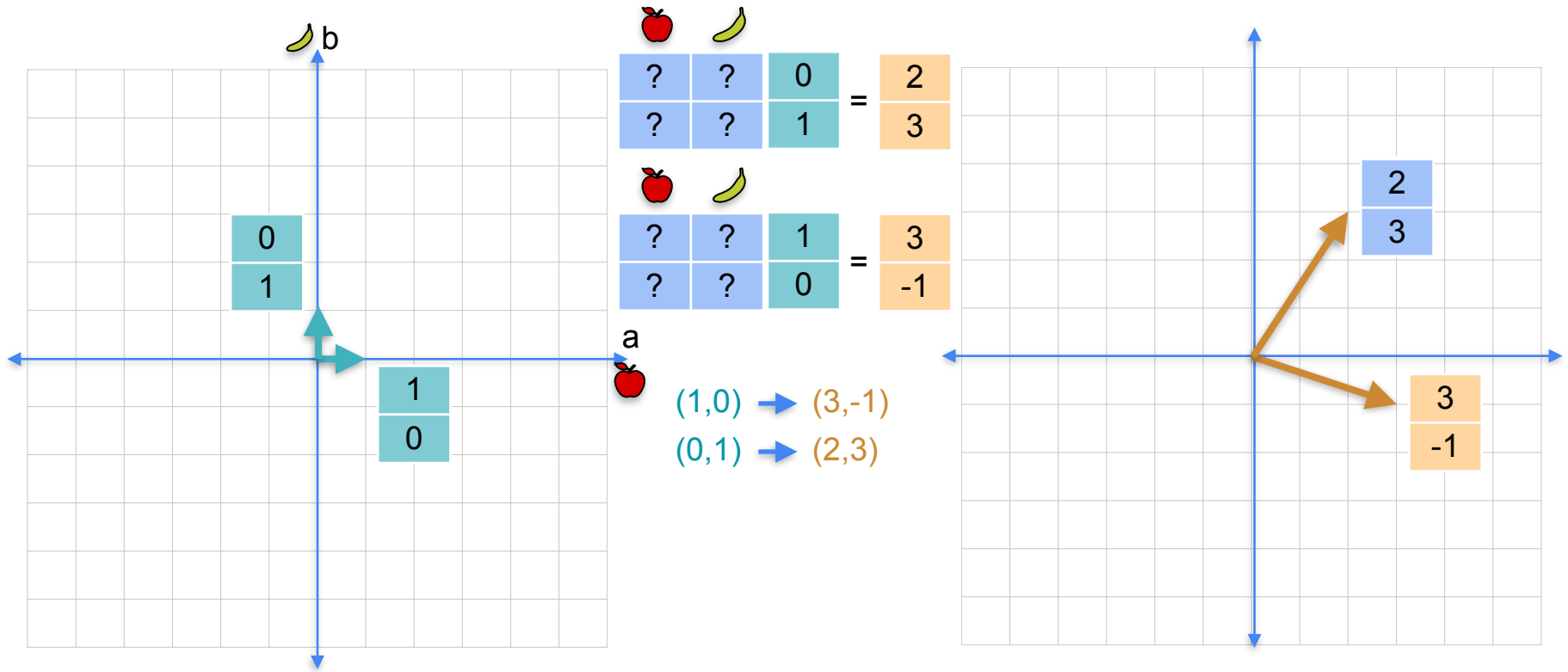
Linear transformations as matrices



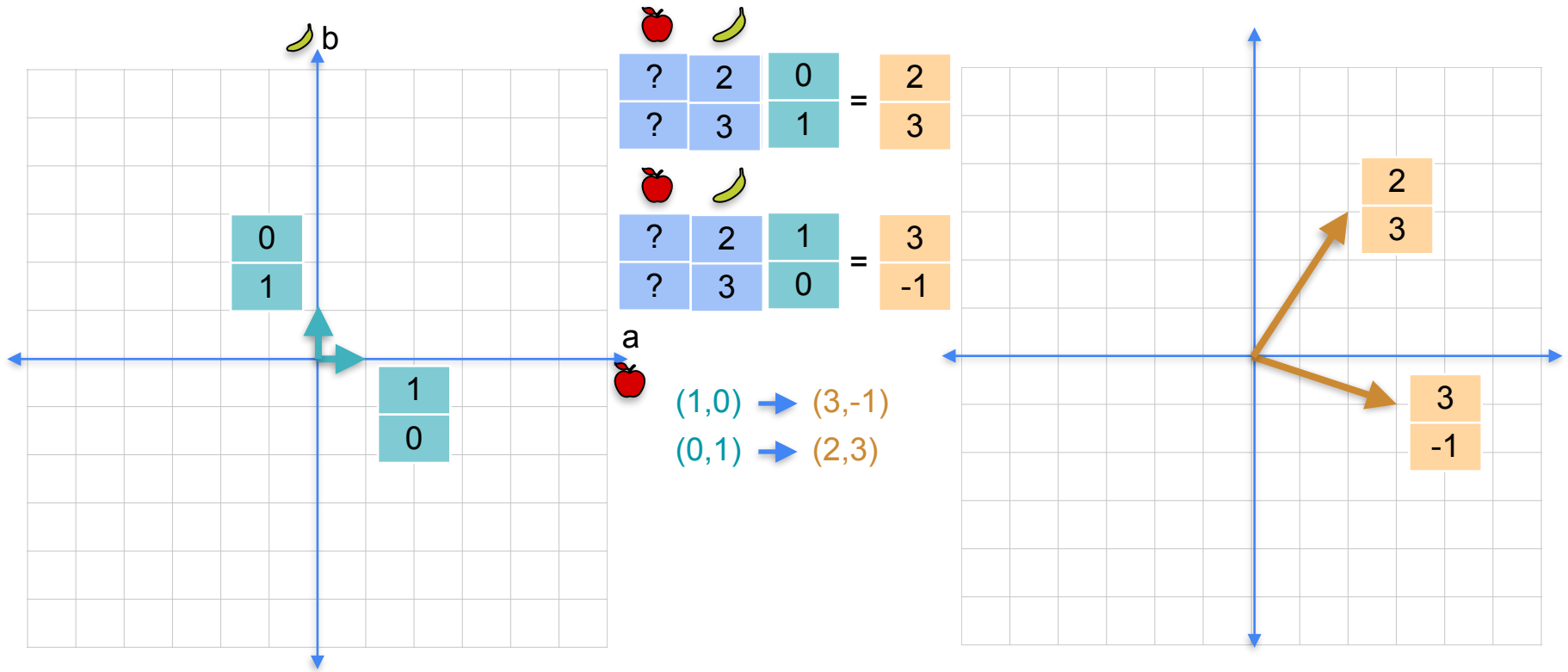
Linear transformations as matrices



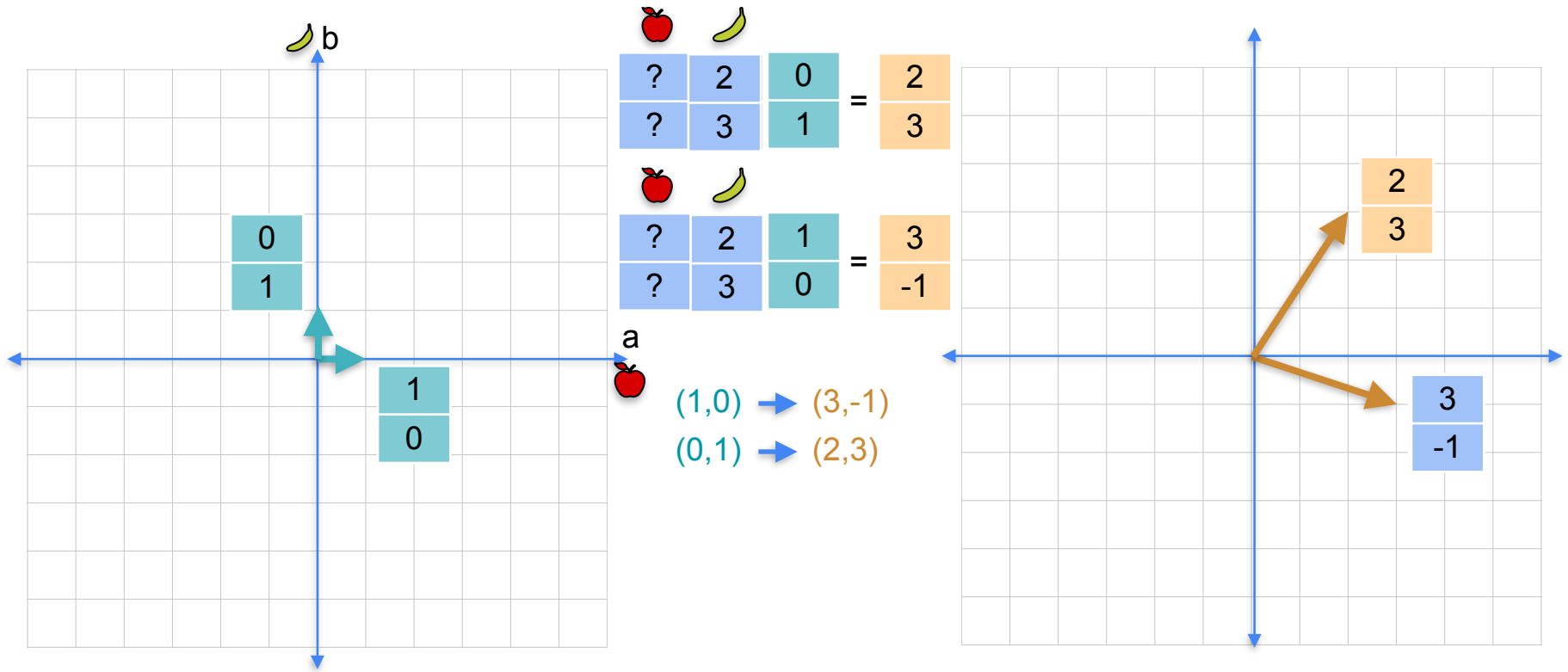
Linear transformations as matrices



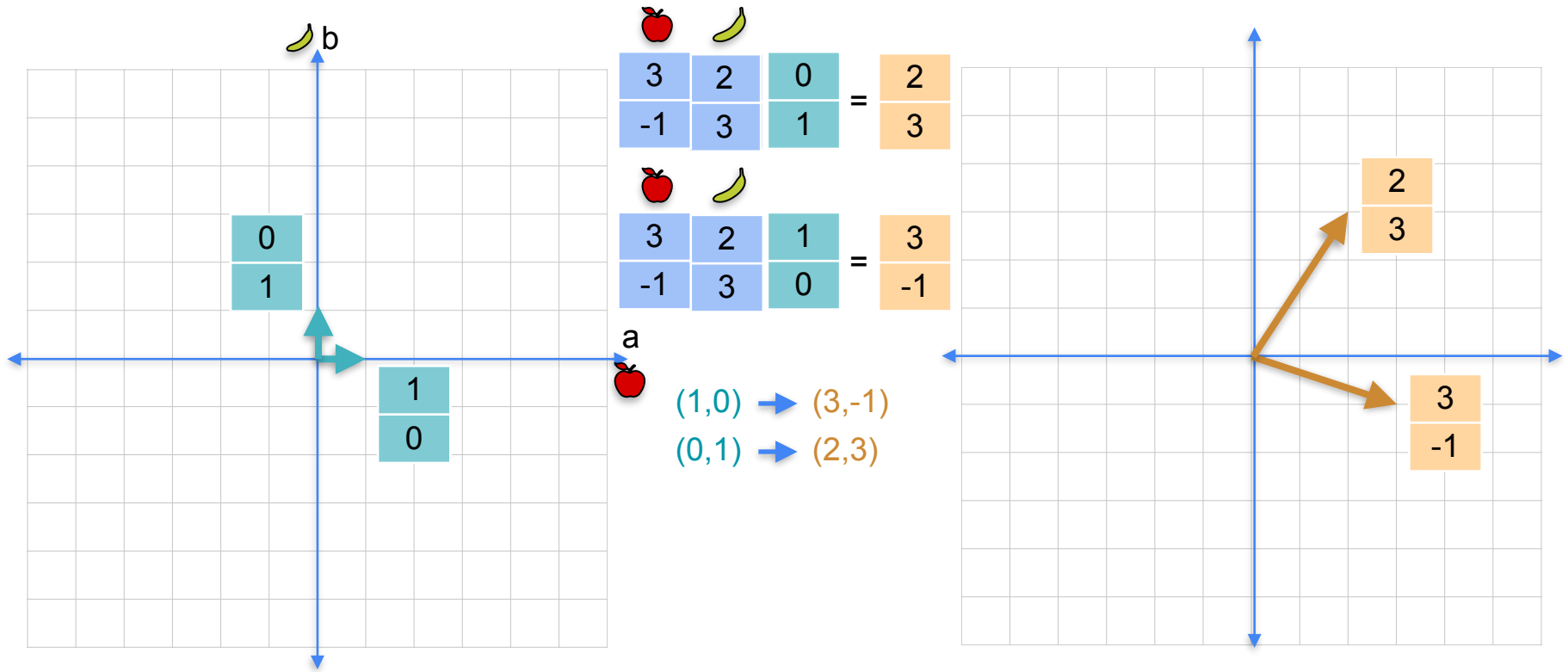
Linear transformations as matrices



Linear transformations as matrices



Linear transformations as matrices



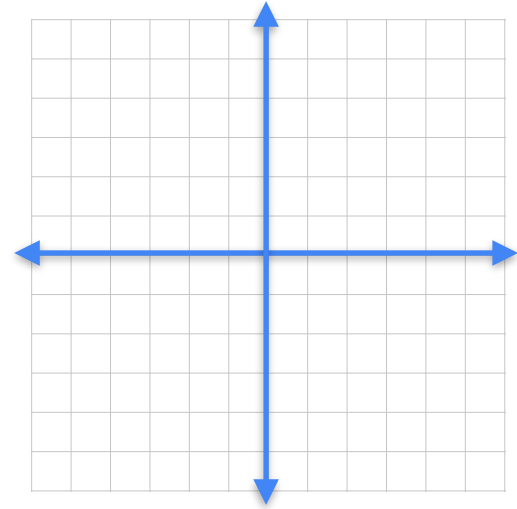
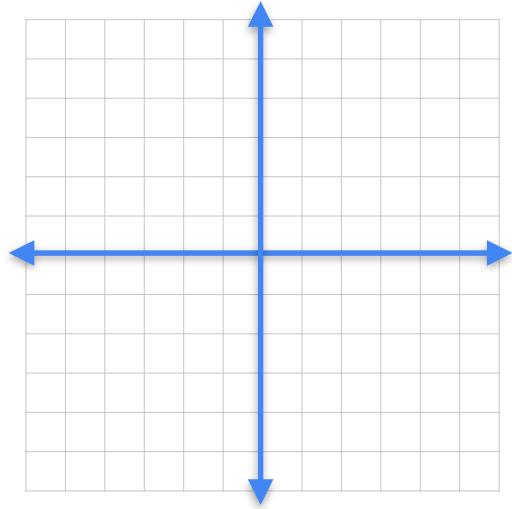


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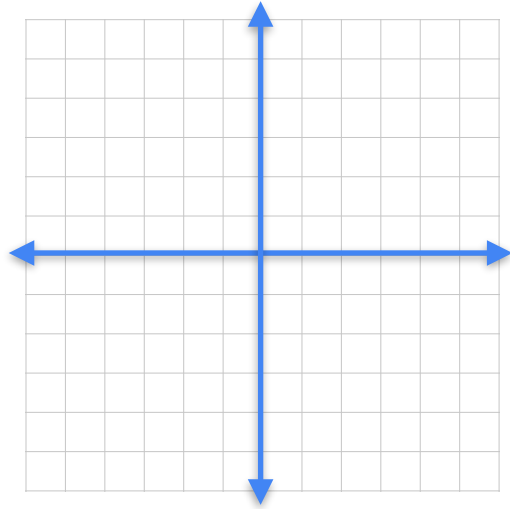
Vectors and Linear Transformations

Matrix multiplication

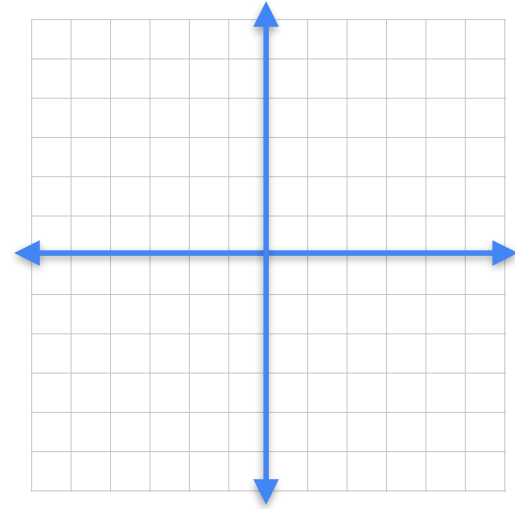
Combining linear transformations



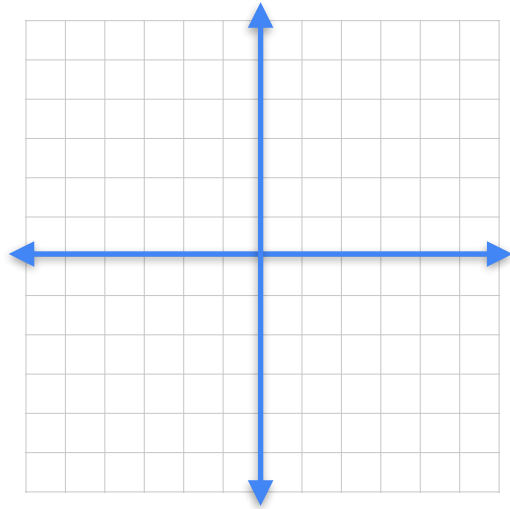
Combining linear transformations



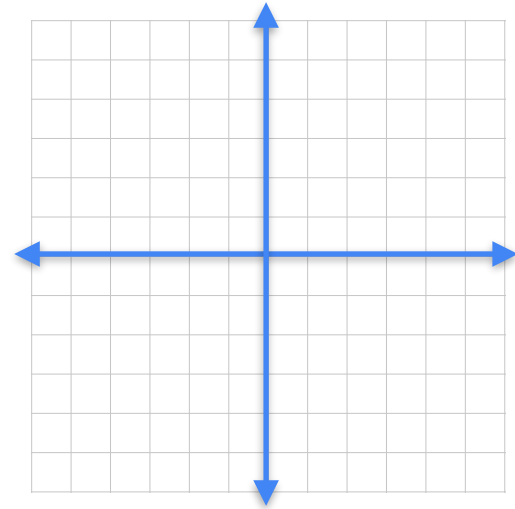
3	1
1	2



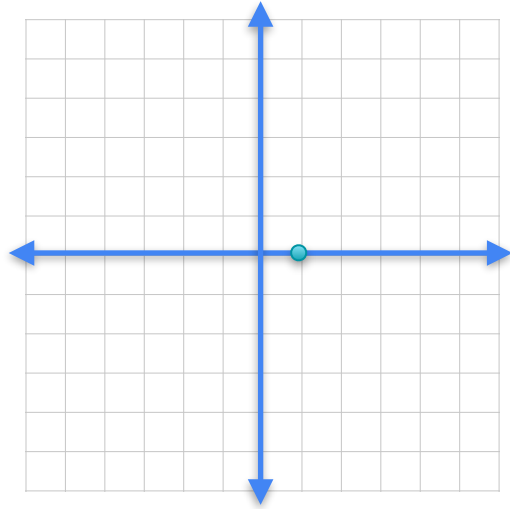
Combining linear transformations



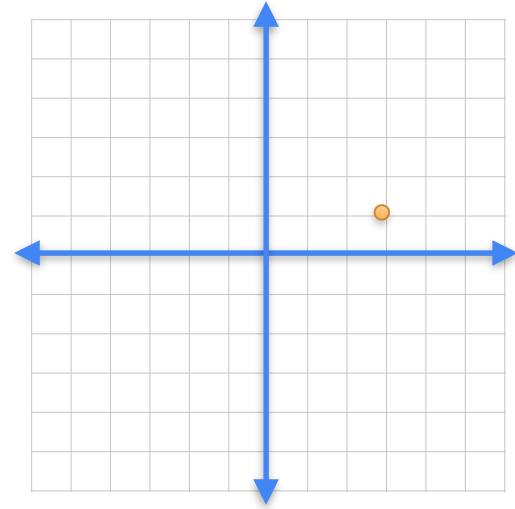
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



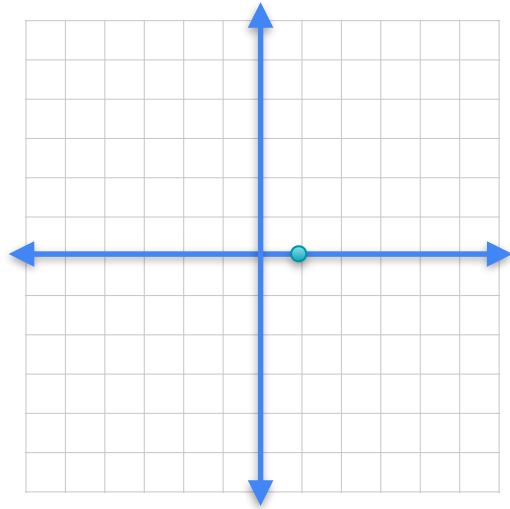
Combining linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

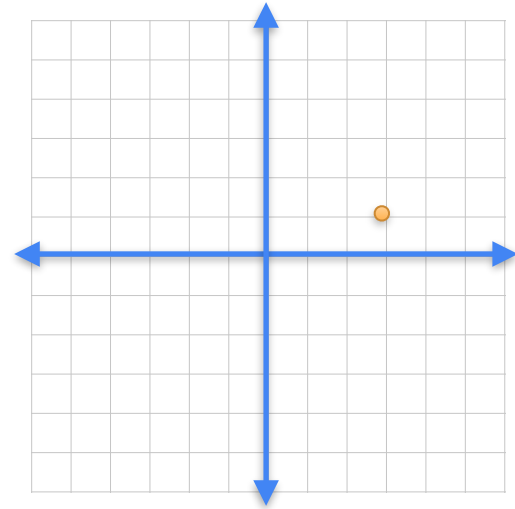


Combining linear transformations

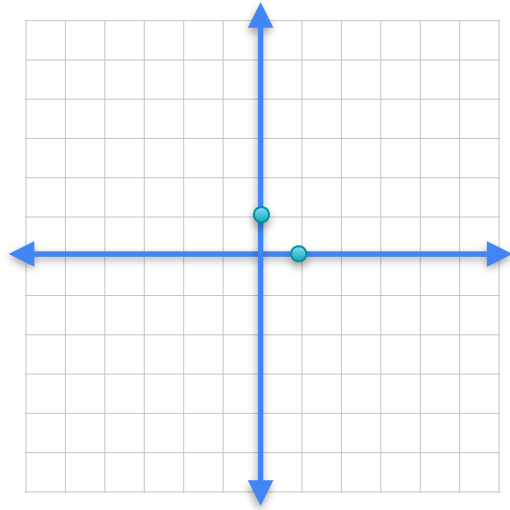


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

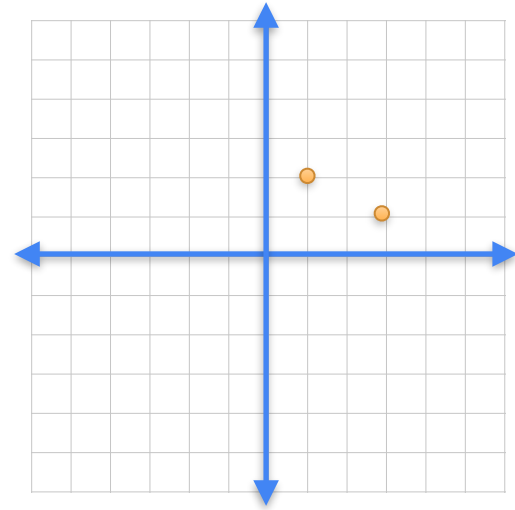


Combining linear transformations

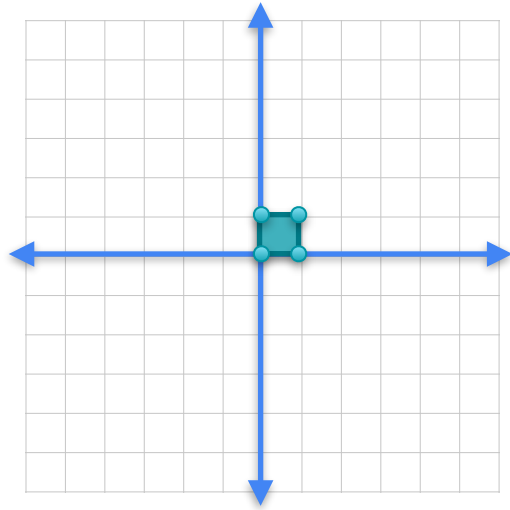


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

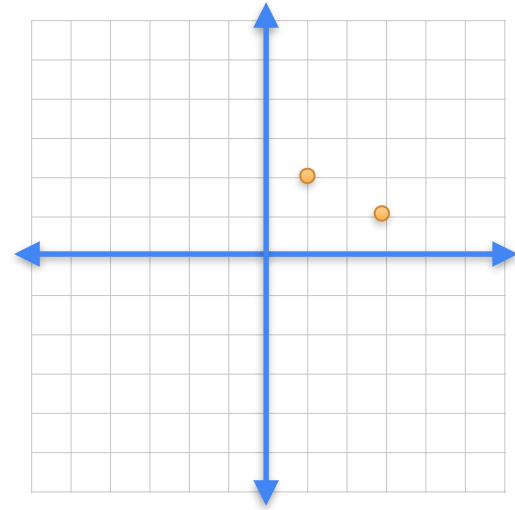


Combining linear transformations

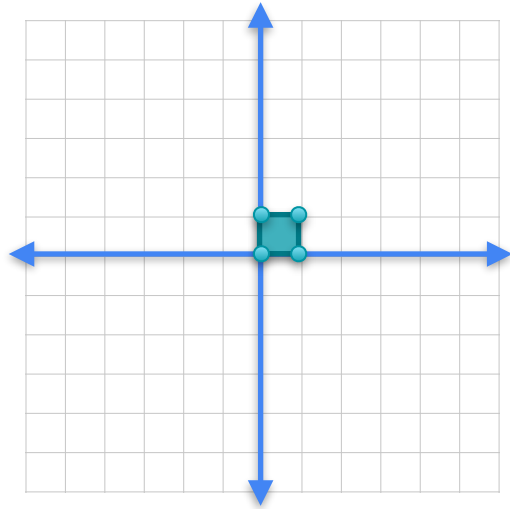


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

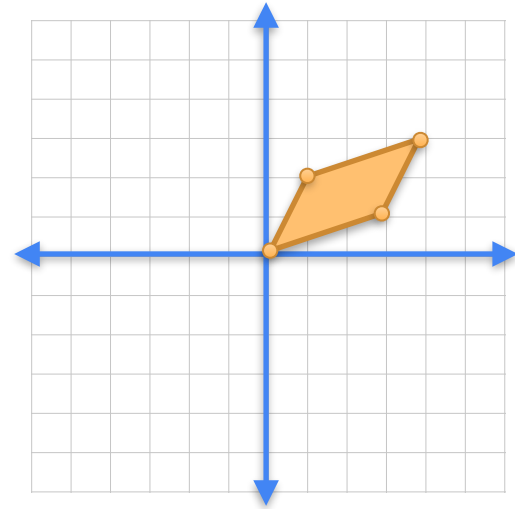


Combining linear transformations

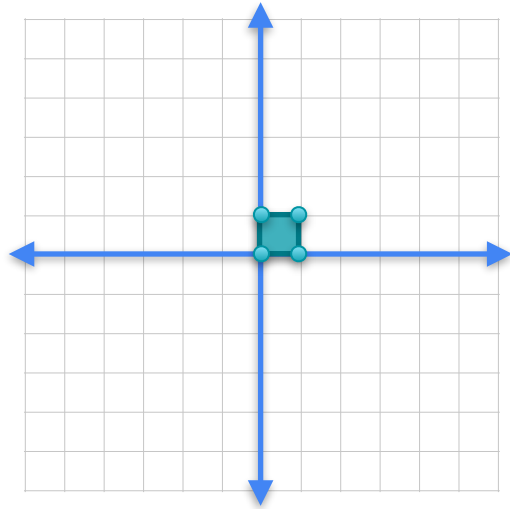


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

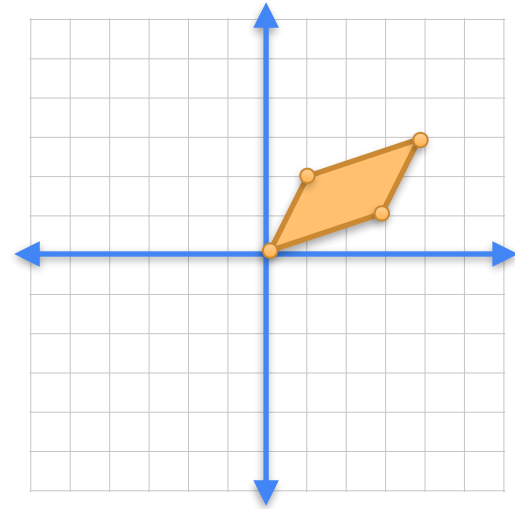


Combining linear transformations

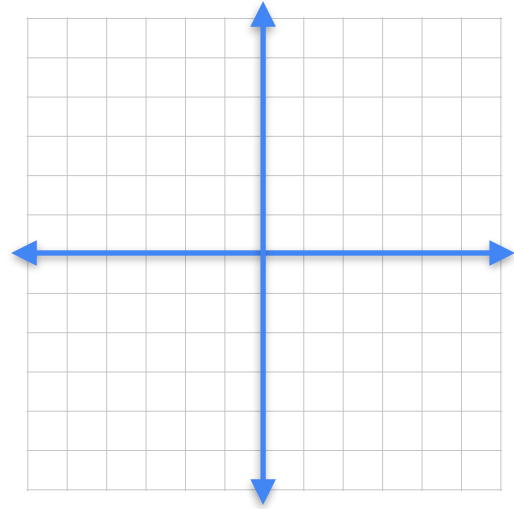
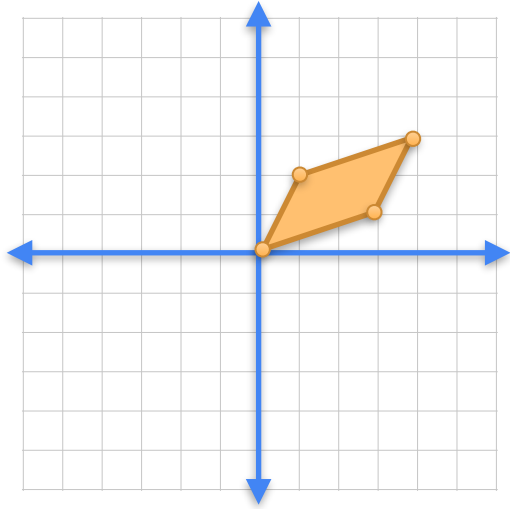


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

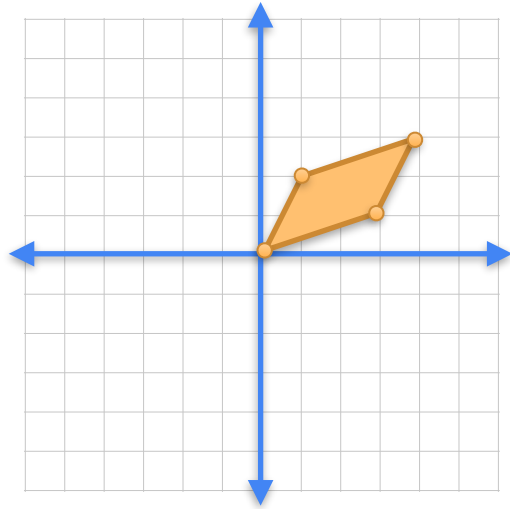
$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



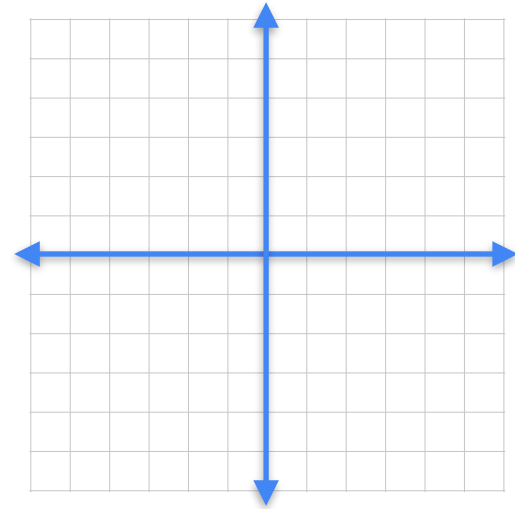
Combining linear transformations



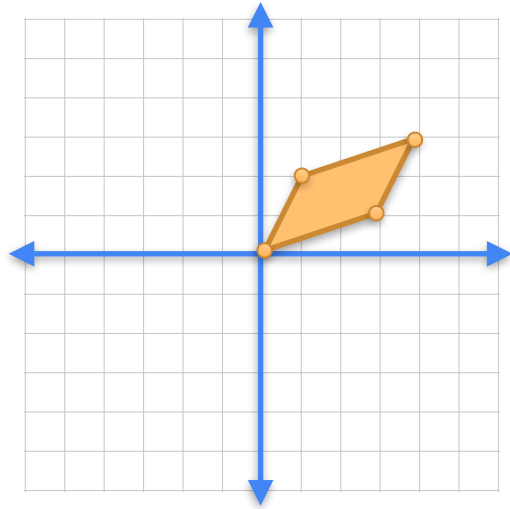
Combining linear transformations



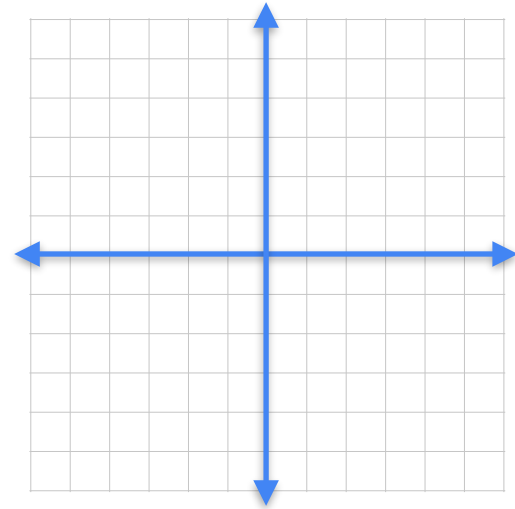
2	-1
0	2



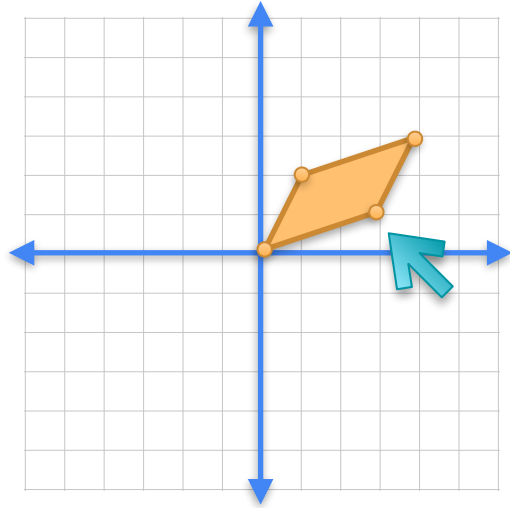
Combining linear transformations



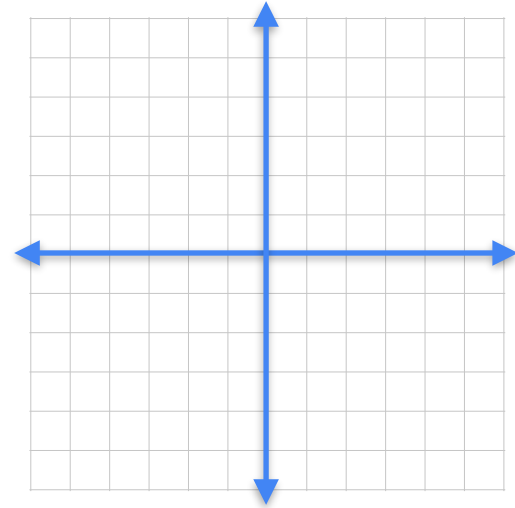
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



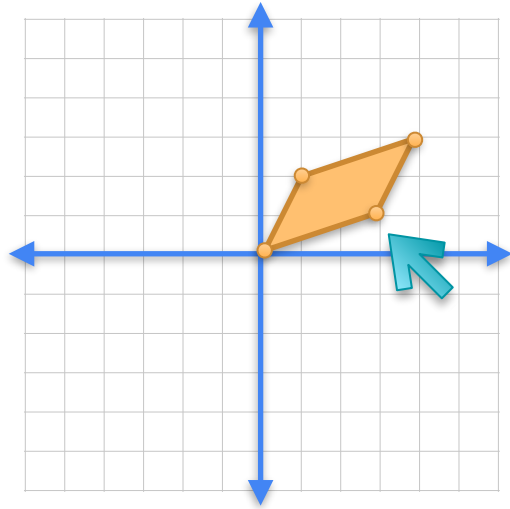
Combining linear transformations



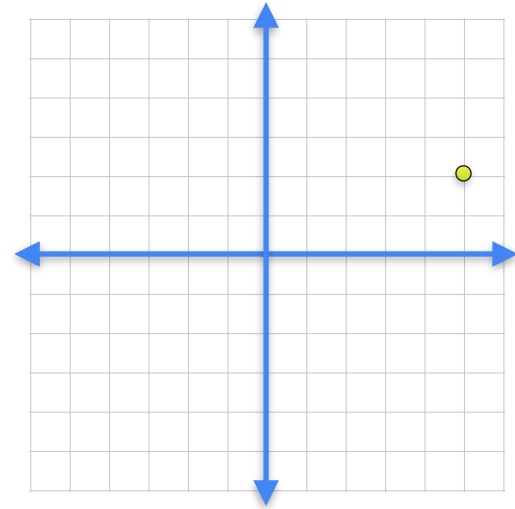
$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



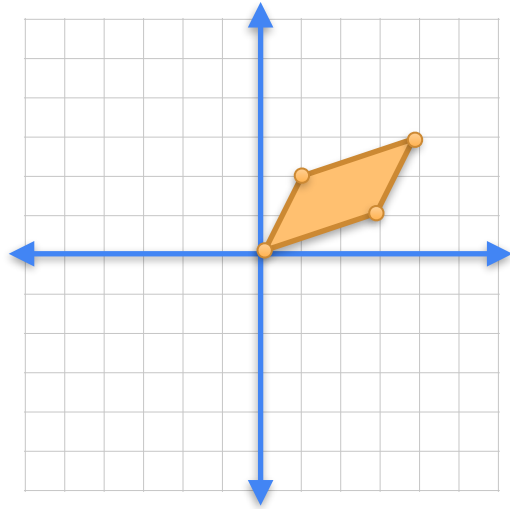
Combining linear transformations



$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

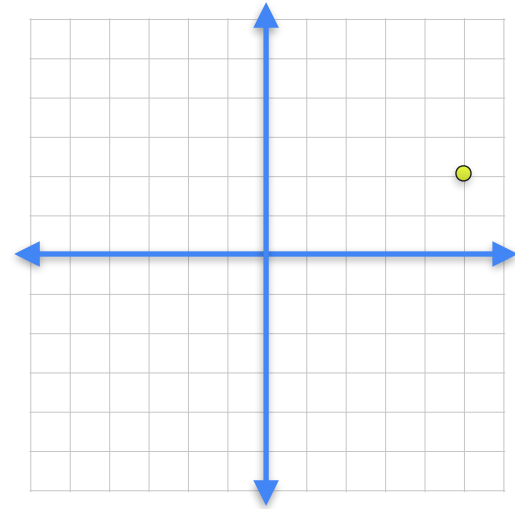


Combining linear transformations

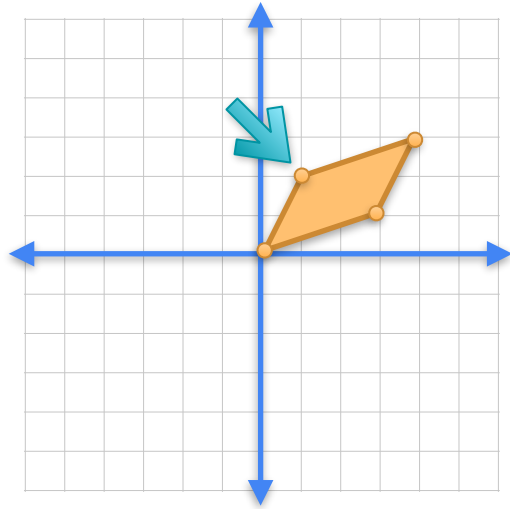


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

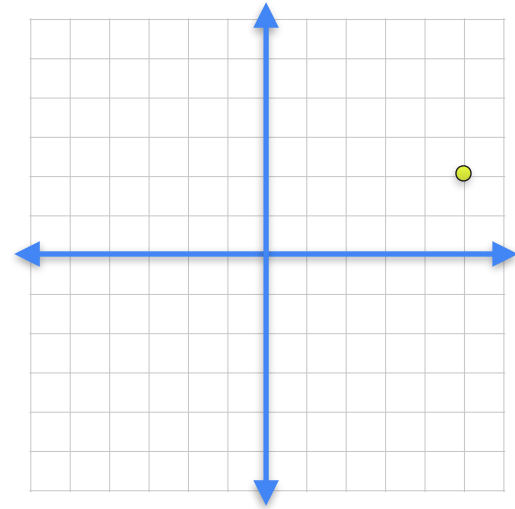


Combining linear transformations

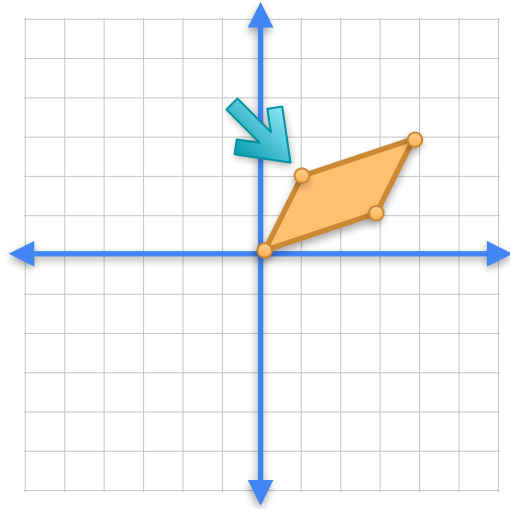


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

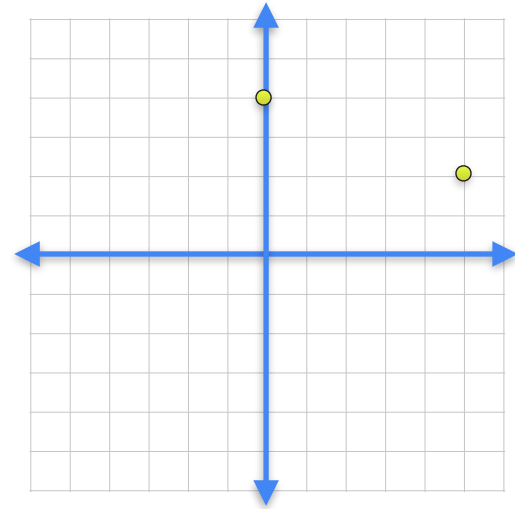


Combining linear transformations

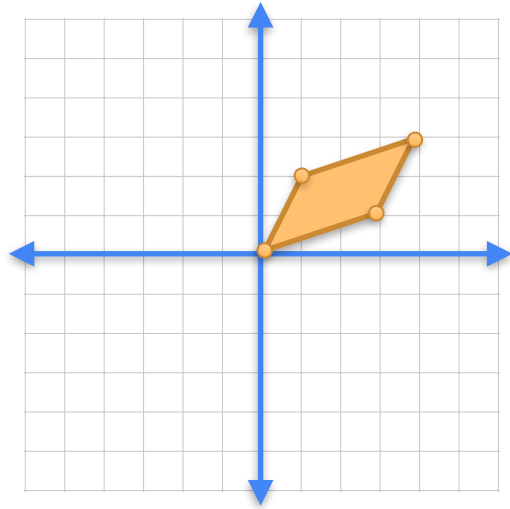


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

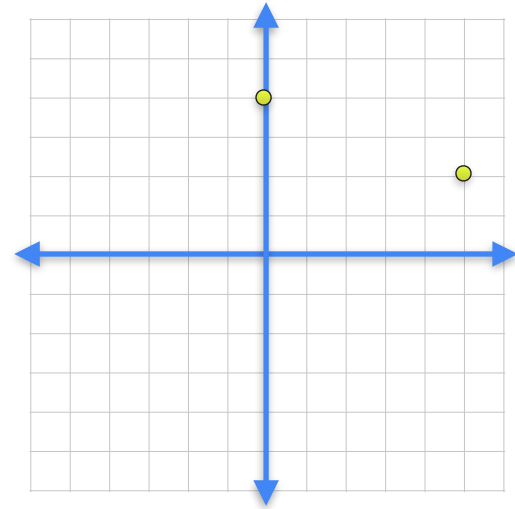


Combining linear transformations

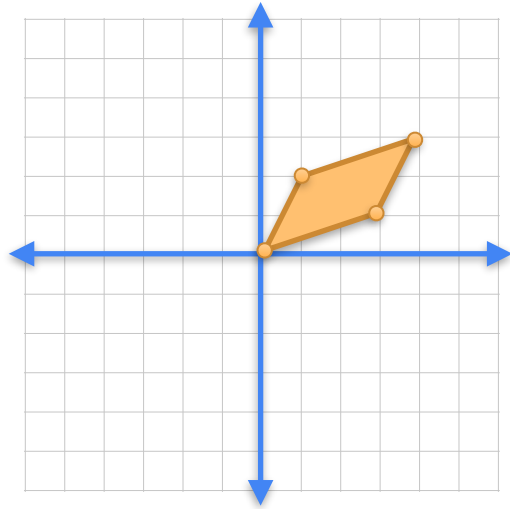


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

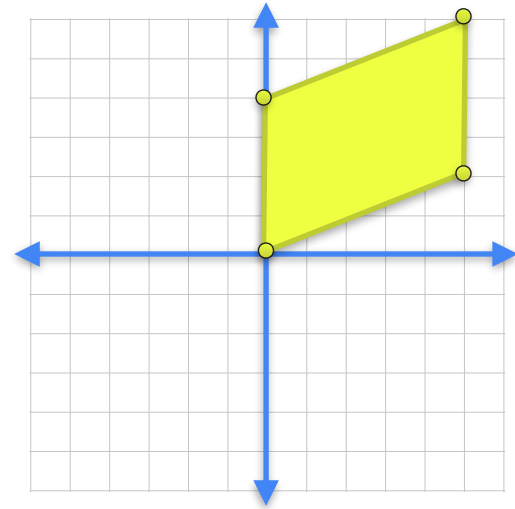


Combining linear transformations

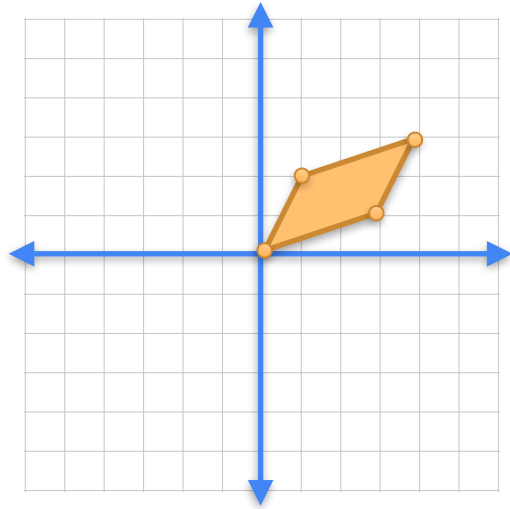


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

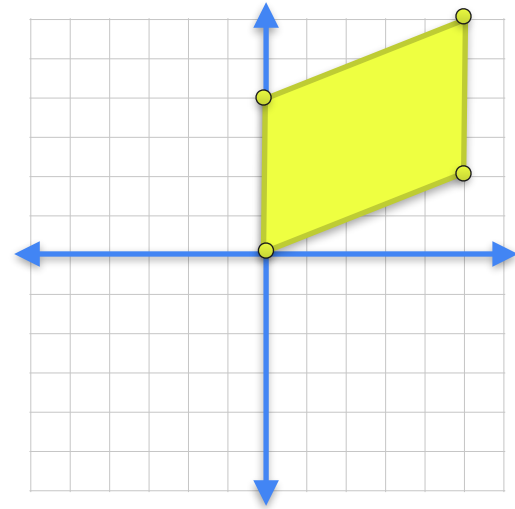


Combining linear transformations

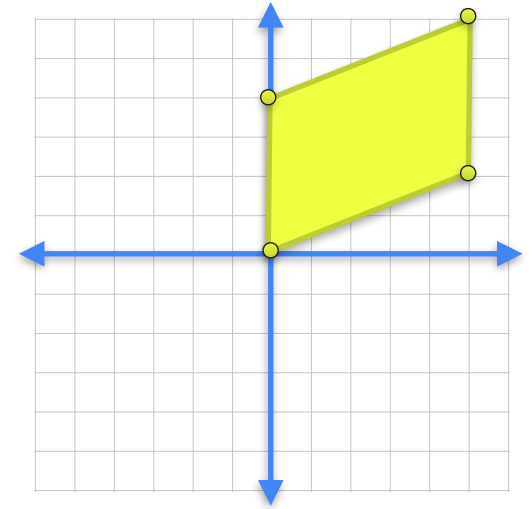
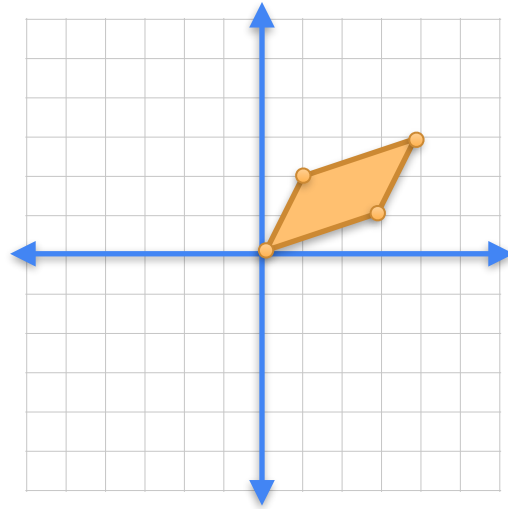
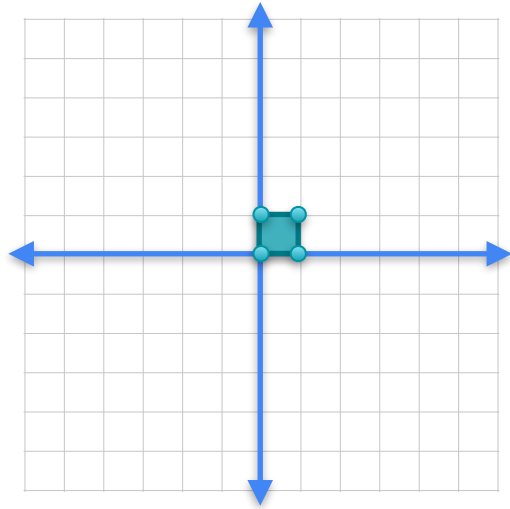


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

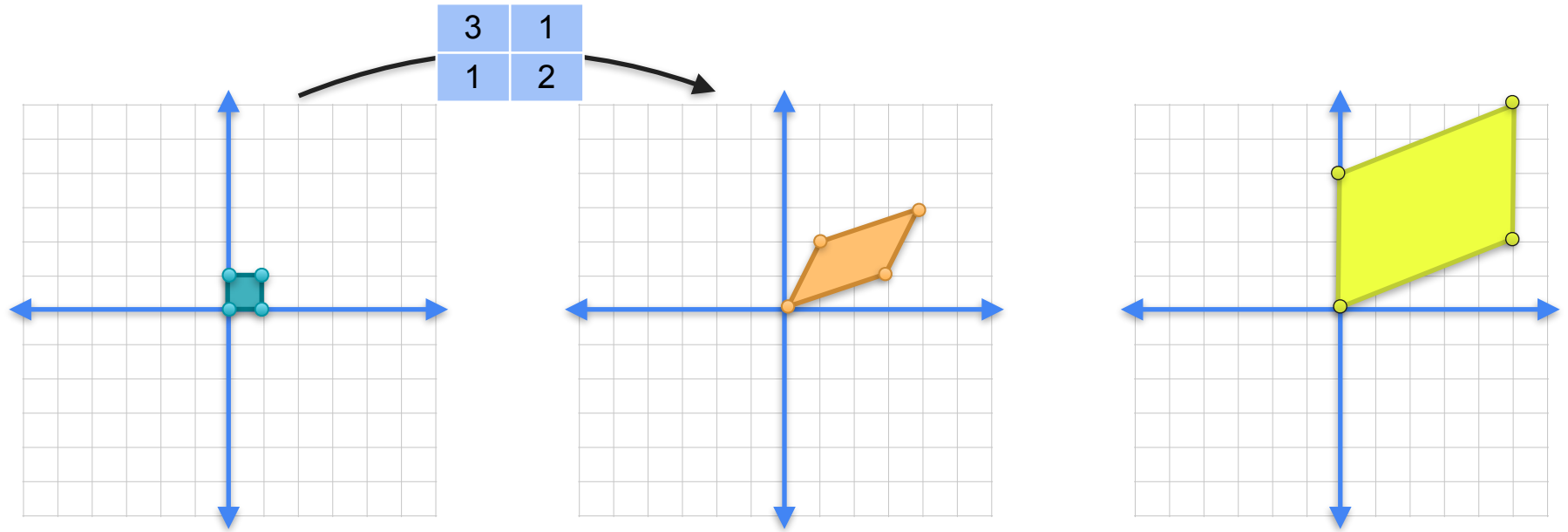
$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$



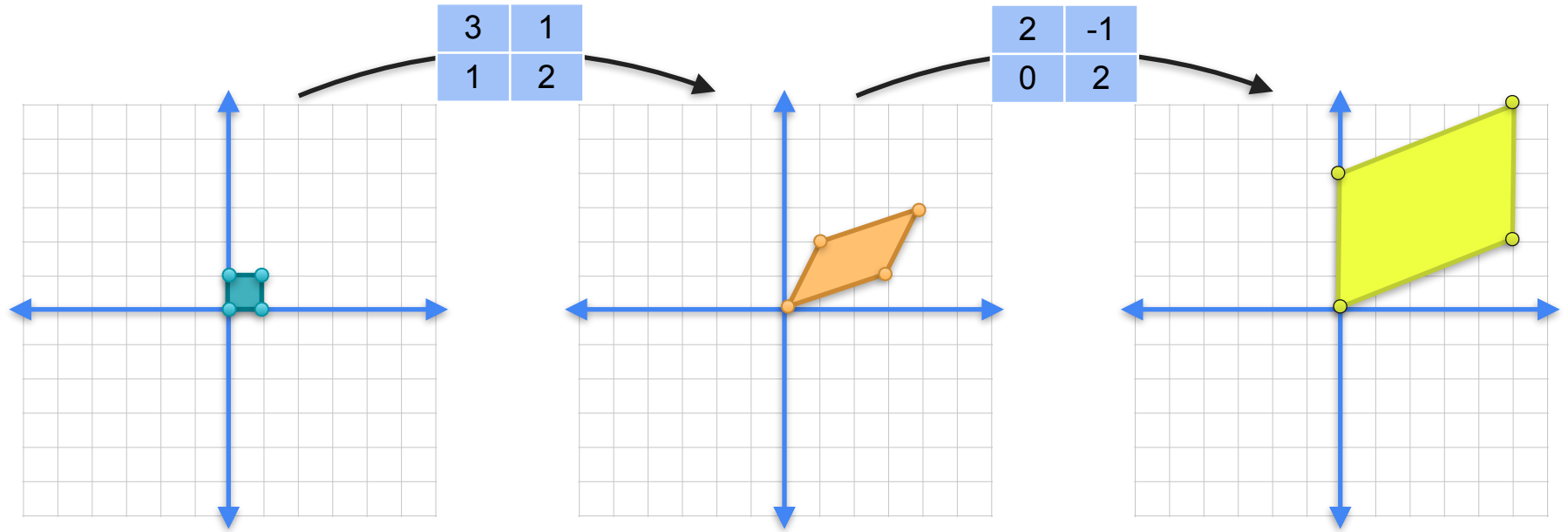
Combining linear transformations



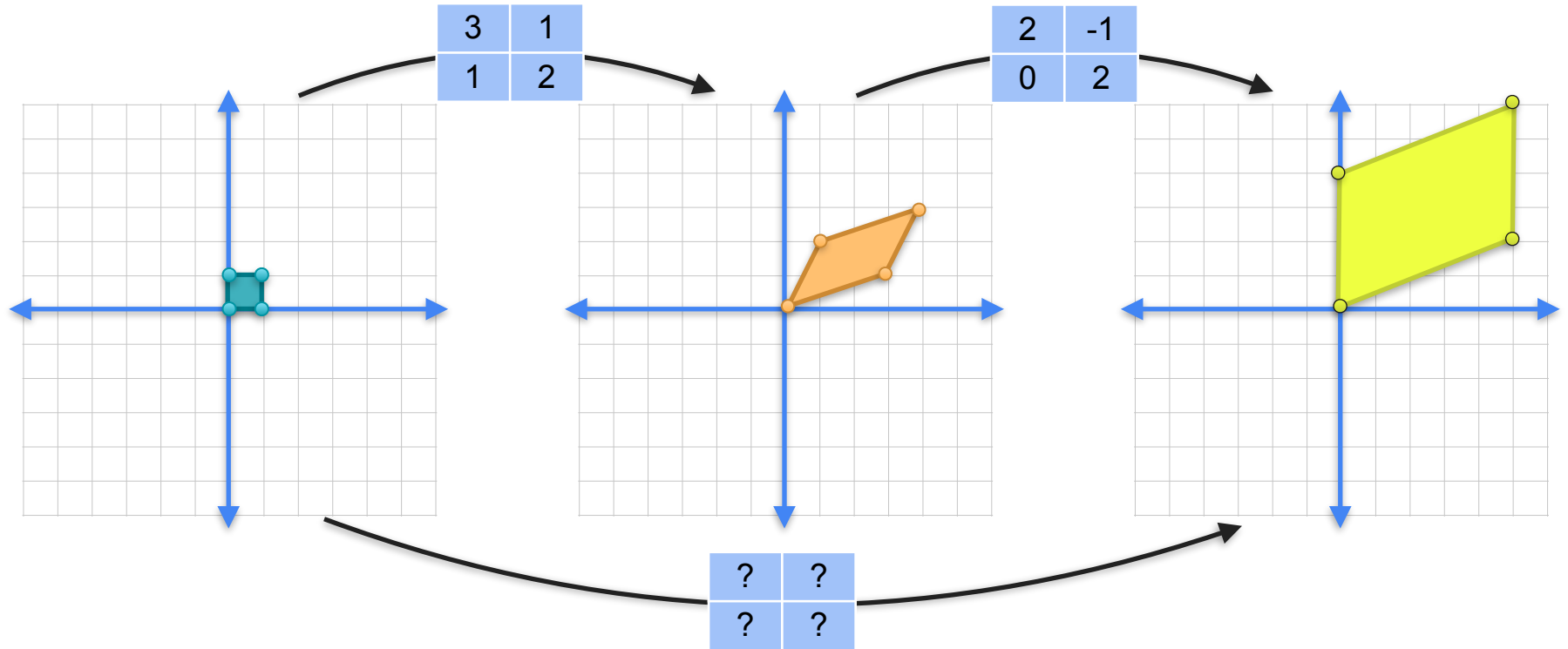
Combining linear transformations



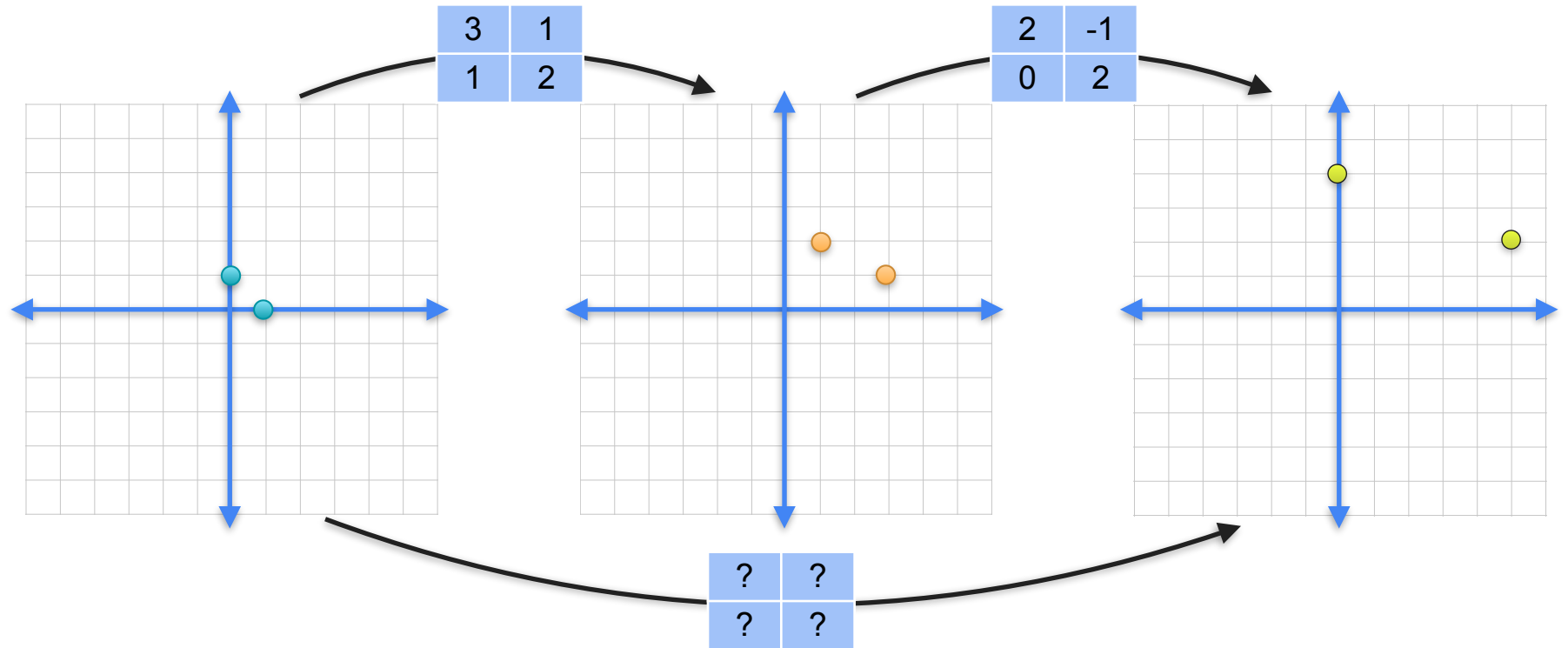
Combining linear transformations



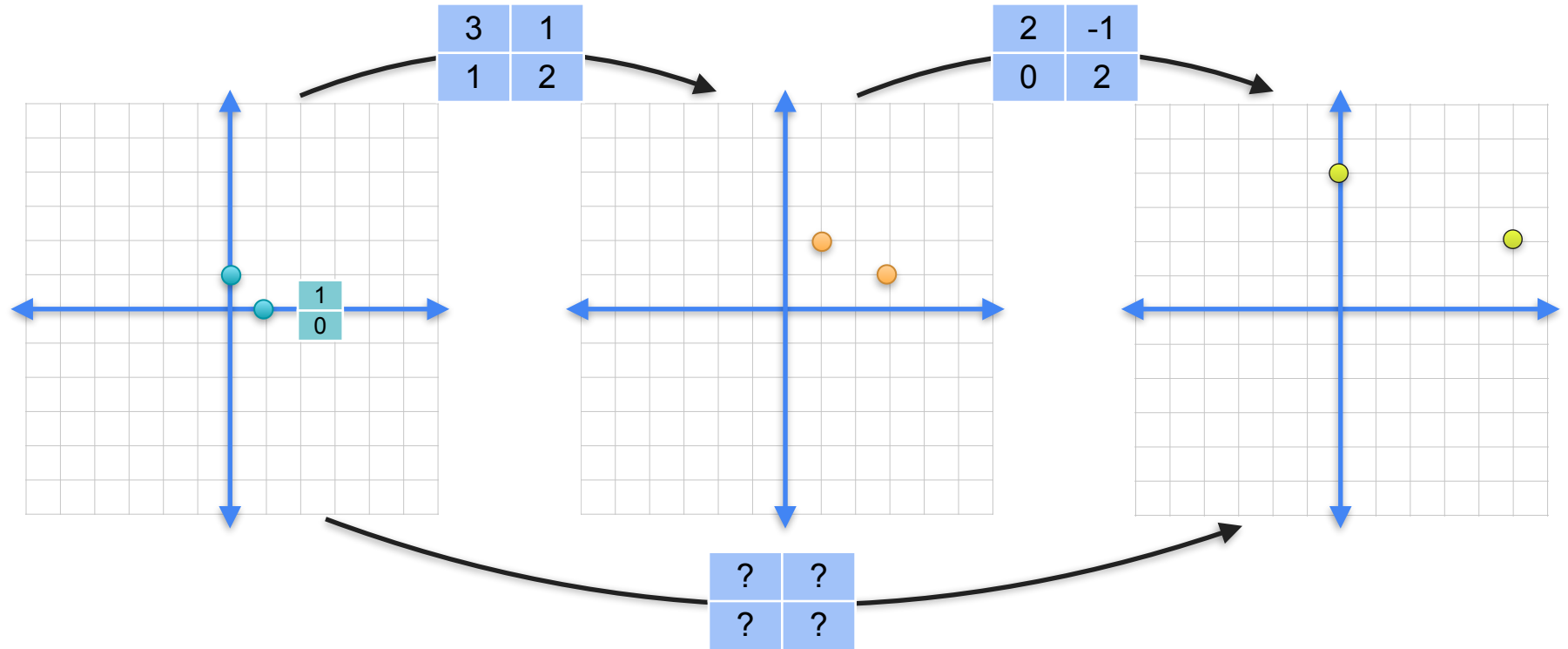
Combining linear transformations



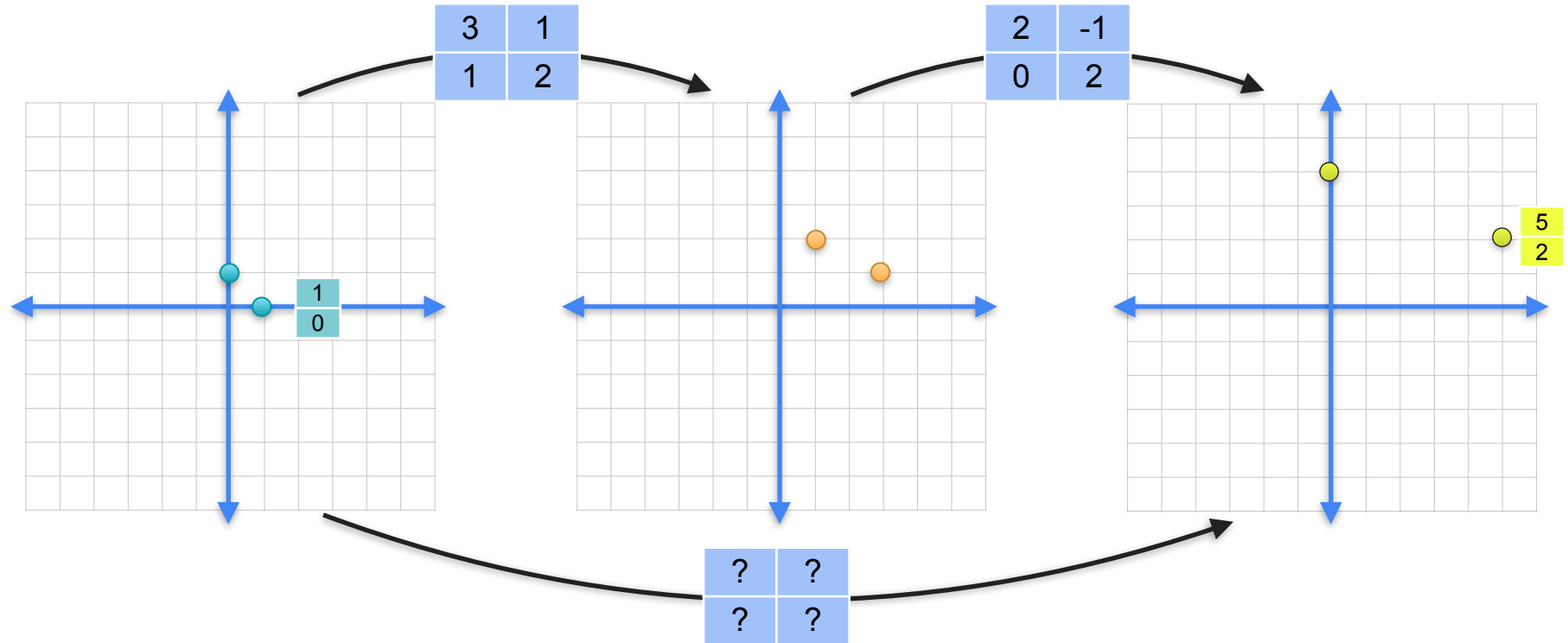
Combining linear transformations



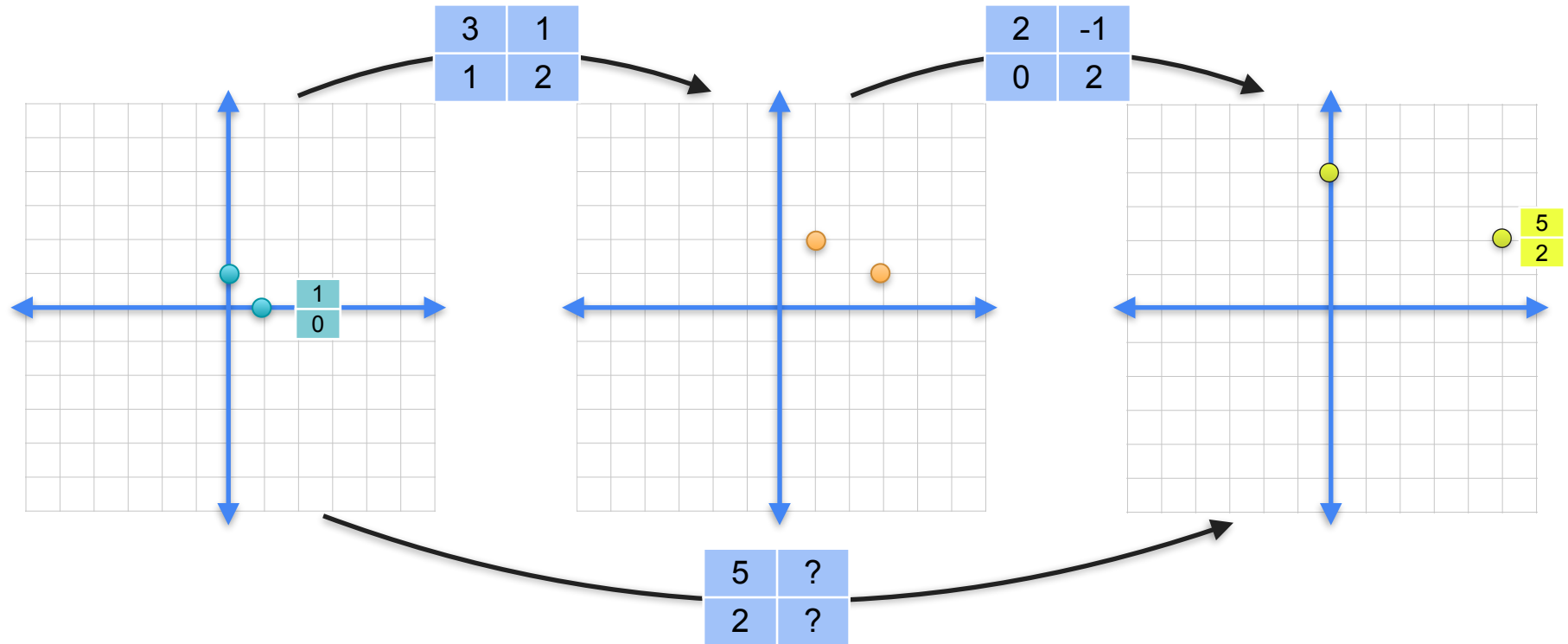
Combining linear transformations



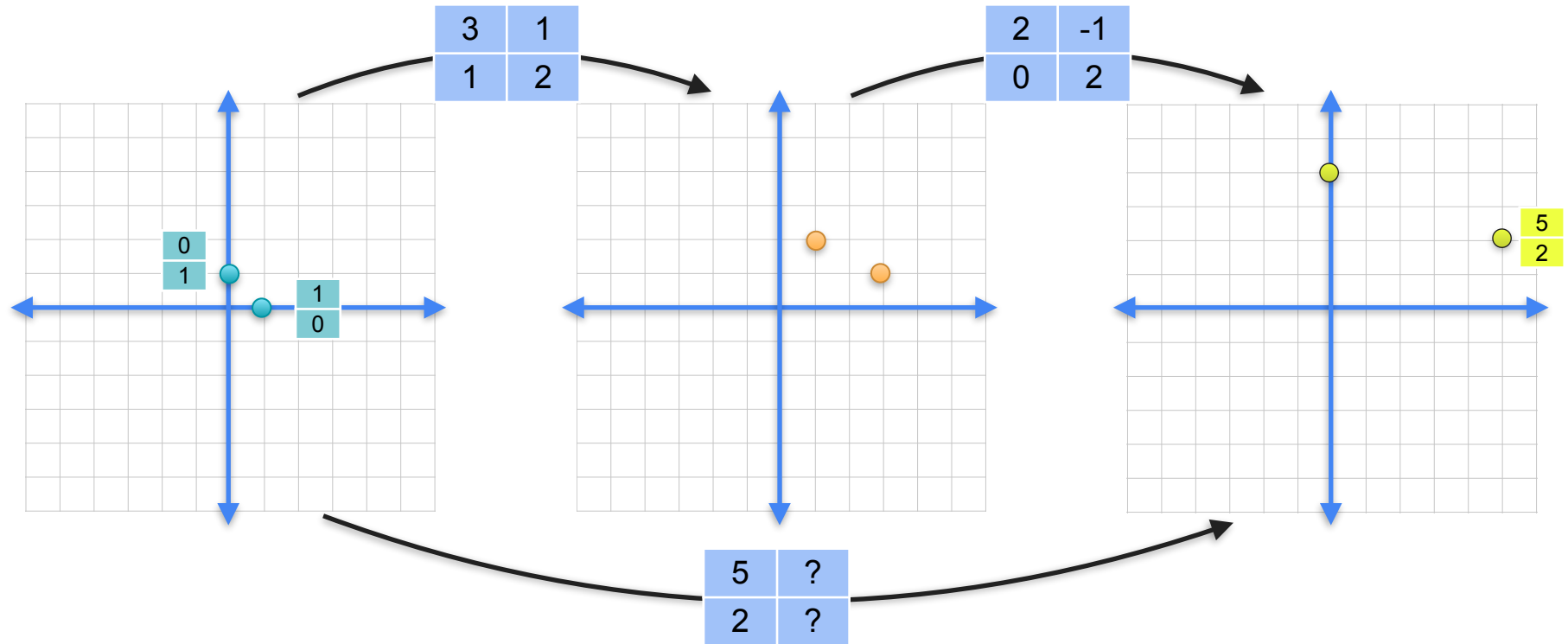
Combining linear transformations



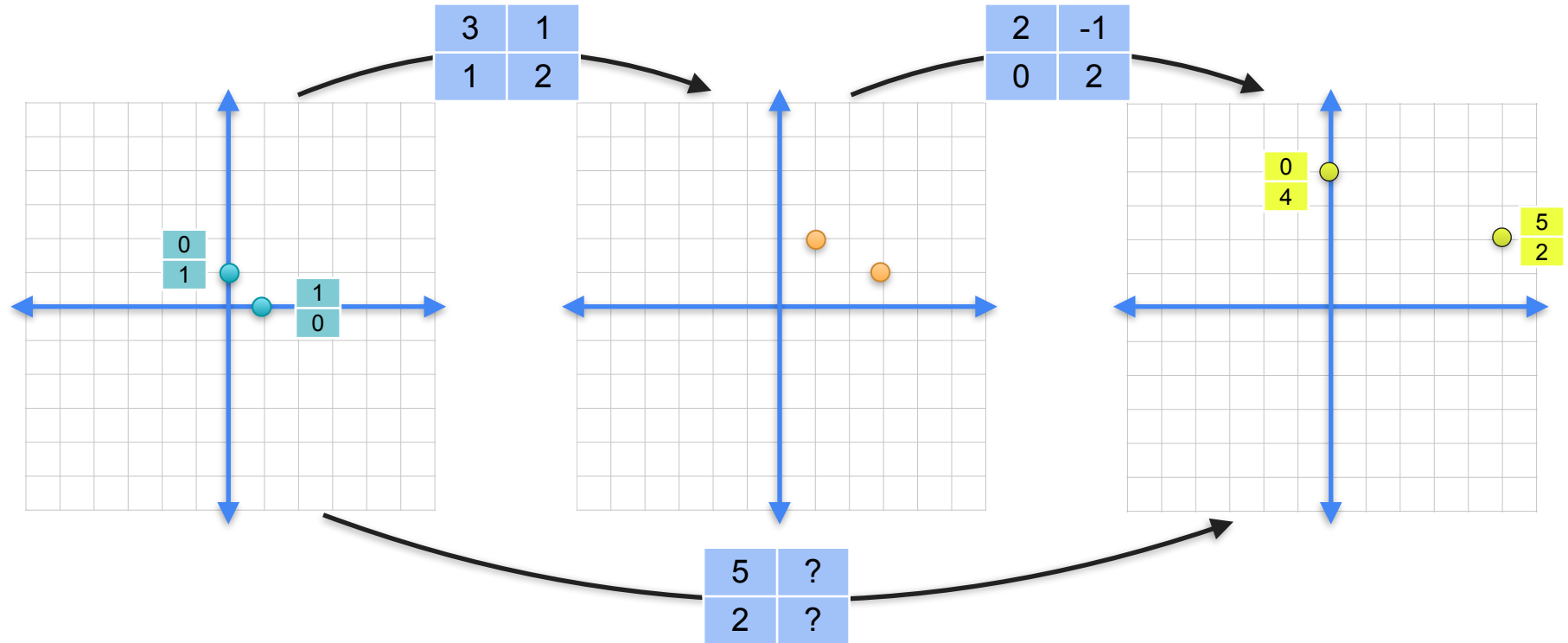
Combining linear transformations



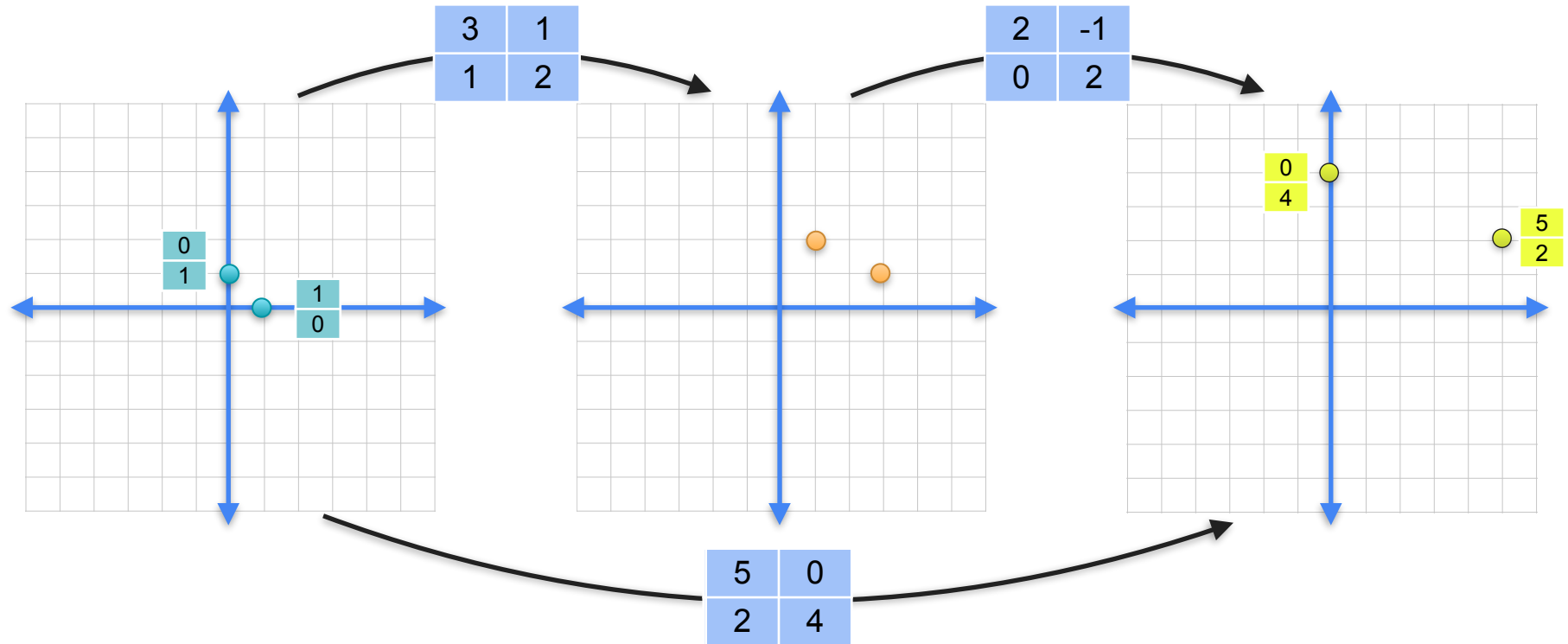
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

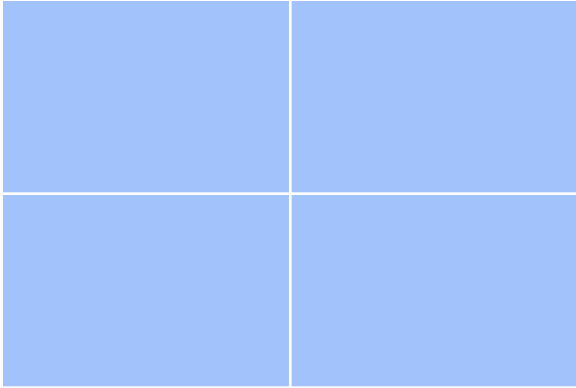
First
↓

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$


Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 0 & 2 & 3 & 1 \\ 2 & -1 & 1 & 2 \\ 0 & 2 & 1 & 2 \end{bmatrix}$$

Multiplying matrices

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix (teal) is multiplied by the second matrix (orange) to produce a 2x2 result matrix (blue). The result matrix contains the value 5 in the top-left cell, and the original matrices are shown as sub-components within the other cells.

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

The diagram illustrates the calculation of the product matrix. The top-left element of the product matrix is 5, which is the dot product of the first row of the teal matrix (2, -1) and the first column of the orange matrix (3, 1). The top-right element is 0, which is the dot product of the first row of the teal matrix (2, -1) and the second column of the orange matrix (1, 2). The bottom row of the product matrix (0, 2) is not explicitly calculated in the diagram, but it is shown as the result of dot products of the second row of the teal matrix (0, 2) with the columns of the orange matrix.

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$
The diagram illustrates the multiplication of two 2x2 matrices. The first matrix, with a light blue background, contains the values 2, -1, 0, and 2. The second matrix, with a light orange background, contains the values 3, 1, 1, and 2. An equals sign follows, leading to the resulting 2x2 matrix with a light blue background, which contains the values 5, 0, 2, and 4.



DeepLearning.AI

Vectors and Linear Transformations

The identity matrix

The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

The identity matrix

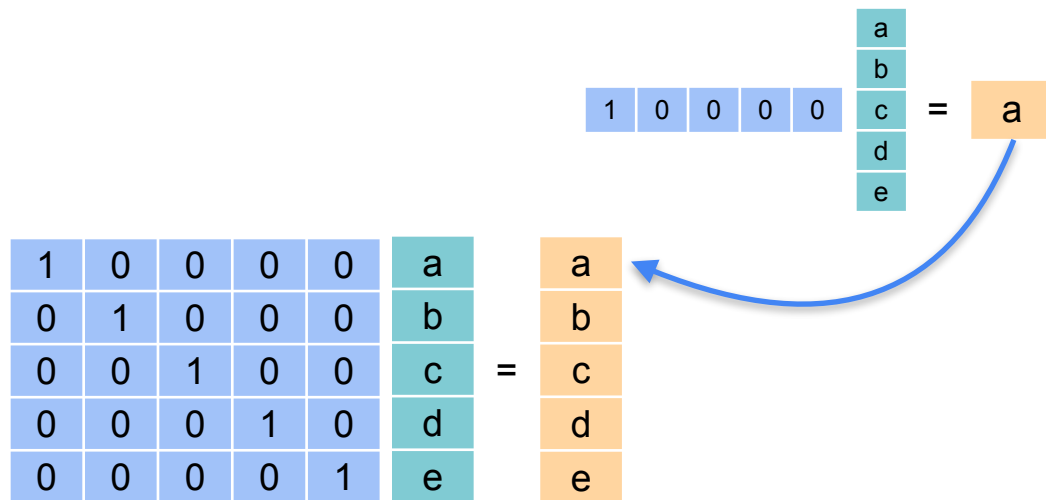
1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

The identity matrix

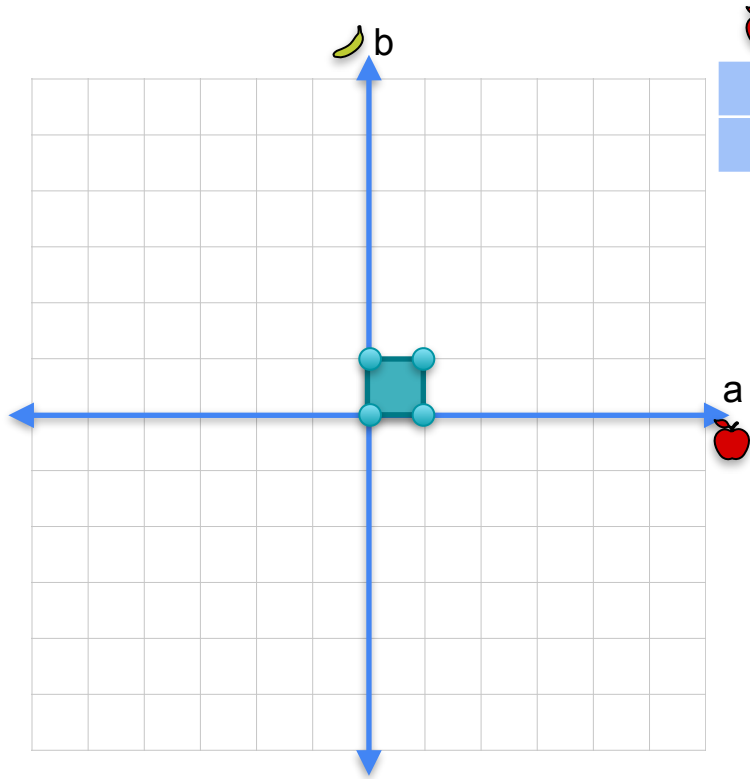
1	0	0	0	0	a	a
0	1	0	0	0	b	b
0	0	1	0	0	c	c
0	0	0	1	0	d	d
0	0	0	0	1	e	e

=

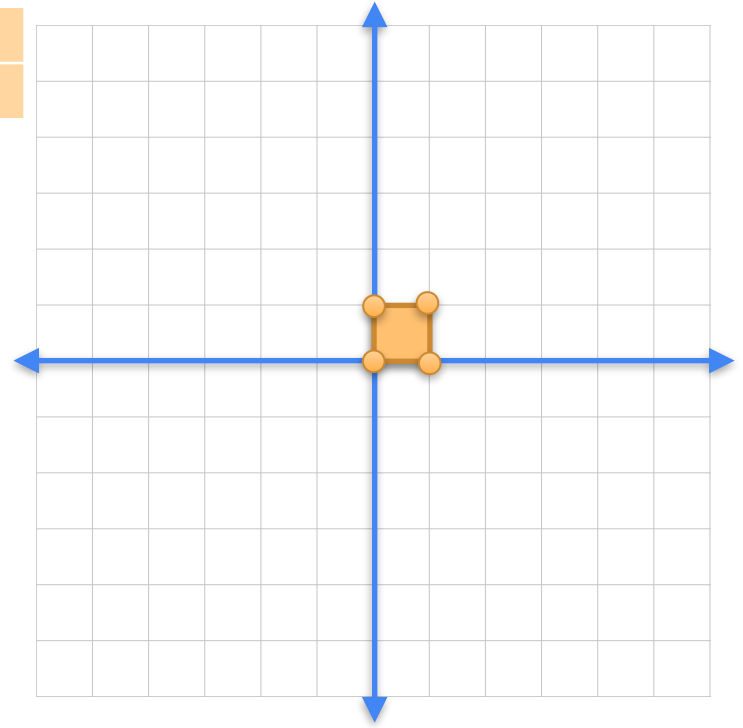
The identity matrix



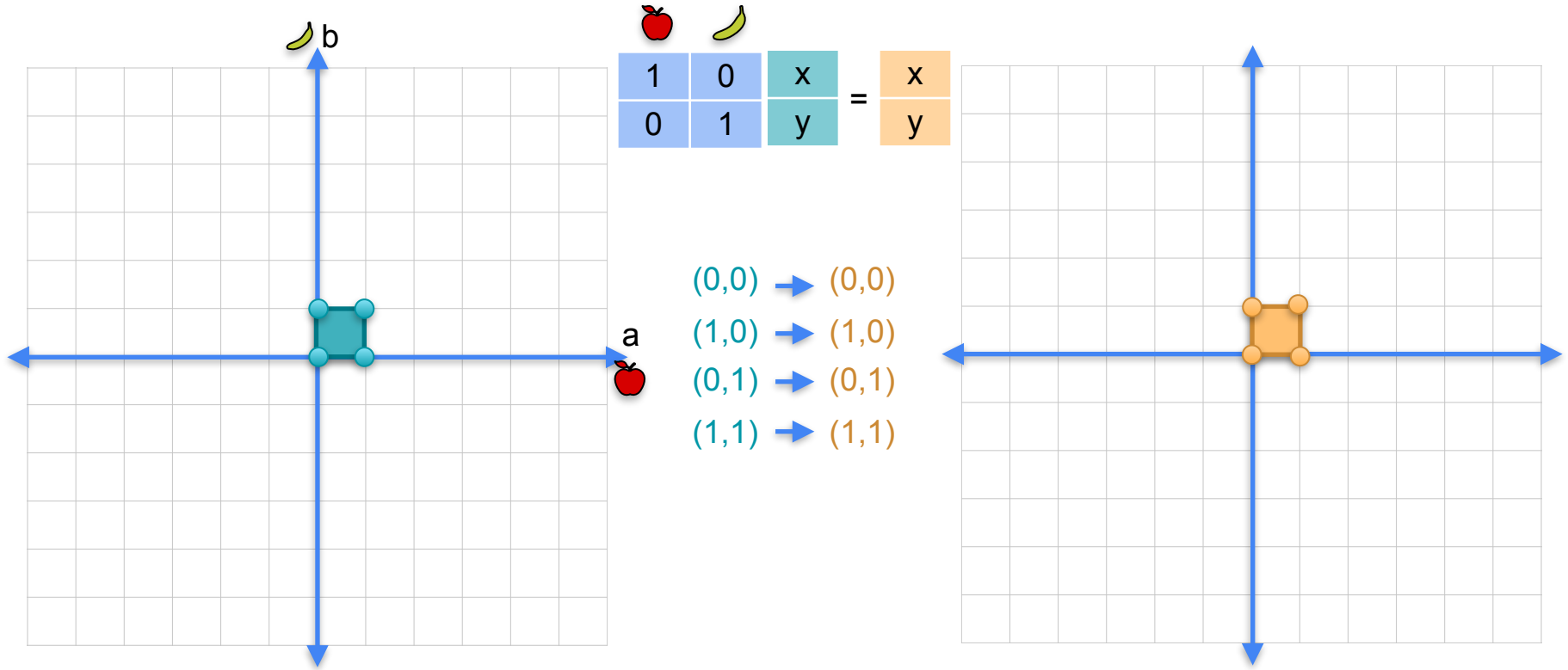
The identity matrix



1	0	x	=	x
0	1	y	=	y



The identity matrix





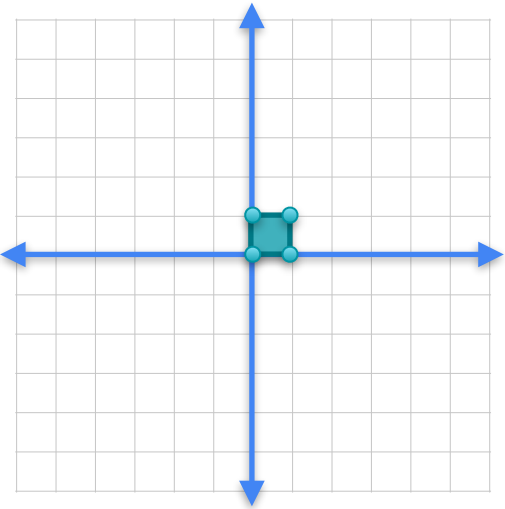
DeepLearning.AI

Vectors and Linear Transformations

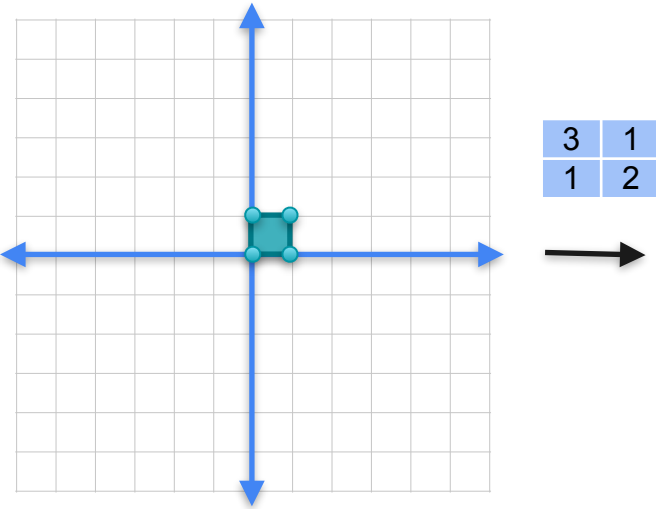
Matrix inverse

Matrix inverses

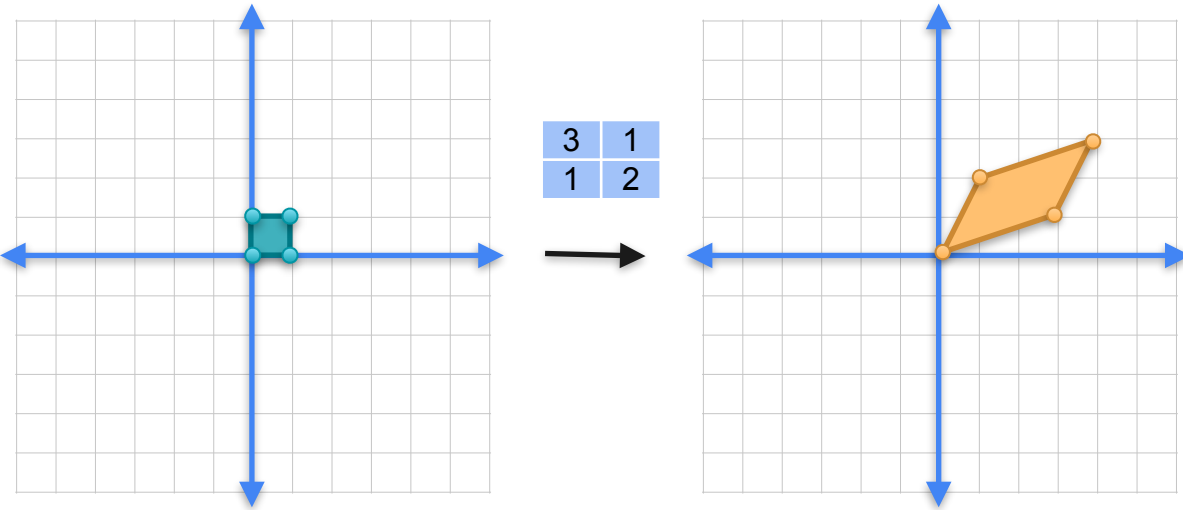
Matrix inverses



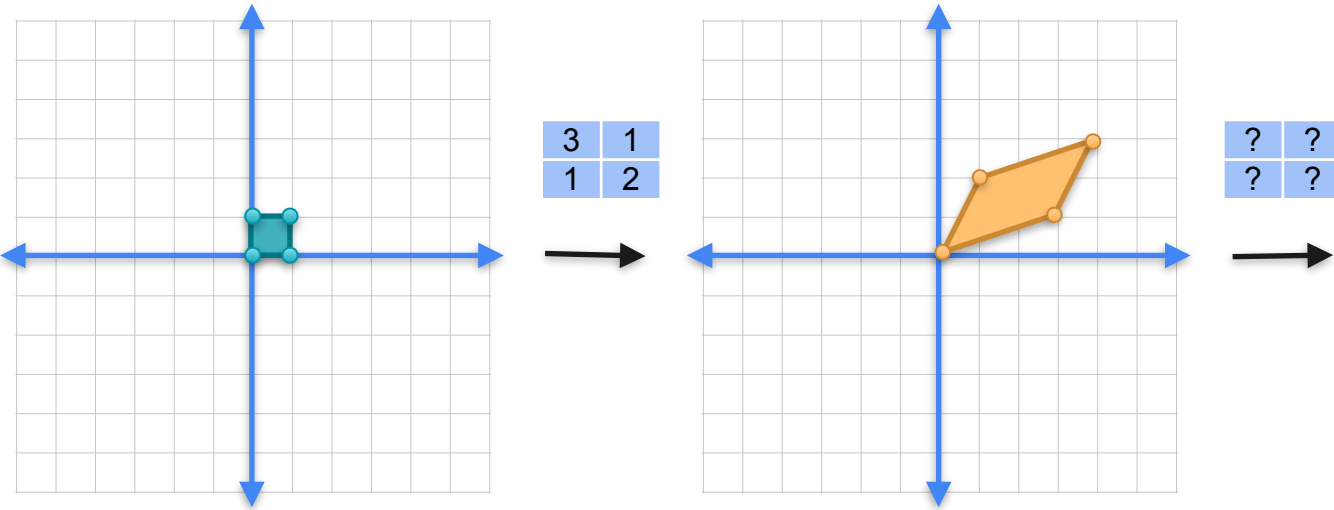
Matrix inverses



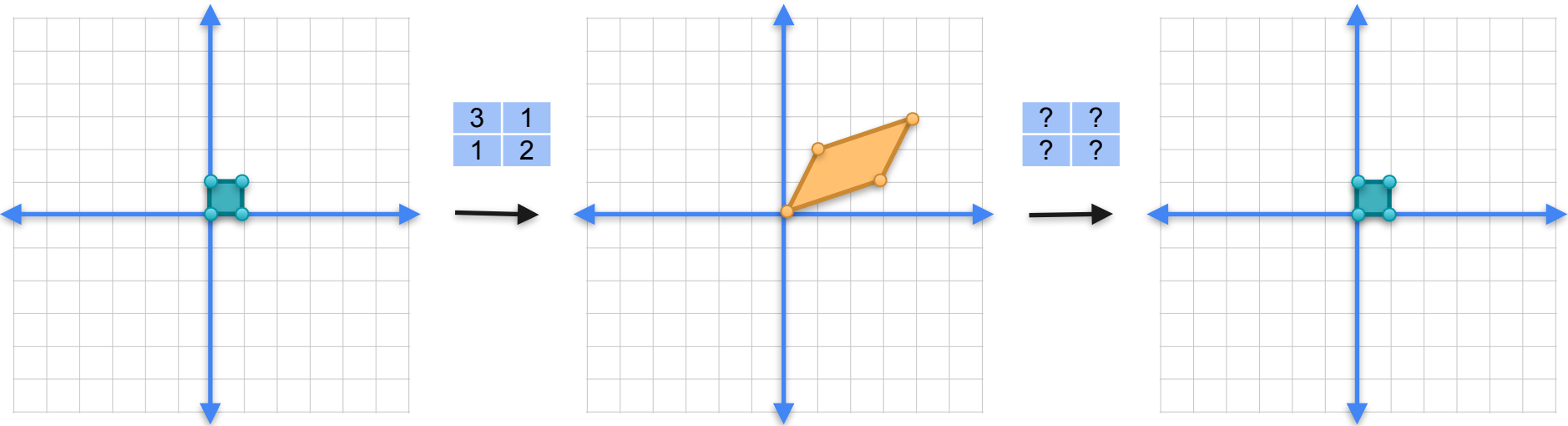
Matrix inverses



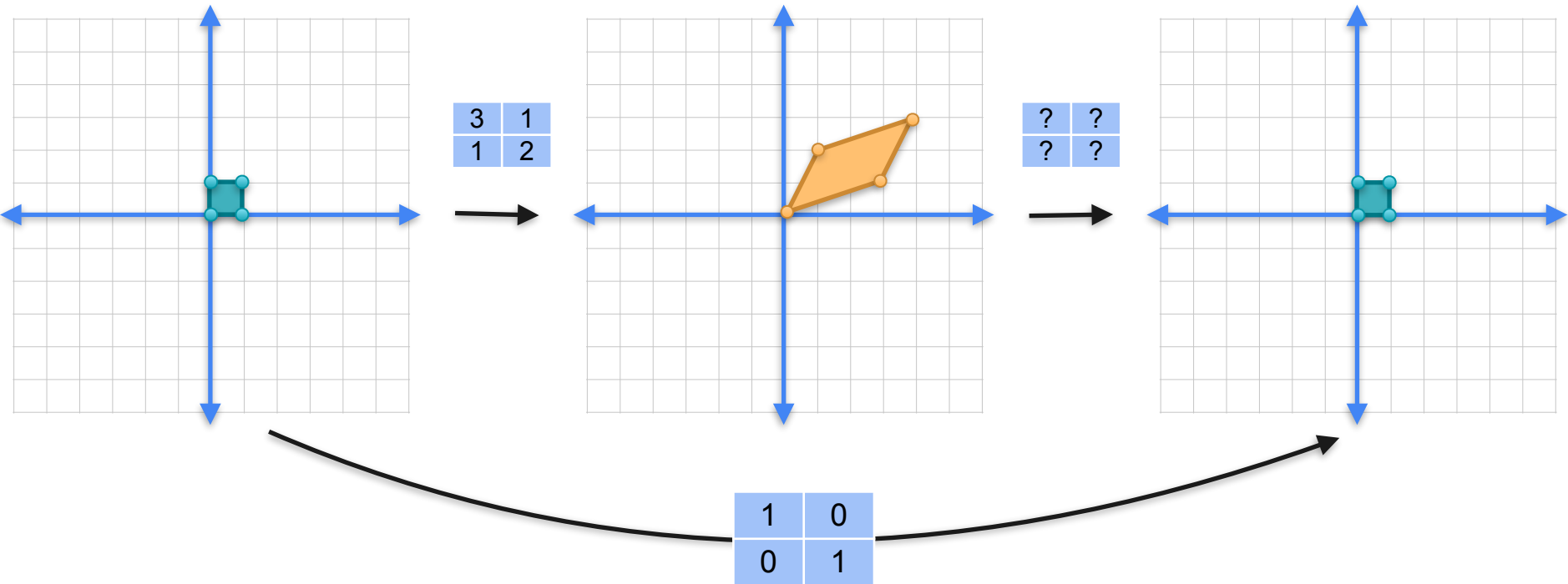
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

a	b
c	d

Multiplying matrices


$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1 \\ \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0 \\ \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 \\ \begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \end{array}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ & \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$3a + 1b = 1$$

$$\begin{bmatrix} a & b \\ & \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$1a + 2b = 0$$

$$\begin{bmatrix} c & d \\ & \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$3c + 1d = 0$$

$$\begin{bmatrix} c & d \\ & \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$1c + 2d = 1$$

How to find an inverse?

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \begin{array}{|c|} \hline 3 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$3a + 1b = 1$$

$$a = \frac{2}{5}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$1a + 2b = 0$$

$$b = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \begin{array}{|c|} \hline 3 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$3c + 1d = 0$$

$$c = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline c & d \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$1c + 2d = 1$$

$$d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1 \\ \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0 \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1 \end{array}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} & \\ 1 \\ & \end{bmatrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} & \\ 1 & \end{bmatrix}$$

$$\bullet b + 2d = 1$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{bmatrix} 5 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ & \end{bmatrix}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{bmatrix} 1 & 2 \\ & \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ & \end{bmatrix}$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.



DeepLearning.AI

Vectors and Linear Transformations

**Which matrices have an
inverse?**

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

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$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

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$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2$$

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Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

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Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

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$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

$$\text{Det} = 5$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

$$\text{Det} = 5$$

$$\text{Det} = 8$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

$$\text{Det} = 5$$

$$\text{Det} = 8$$

$$\text{Det} = 0$$

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5

Det = 8

Det = 0

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix
Invertible

Non-singular matrix
Invertible

Singular matrix
Non-invertible

Det = 5

Det = 8

Det = 0

Non-zero determinants

Zero determinant

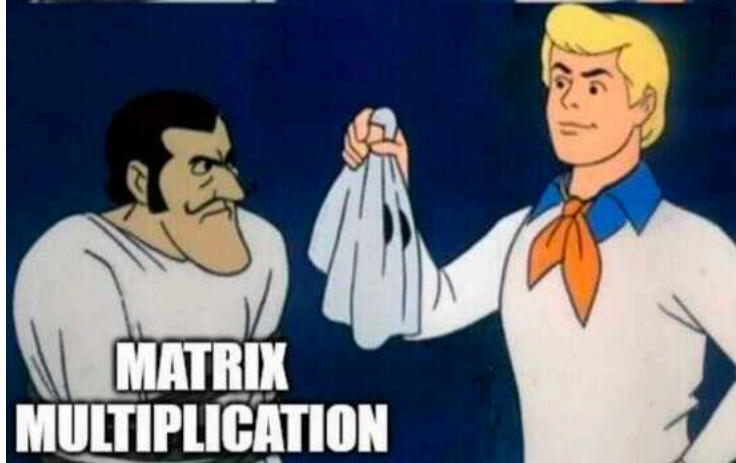


DeepLearning.AI

Vectors and Linear Transformations

**Neural networks and
matrices**

AI, ML, DL, RL



**MATRIX
MULTIPLICATION**

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ____ points

Win: ____ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than _____, then the email is spam.

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

Rule:

If the number of points of the sentence is bigger than ____, then the email is spam.

Goal: Find the best points and threshold

Lottery: ___ point

Win: ___ point

Threshold: ___ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

Solution:

Lottery: 1 point

Win: 1 point

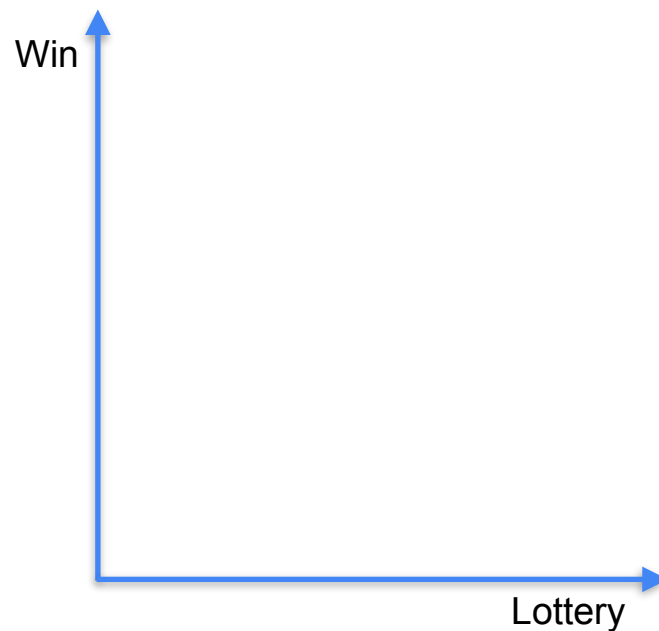
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

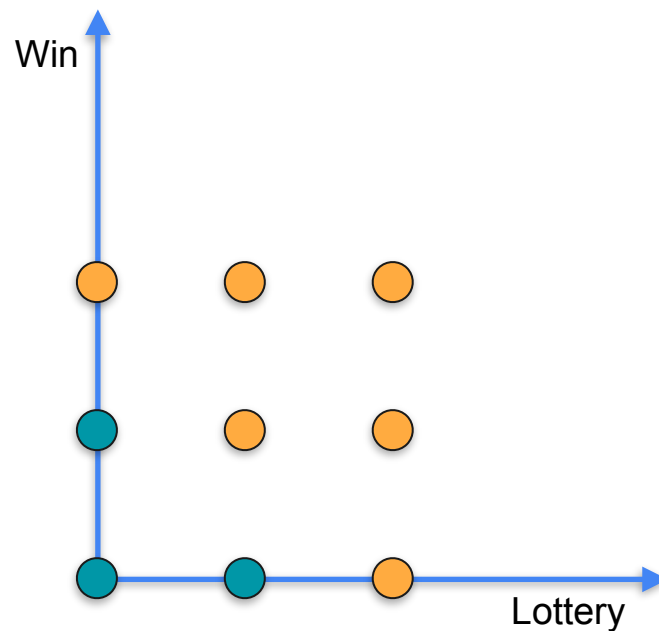
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

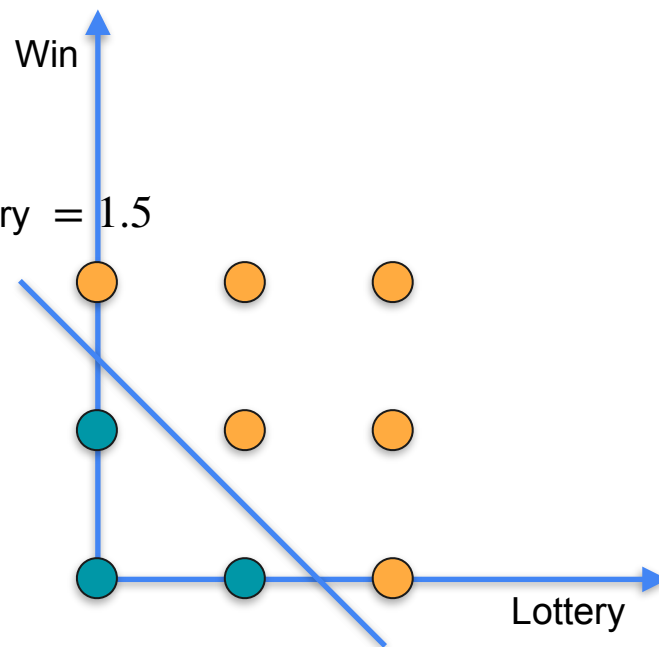


Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$

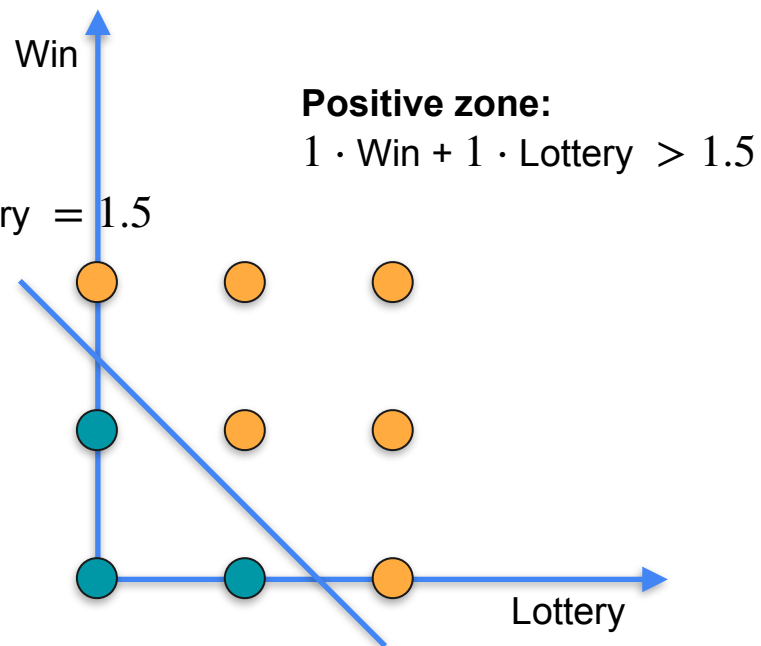


Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

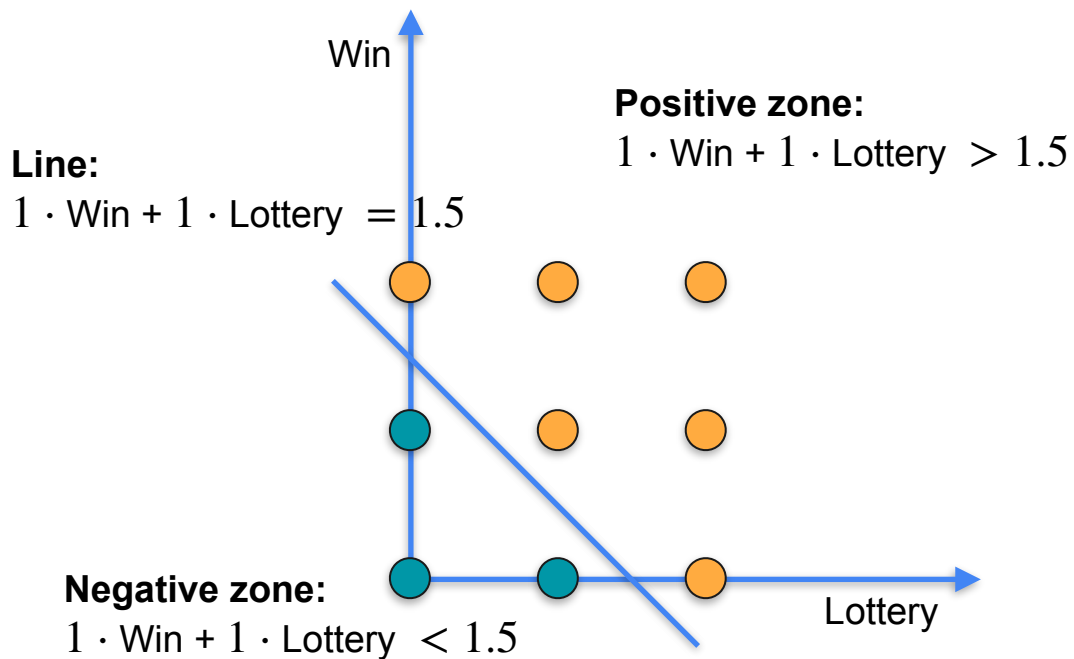
Line:

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$$



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Graphical natural language processing

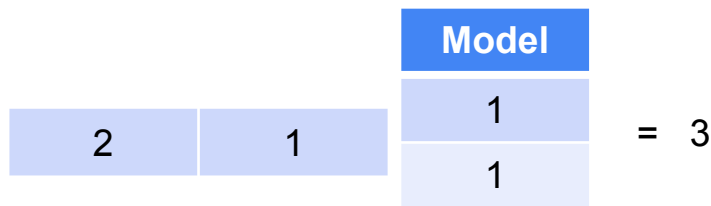
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: > 1.5?

Graphical natural language processing

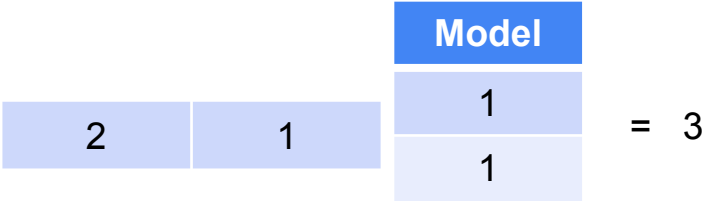
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: > 1.5?



Spam

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Dot product between vectors

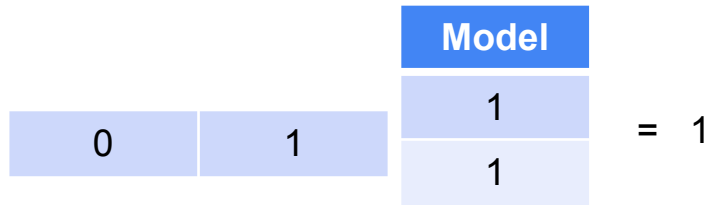
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1	Model
		1
		1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$$

Check: > 1.5?



Not spam

Matrix multiplication

Spam	Lottery	Win	
Yes	1	1	
Yes	2	1	
No	0	0	
Yes	0	2	Model
No	0	1	1
No	1	0	
Yes	2	2	
Yes	2	0	
Yes	1	2	

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?



Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

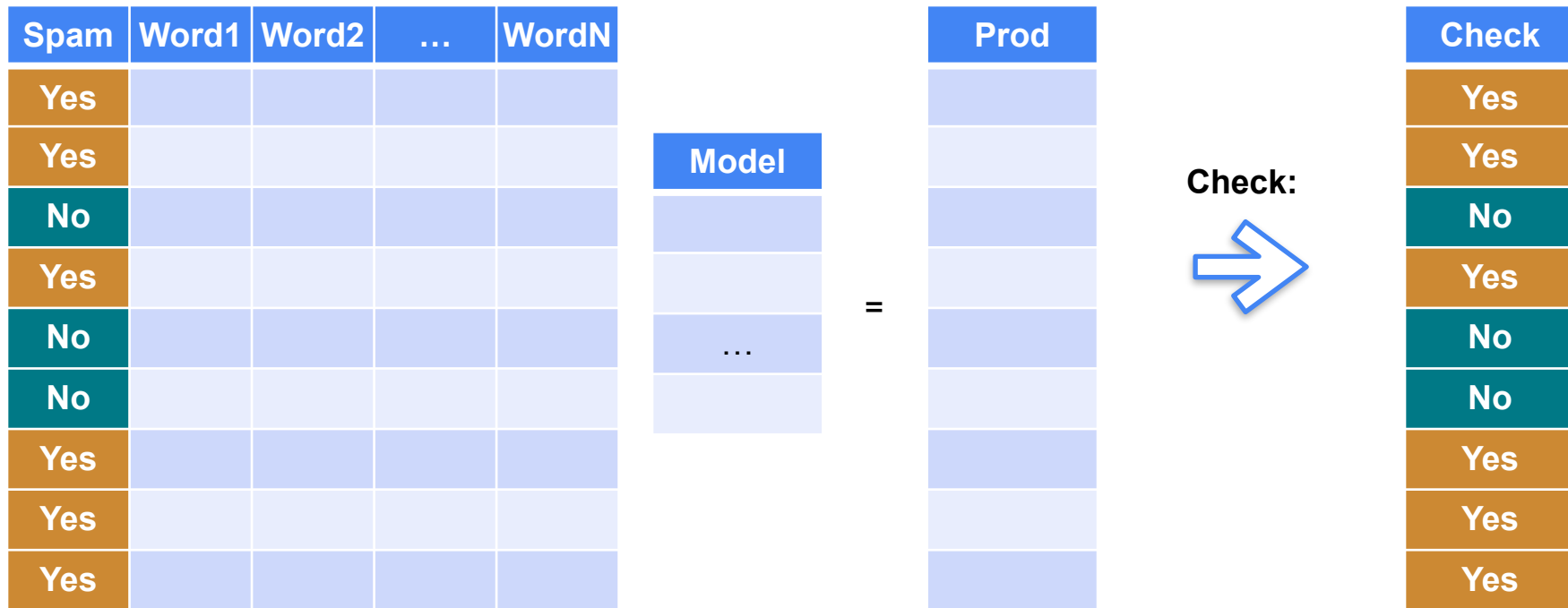
Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?



Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Perceptrons



Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1
-1.5

Bias

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$
$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Model
1
1
-1.5

Check: > 0?

Bias

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

The AND operator

AND	x	y	
No	0	0	
No	1	0	
No	0	1	
Yes	1	1	

Model
1
1

The AND operator

AND	x	y		Dot prod
No	0	0	Model	0
No	1	0		1
No	0	1		1
Yes	1	1		2

=

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

Check: >1.5?



The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

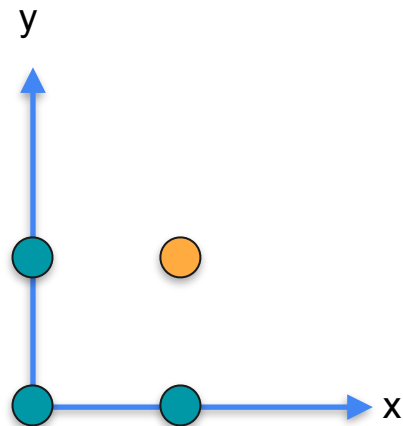
Check: >1.5?



Check
No
No
No
Yes

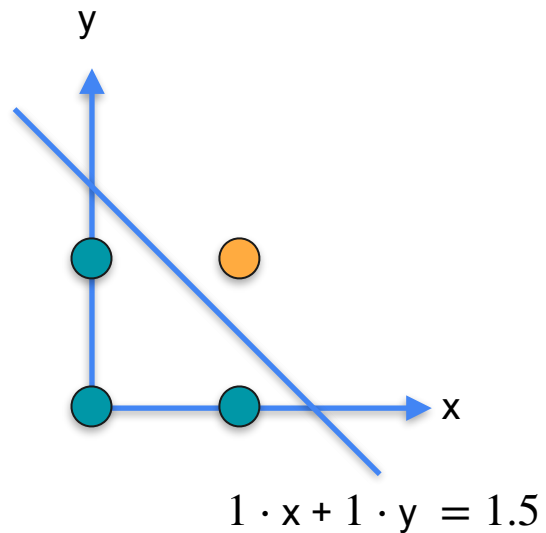
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

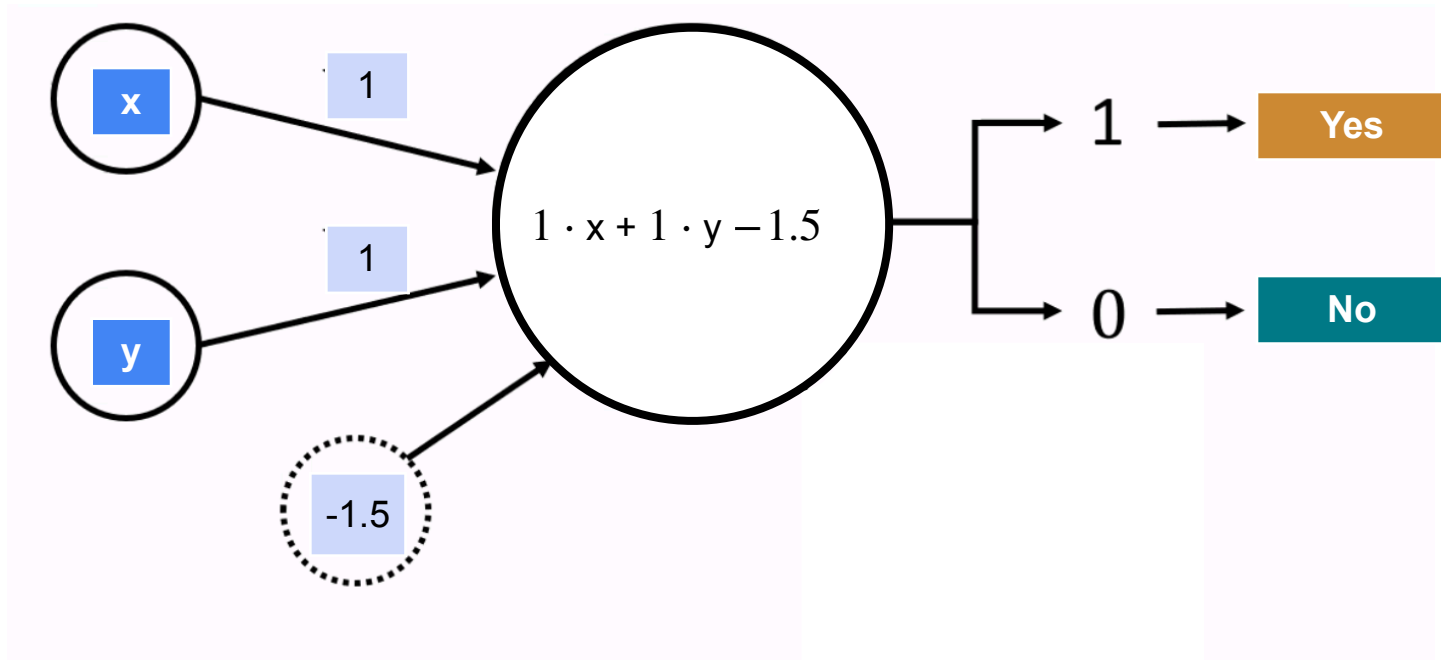


The AND operator

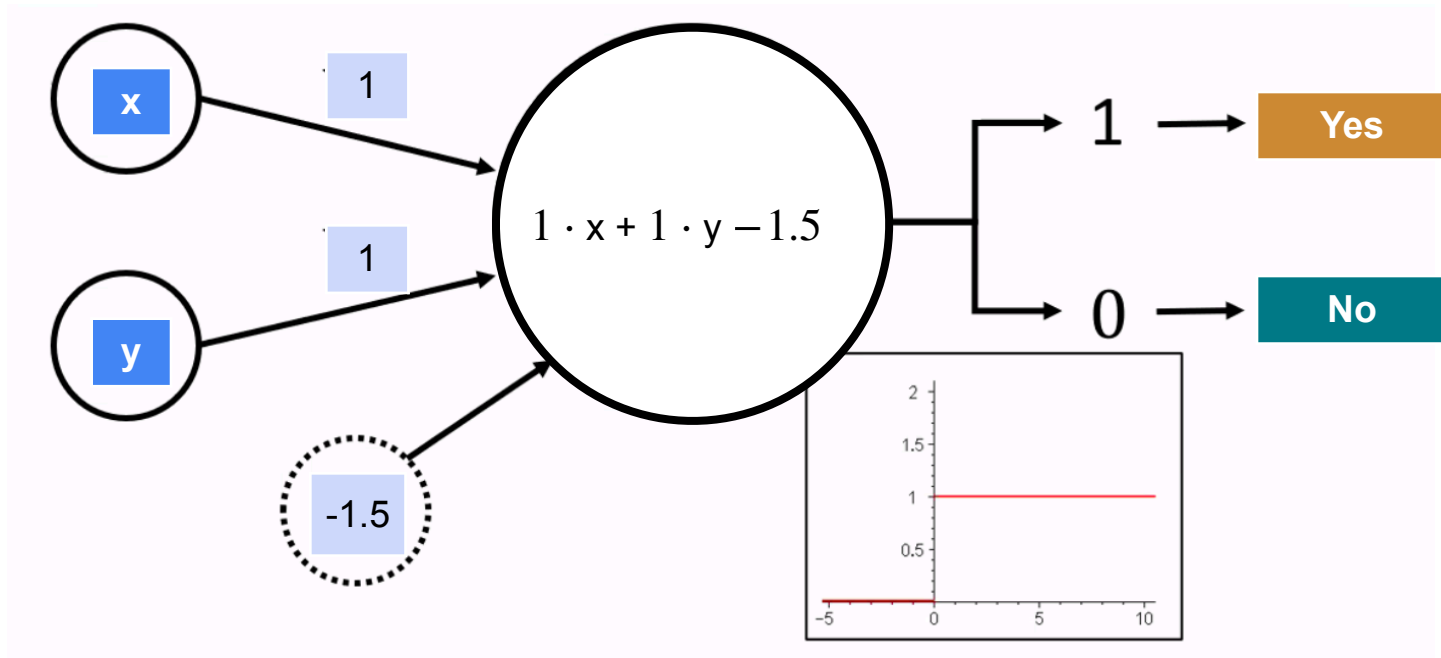
AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

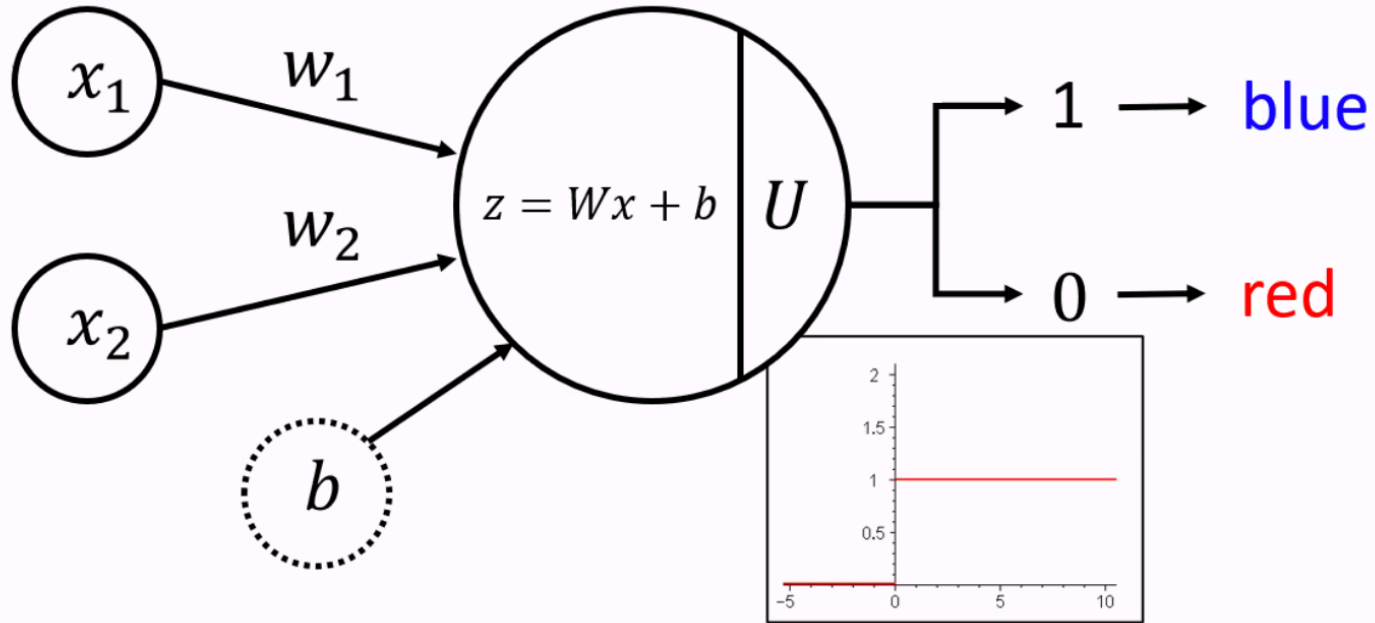


The perceptron



The perceptron







DeepLearning.AI

Vectors and Linear Transformations

Conclusion