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Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations

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Vectors and Linear Transformations

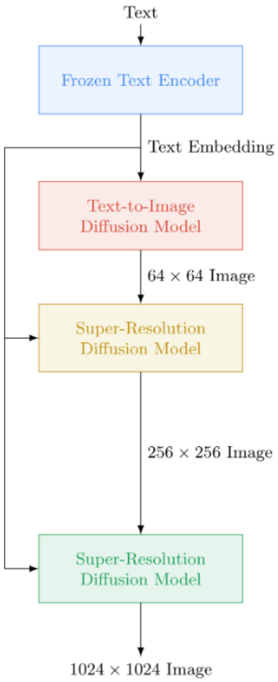
Machine Learning motivation

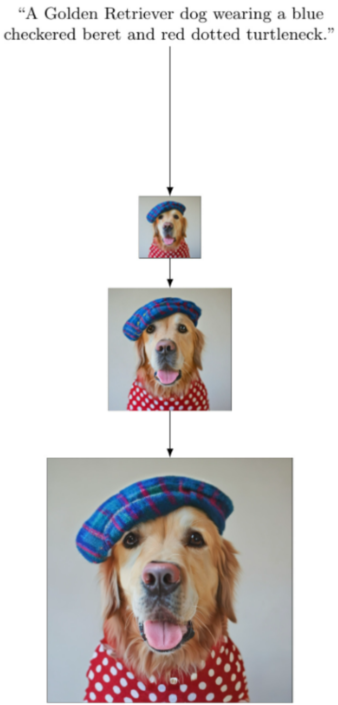
Neural Networks - AI generated images

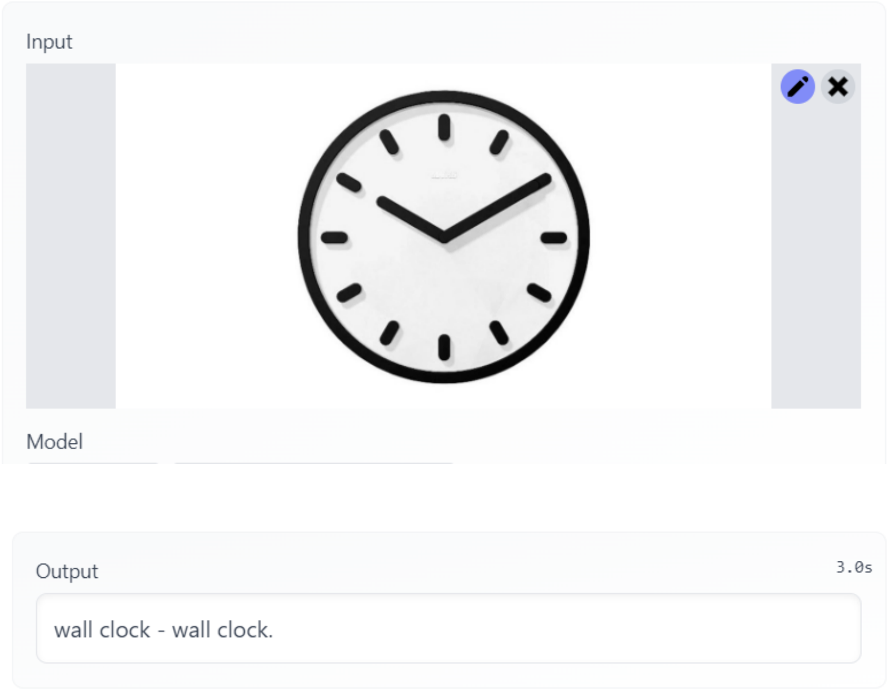
AI-generated human faces.

- Generative learning: Generating realistic looking images.

Text-to-image and image-to-text generation







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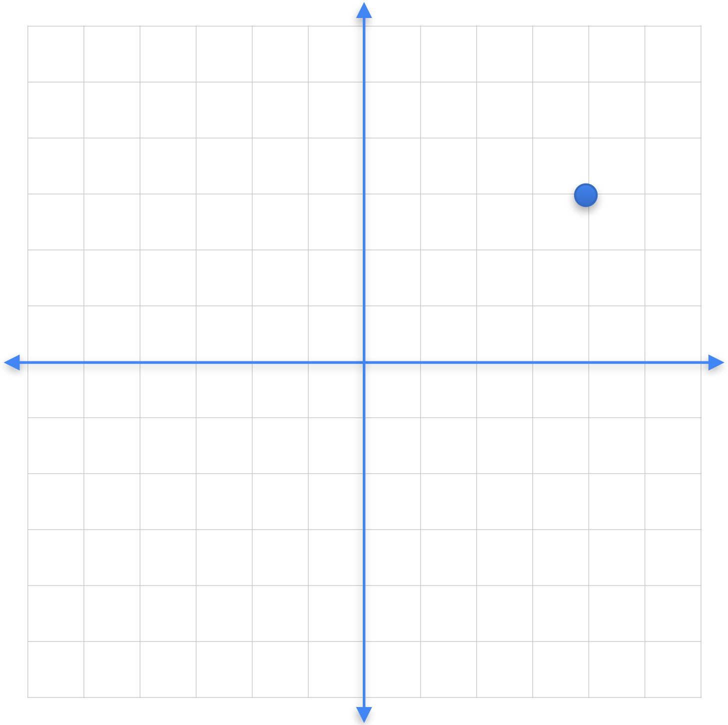
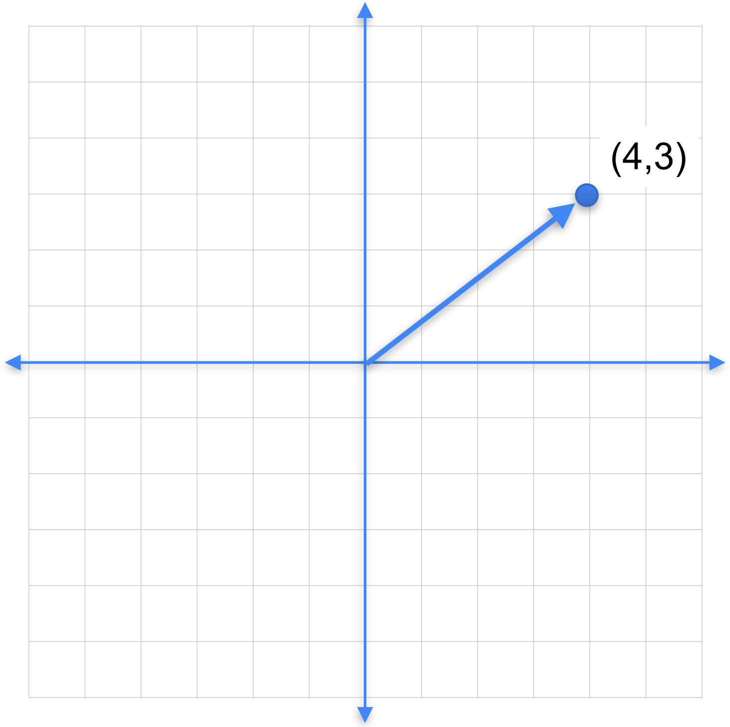
Vectors and Linear Transformations

Vectors and their properties

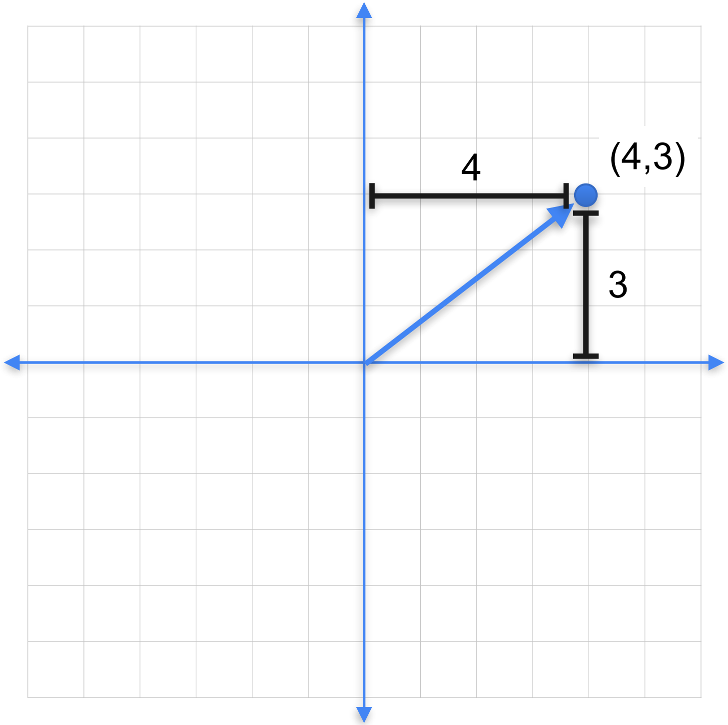
Vectors



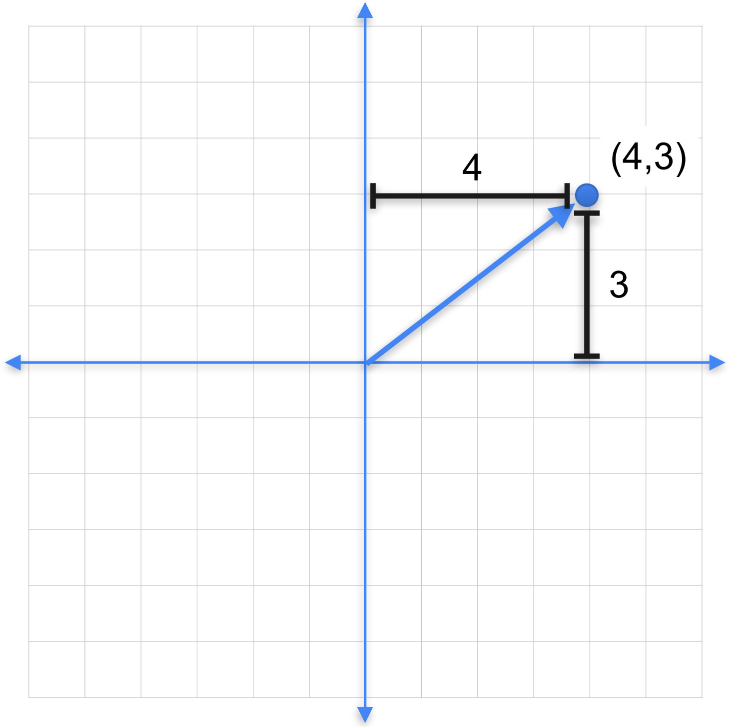
Vectors

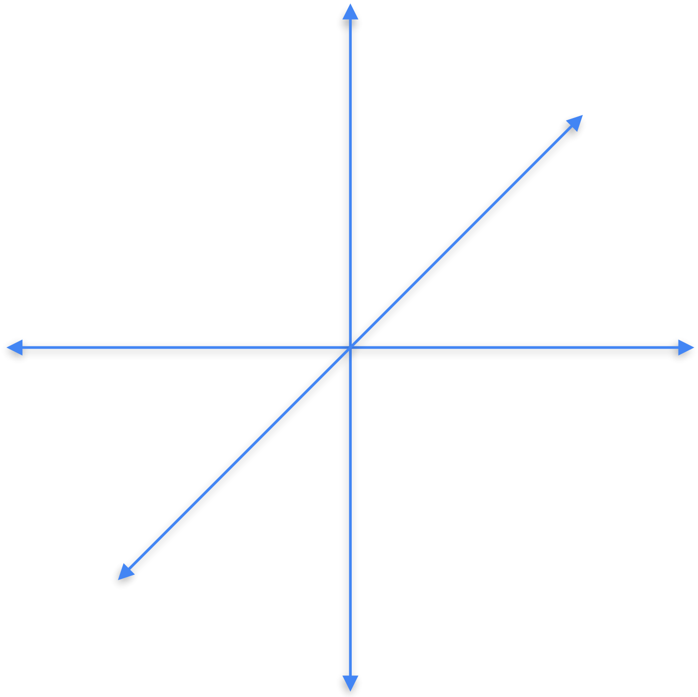


Vectors

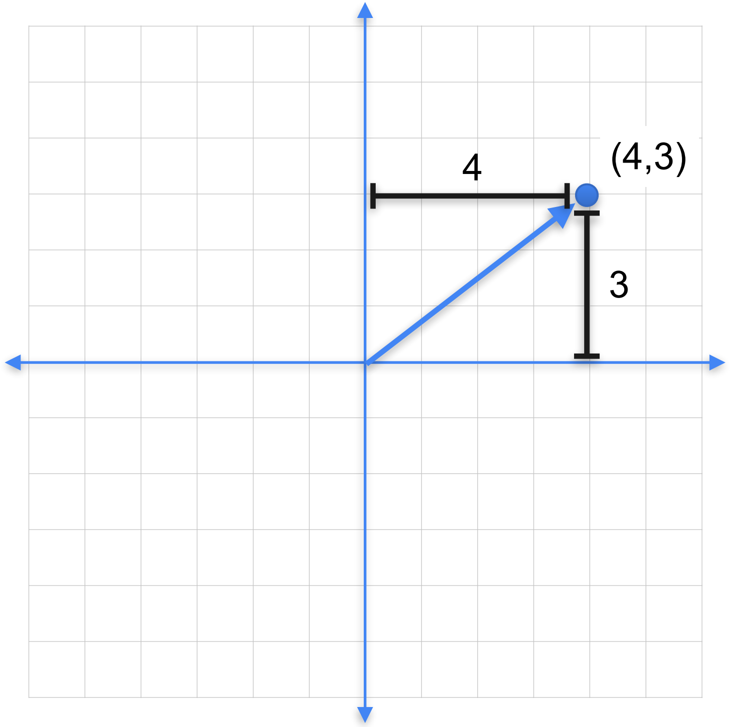


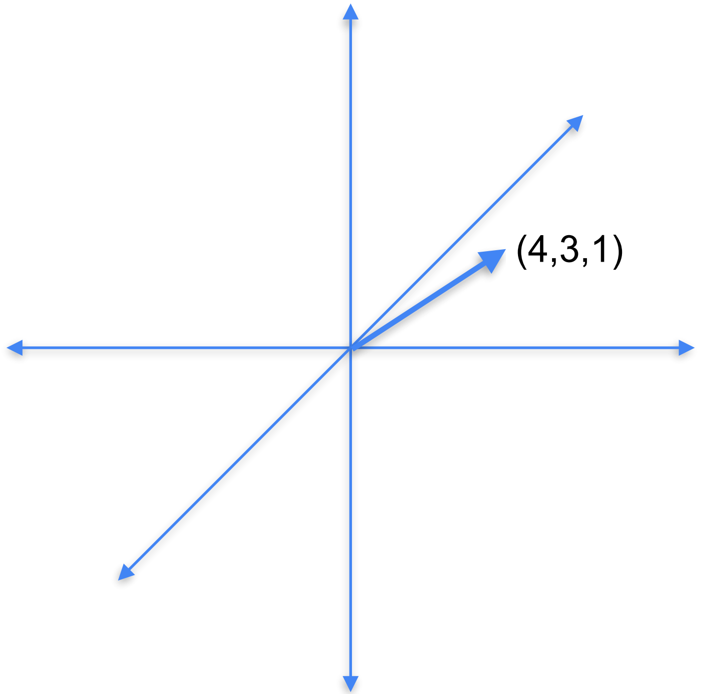
Vectors



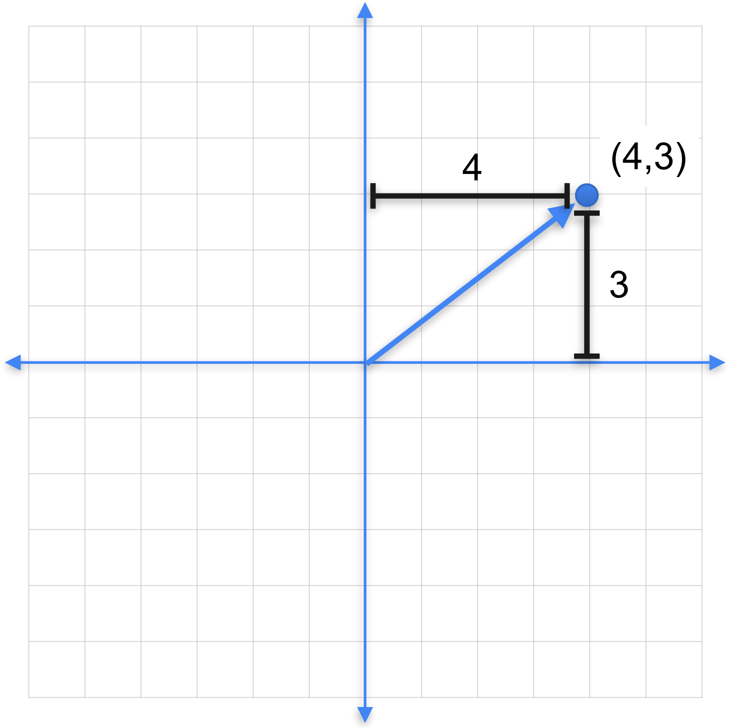


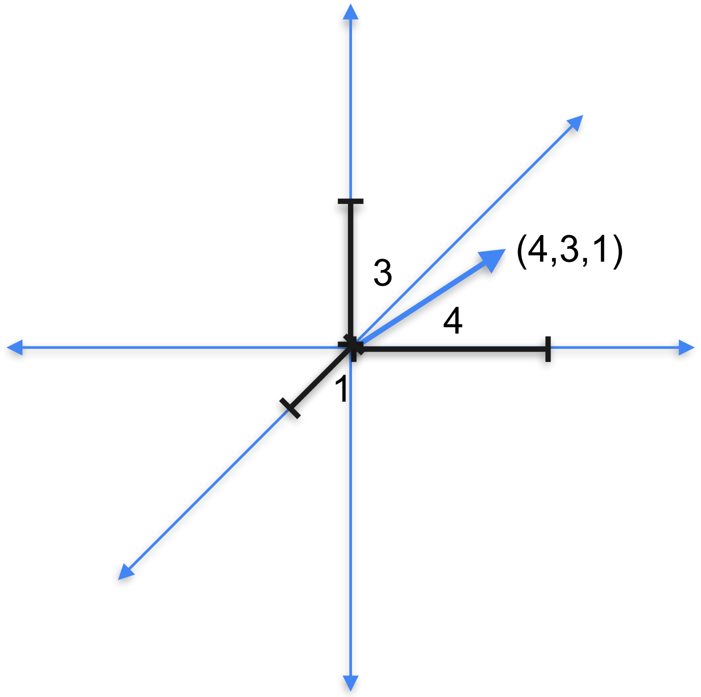
Vectors





Vectors





How to get from point A to point B?

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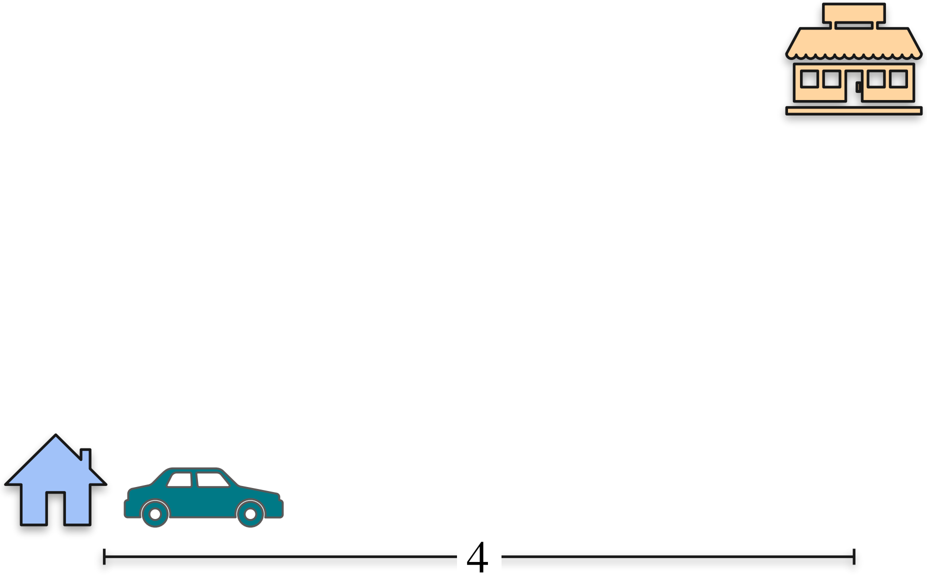
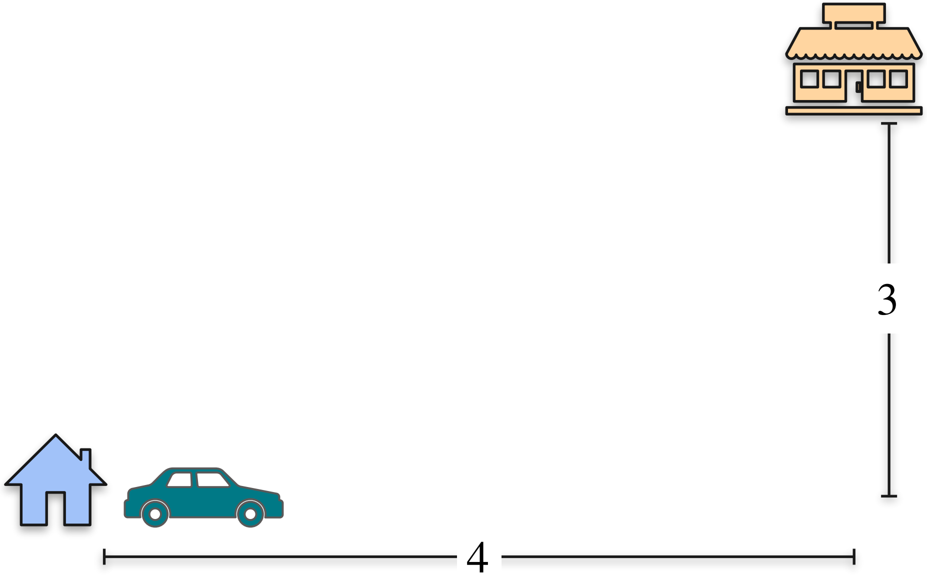
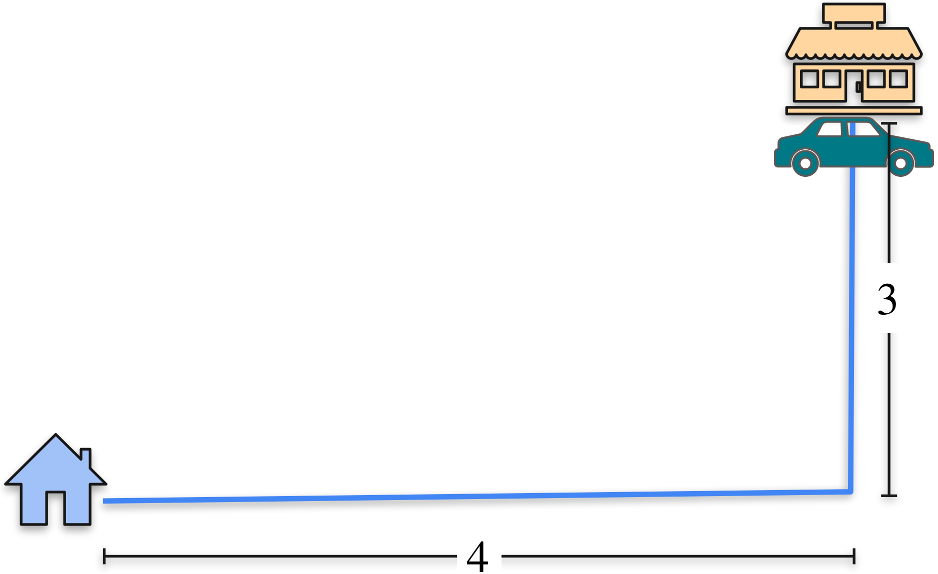
How to get from point A to point B?



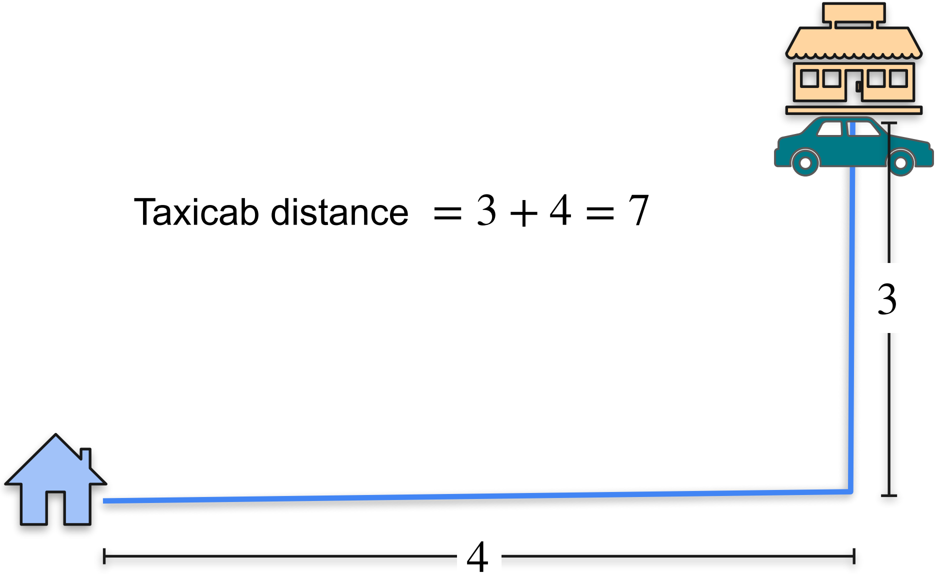
How to get from point A to point B?



How to get from point A to point B?

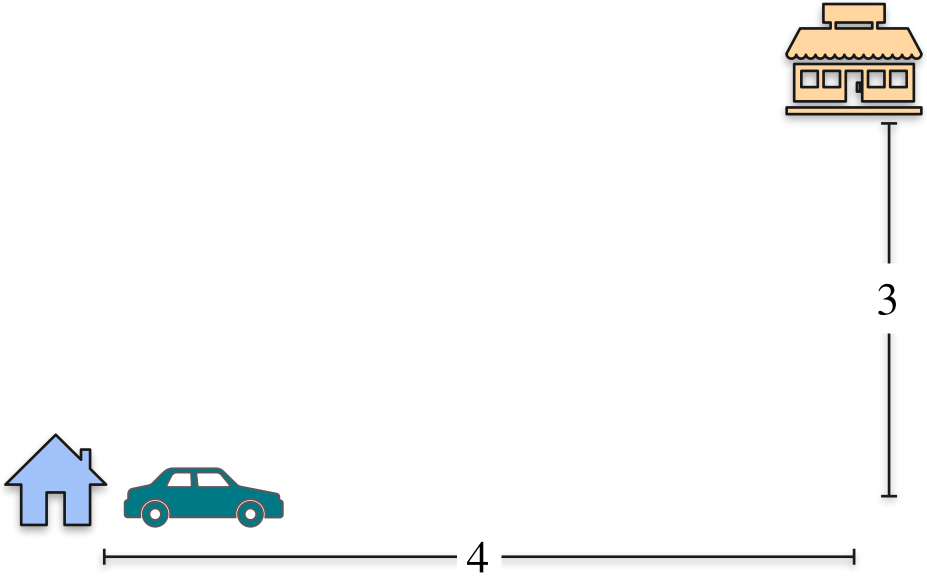


How to get from point A to point B?

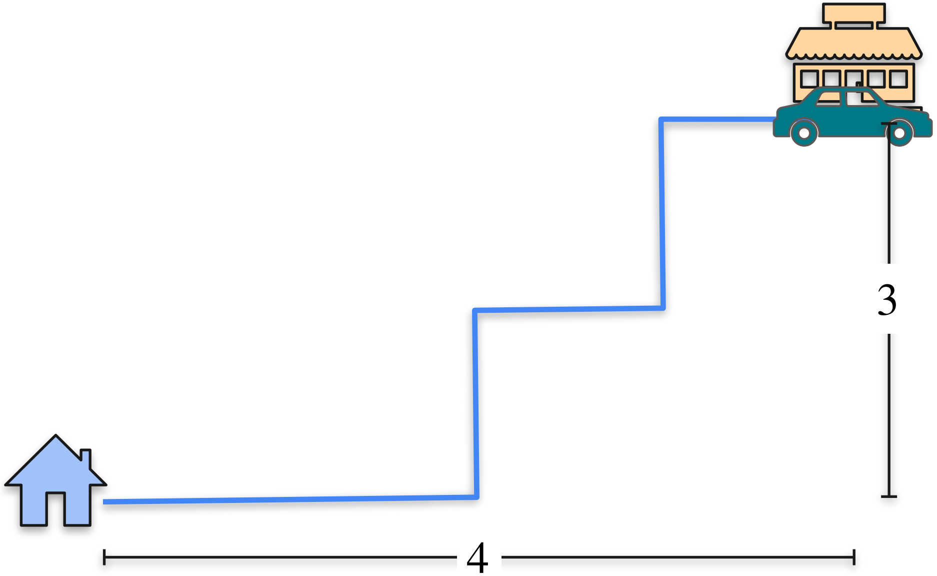


Taxicab distance $= 3 + 4 = 7$

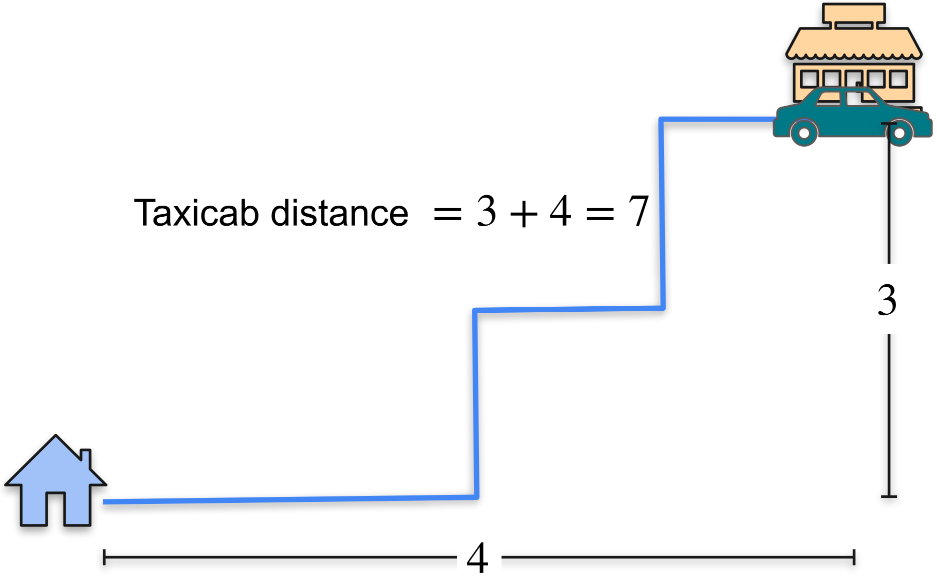
How to get from point A to point B?



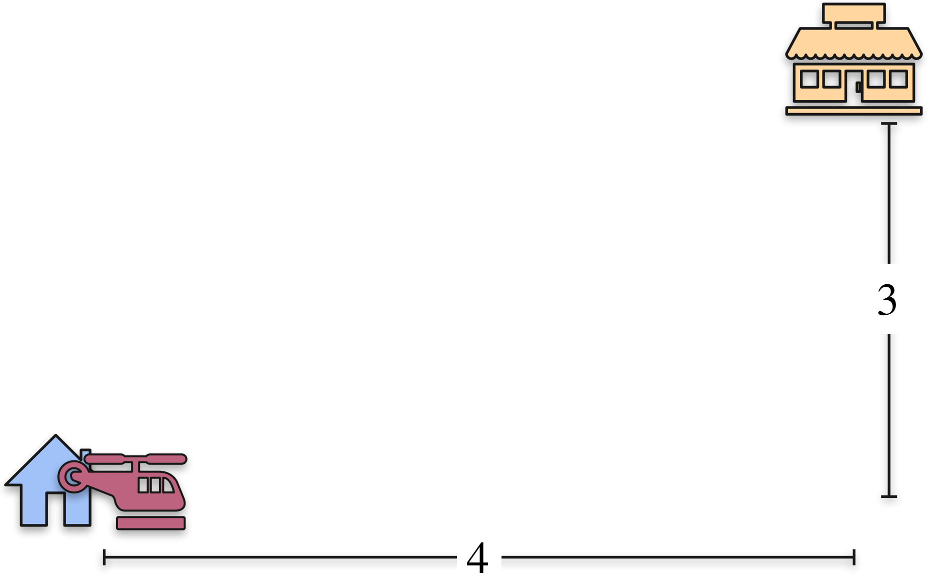
How to get from point A to point B?



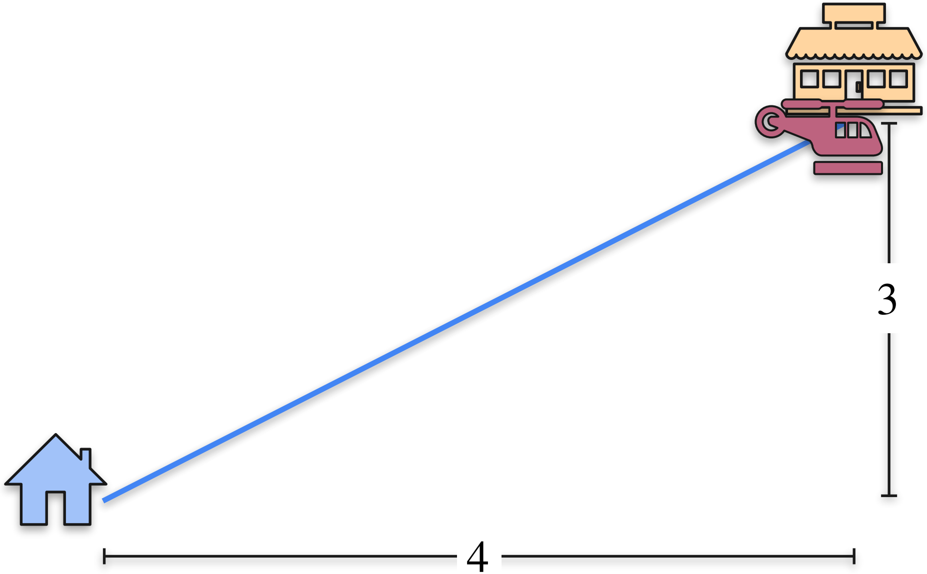
How to get from point A to point B?



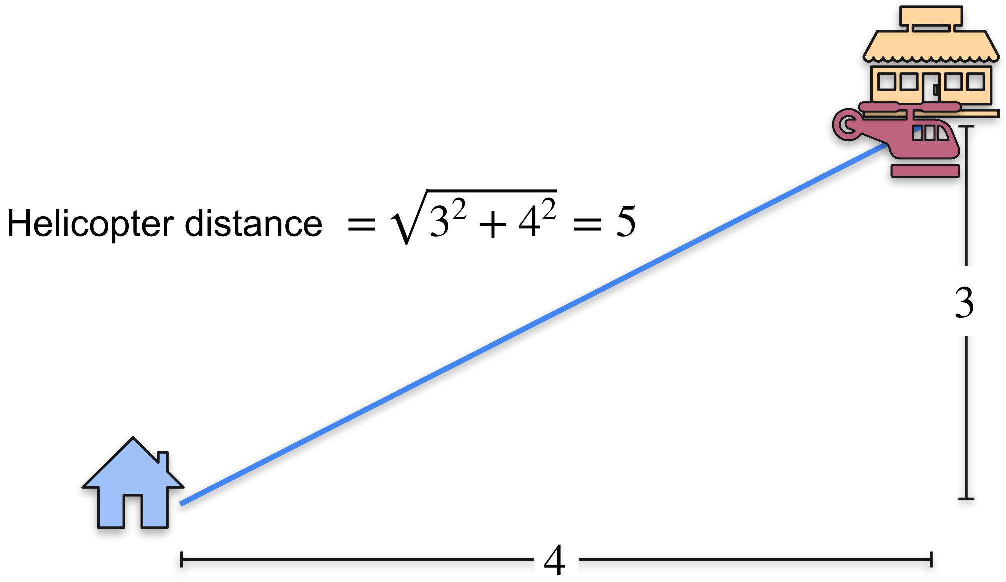
How to get from point A to point B?



How to get from point A to point B?

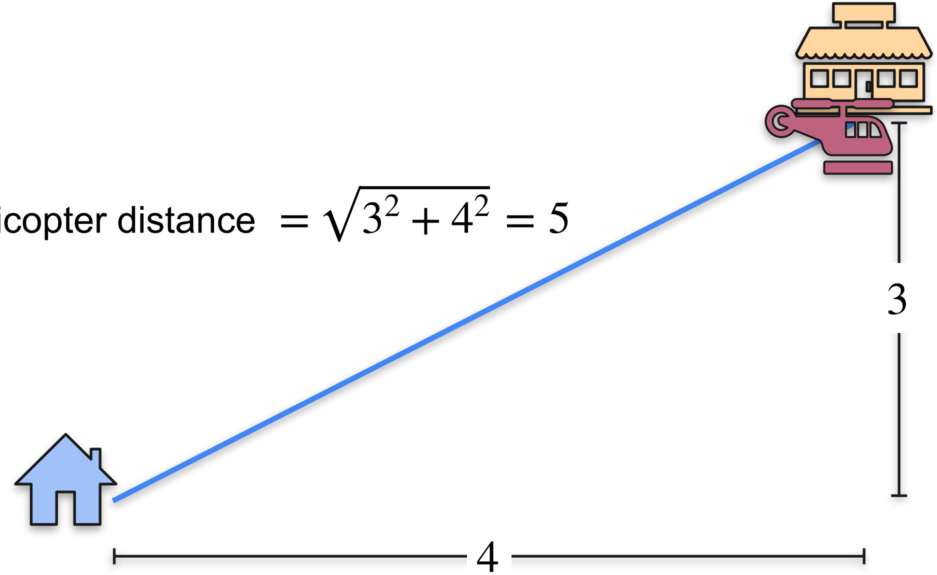


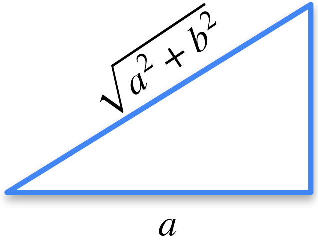
How to get from point A to point B?



How to get from point A to point B?

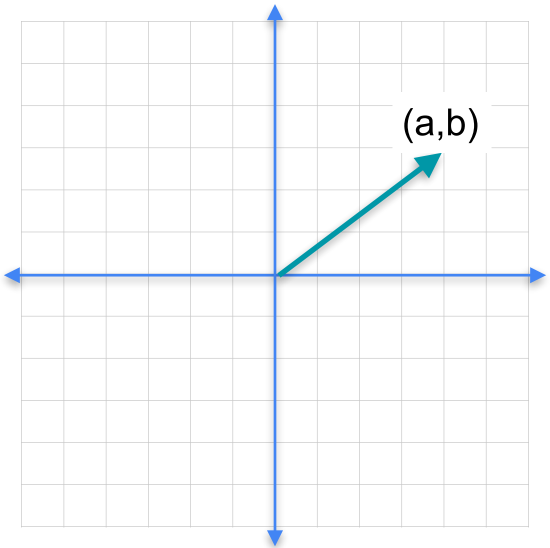
Helicopter distance $= \sqrt{3^2 + 4^2} = 5$



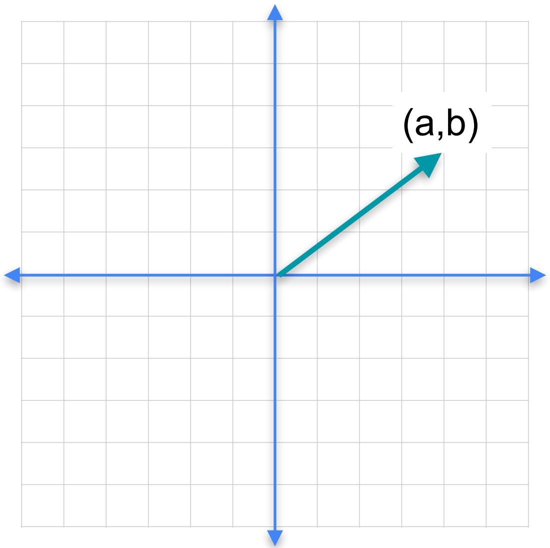


Pythagorean Theorem

Norms

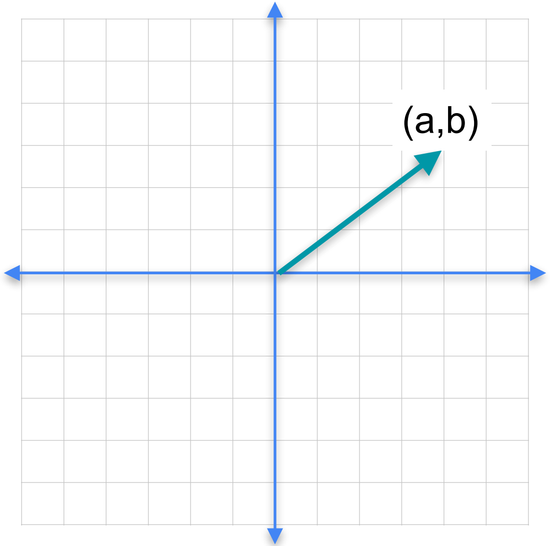


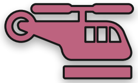
Norms



L1-norm $= |(a, b)|_1 = |a| + |b|$

Norms

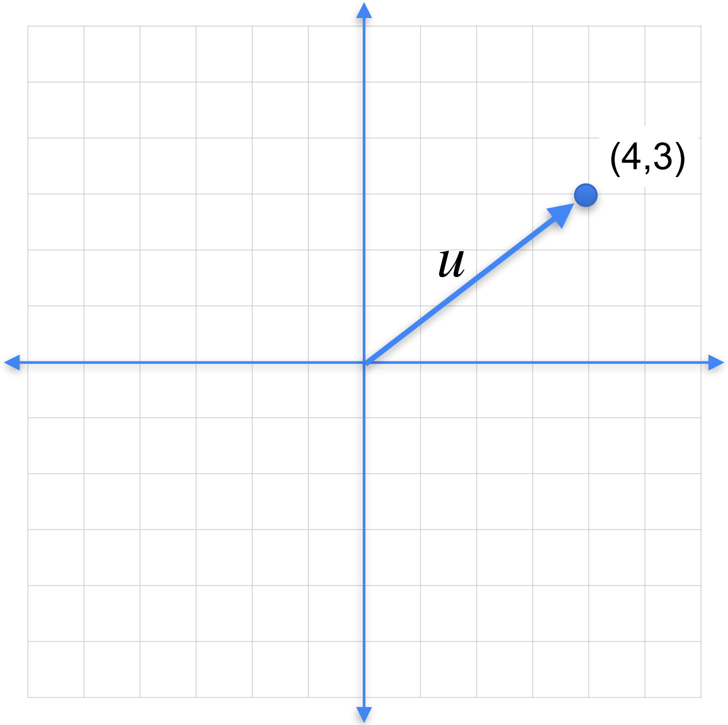




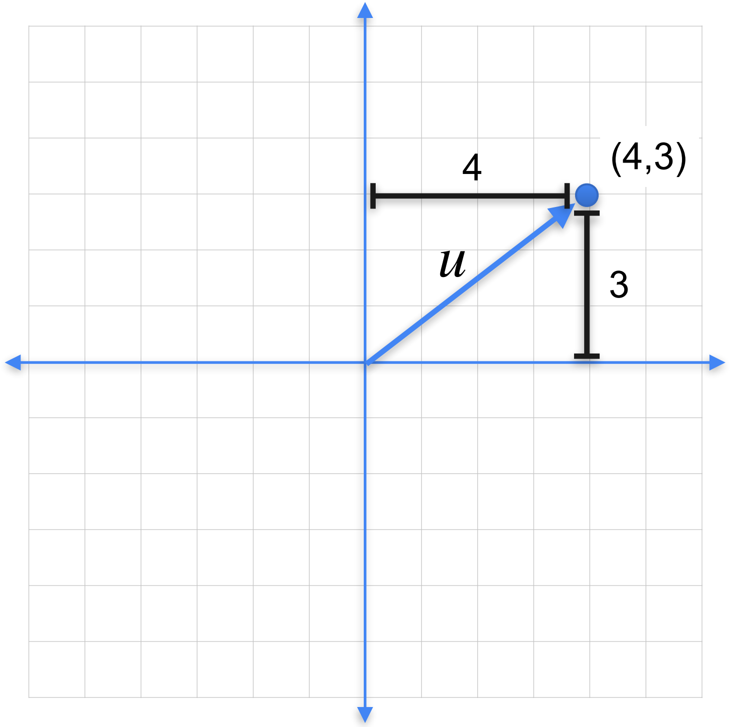
L1-norm $= |(a, b)|_1 = |a| + |b|$

L2-norm $= |(a, b)|_2 = \sqrt{a^2 + b^2}$

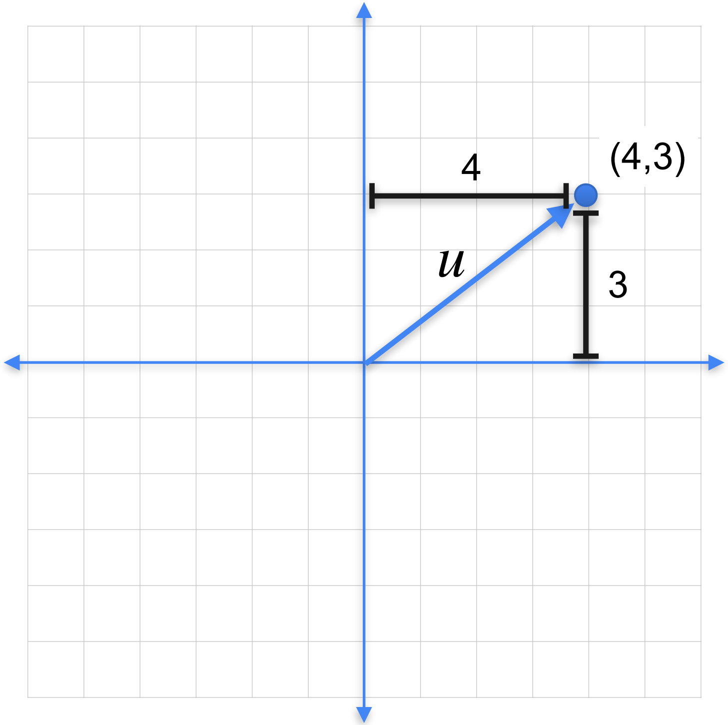
Norm of a vector



Norm of a vector

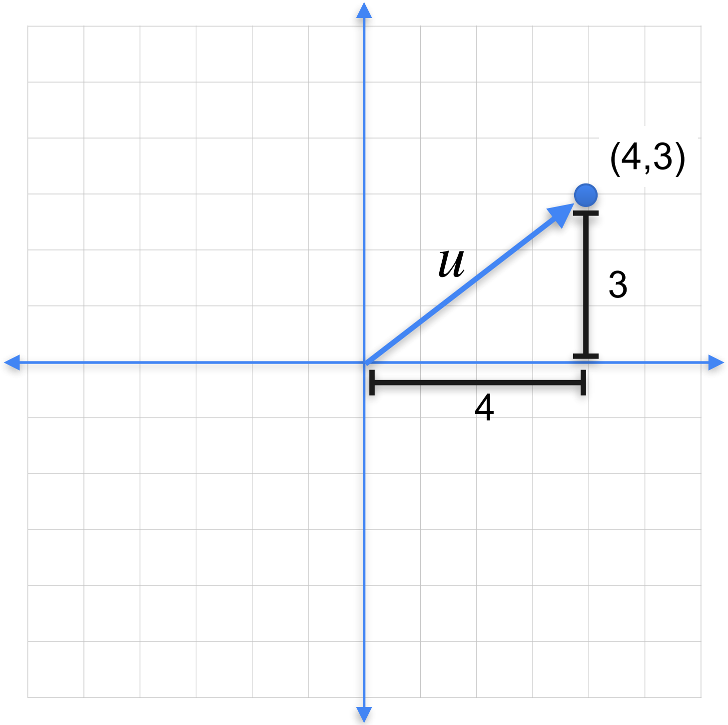


Norm of a vector

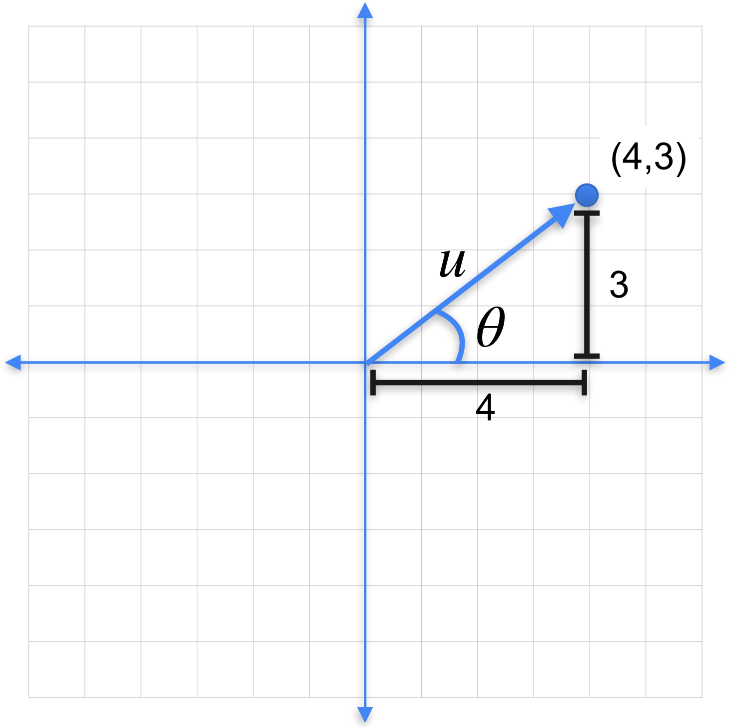


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

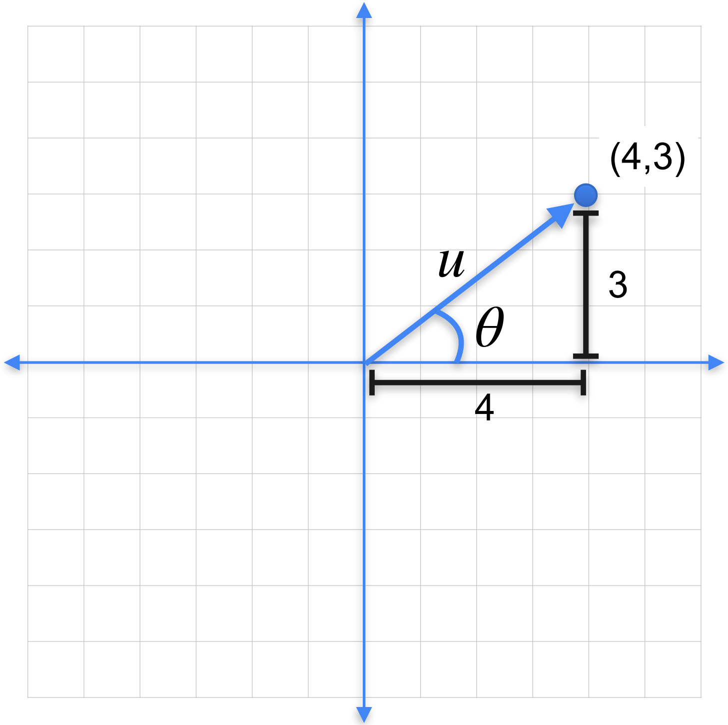
Direction of a vector



Direction of a vector

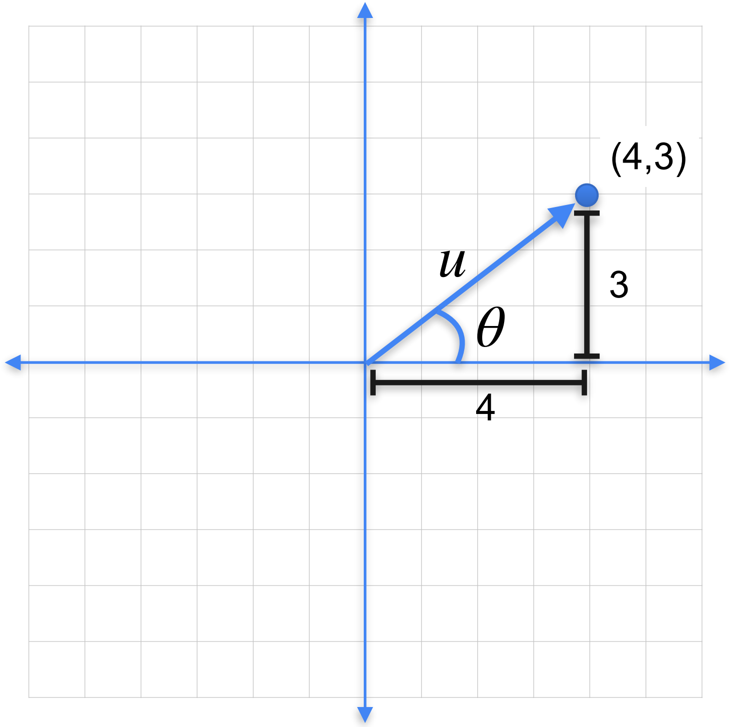


Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

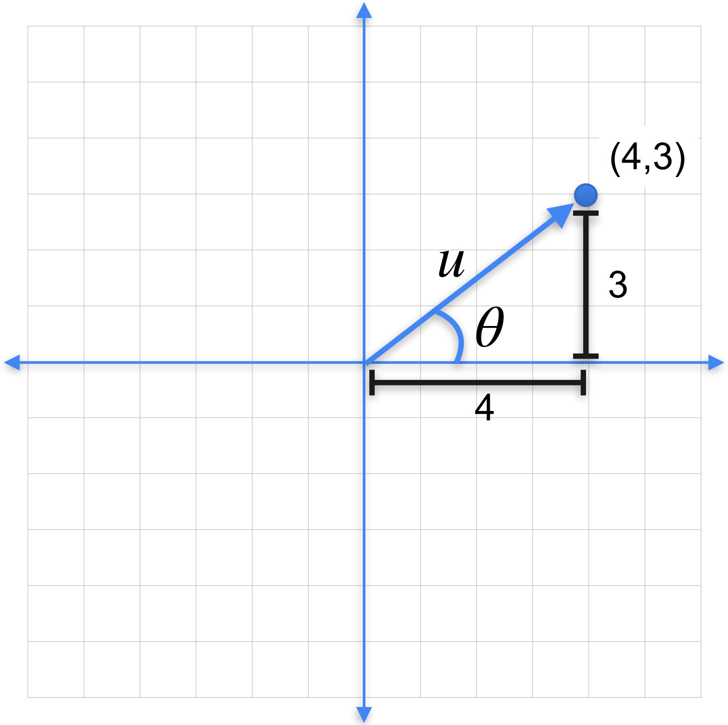
Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64$$

Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 \; = 36.87°$$

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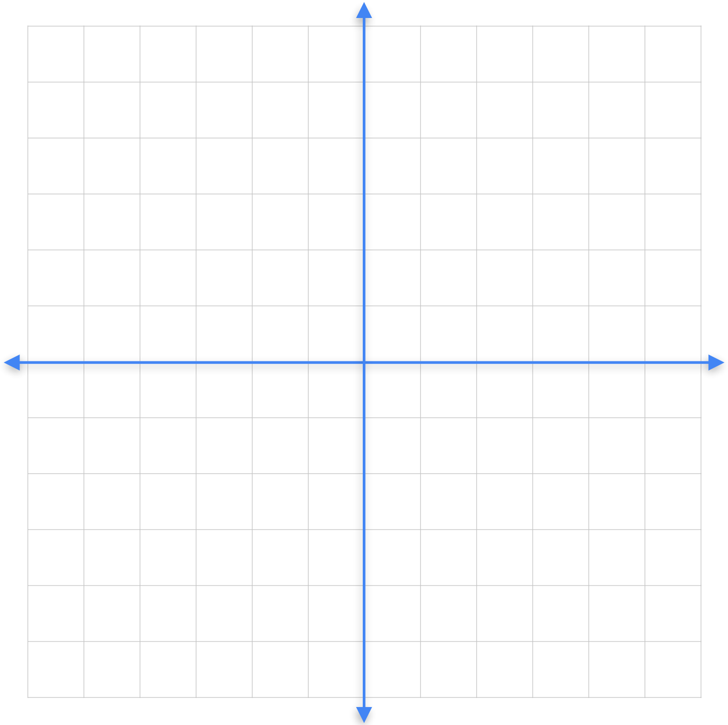
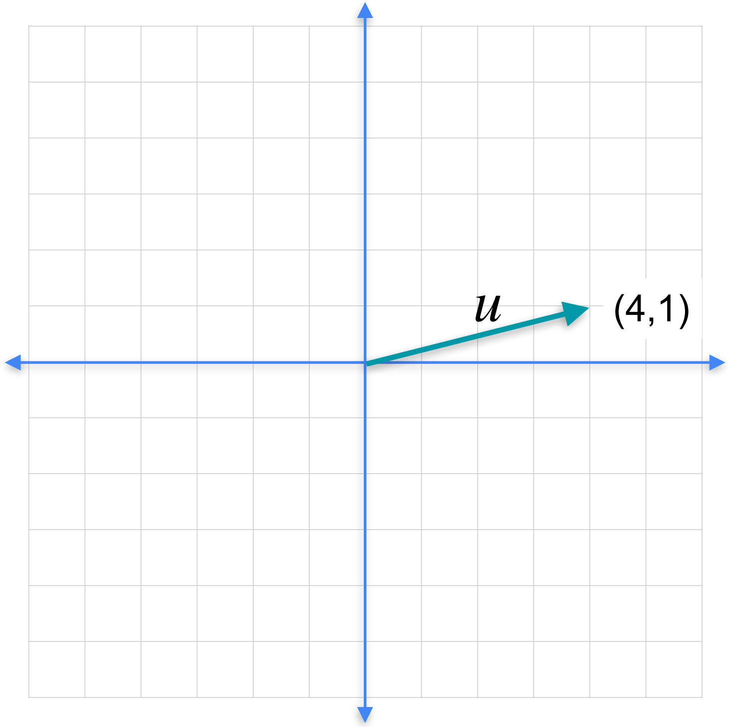
Vectors and Linear Transformations

Sum and difference of vectors

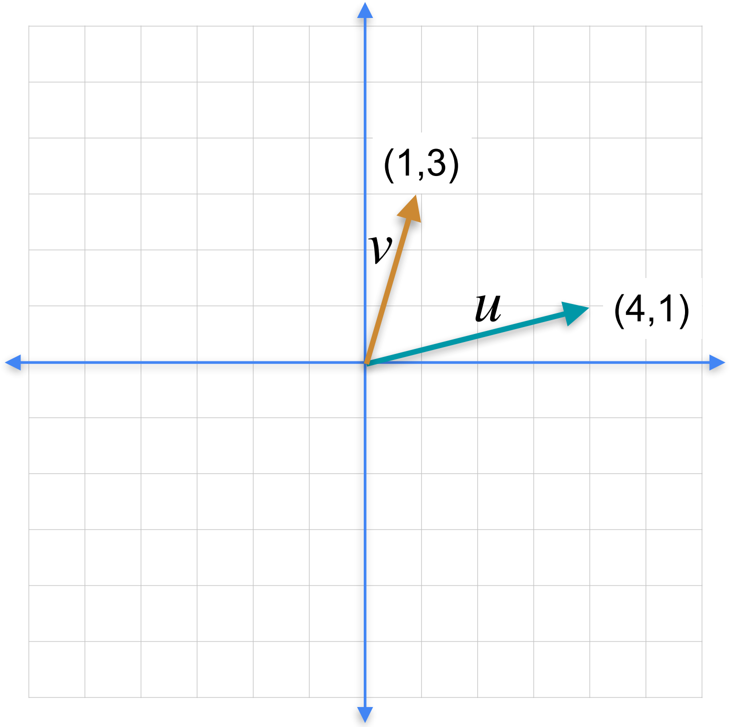
Sum of vectors



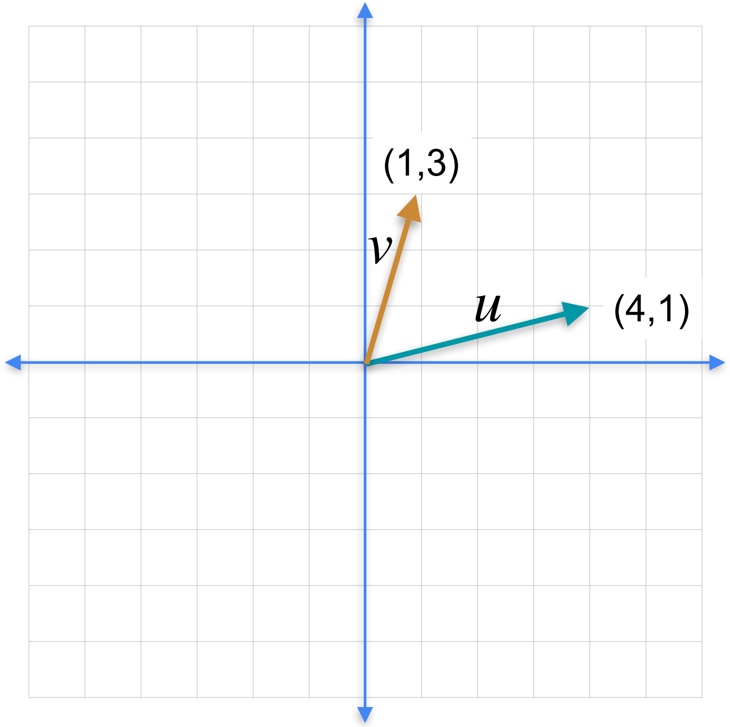
Sum of vectors



Sum of vectors

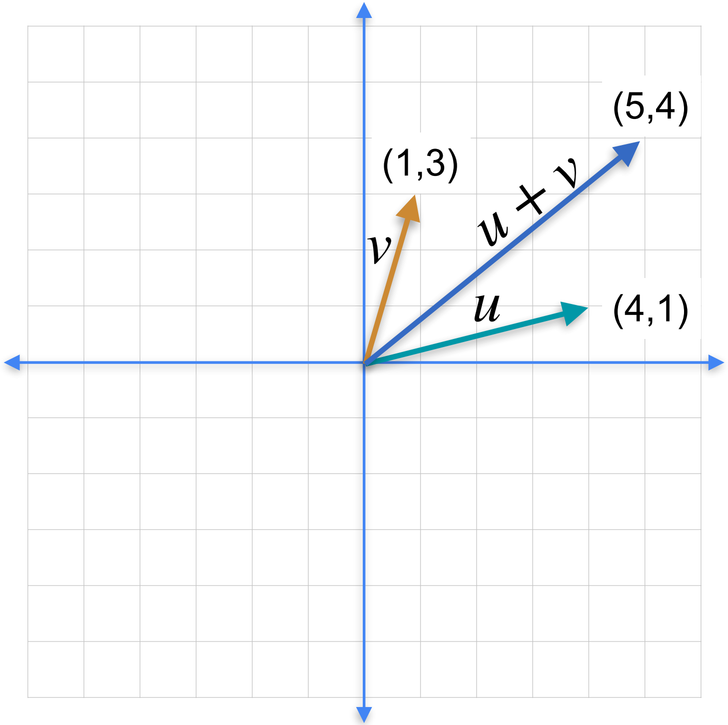


Sum of vectors



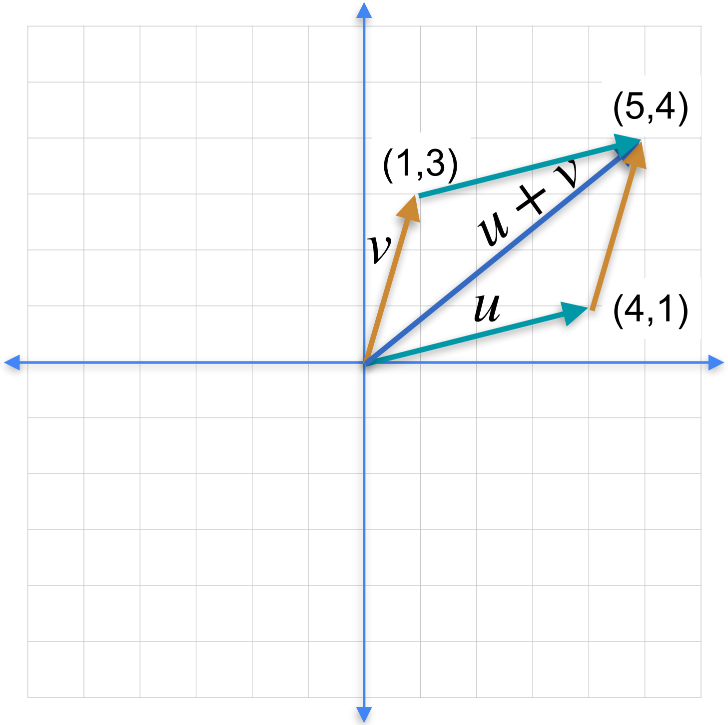
$$u + v = (4 + 1, 1 + 3) = (5,4)$$

Sum of vectors



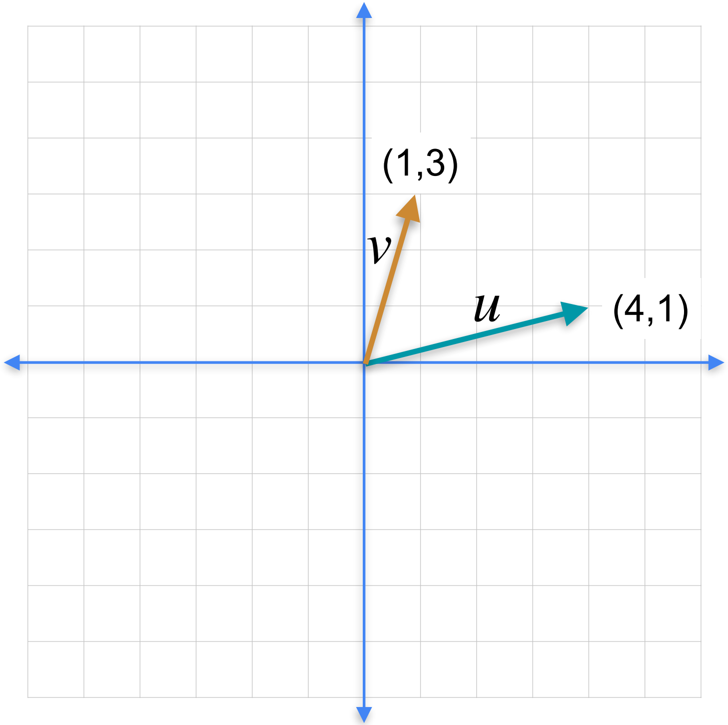
$$u + v = (4 + 1, 1 + 3) = (5,4)$$

Sum of vectors

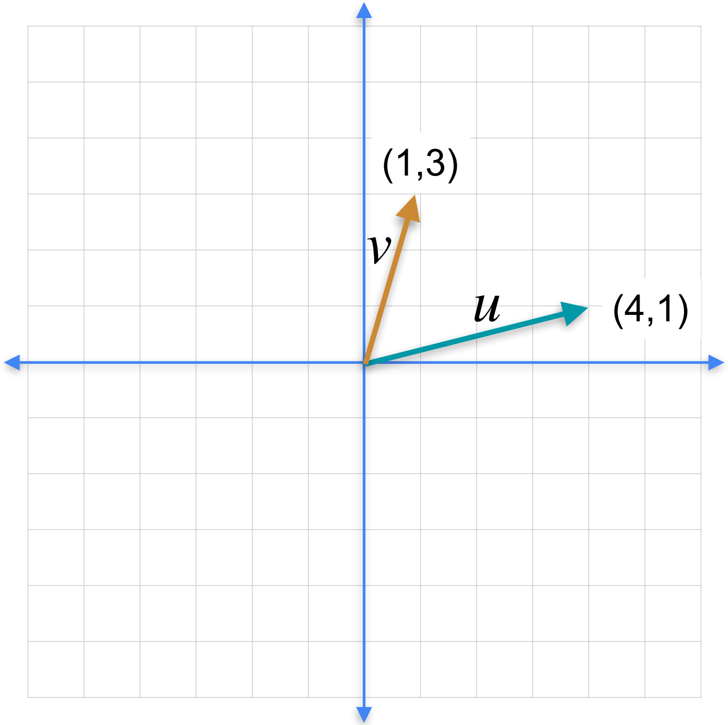


$$u + v = (4 + 1, 1 + 3) = (5,4)$$

Difference of vectors

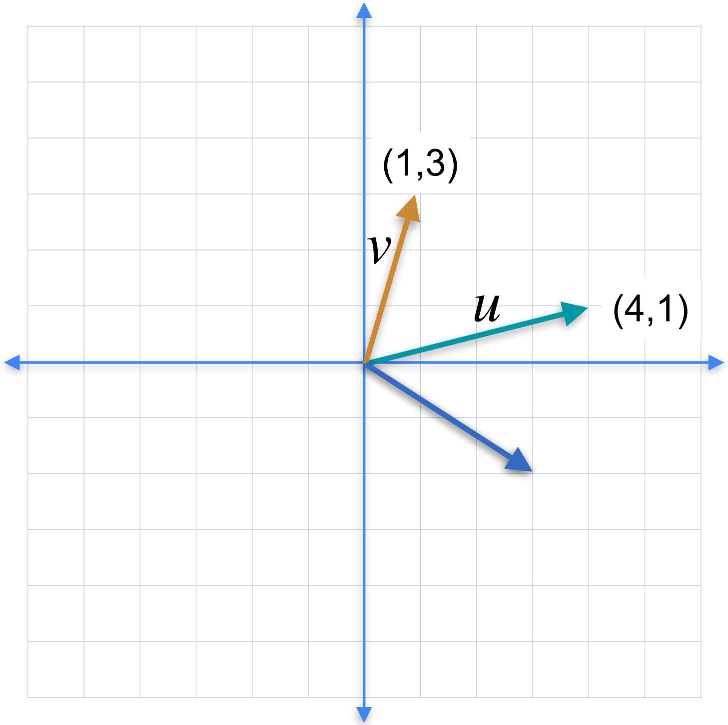


Difference of vectors



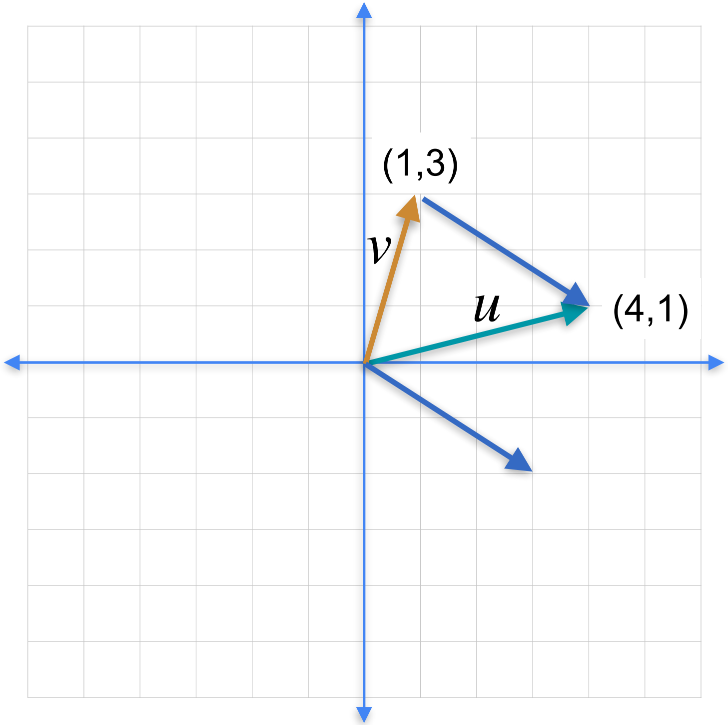
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



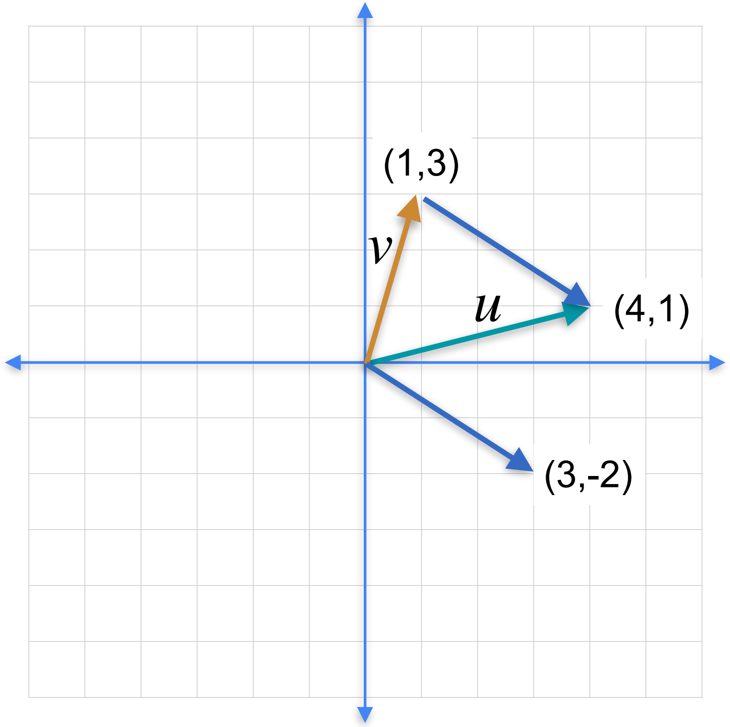
$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

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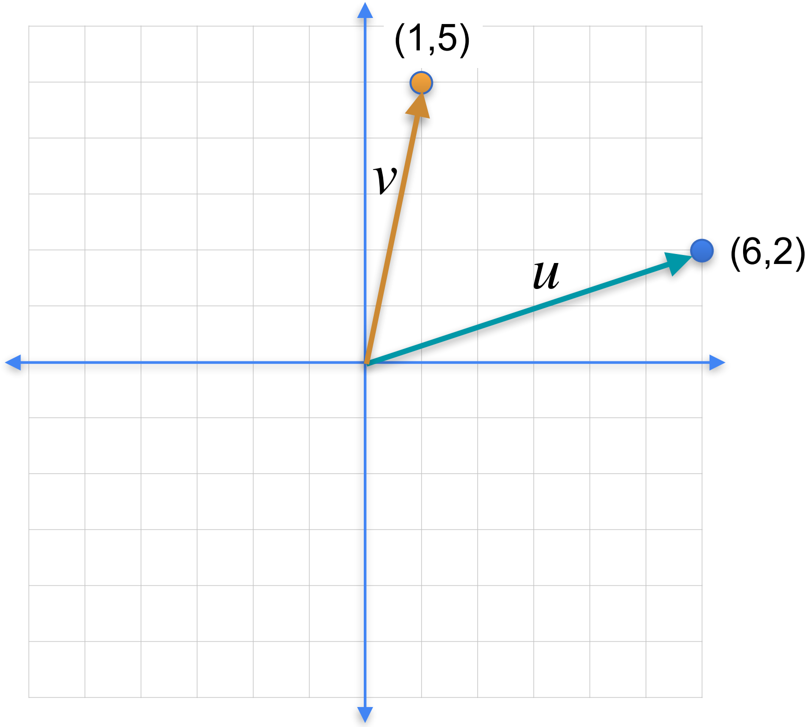
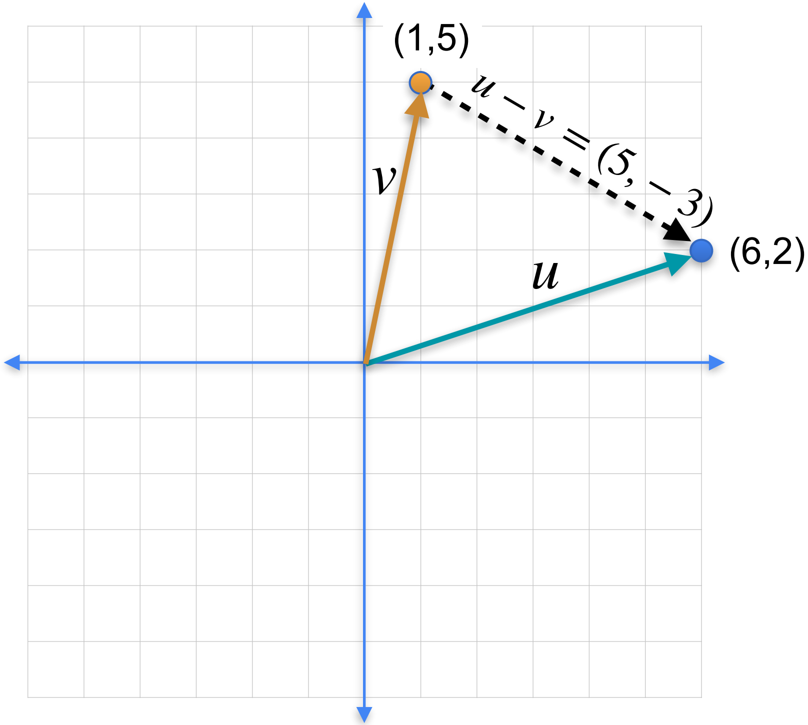
Vectors and Linear Transformations

Distance between vectors

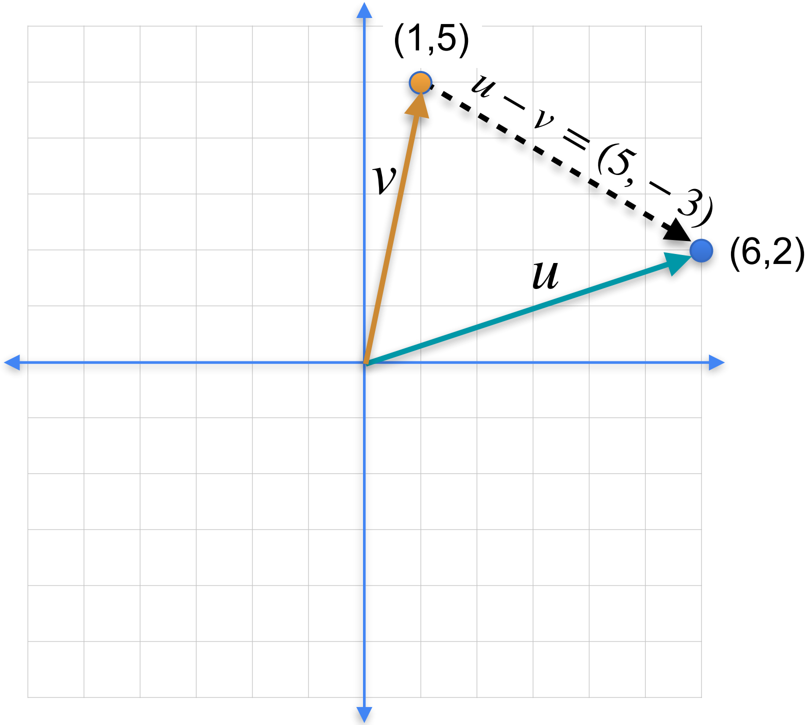
Distances



Distances



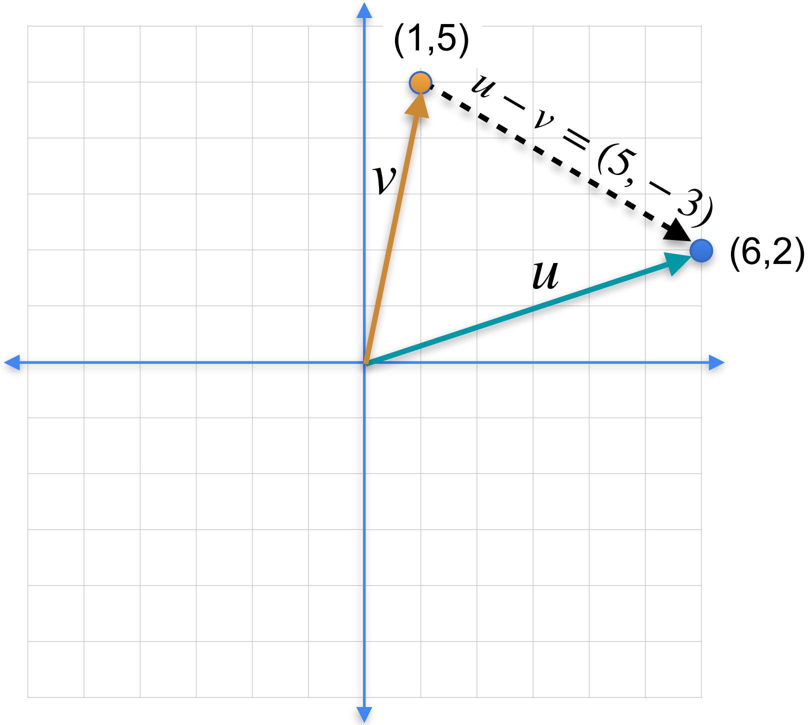
Distances



L1-distance

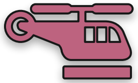
$$|u - v|_1 = |5| + |-3| = 8$$

Distances



L1-distance

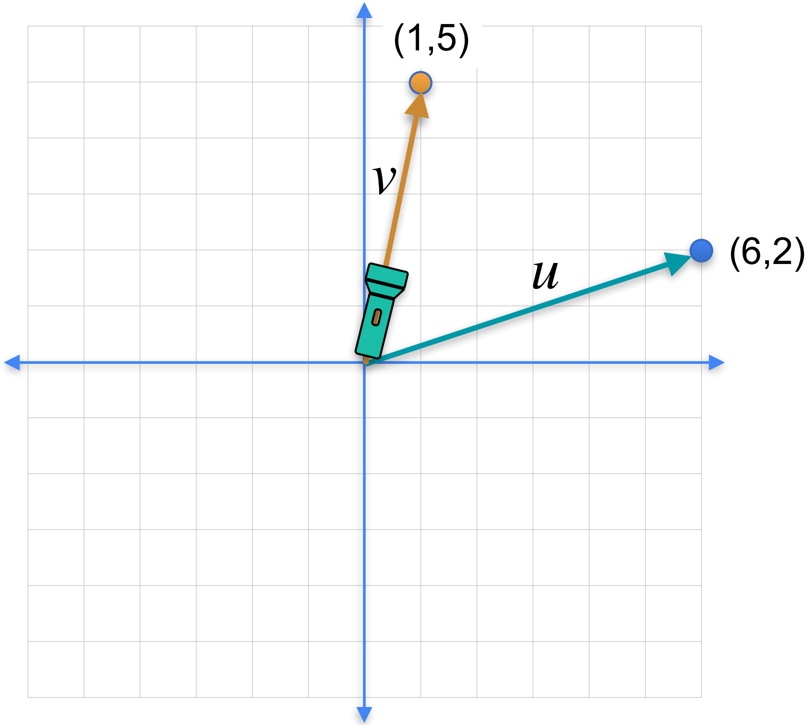
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

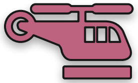
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

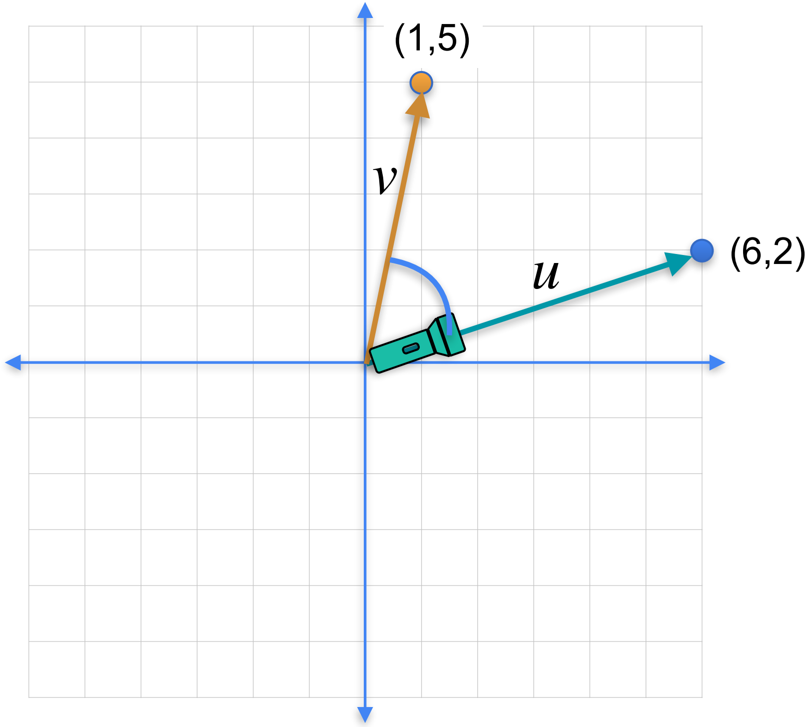
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

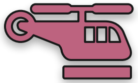
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



L1-distance

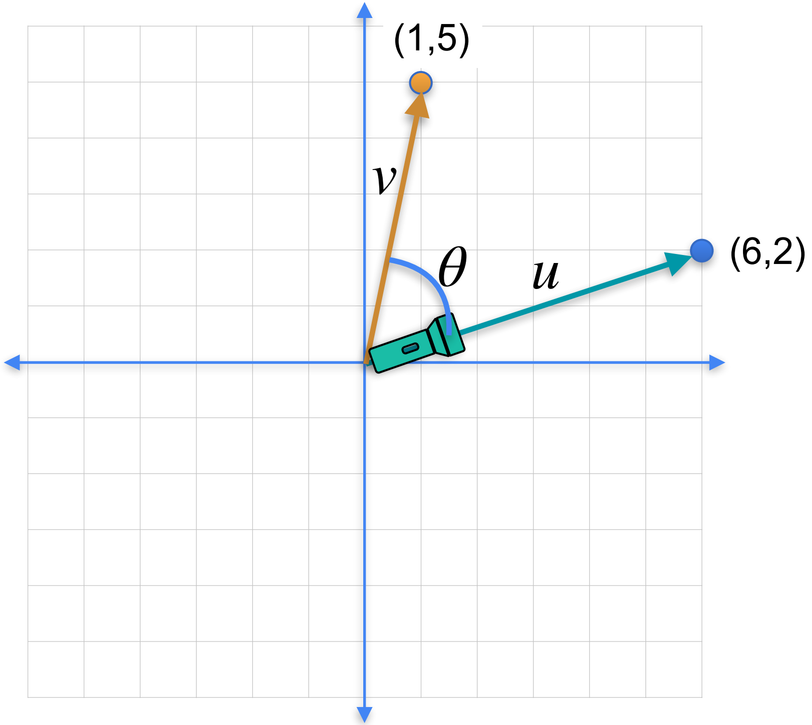
$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



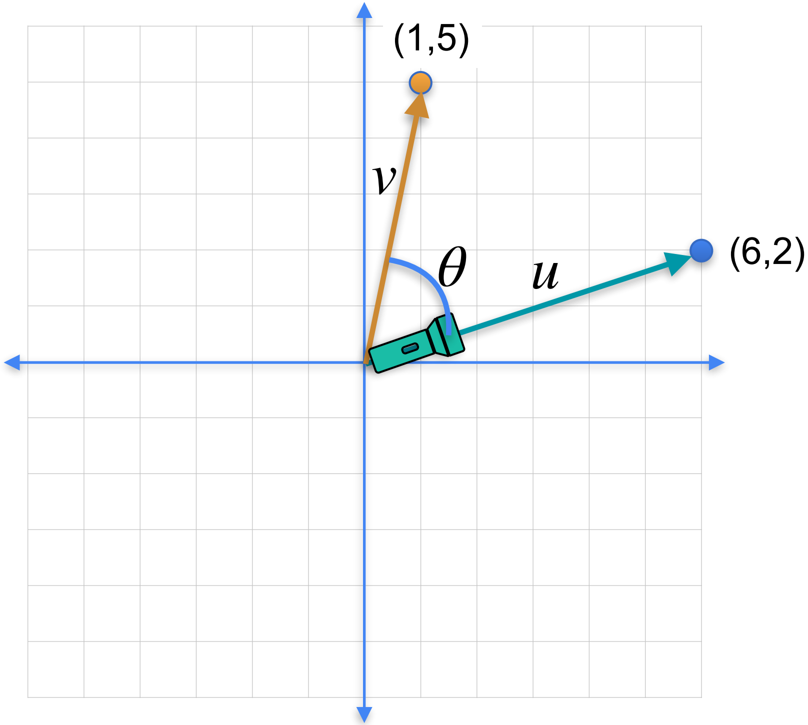
L1-distance

$$|u - v|_1 = |5| + |-3| = 8$$

L2-distance

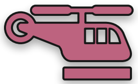
$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

Distances



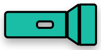
L1-distance

$$|u - v|_1 = |5| + |-3| = 8$$



L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$



Cosine distance

$$\cos(\theta)$$

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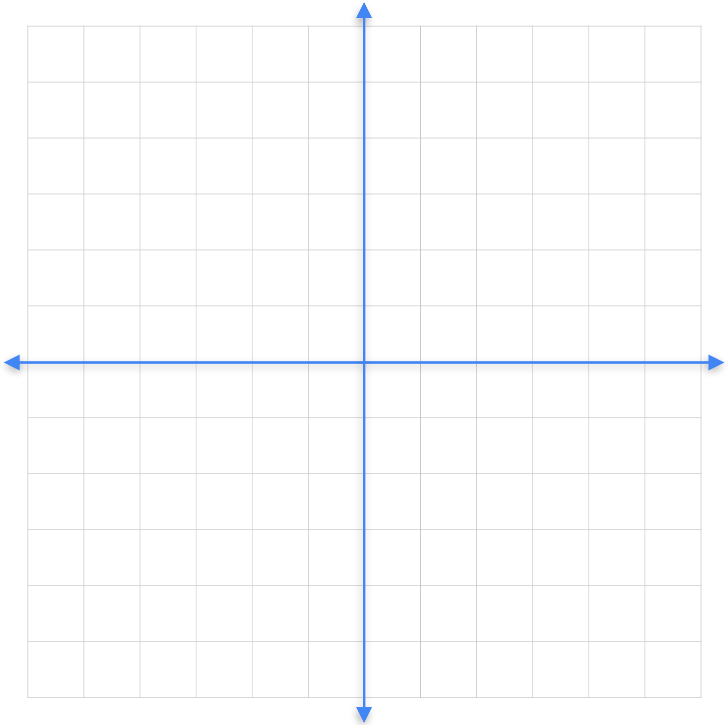
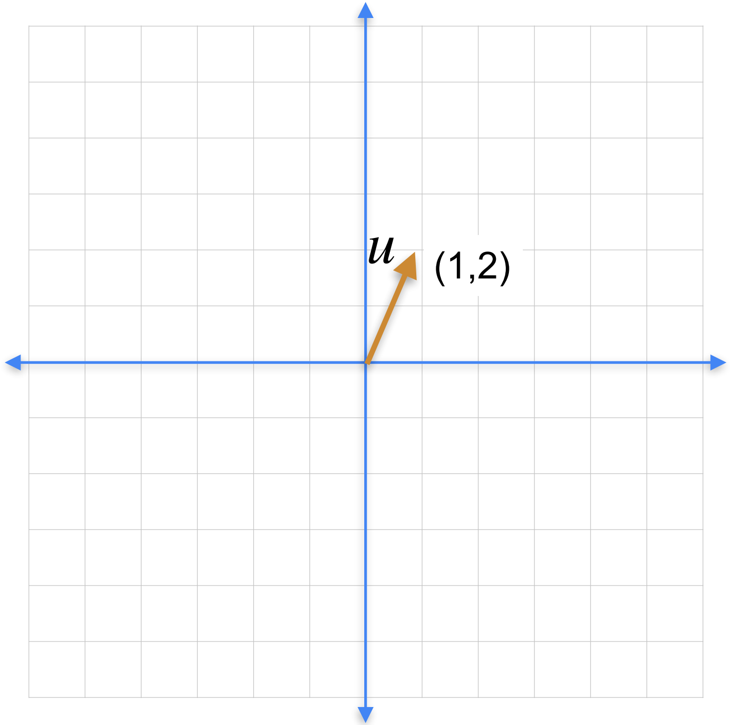
Vectors and Linear Transformations

Multiplying a vector by a scalar

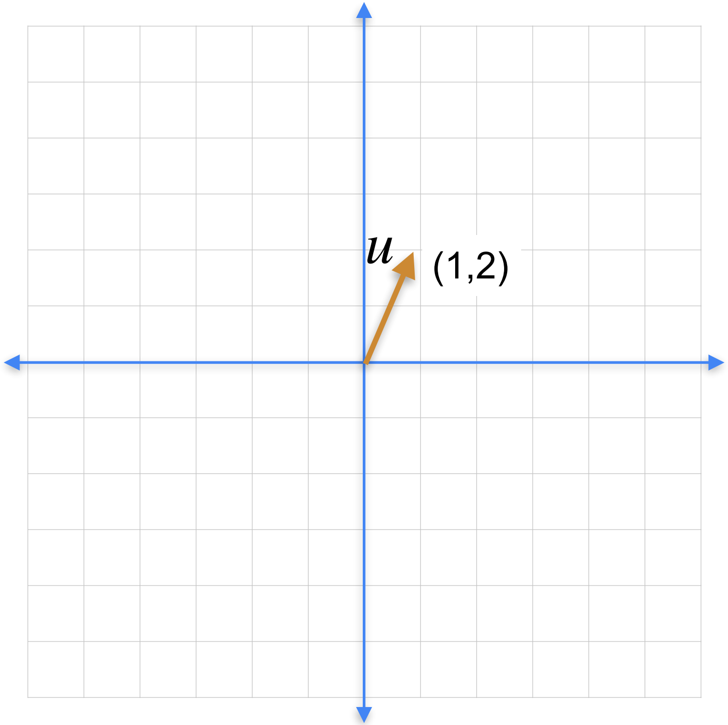
Multiplying a vector by a scalar



Multiplying a vector by a scalar

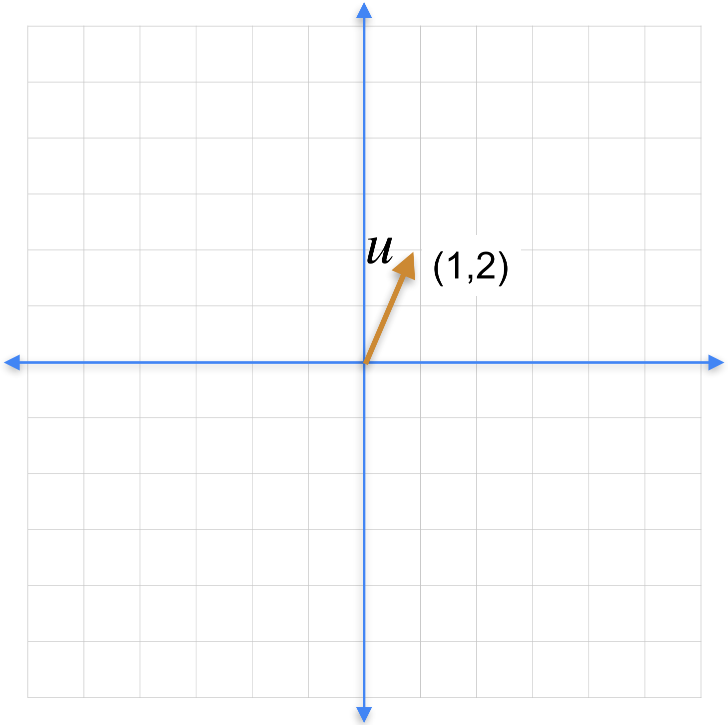


Multiplying a vector by a scalar



$$u = (1,2)$$

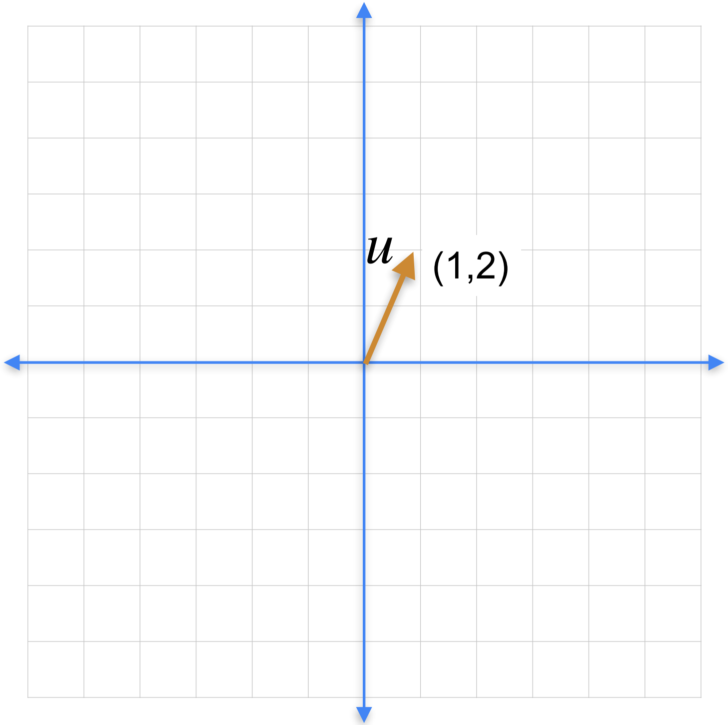
Multiplying a vector by a scalar



$u = (1,2)$

$\lambda = 3$

Multiplying a vector by a scalar

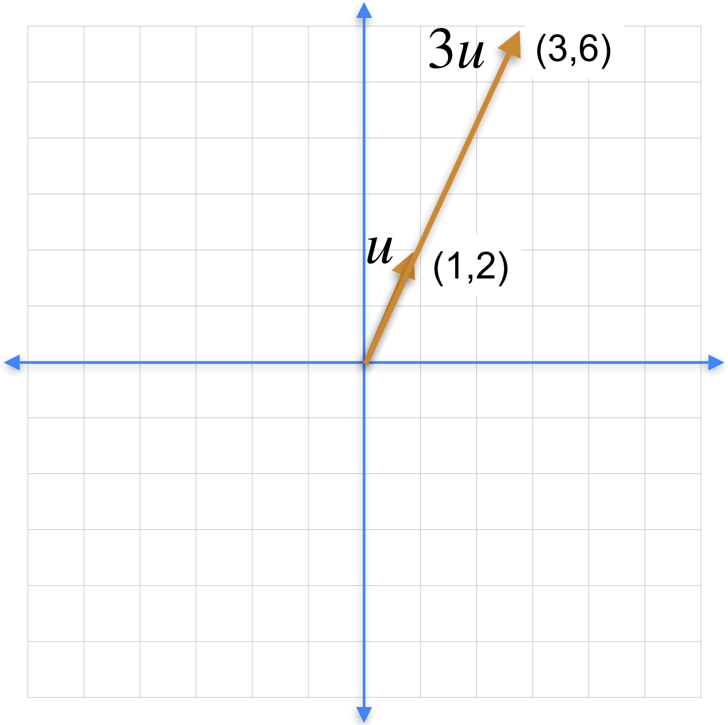


$u = (1,2)$

$\lambda = 3$

$\lambda u = (3,6)$

Multiplying a vector by a scalar

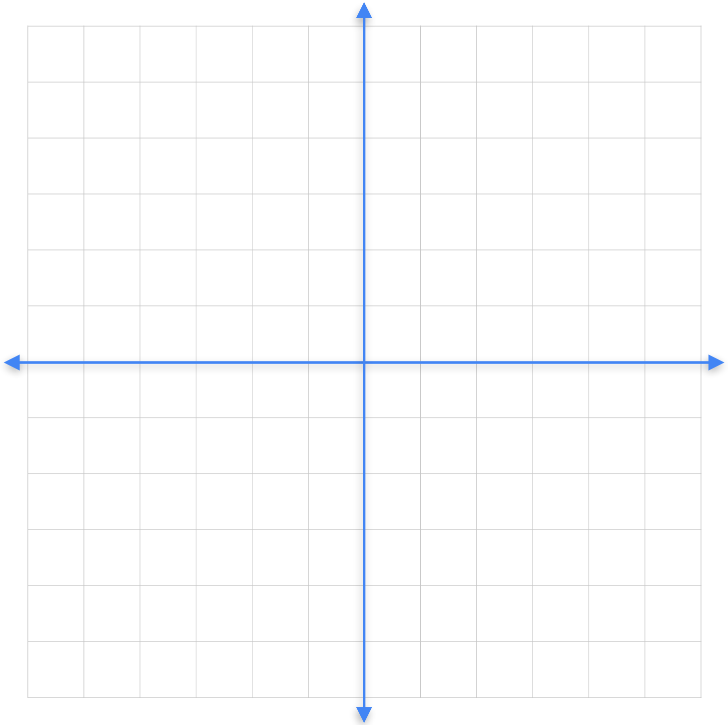


$$u = (1,2)$$

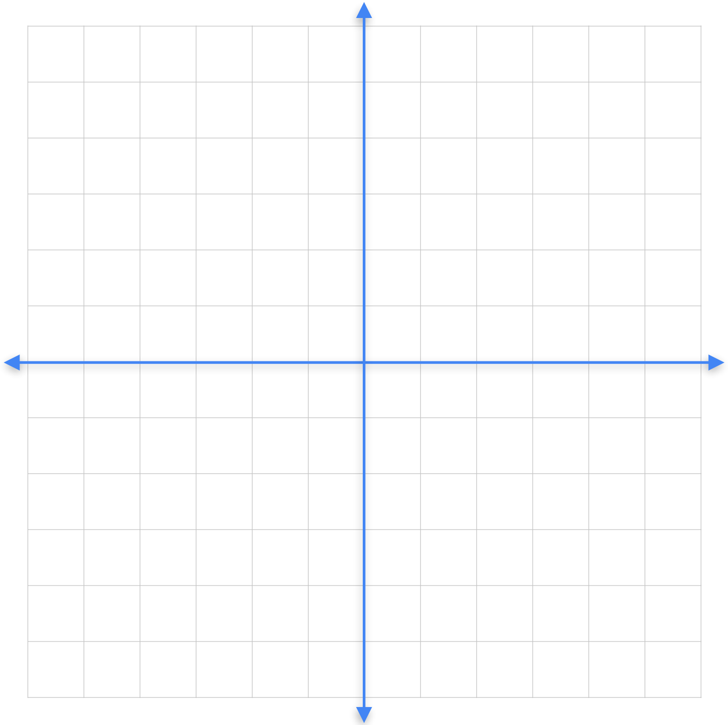
$$\lambda = 3$$

$$\lambda u = (3,6)$$

If the scalar is negative

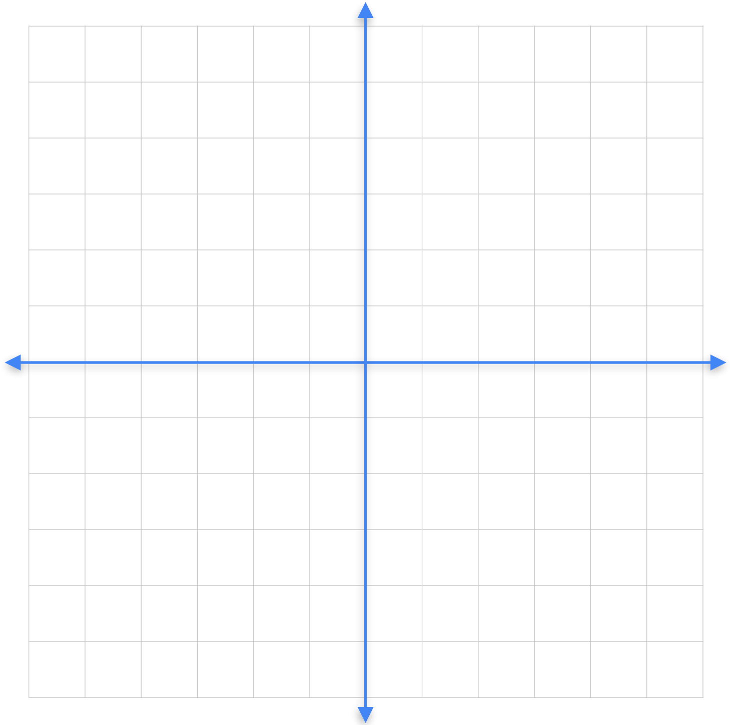


If the scalar is negative



$u = (1,2)$

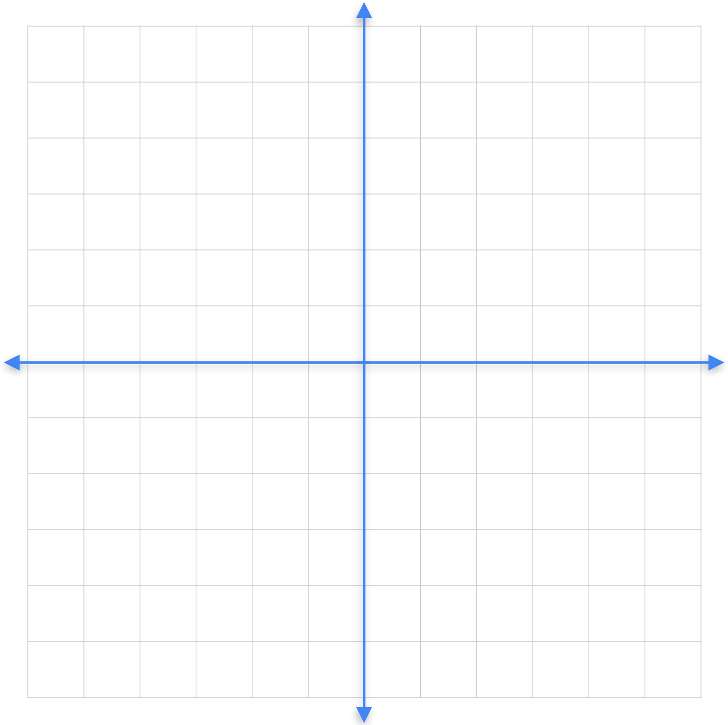
If the scalar is negative



$$u = (1,2)$$

$$\lambda = -2$$

If the scalar is negative

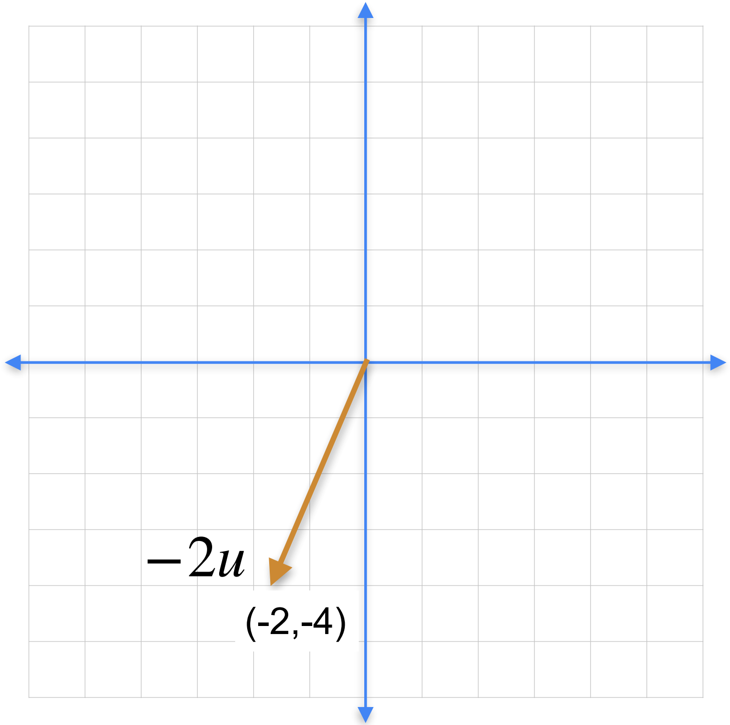


$u = (1,2)$

$\lambda = -2$

$\lambda u = (-2, -4)$

If the scalar is negative

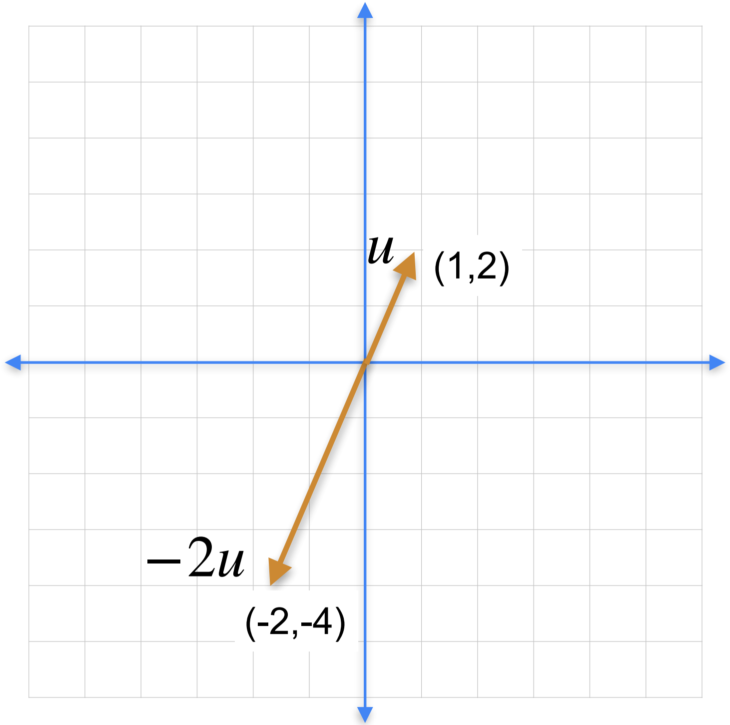


$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

If the scalar is negative



$u = (1,2)$

$\lambda = -2$

$\lambda u = (-2, -4)$

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Vectors and Linear Transformations

The dot product

A shortcut for linear operations

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

A shortcut for linear operations

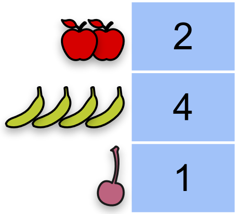
Quantities	Prices
2 apples	apples: $3
4 bananas	bananas: $5
1 cherry	cherries: $2

A shortcut for linear operations

Quantities	Prices	Total price
2 apples	apples: $3	
4 bananas	bananas: $5	
1 cherry	cherries: $2	

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry



Prices
apples: $3
bananas: $5
cherries: $2

Total price

A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

A shortcut for linear operations

Quantities
2 apples
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apples: $3
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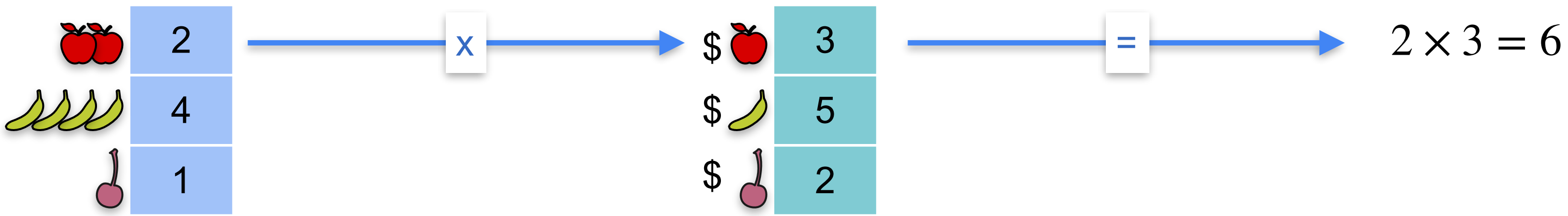
Total price



A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
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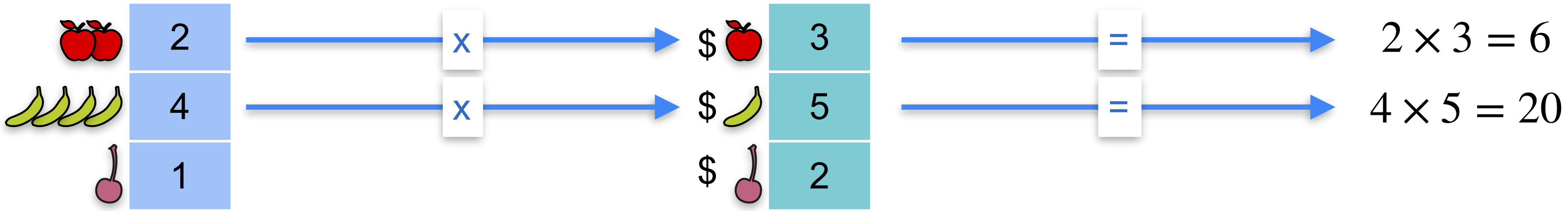
Total price

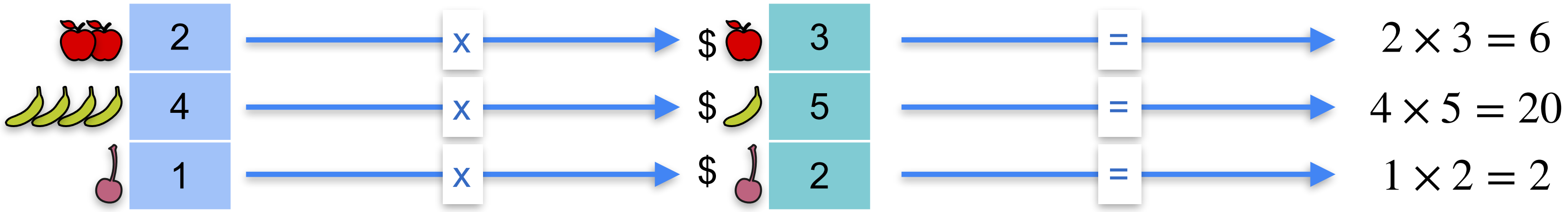


A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

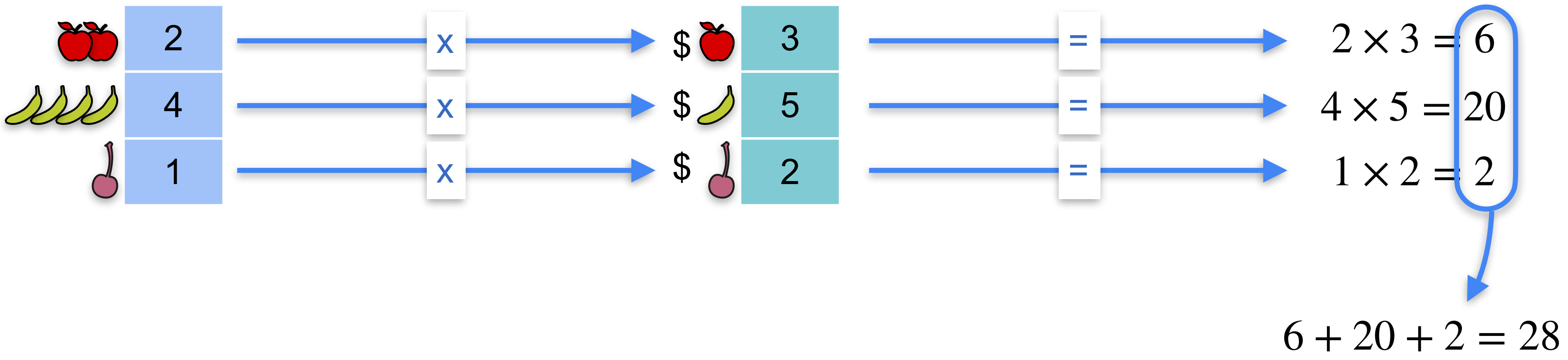


A shortcut for linear operations

Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

Total price

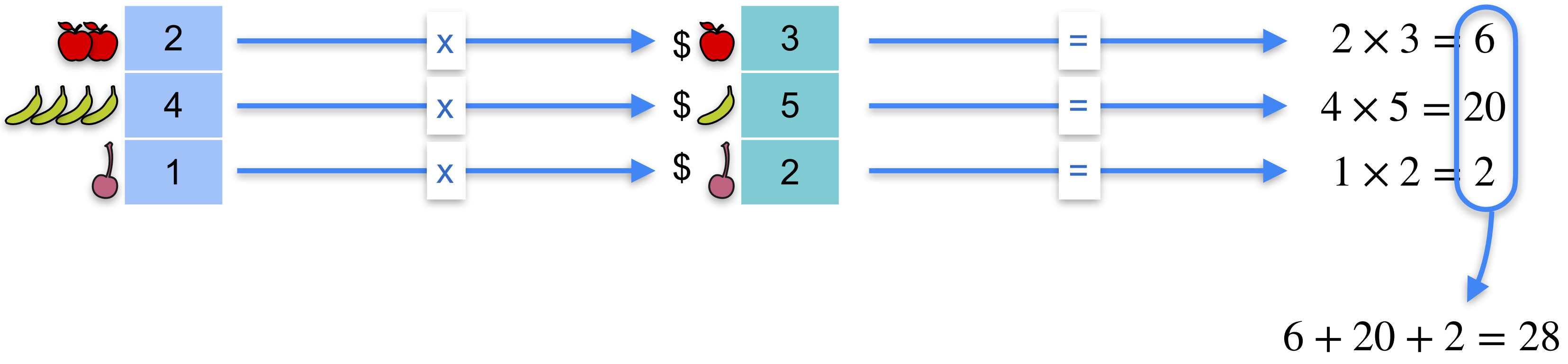


A shortcut for linear operations

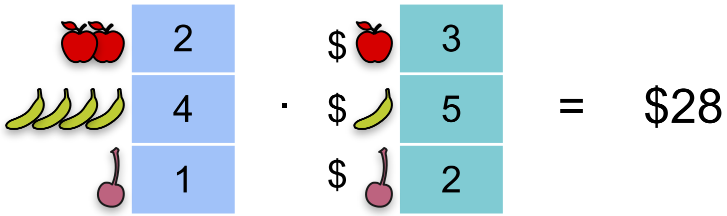
Quantities
2 apples
4 bananas
1 cherry

Prices
apples: $3
bananas: $5
cherries: $2

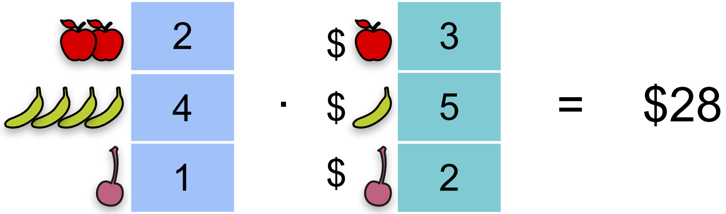
Total price
$28



The dot product



The dot product



$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

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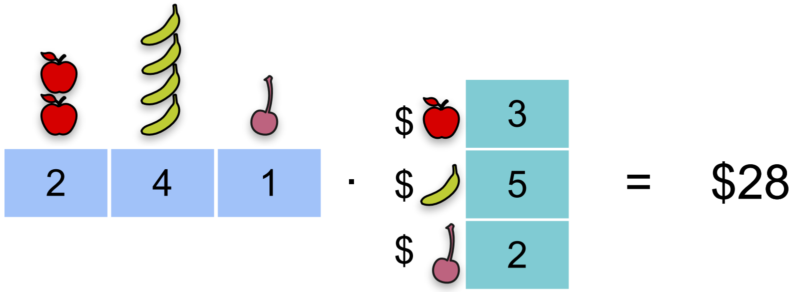
The dot product



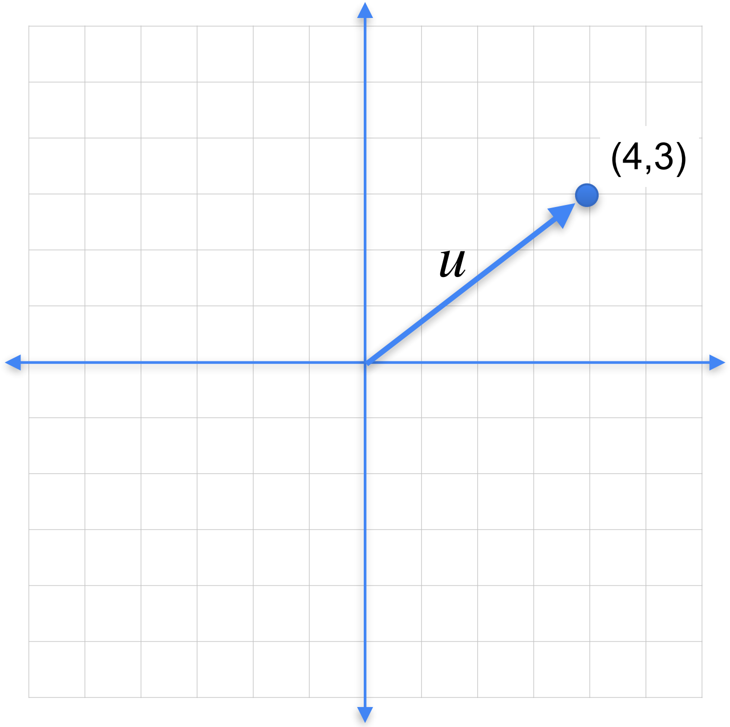
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

The dot product

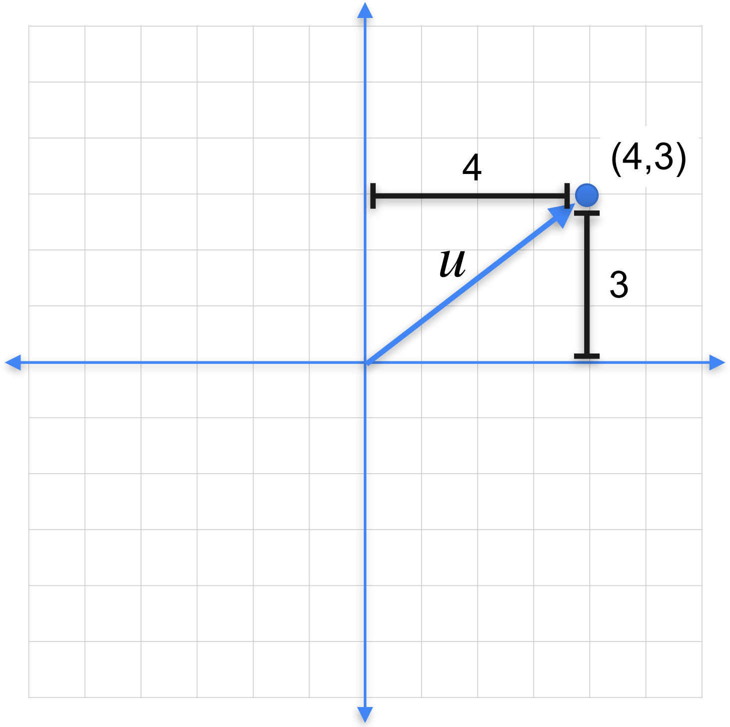
$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

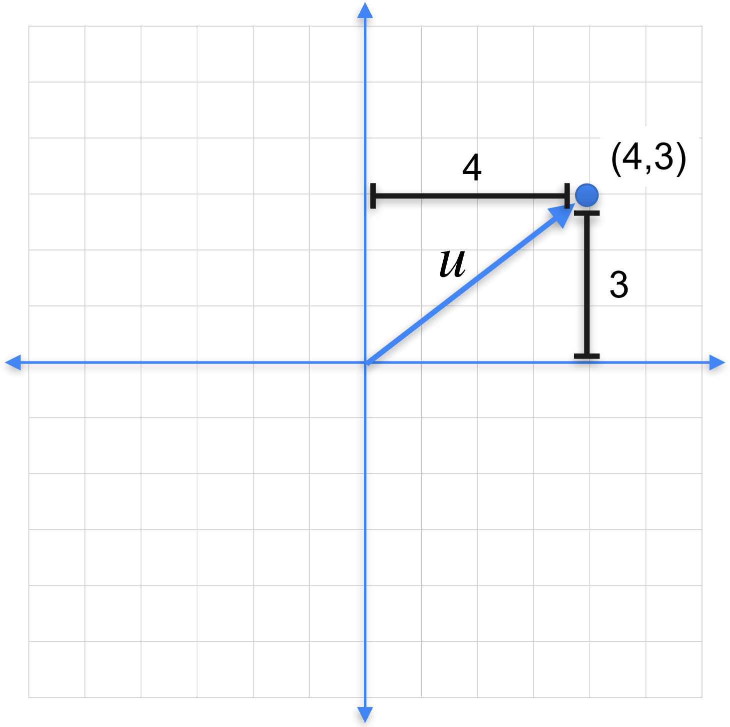
Norm of a vector using dot product



Norm of a vector using dot product

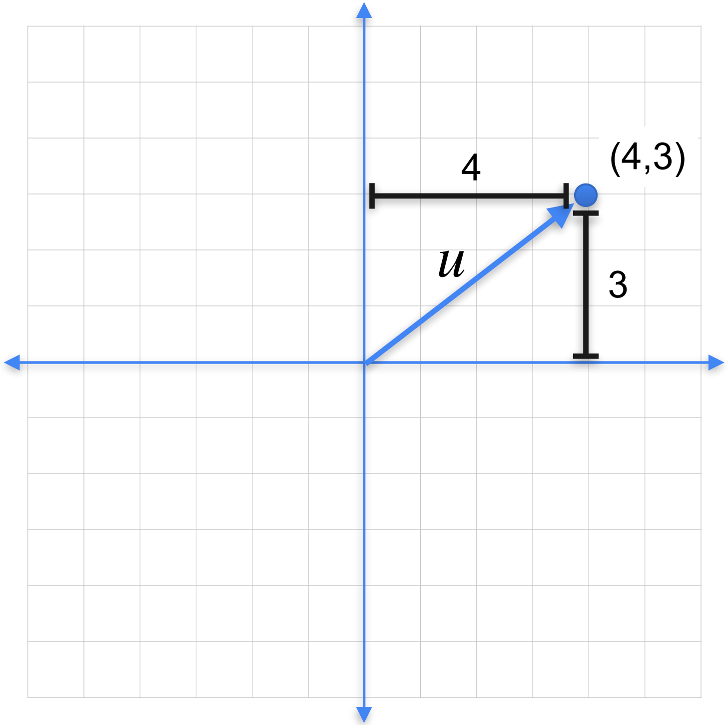


Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

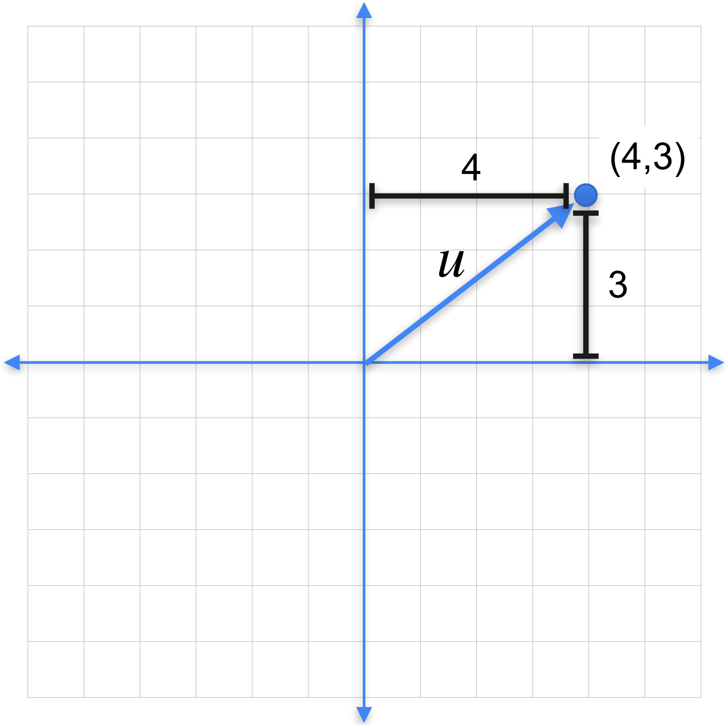
Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

Norm of a vector using dot product

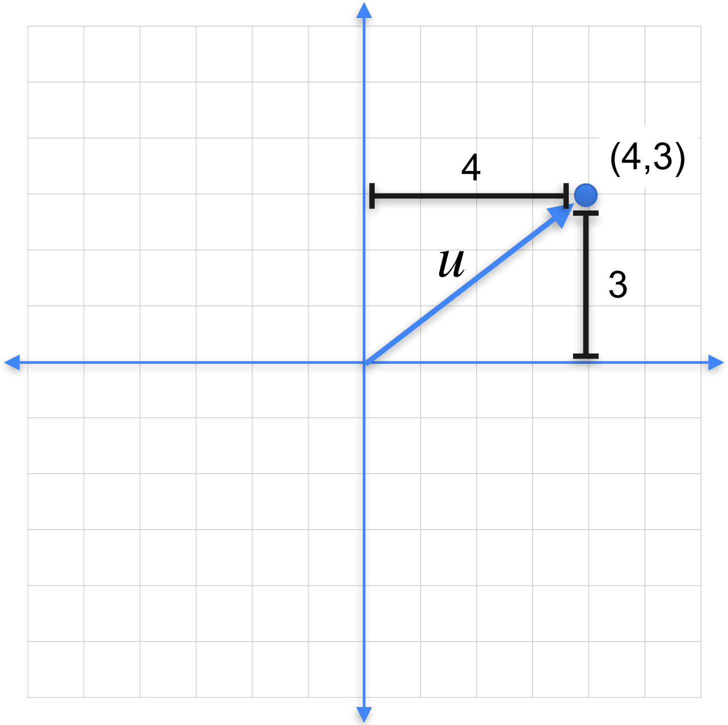


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{dot \ \ product(u, u)}$$

Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

4	3	4 / 3	=	25

$$L2 - norm = \sqrt{dot\ \ product(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$

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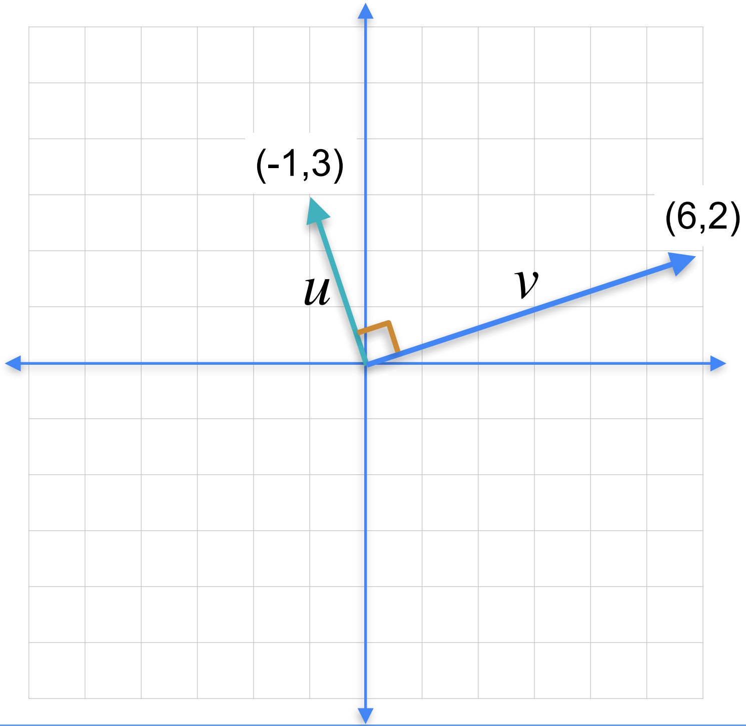
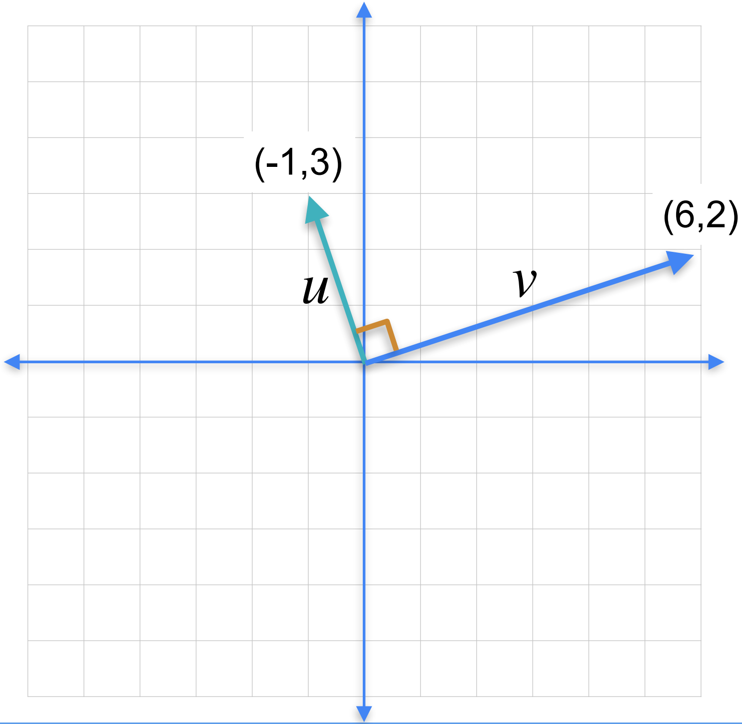
Vectors and Linear Transformations

Geometric dot product

Orthogonal vectors have dot product 0

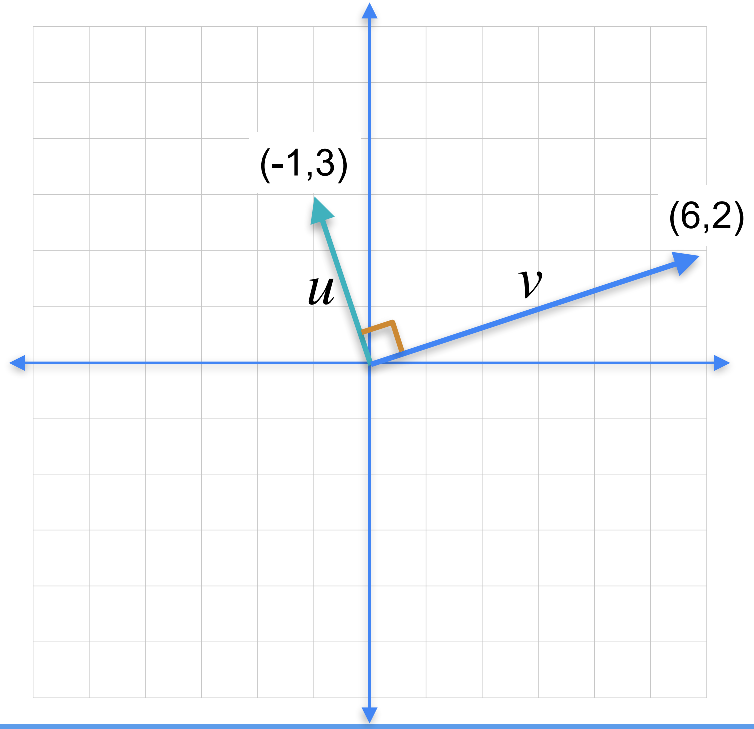


Orthogonal vectors have dot product 0



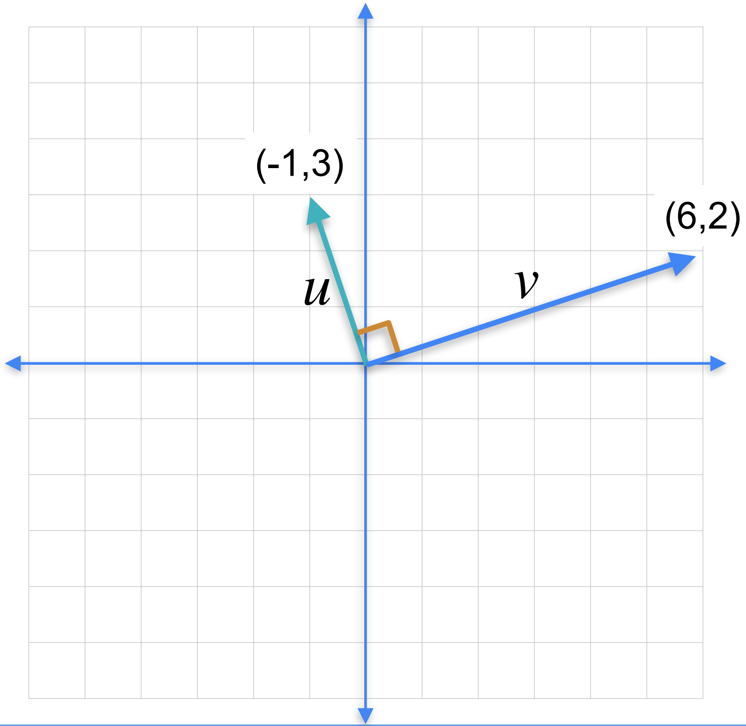
6	2

Orthogonal vectors have dot product 0



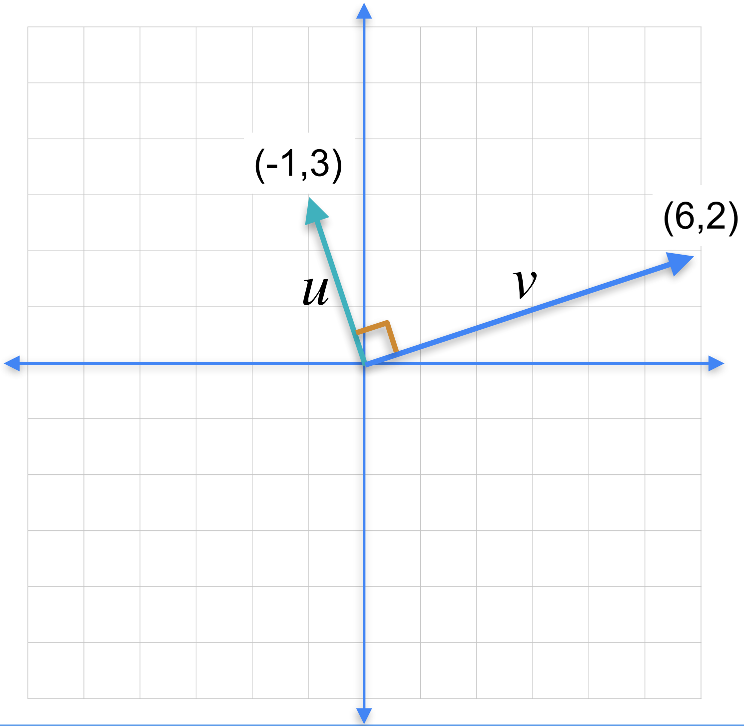
6	2	-1
		3

Orthogonal vectors have dot product 0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Orthogonal vectors have dot product 0



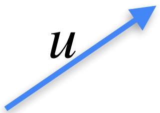
6	2	

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

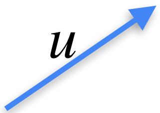
$$\langle u, v \rangle = 0$$

The dot product

The dot product

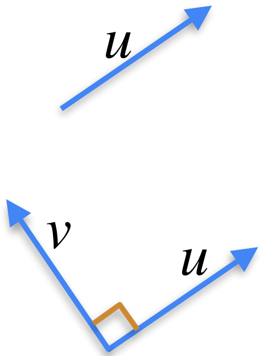


The dot product



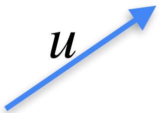
$$\langle u, u \rangle = |u|^2$$

The dot product

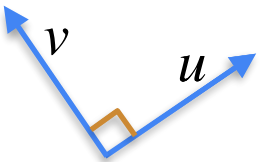


$$\langle u, u \rangle = |u|^2$$

The dot product

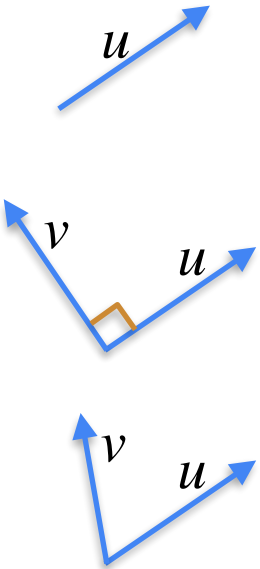


$$\langle u, u \rangle = |u|^2$$



$$\langle u, v \rangle = 0$$

The dot product



$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

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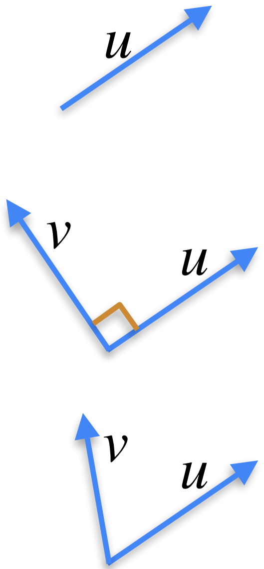
The dot product



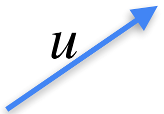
$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

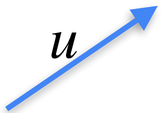
$$\langle u, v \rangle = ?$$

The dot product

The dot product

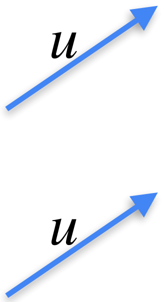


The dot product



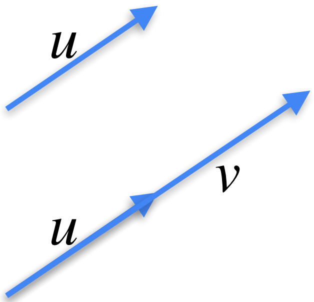
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



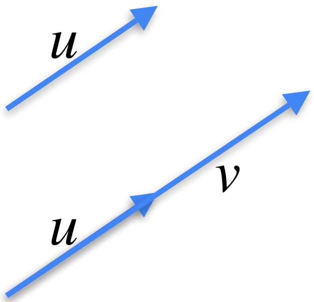
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

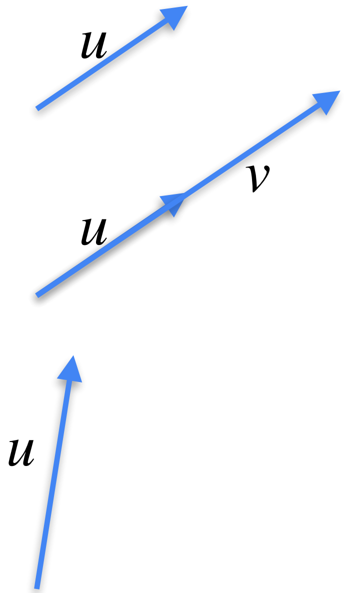
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

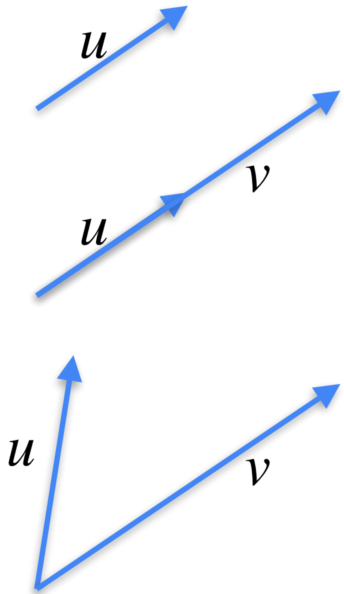
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

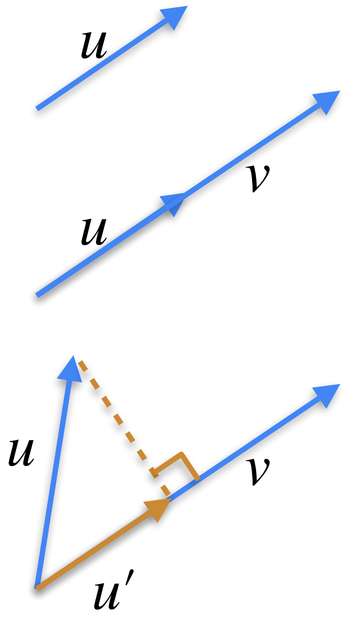
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

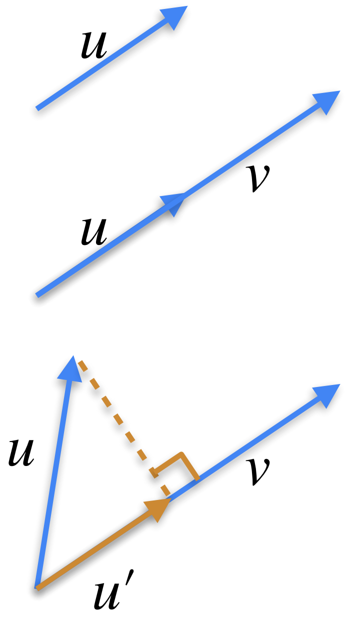
The dot product



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

The dot product

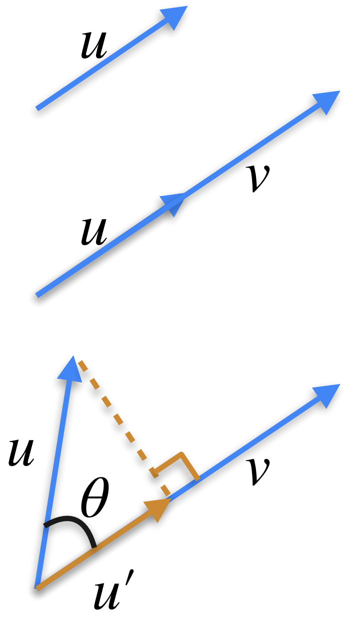


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product

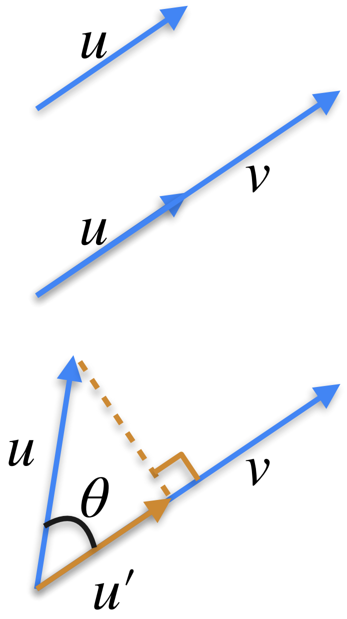


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

$$\langle u, v \rangle = |u'| \cdot |v|$$

The dot product



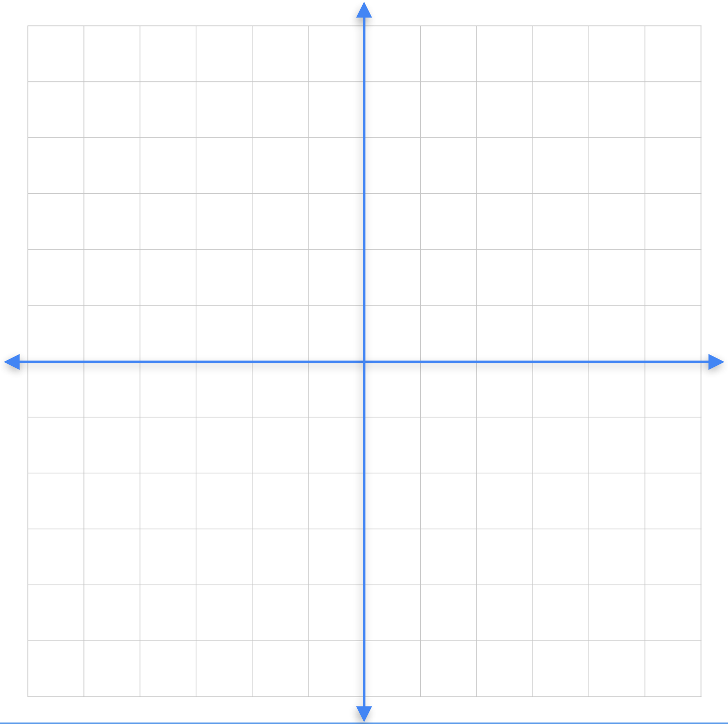
$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

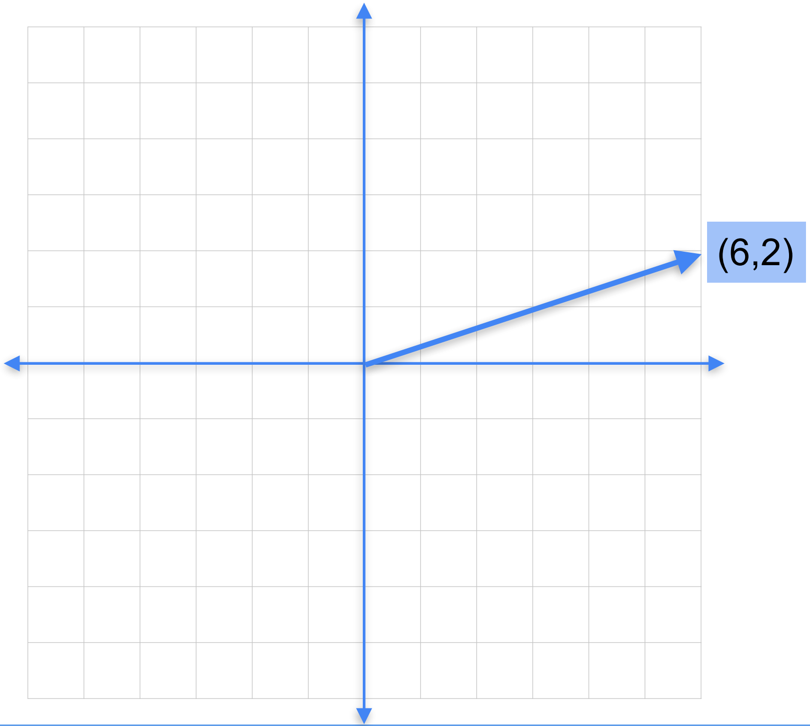
$$\langle u, v \rangle = |u'| \cdot |v|$$

$$= |u||v|\cos(\theta)$$

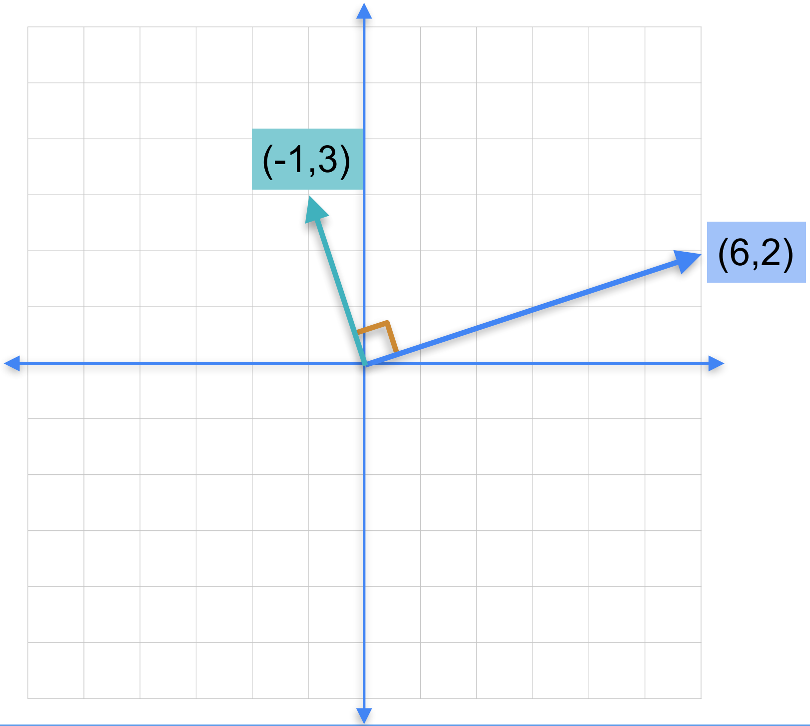
Geometric dot product



Geometric dot product



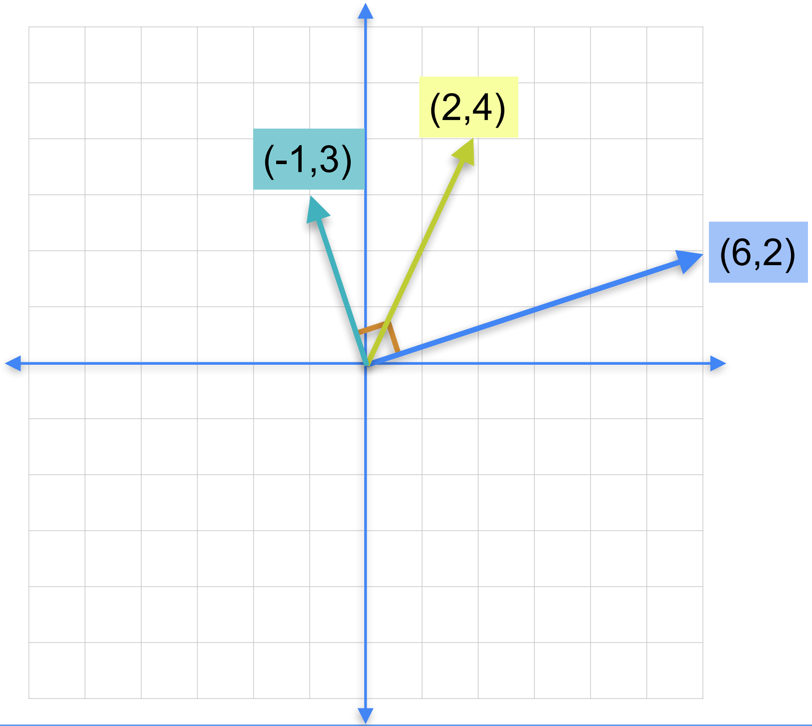
Geometric dot product

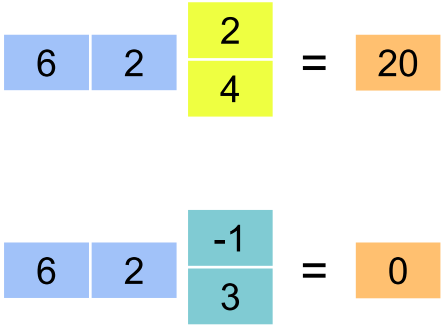


6	2	

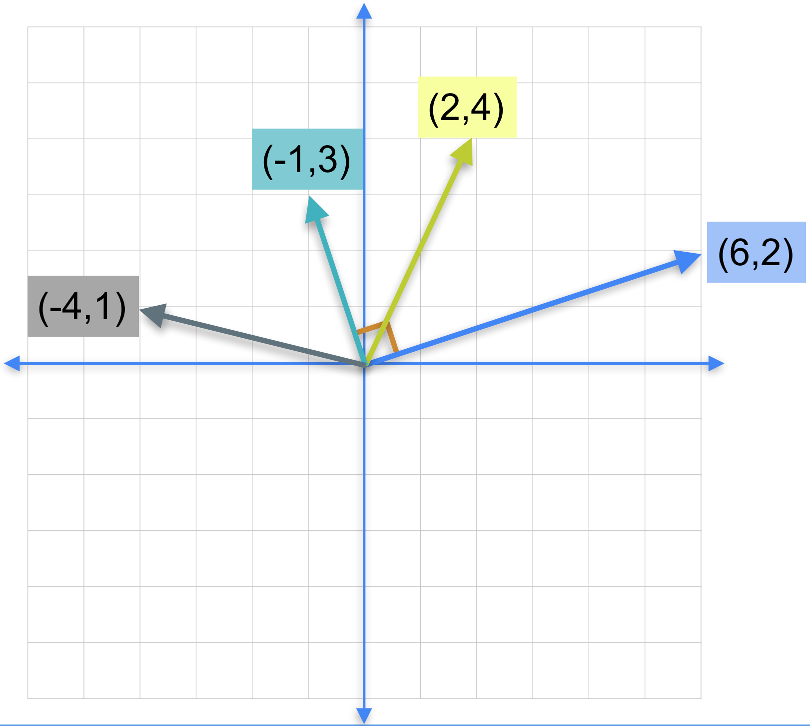
$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

Geometric dot product





Geometric dot product

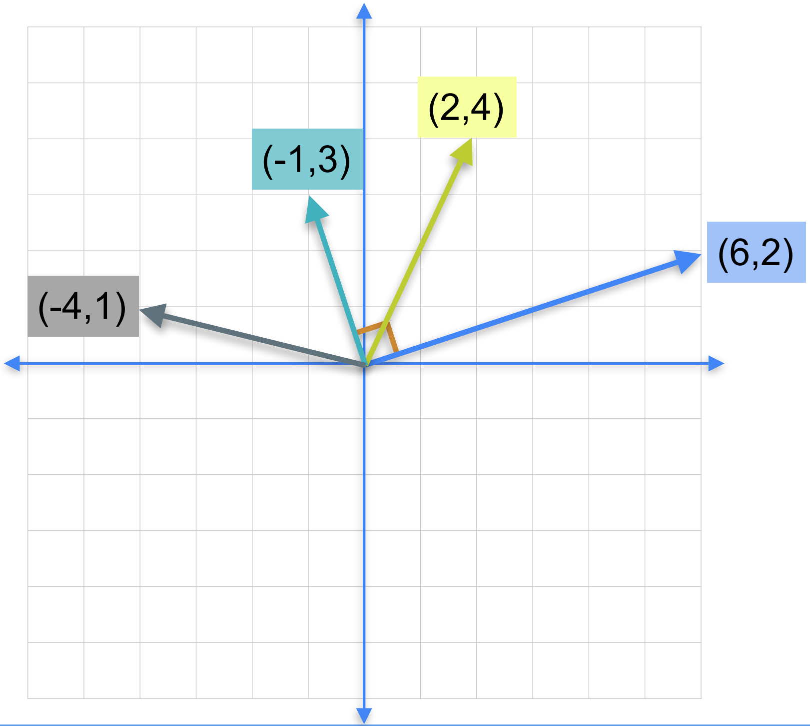


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product

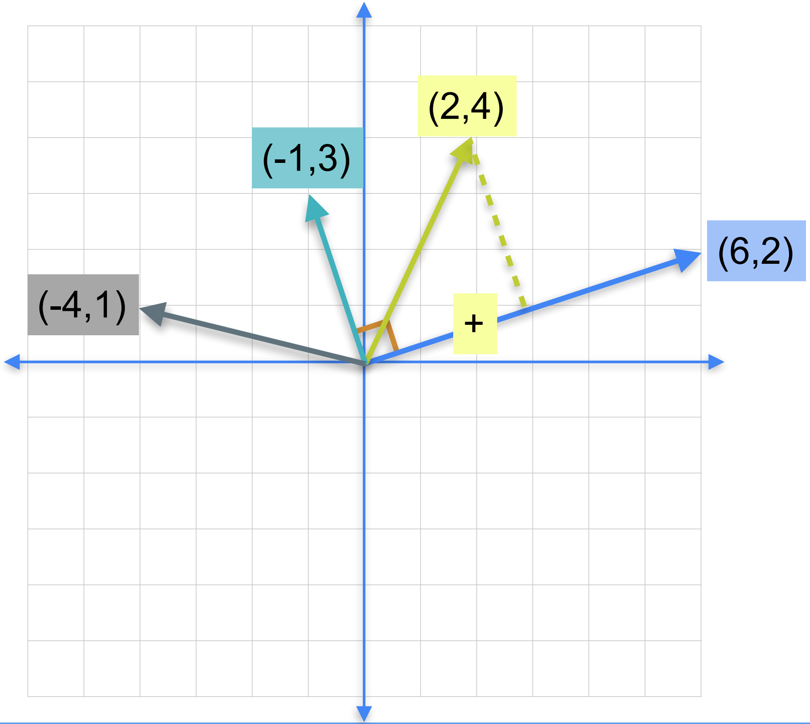


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$ Positive

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

Geometric dot product



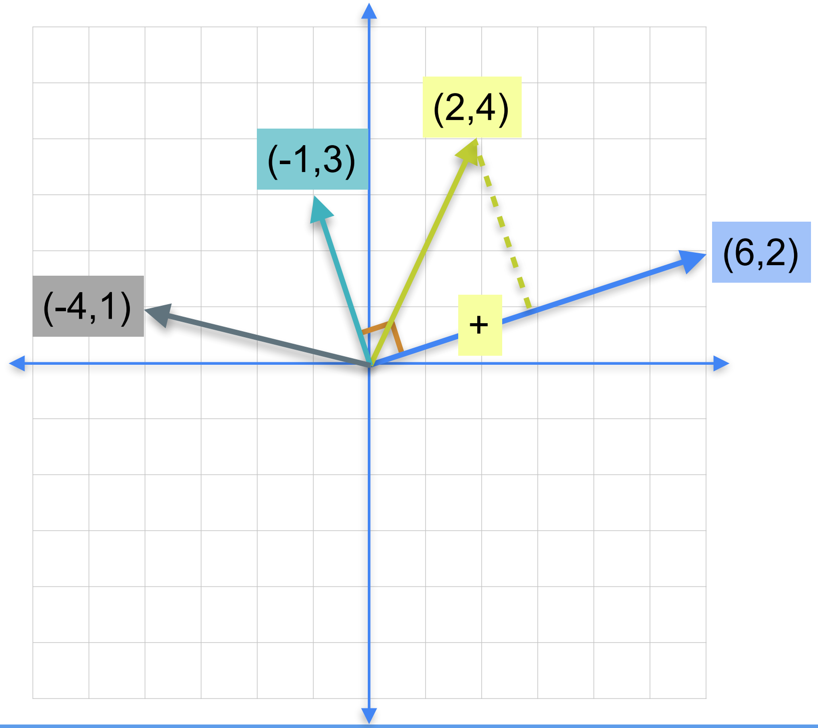
$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix} = 20$$

Positive

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = 0$$

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix} = -22$$

Geometric dot product

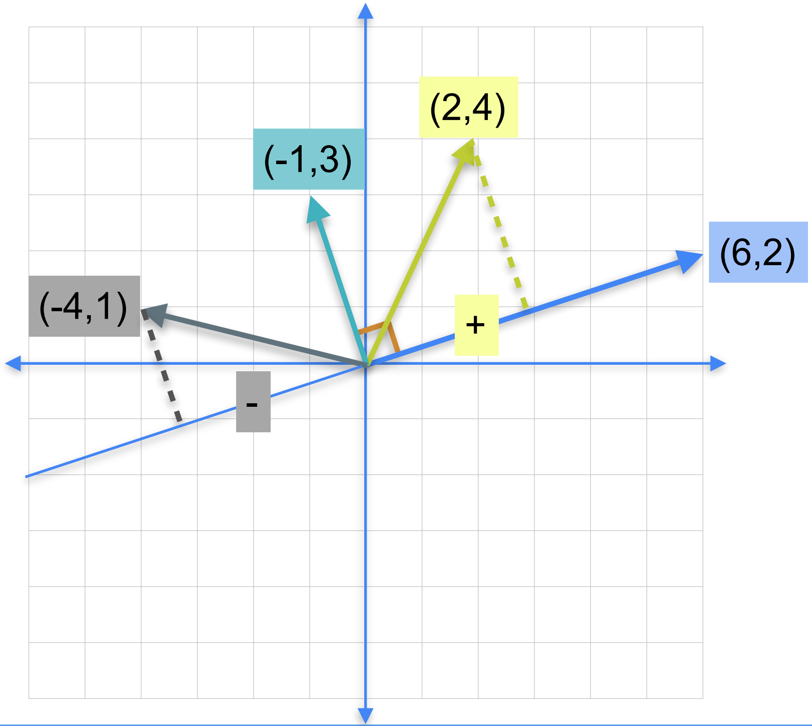


$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$ Positive

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$ Negative

Geometric dot product



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$ Positive

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$ Negative

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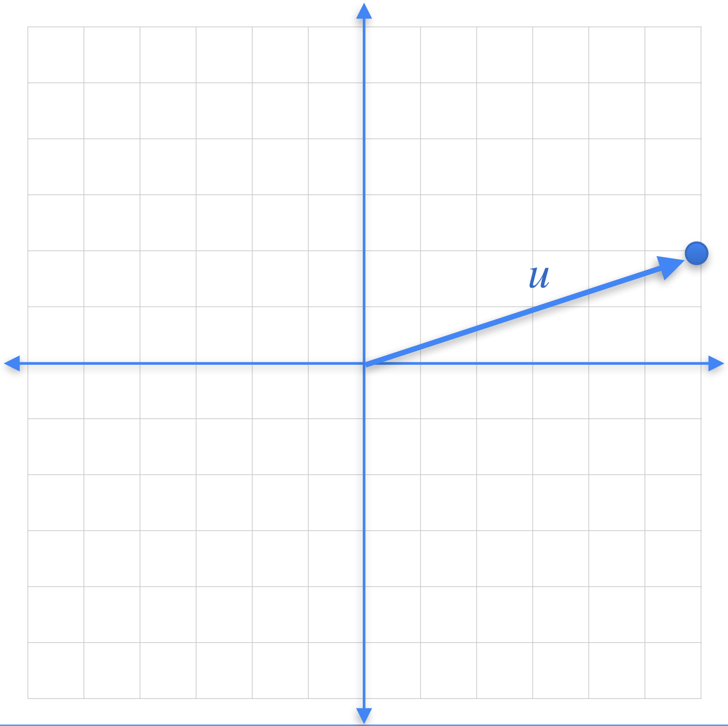
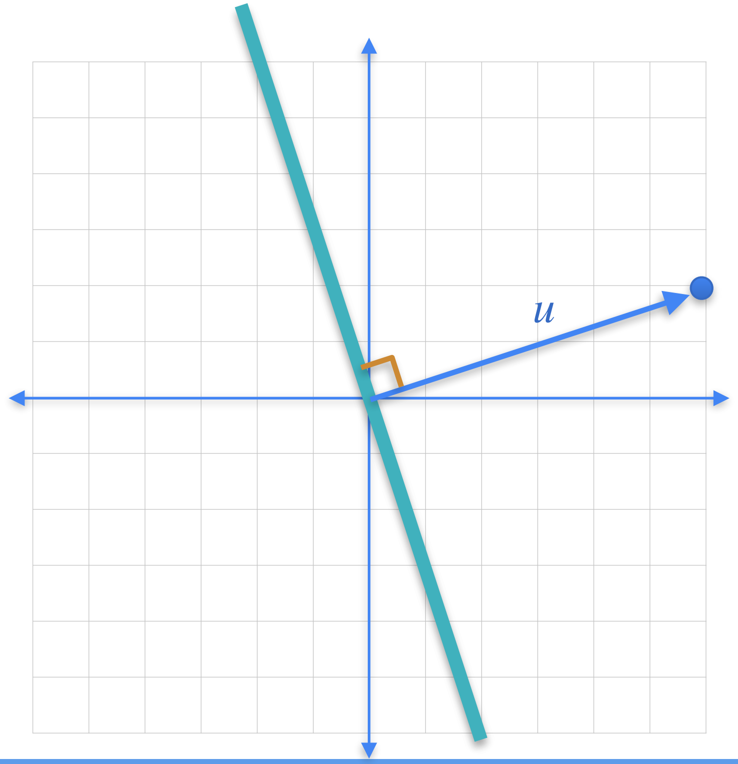
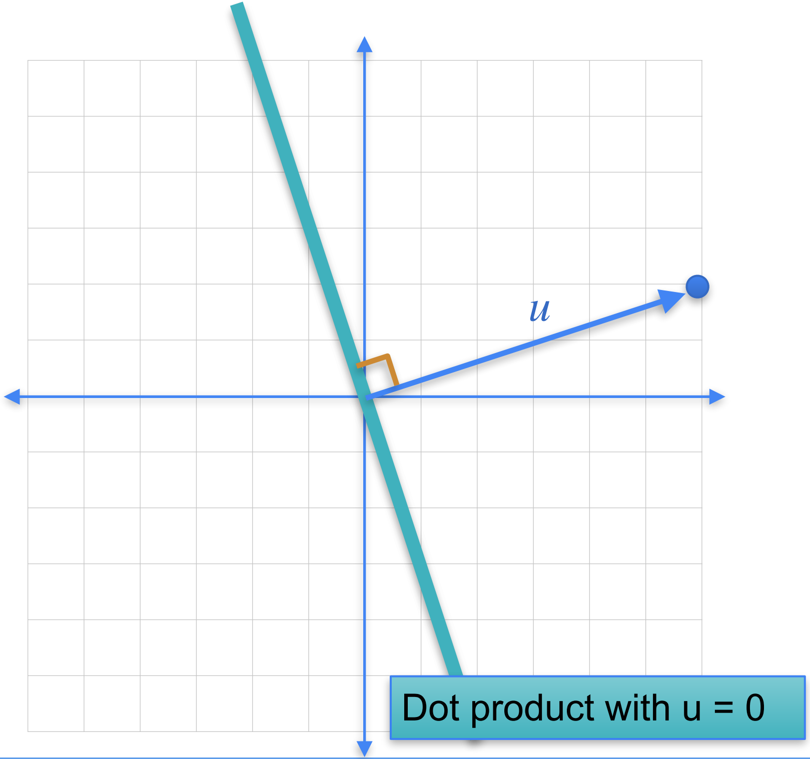
Geometric dot product



Geometric dot product

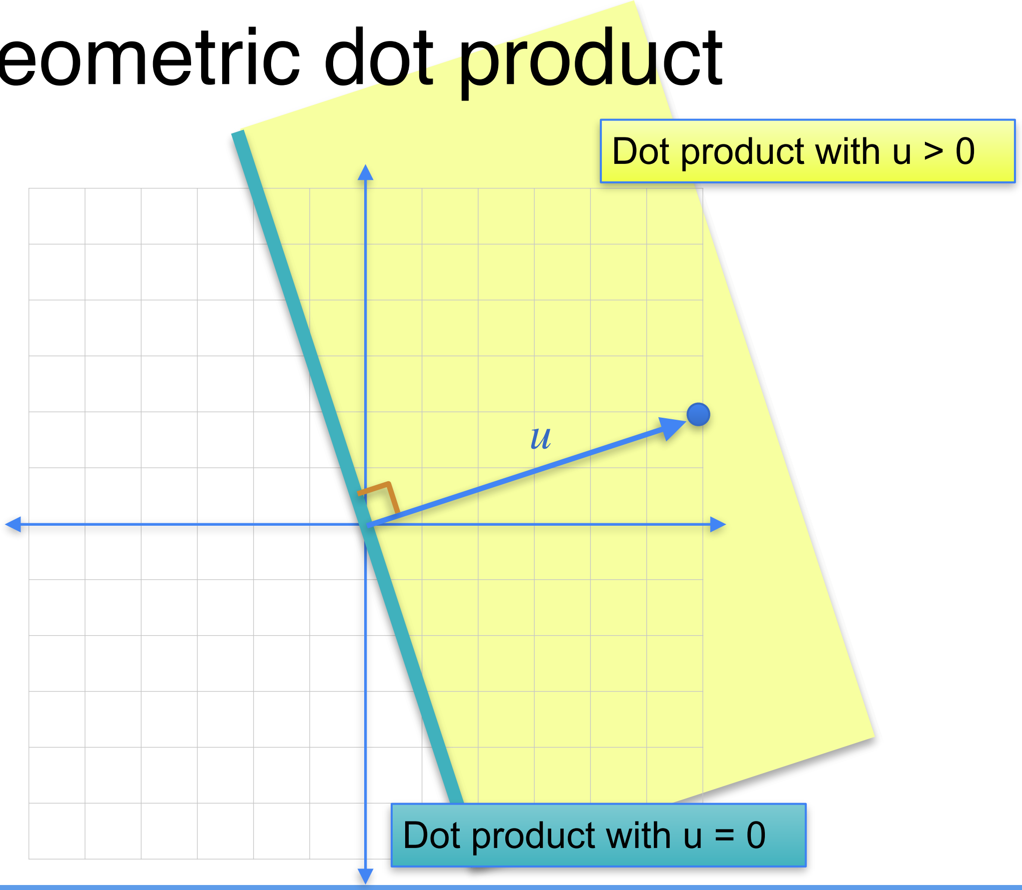


Geometric dot product



$$\langle u, v \rangle = 0$$

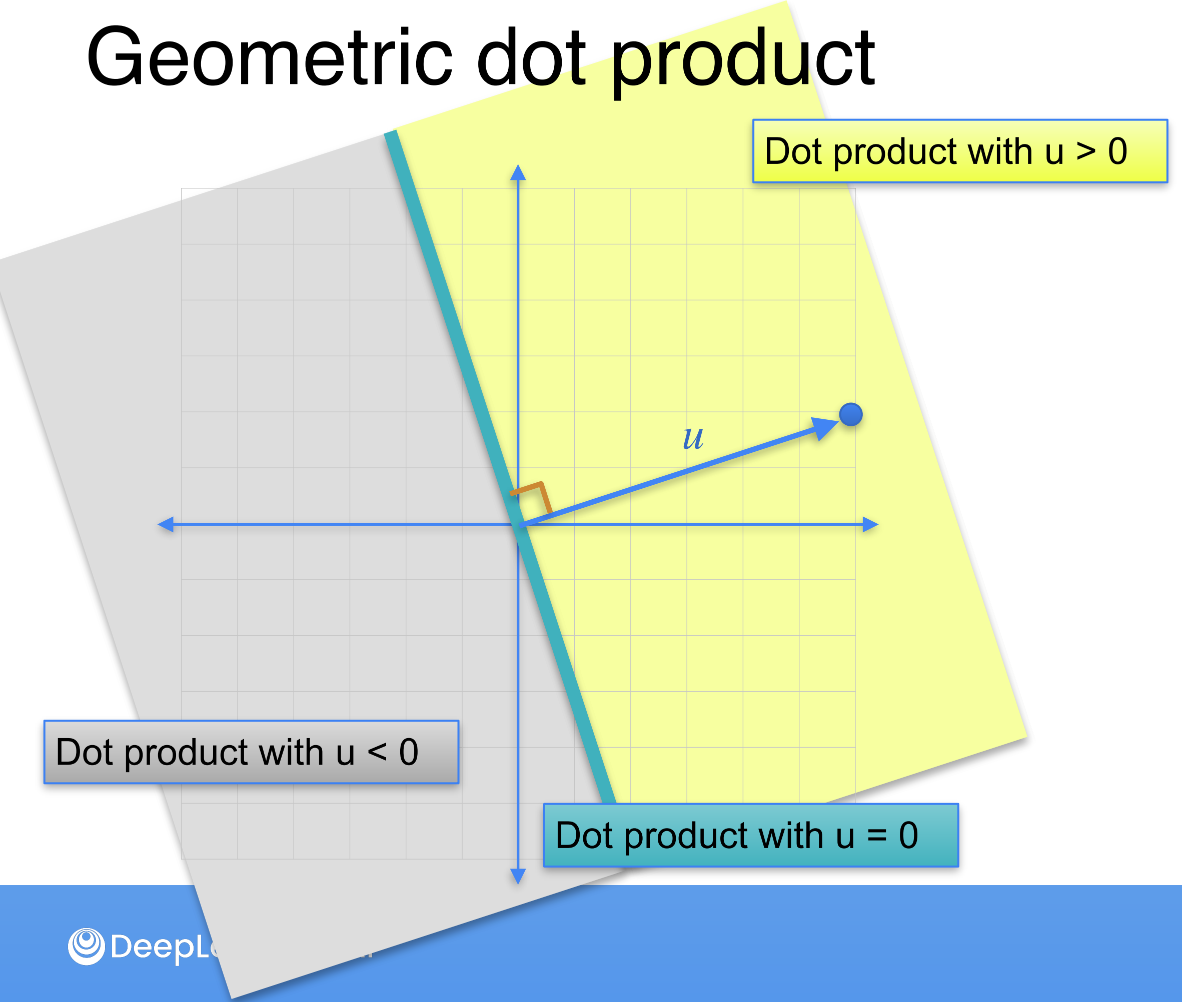
Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

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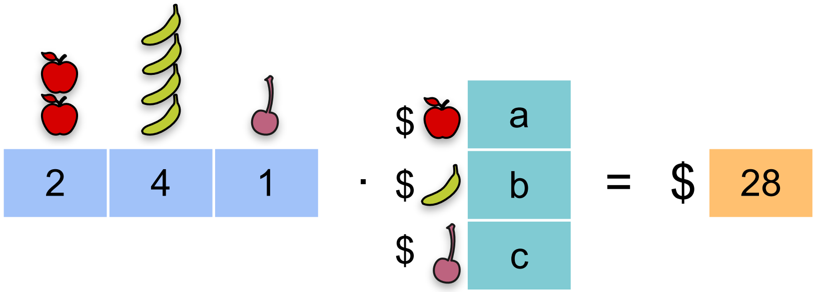
Vectors and Linear Transformations

Multiplying a matrix by a vector

Equations as dot product

$$2a + 4b + c = 28$$



Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

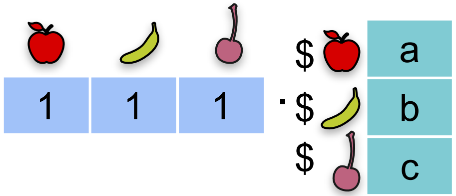
1	1	1

Equations as dot product

a + b + c = 10

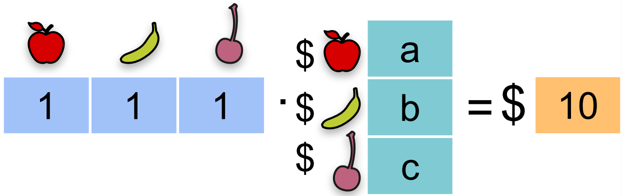
a + 2b + c = 15

a + b + 2c = 12



Equations as dot product

a + b + c = 10



a + 2b + c = 15

a + b + 2c = 12

Equations as dot product

a + b + c = 10

1	1	1

·

$ a
$ b
$ c

= $ 10

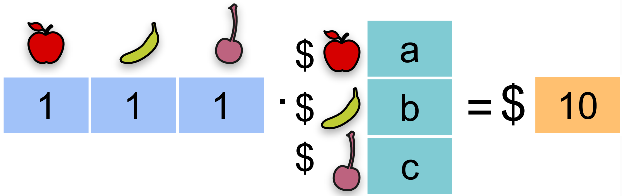
a + 2b + c = 15

1	2	1

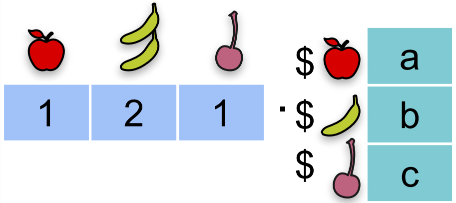
a + b + 2c = 12

Equations as dot product

$a + b + c = 10$



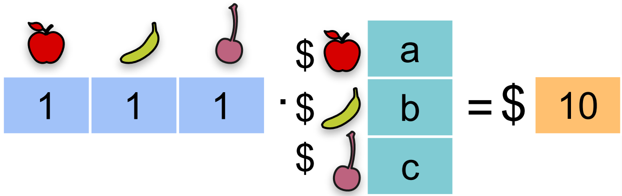
$a + 2b + c = 15$



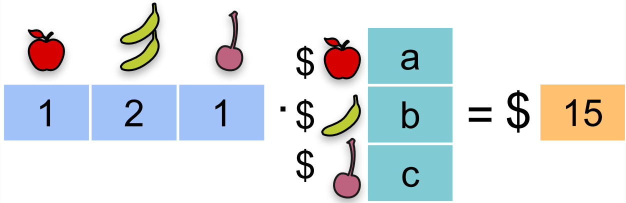
$a + b + 2c = 12$

Equations as dot product

$a + b + c = 10$



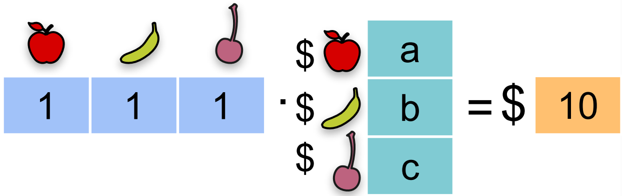
$a + 2b + c = 15$



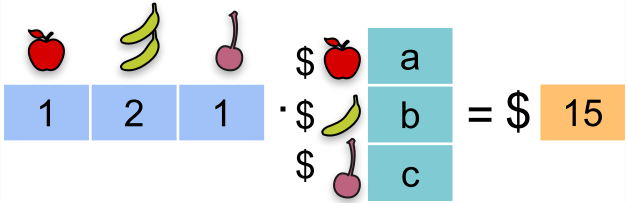
$a + b + 2c = 12$

Equations as dot product

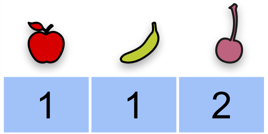
$a + b + c = 10$



$a + 2b + c = 15$



$a + b + 2c = 12$

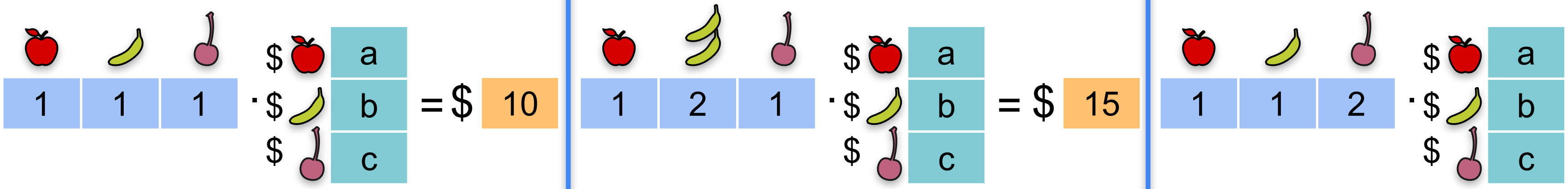


Equations as dot product

$a + b + c = 10$

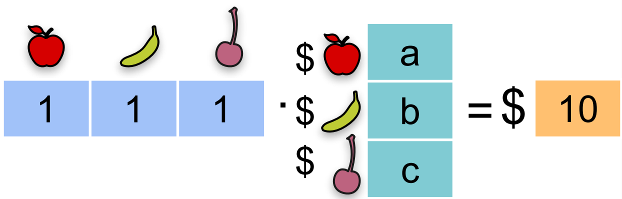
$a + 2b + c = 15$

$a + b + 2c = 12$

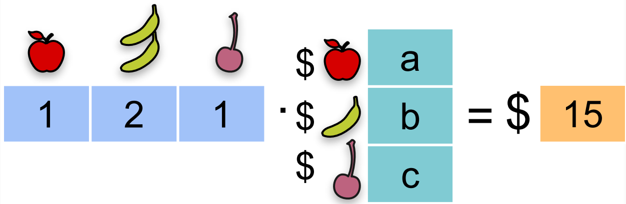


Equations as dot product

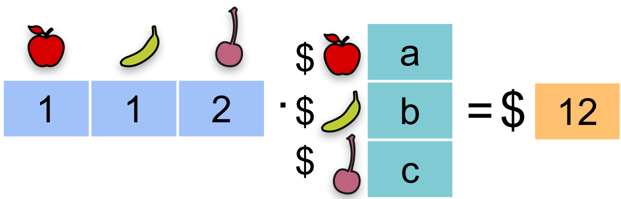
$a + b + c = 10$



$a + 2b + c = 15$



$a + b + 2c = 12$



Equations as dot product

$a + b + c = 10$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ 10$$

$a + 2b + c = 15$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ 15$$

$a + b + 2c = 12$

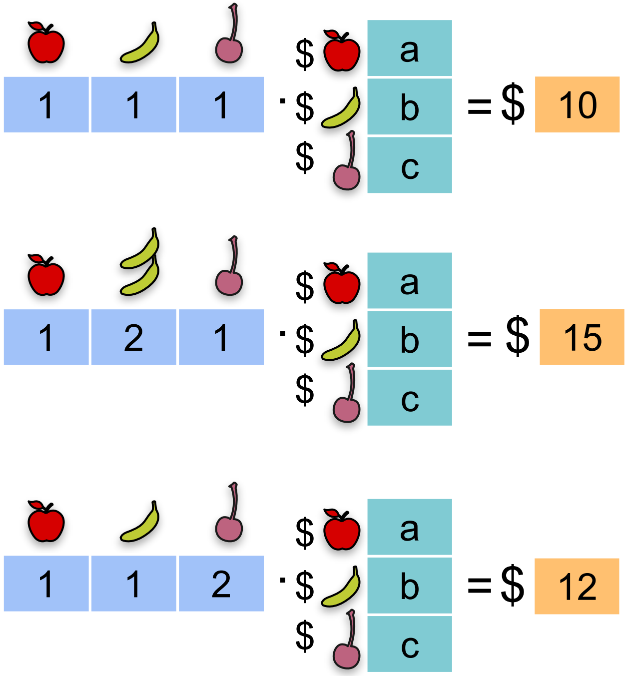
$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$\ 12$$

Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

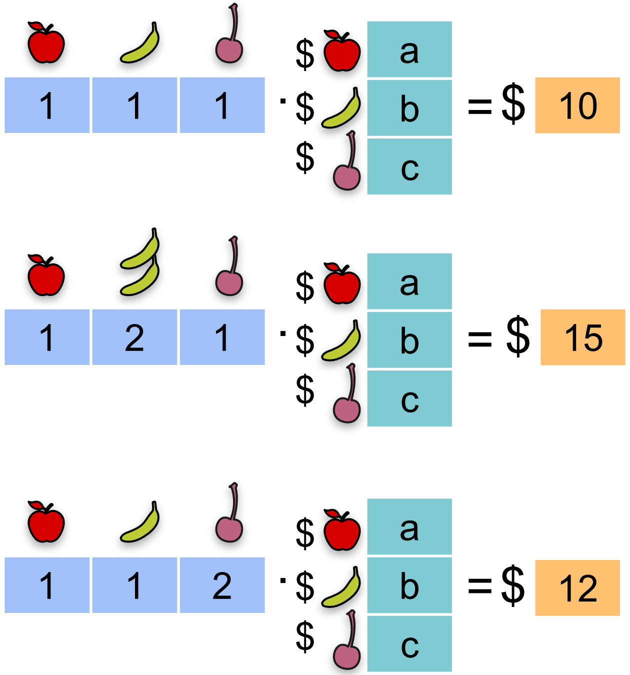


Equations as dot product

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$



Equations as dot product

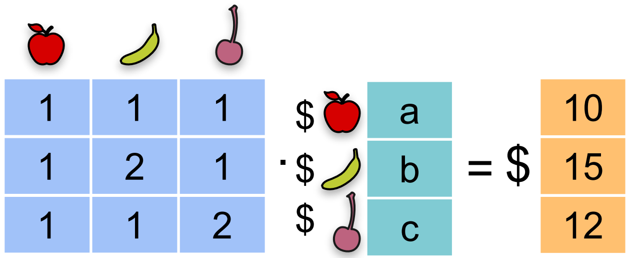
System of equations

$a + b + c = 10$

$a + 2b + c = 15$

$a + b + 2c = 12$

Matrix product



Equations as dot product

System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

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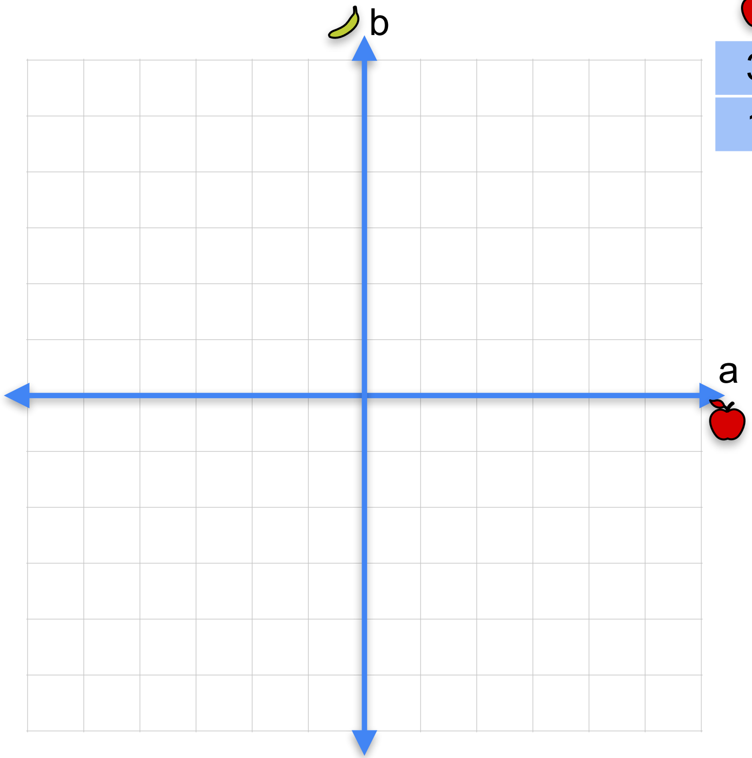
Vectors and Linear Transformations

Matrices as linear transformations

Matrices as linear transformations

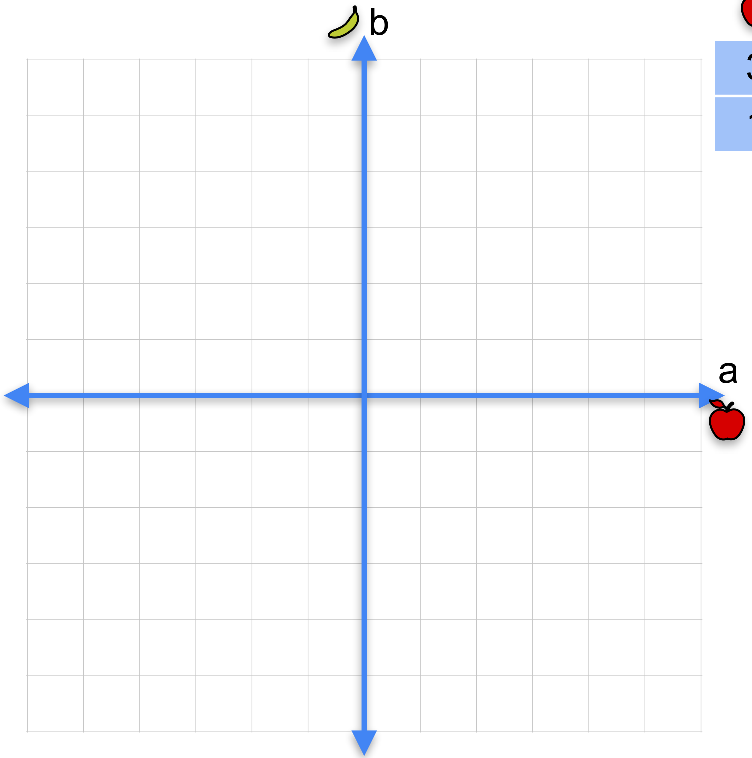
3	1
1	2

Matrices as linear transformations

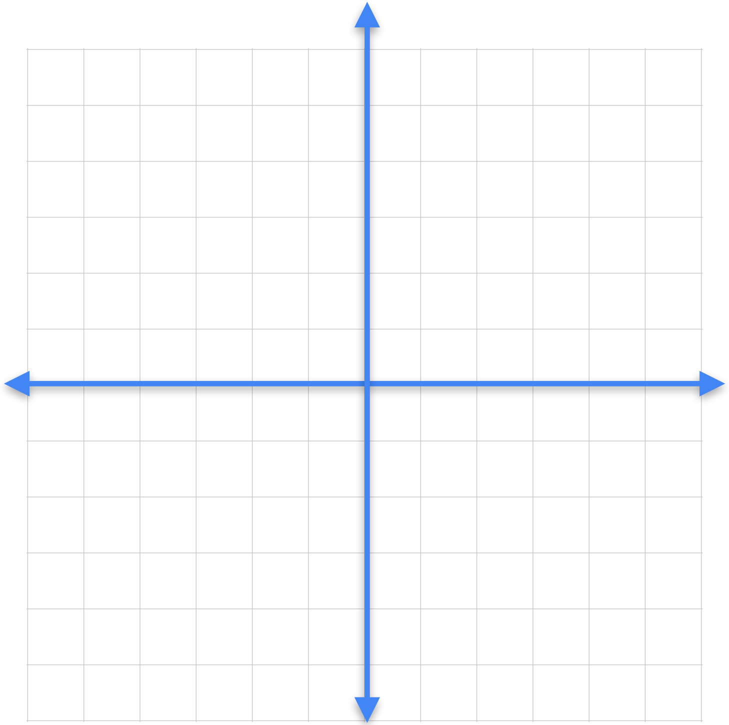


🍎	🍌
3	1
1	2

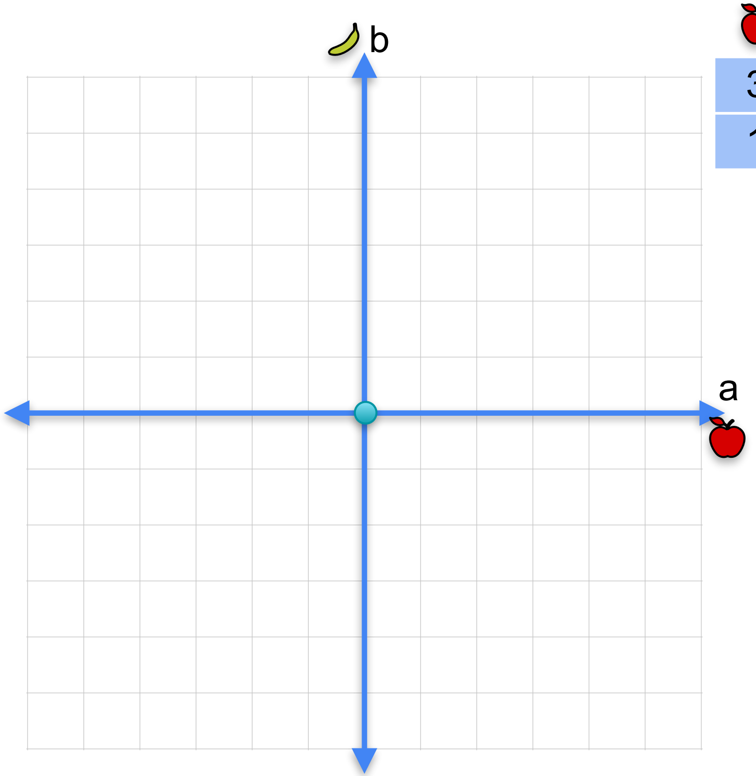
Matrices as linear transformations



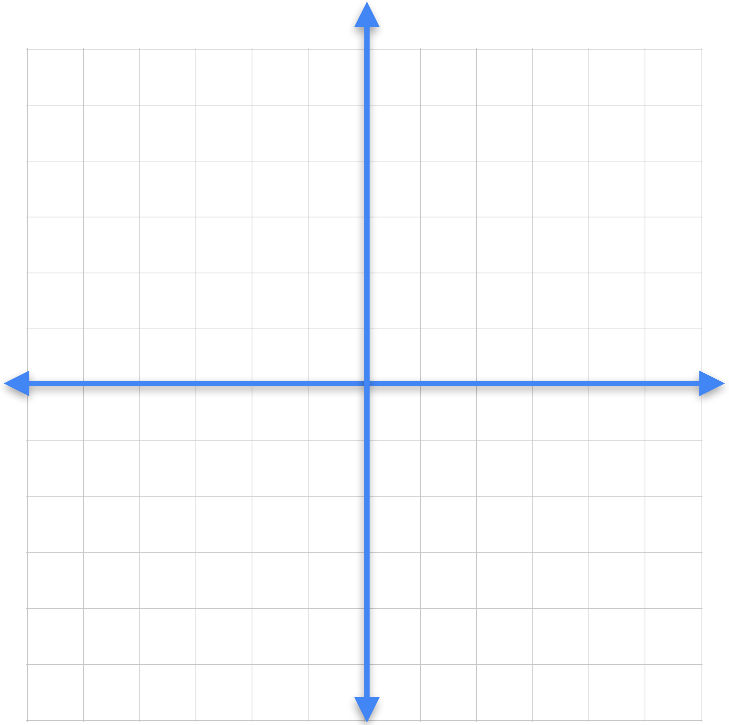
3	1
1	2



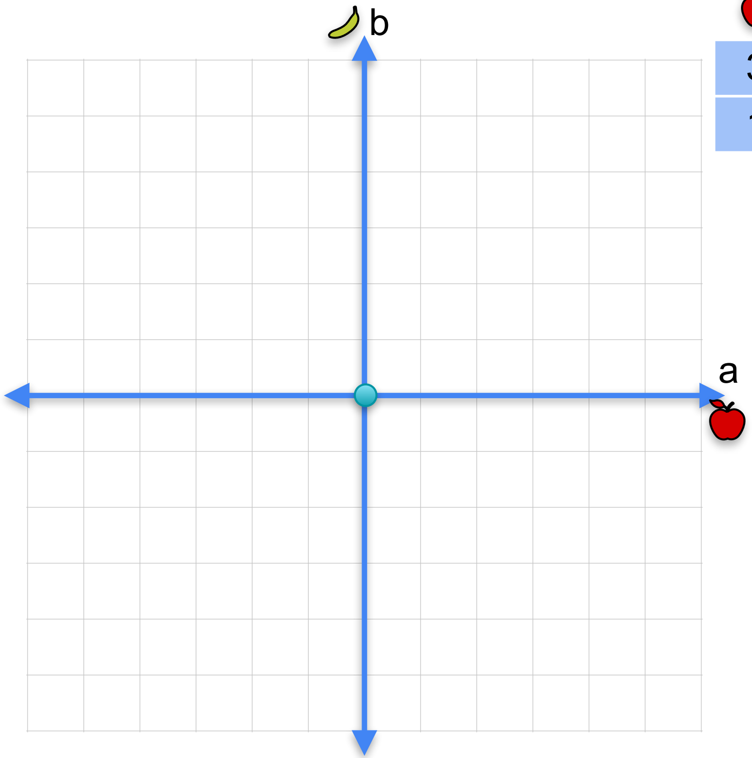
Matrices as linear transformations



🍎	🍌
3	1
1	2

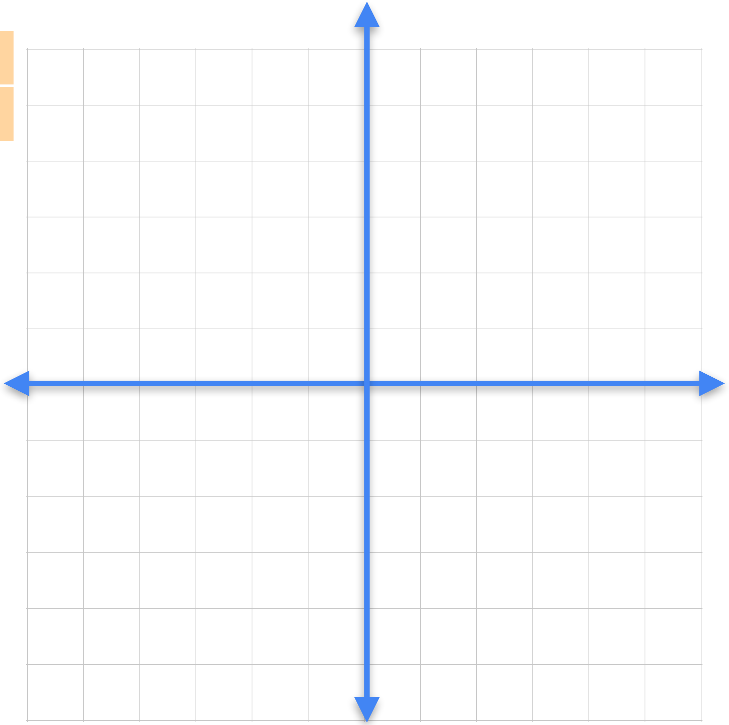


Matrices as linear transformations

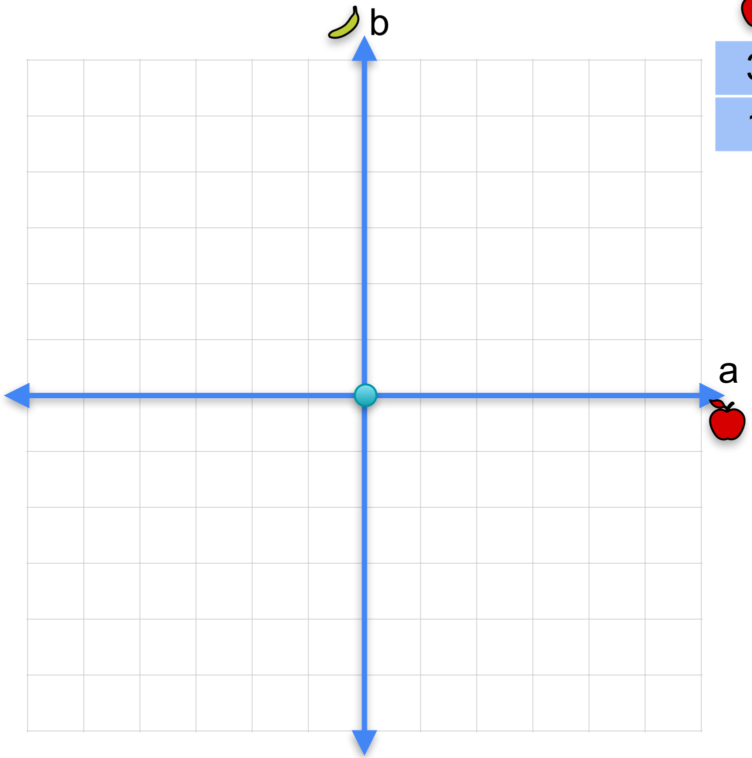


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)

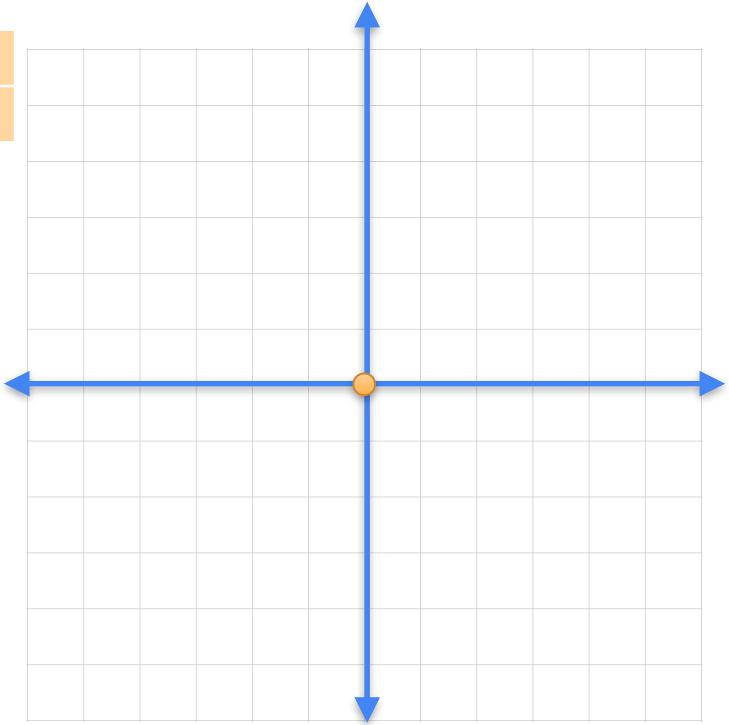


Matrices as linear transformations

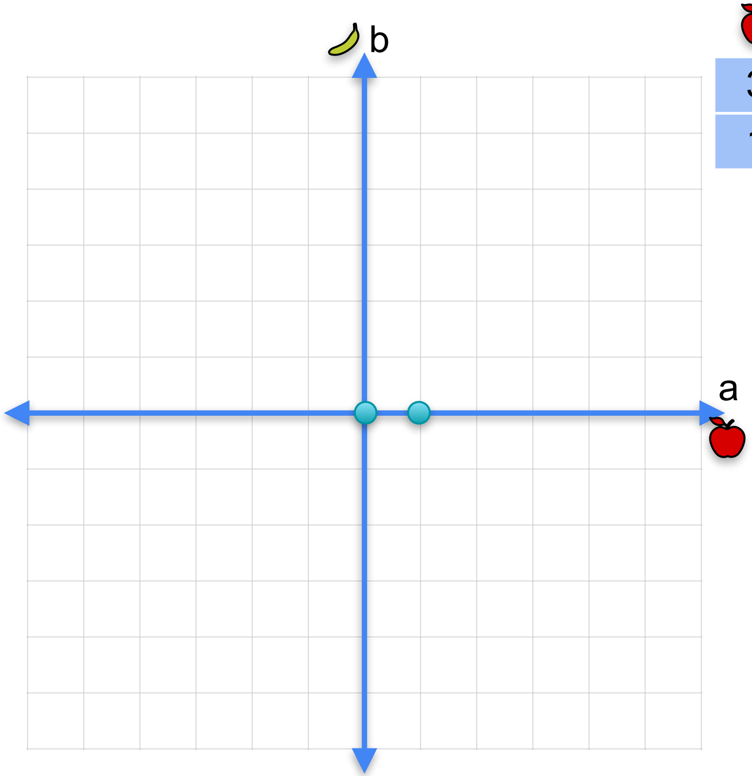


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)

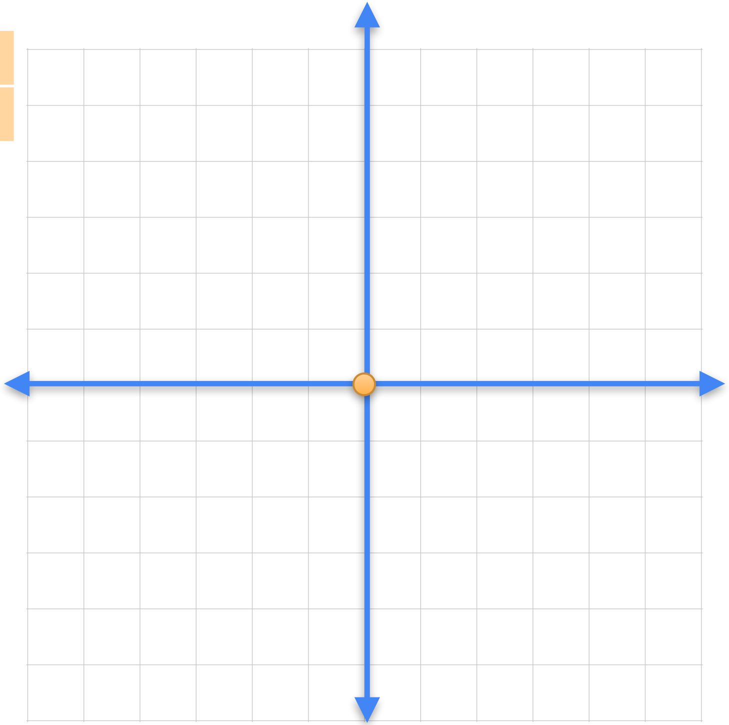


Matrices as linear transformations

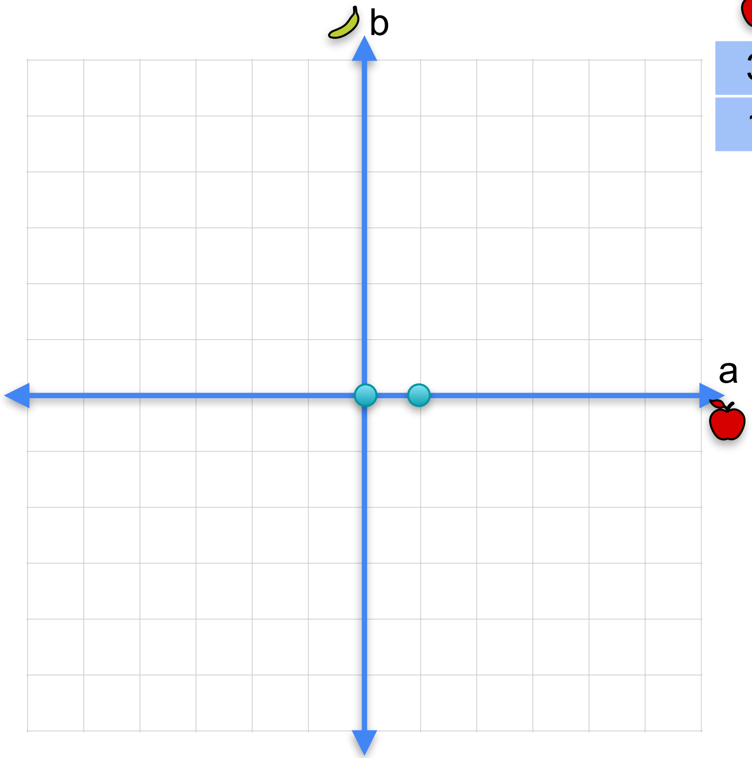


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(0,0) → (0,0)



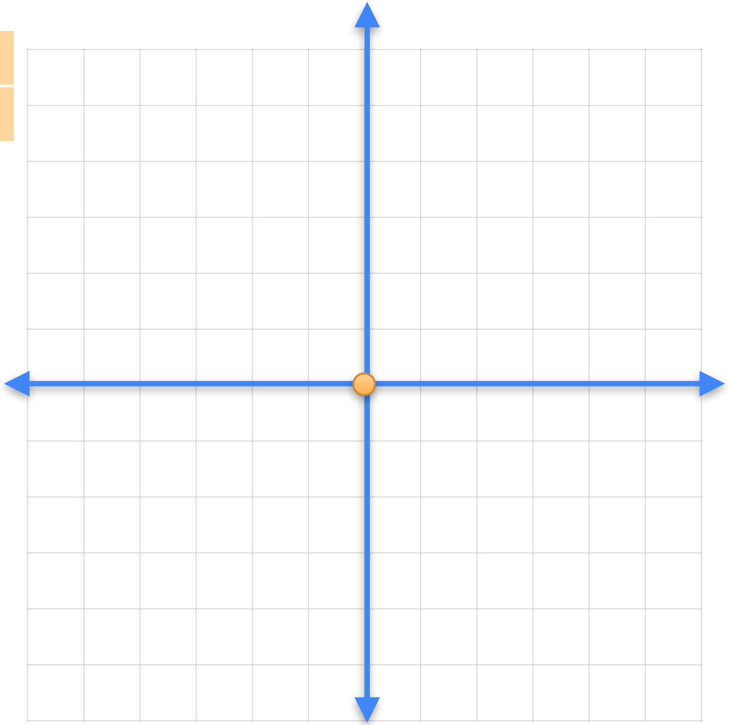
Matrices as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



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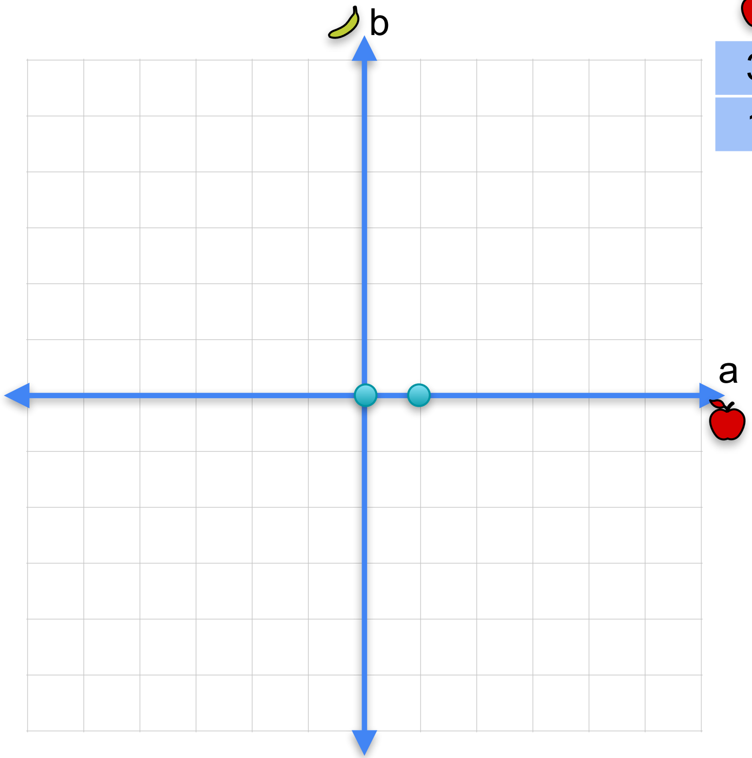
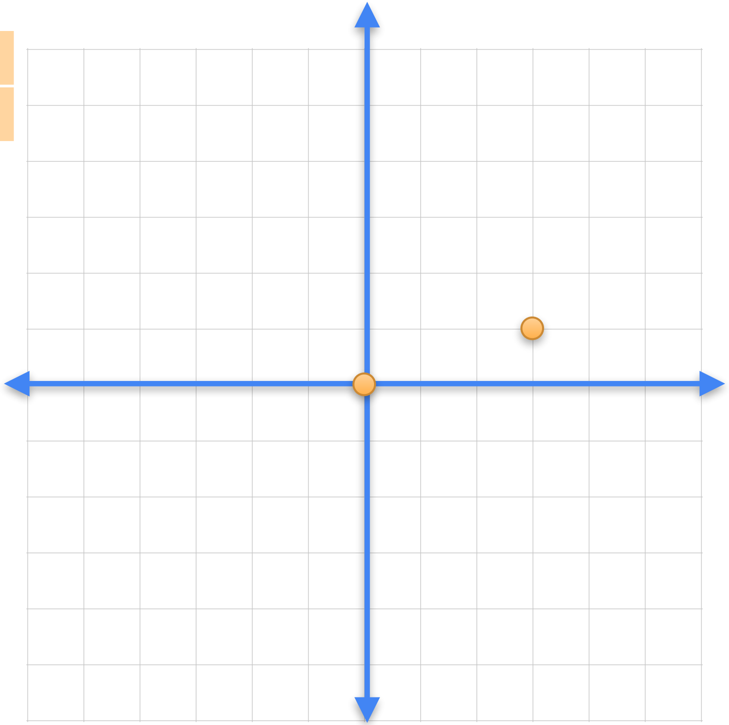
Matrices as linear transformations



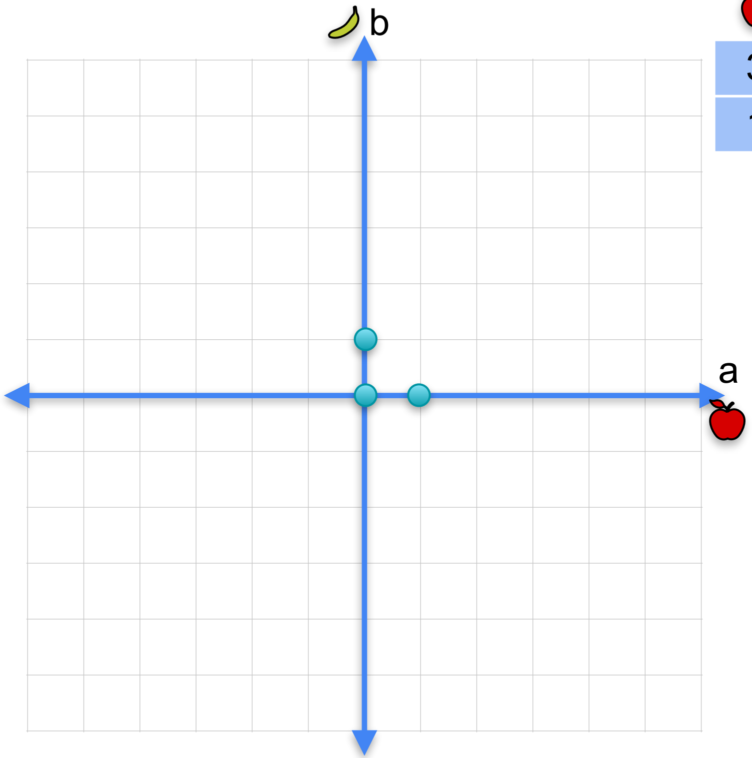
$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}3\\ 1\end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



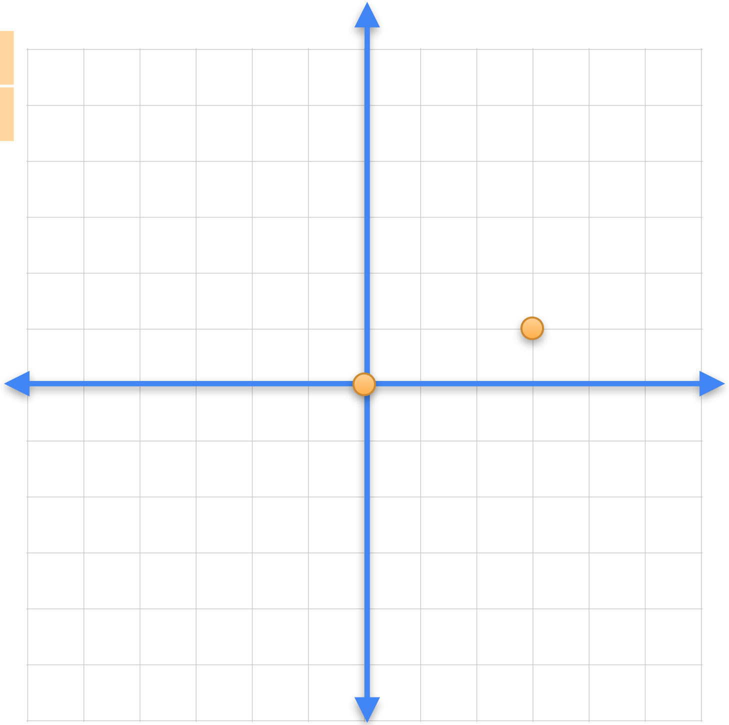
Matrices as linear transformations



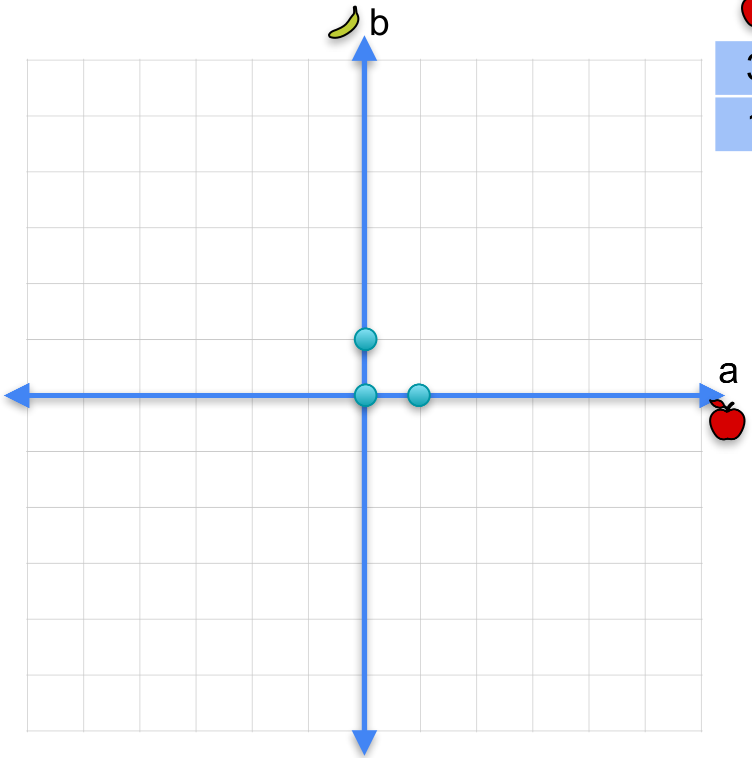
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



Matrices as linear transformations

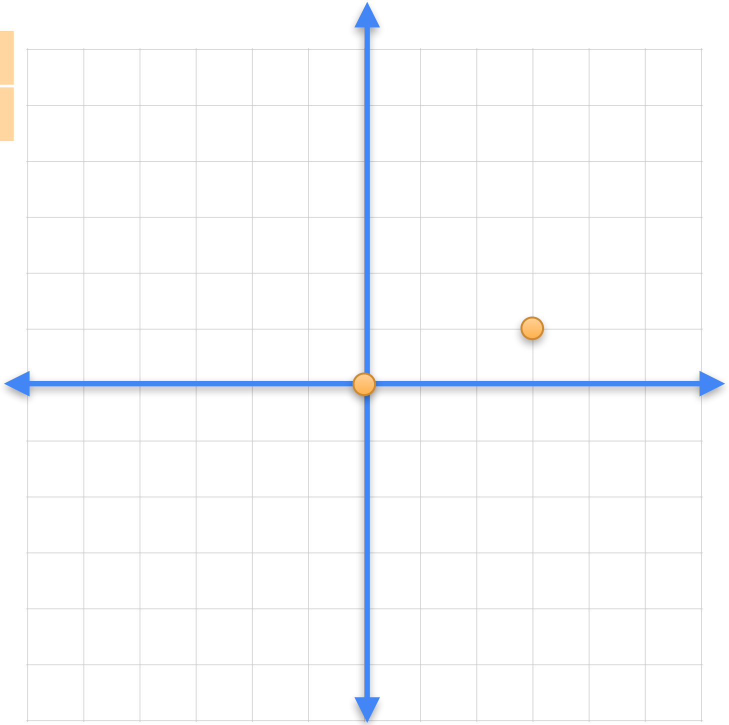


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

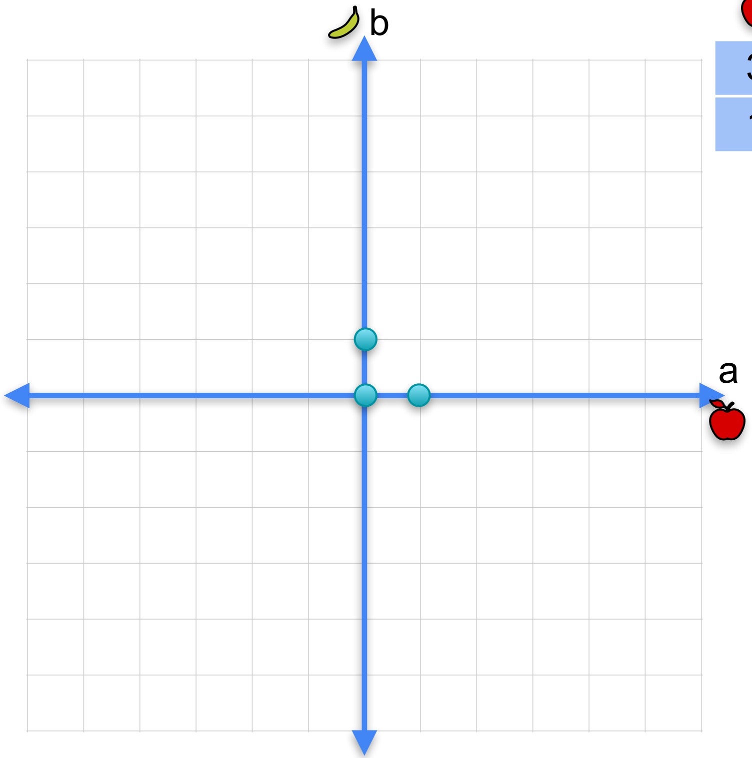
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)



Matrices as linear transformations

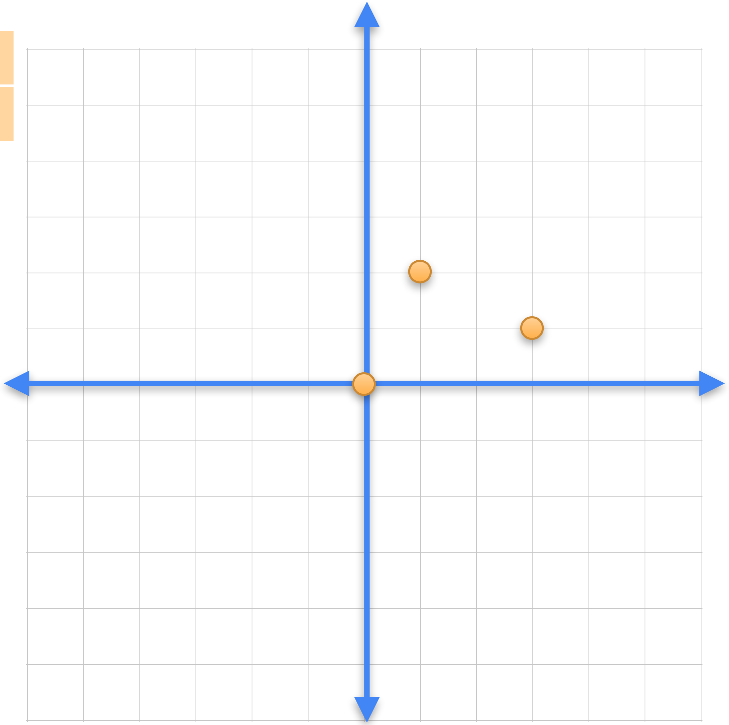


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

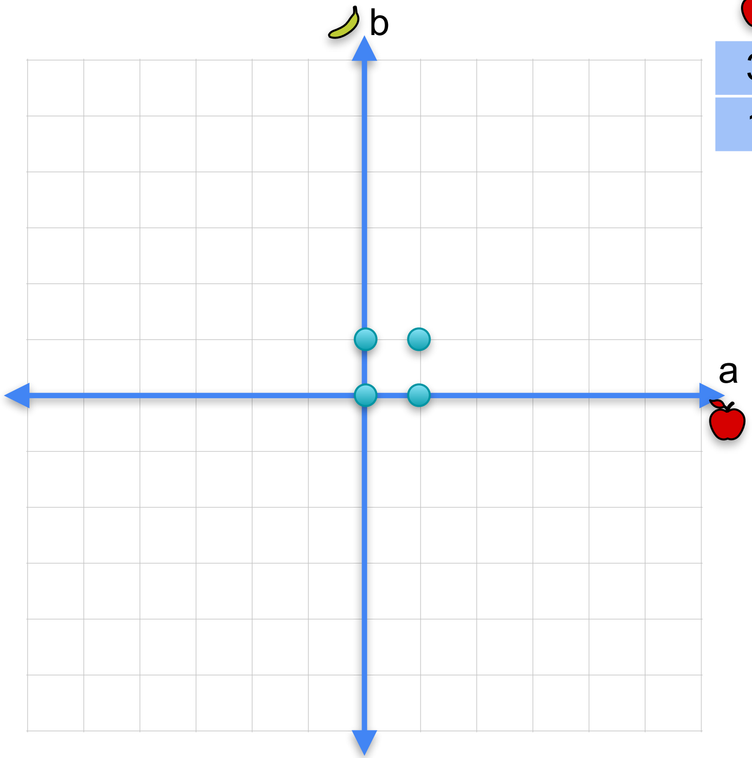
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)



Matrices as linear transformations

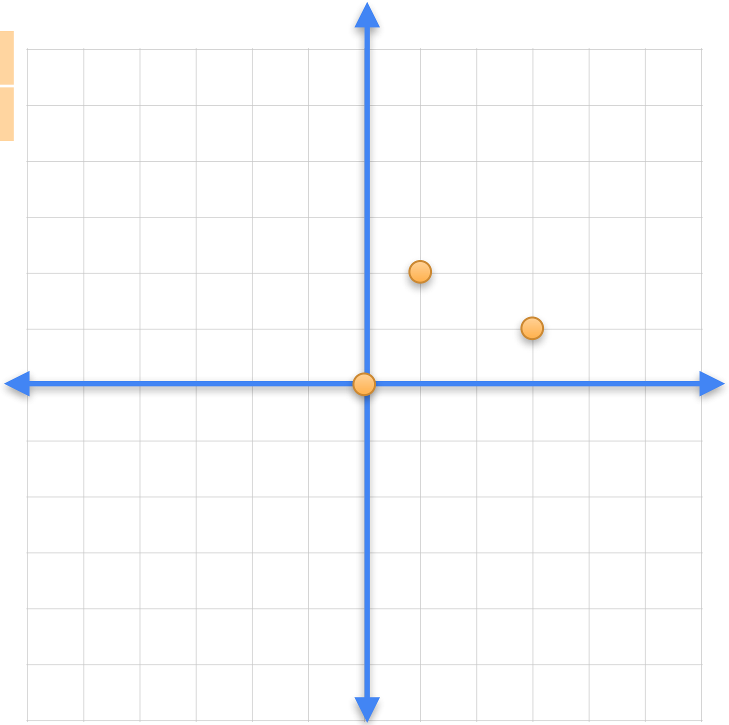


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

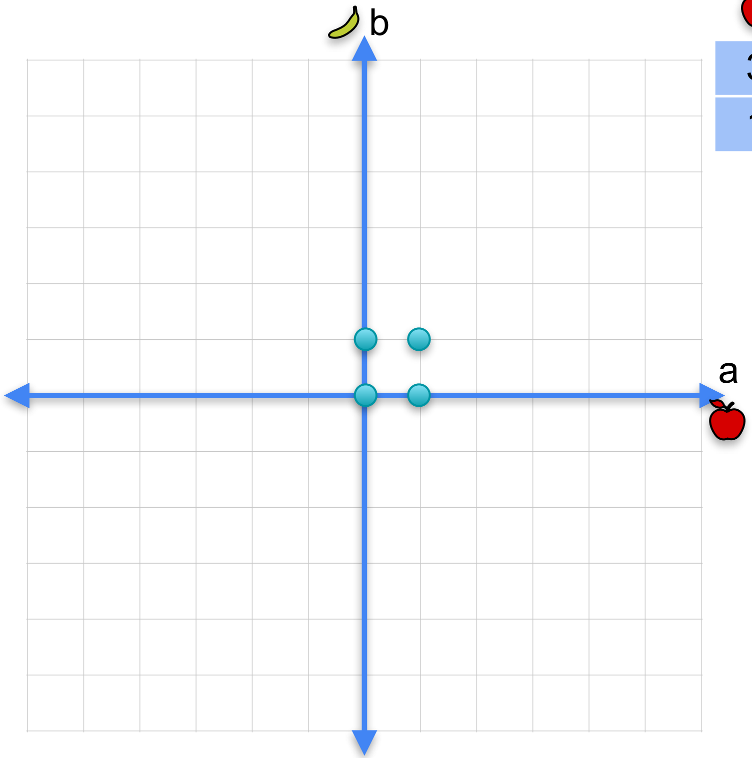
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)



Matrices as linear transformations



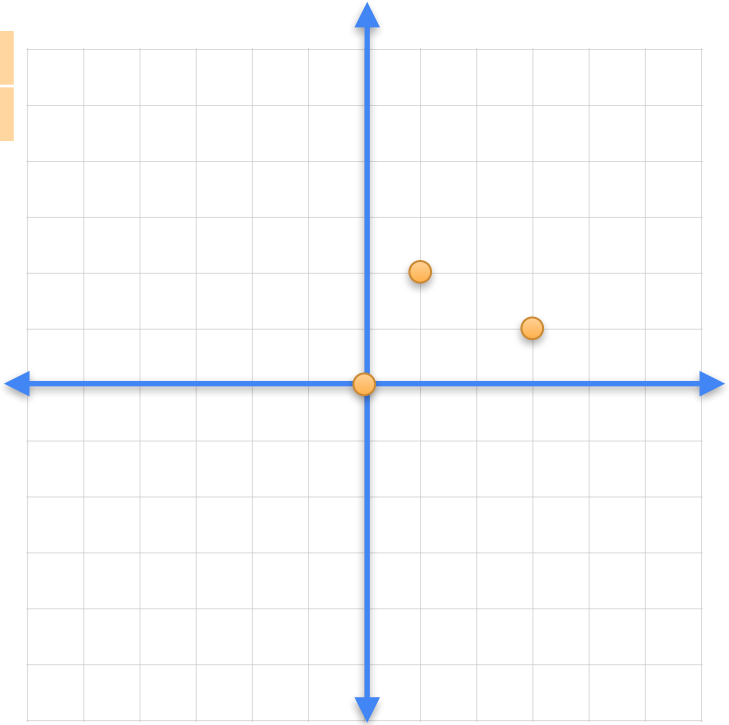
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

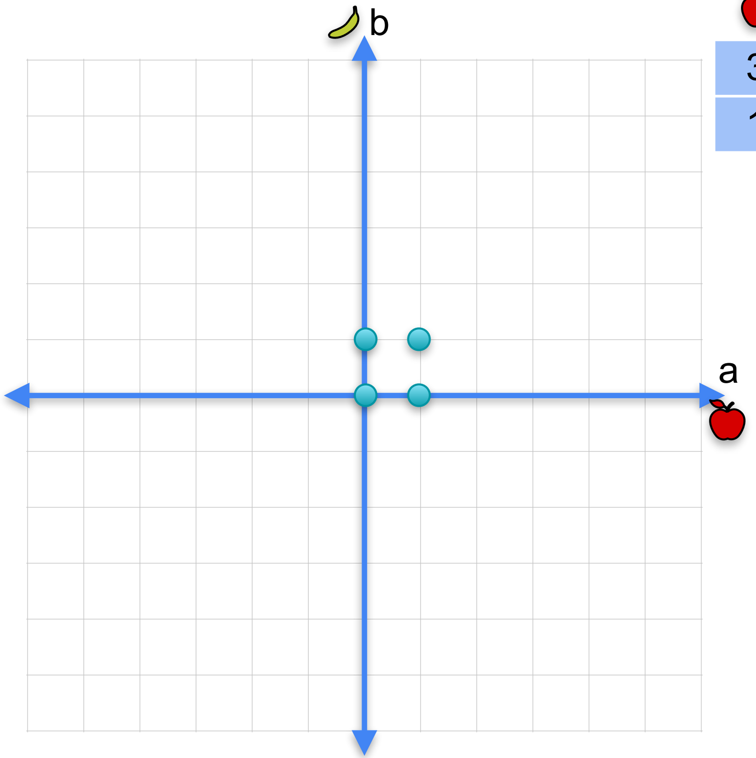
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



Matrices as linear transformations



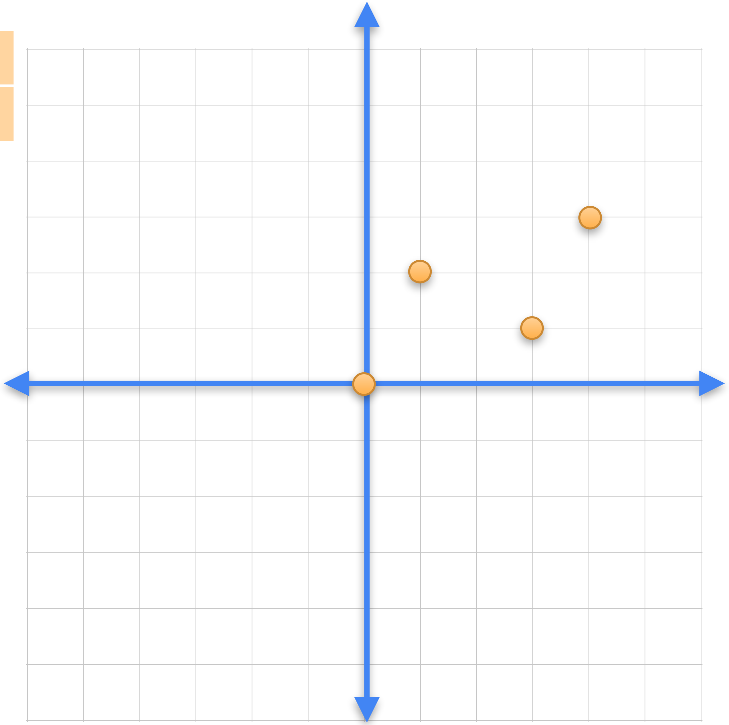
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

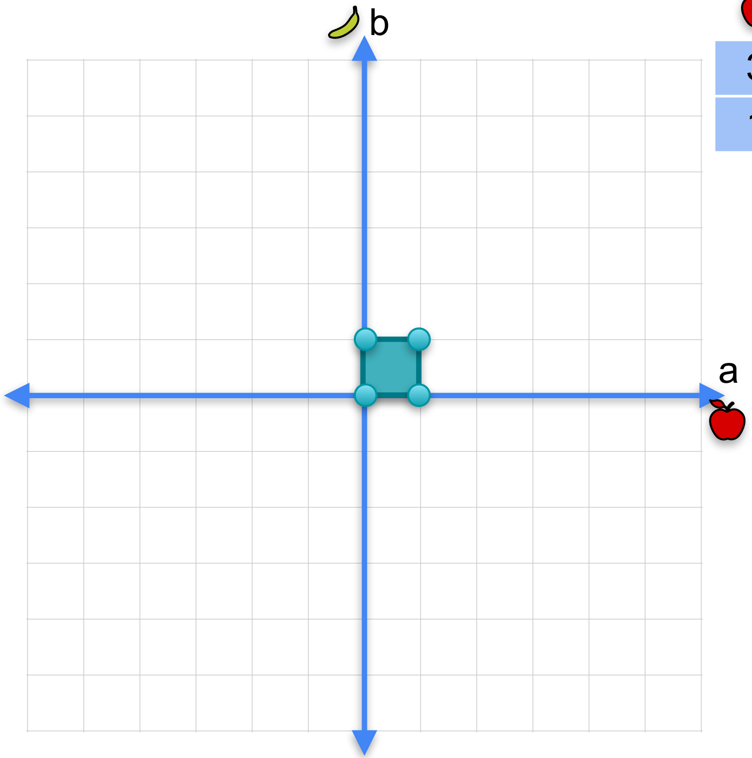
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)

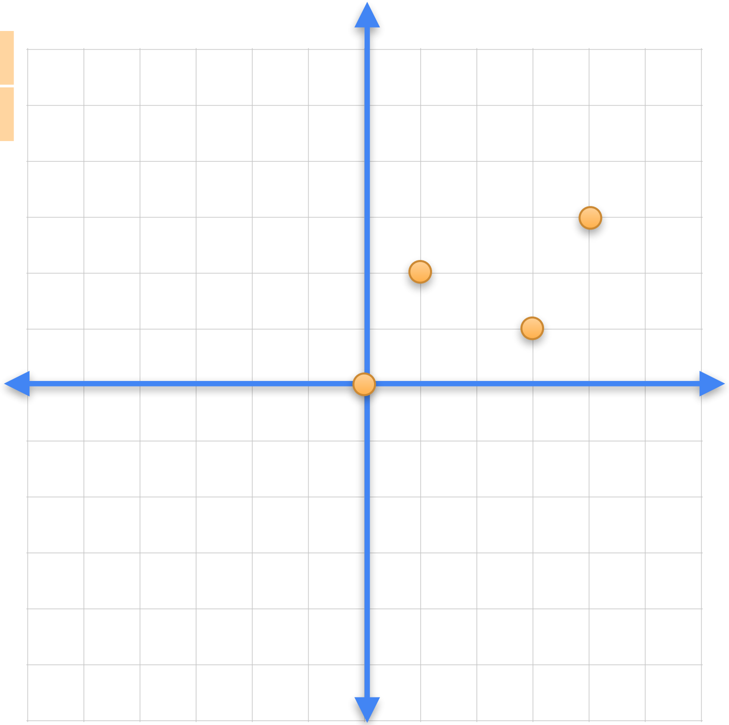


Matrices as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)
(1,0) → (3,1)
(0,1) → (1,2)
(1,1) → (4,3)



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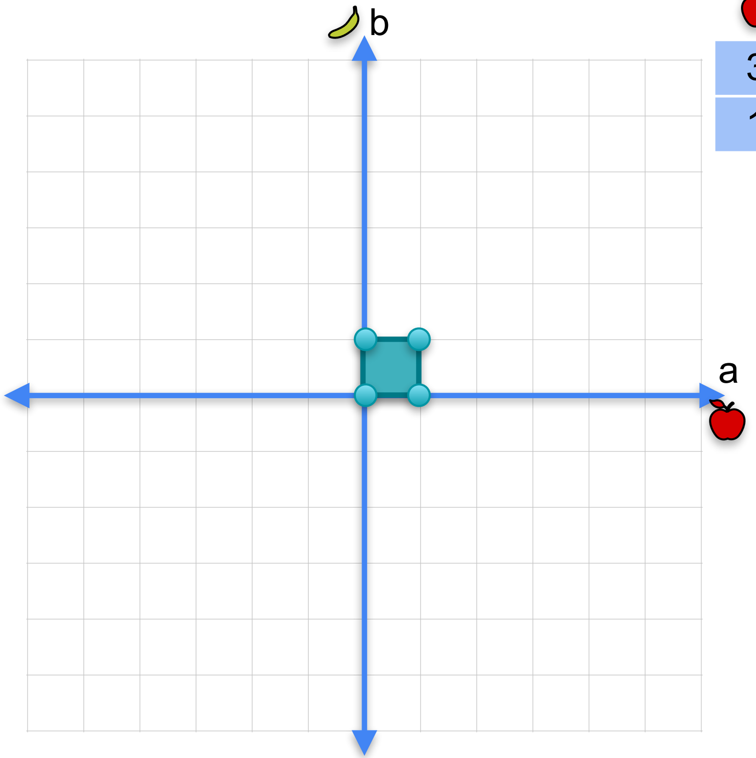
Matrices as linear transformations



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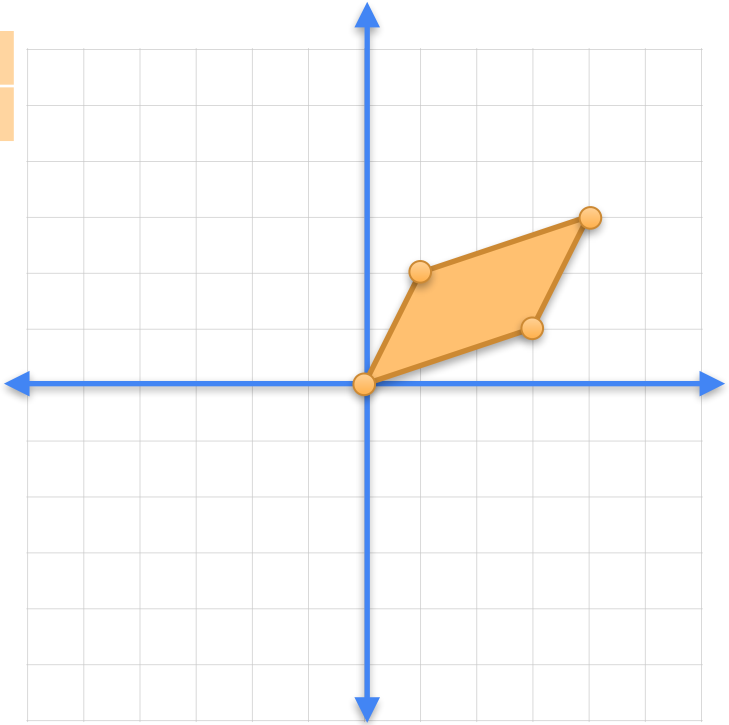
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

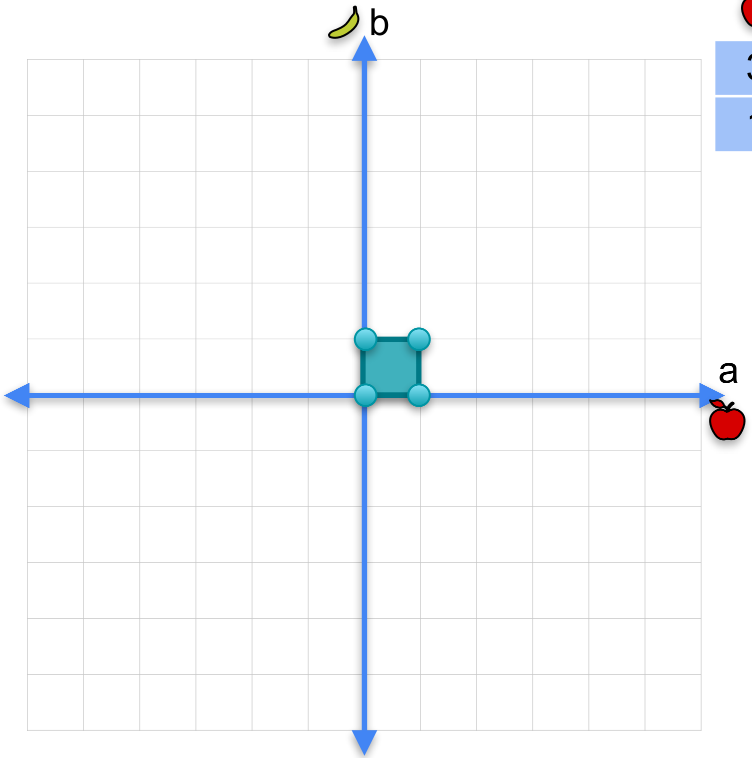
(1,0) → (3,1)

(0,1) → (1,2)

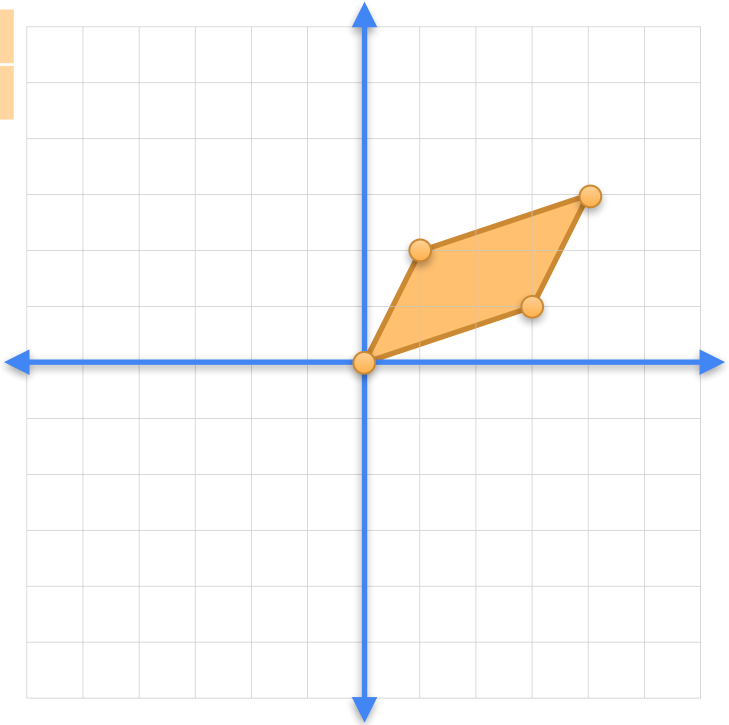
(1,1) → (4,3)



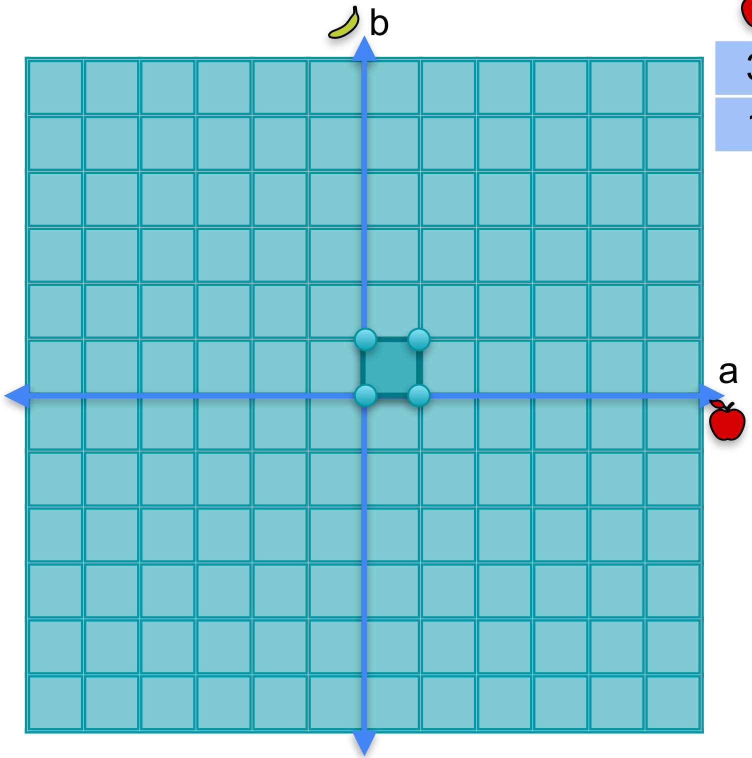
Matrices as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

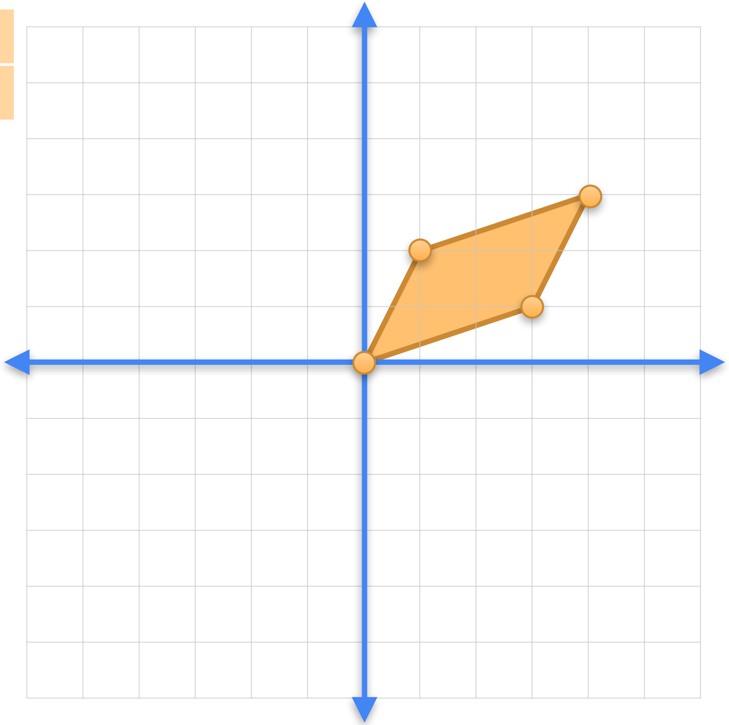


Matrices as linear transformations

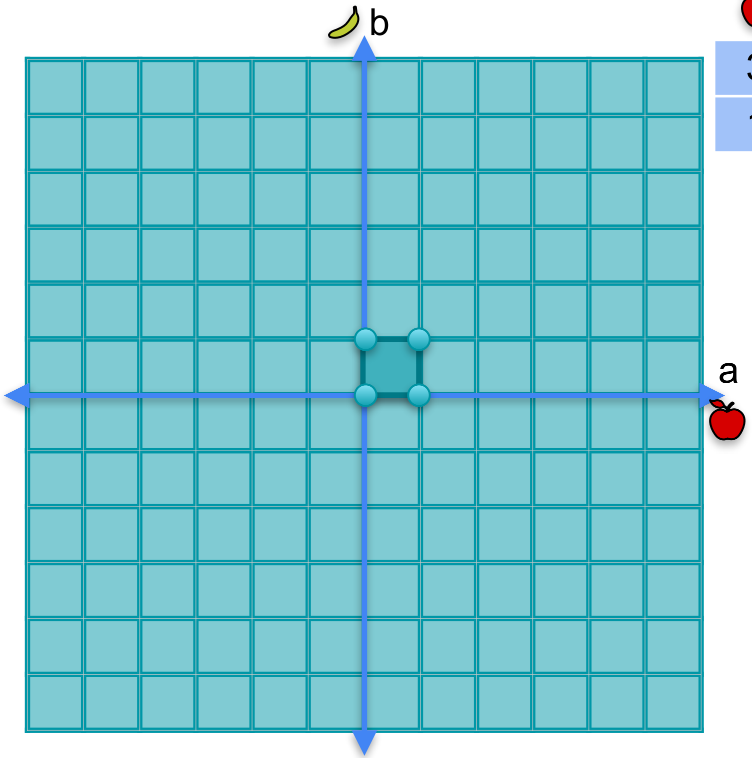


(🍎, 🍌)

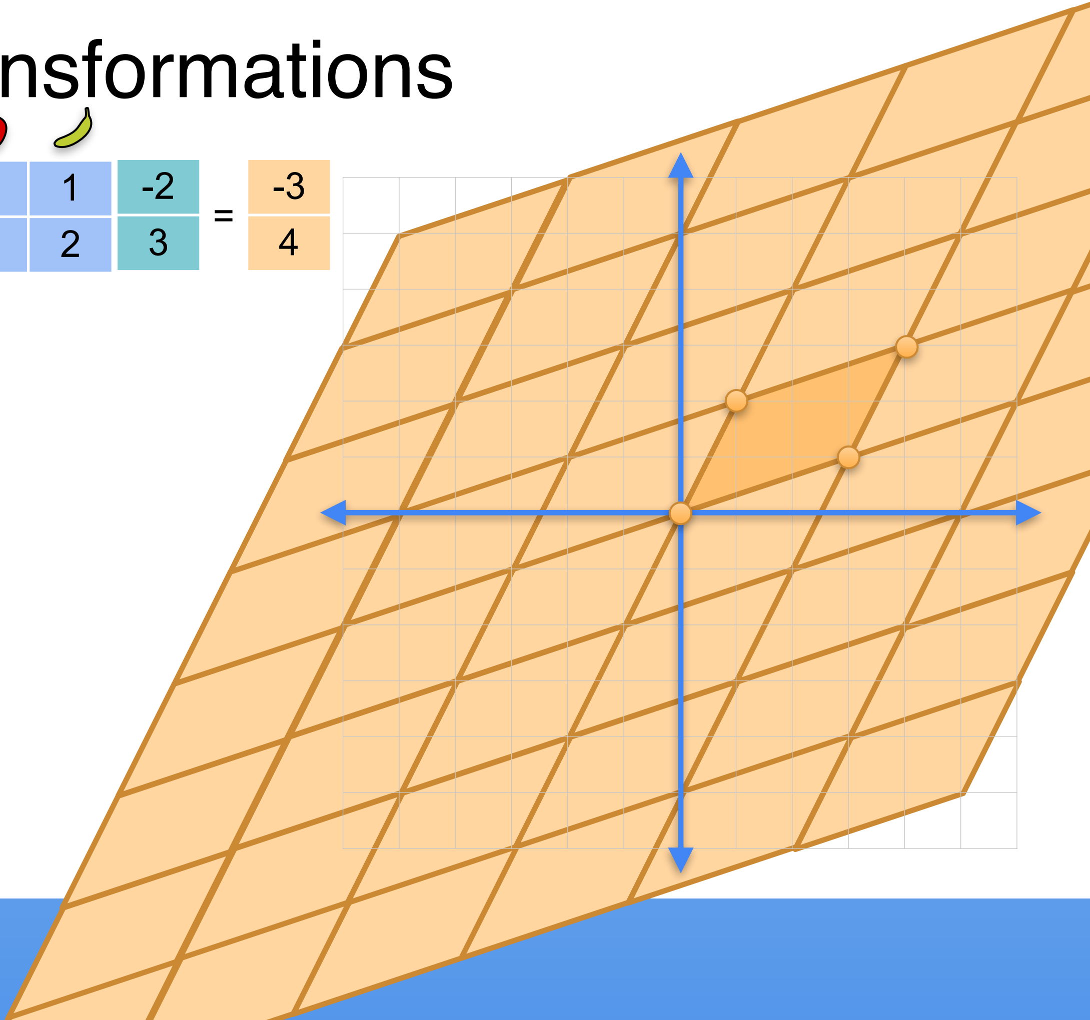
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



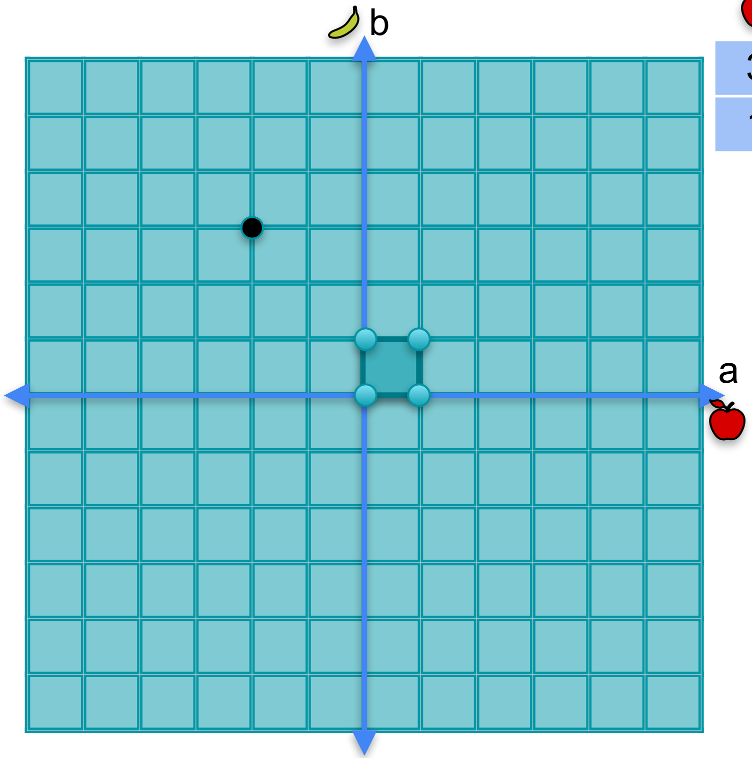
Matrices as linear transformations



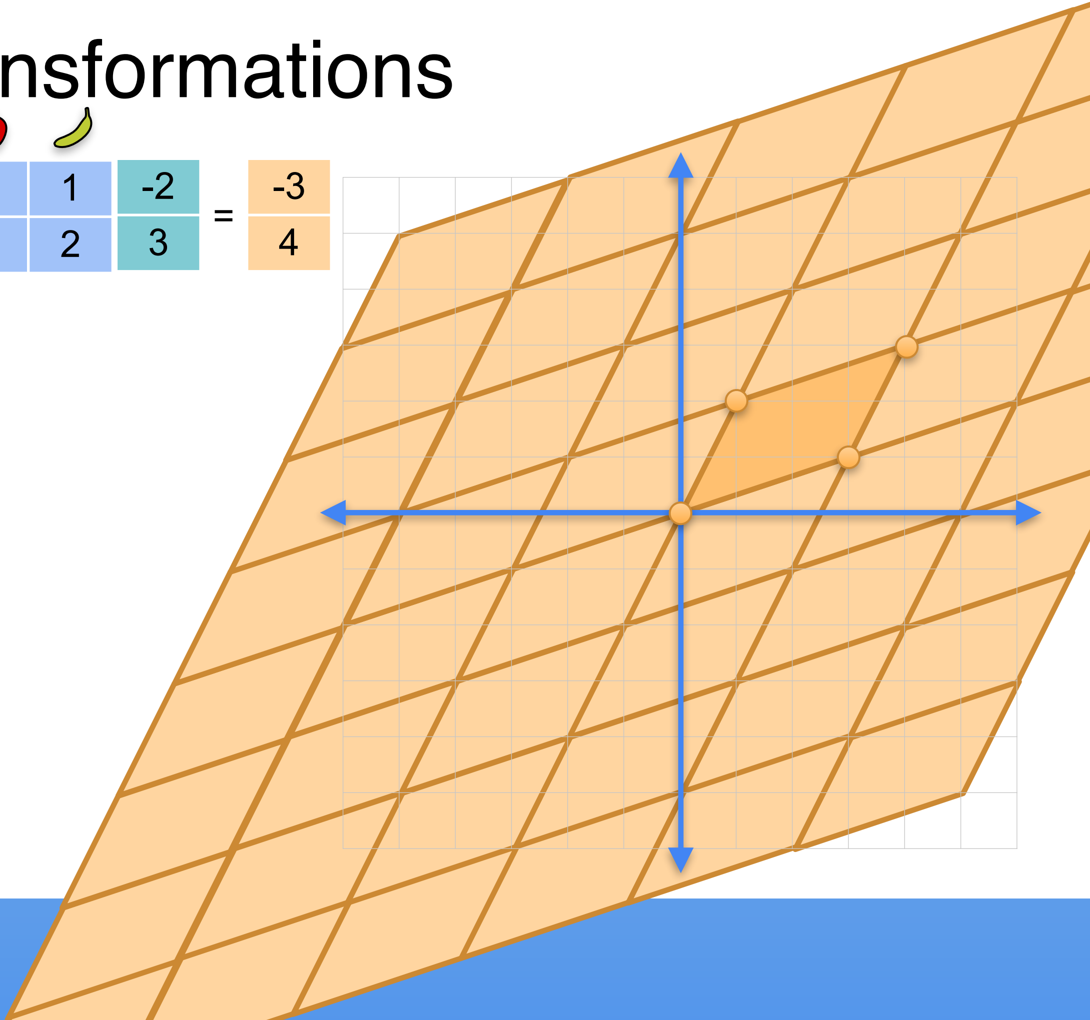
3	1	-2	=	-3
1	2	3		4



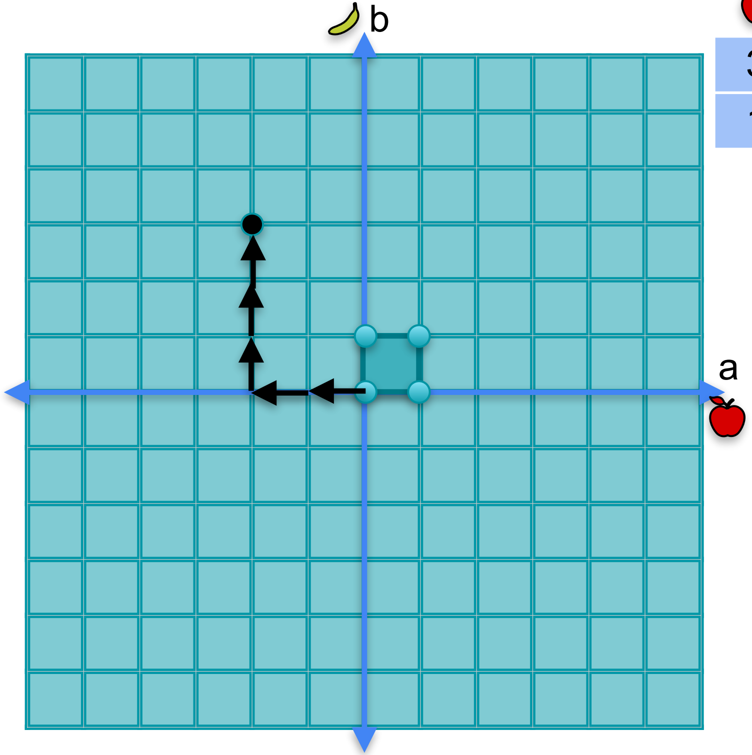
Matrices as linear transformations



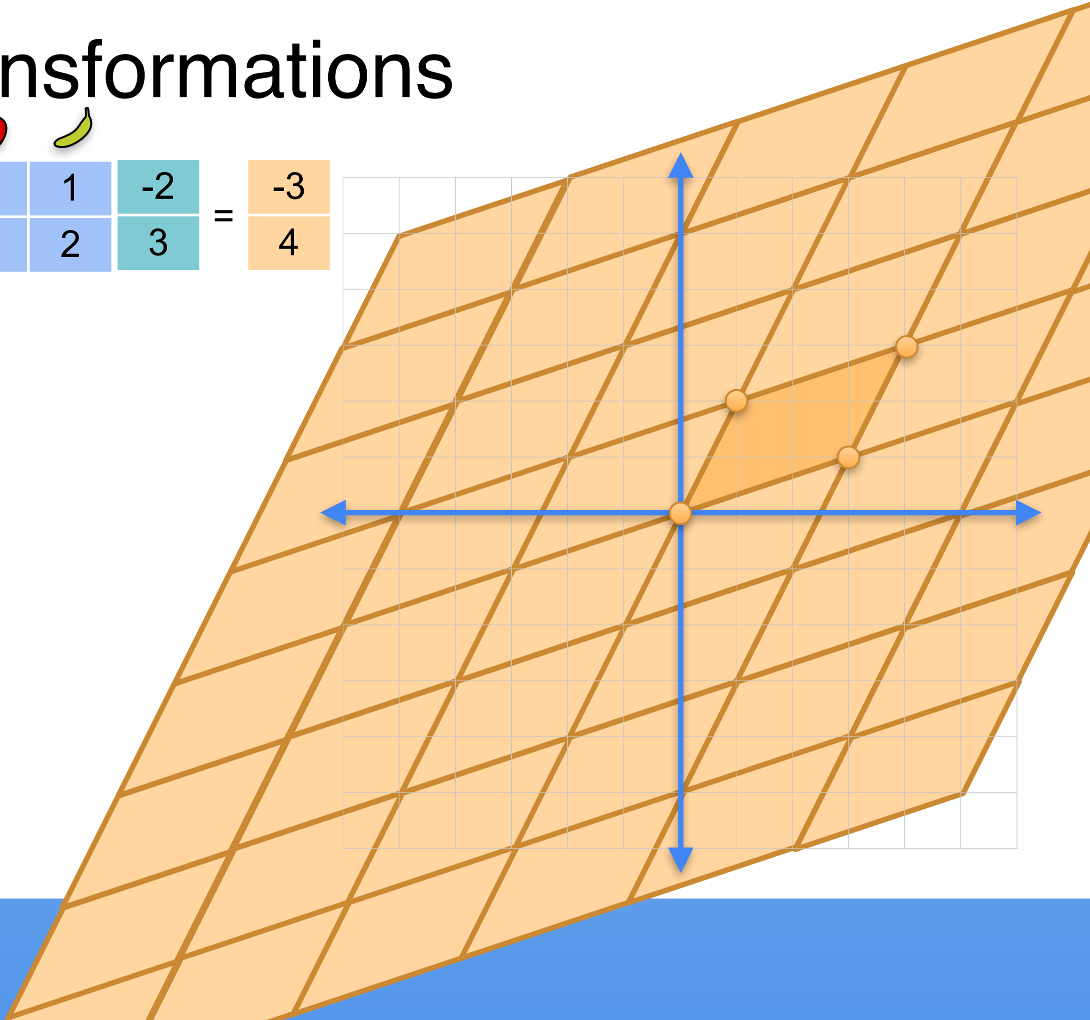
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



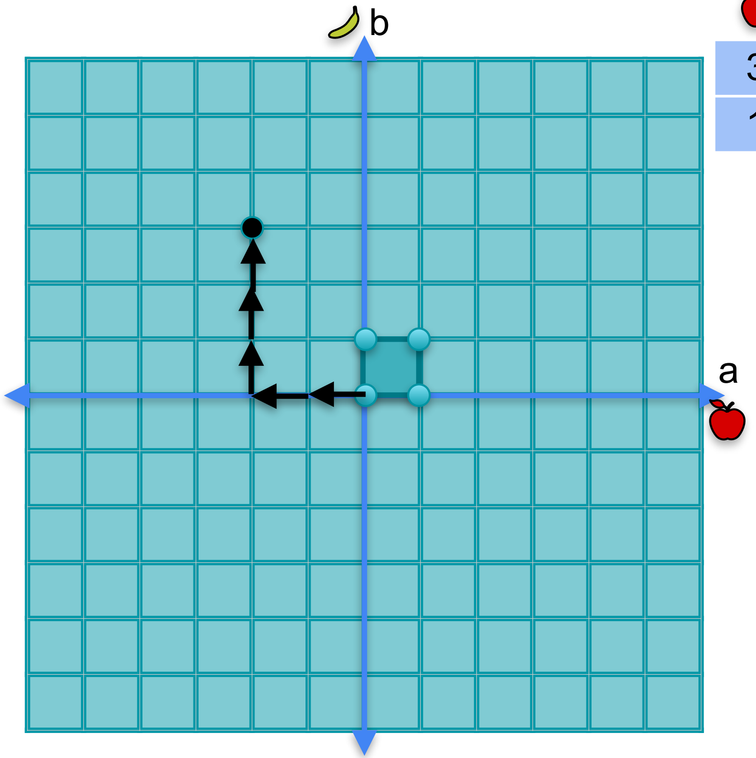
Matrices as linear transformations



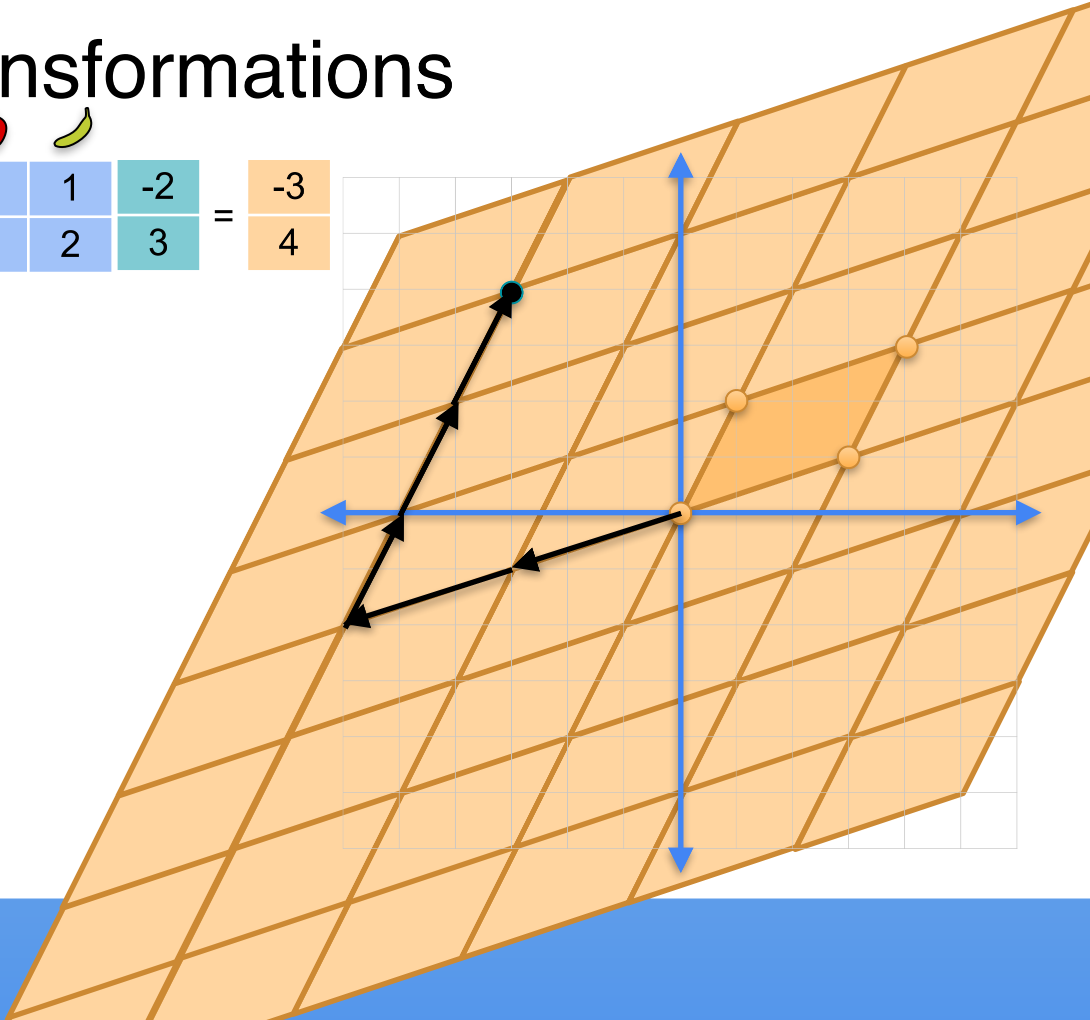
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



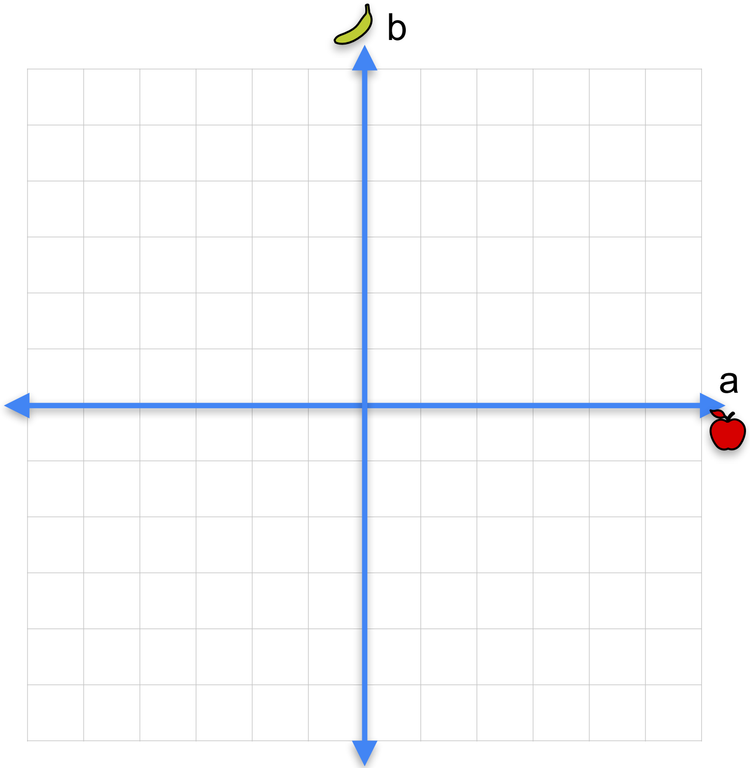
Matrices as linear transformations

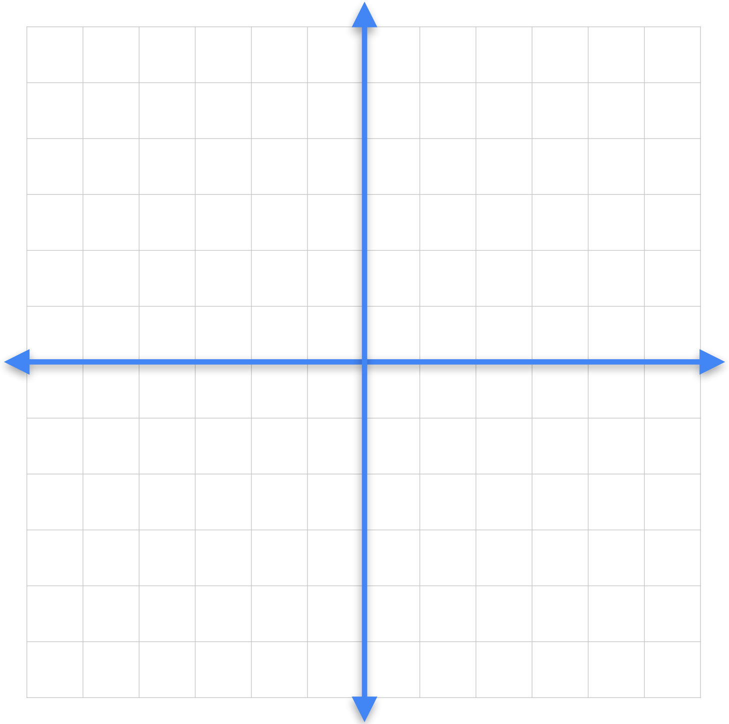


3	1	-2	=	-3
1	2	3		4



Systems of equations as linear transformations





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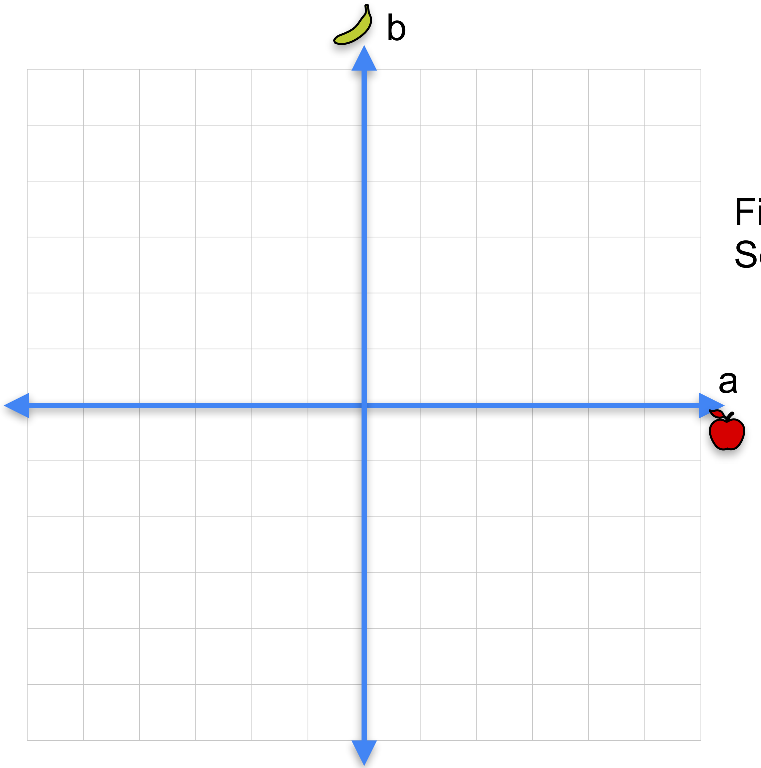
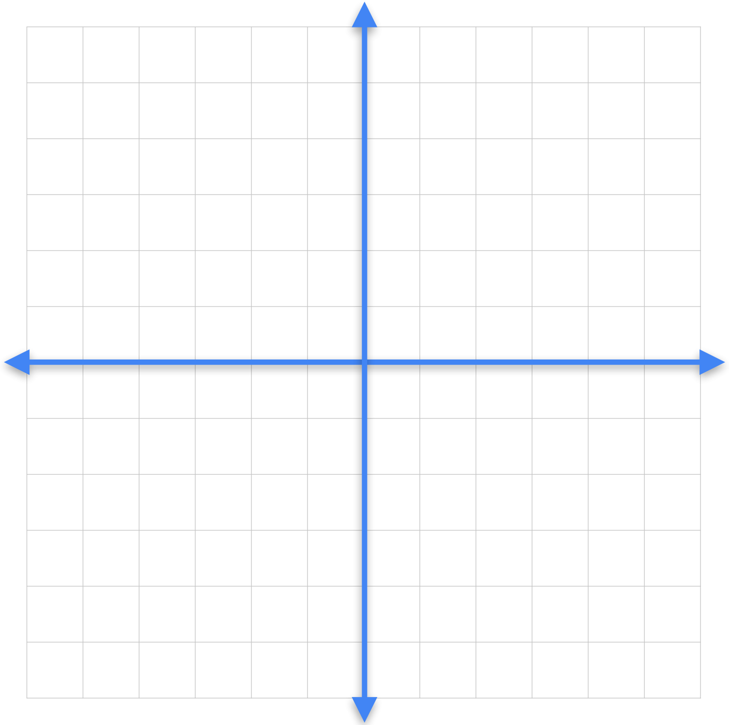
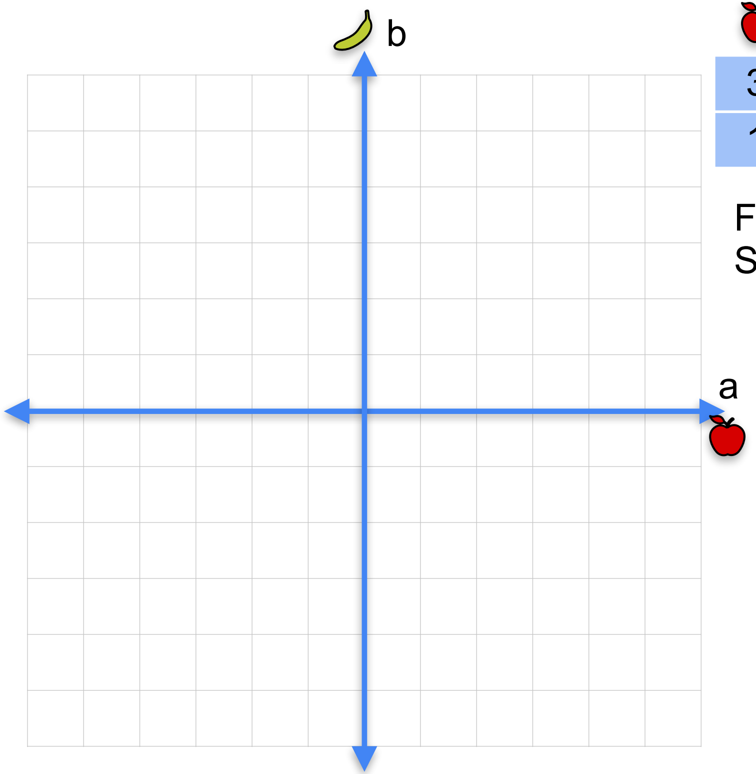
Systems of equations as linear transformations



First day: 3a + b
Second day: a + 2b

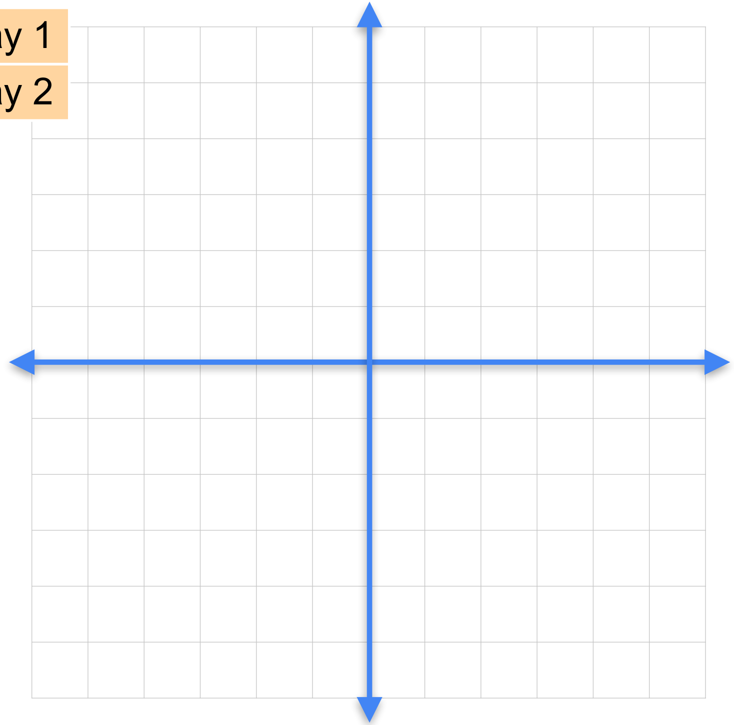


Systems of equations as linear transformations

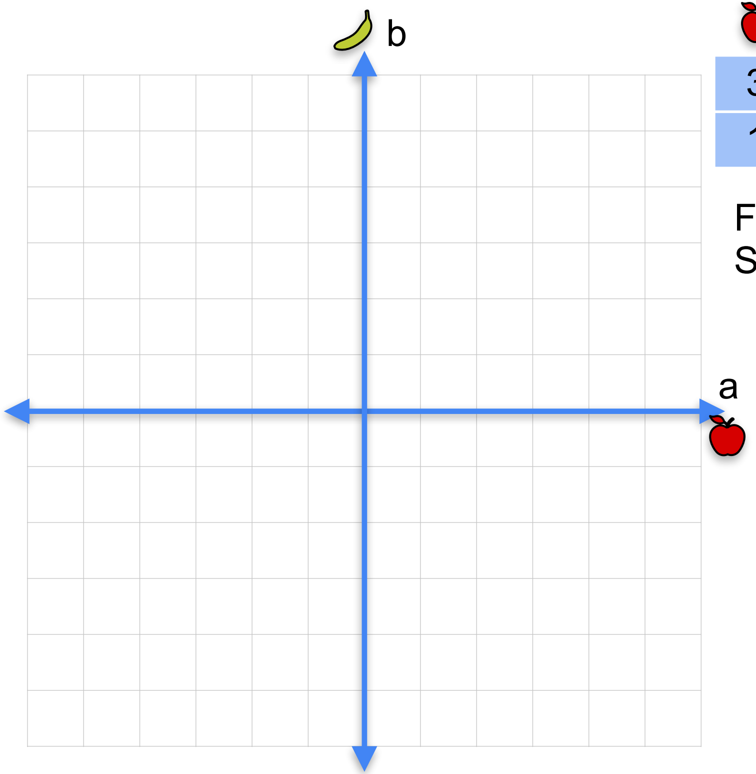


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \text{Day 1} \\ \text{Day 2} \end{bmatrix}$$

First day:	$3a + b$
Second day:	$a + 2b$

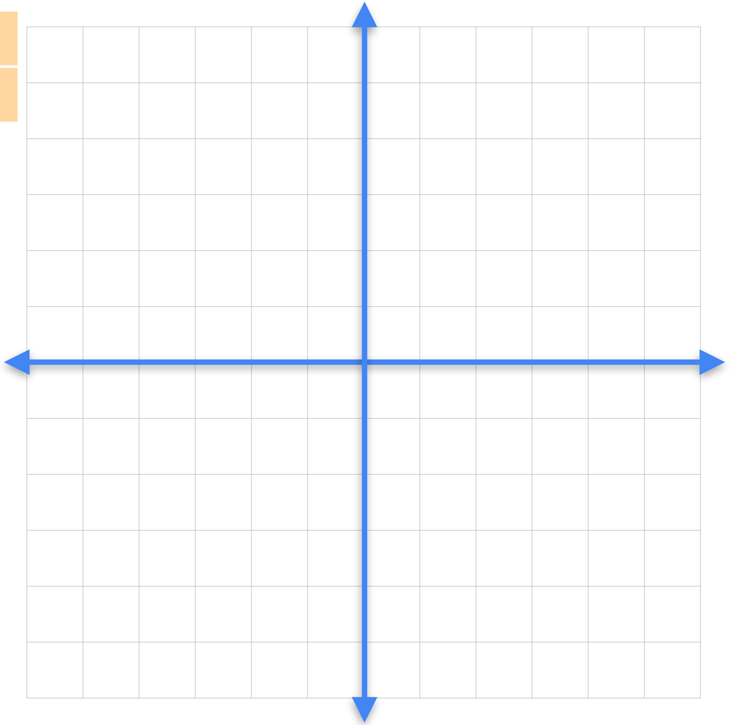


Systems of equations as linear transformations

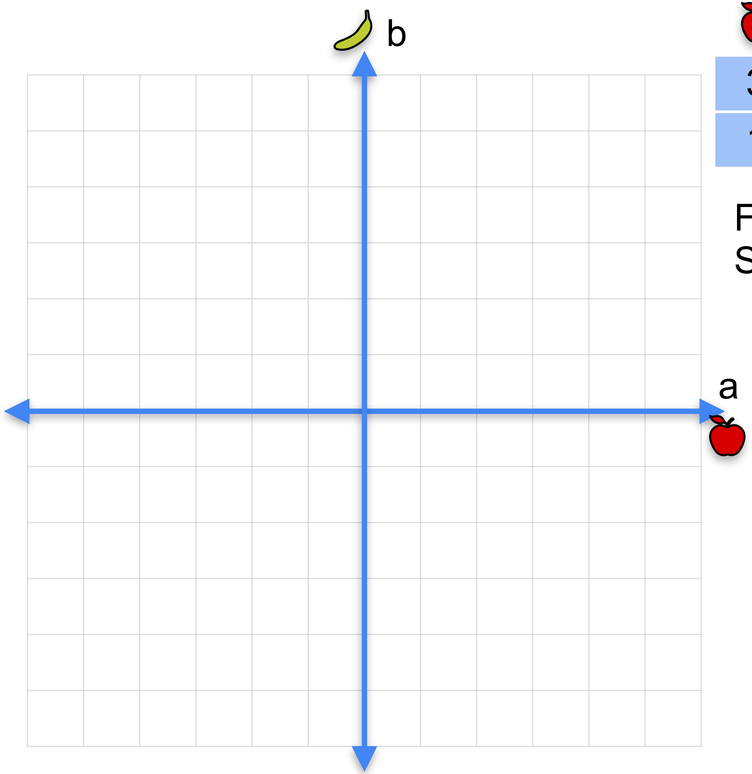


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: $3a + b$
Second day: $a + 2b$



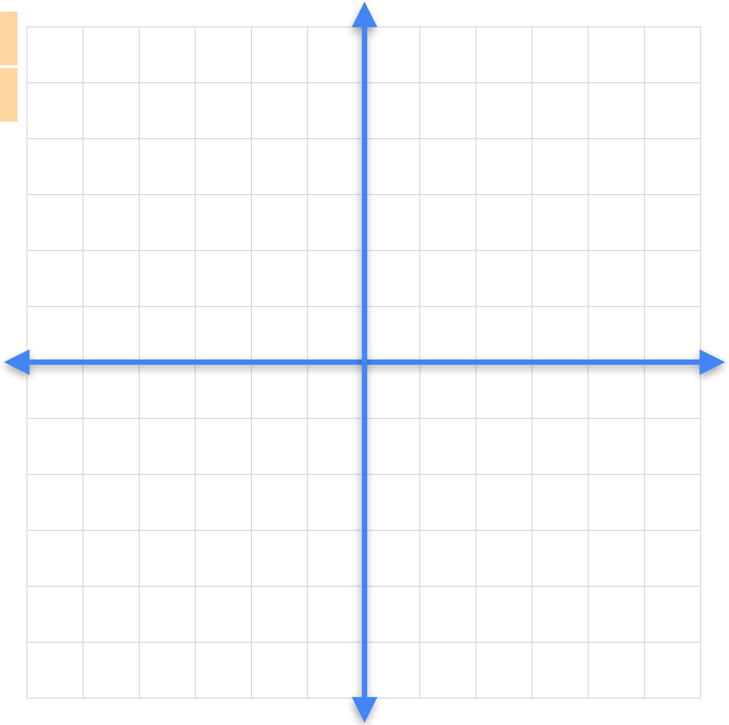
Systems of equations as linear transformations



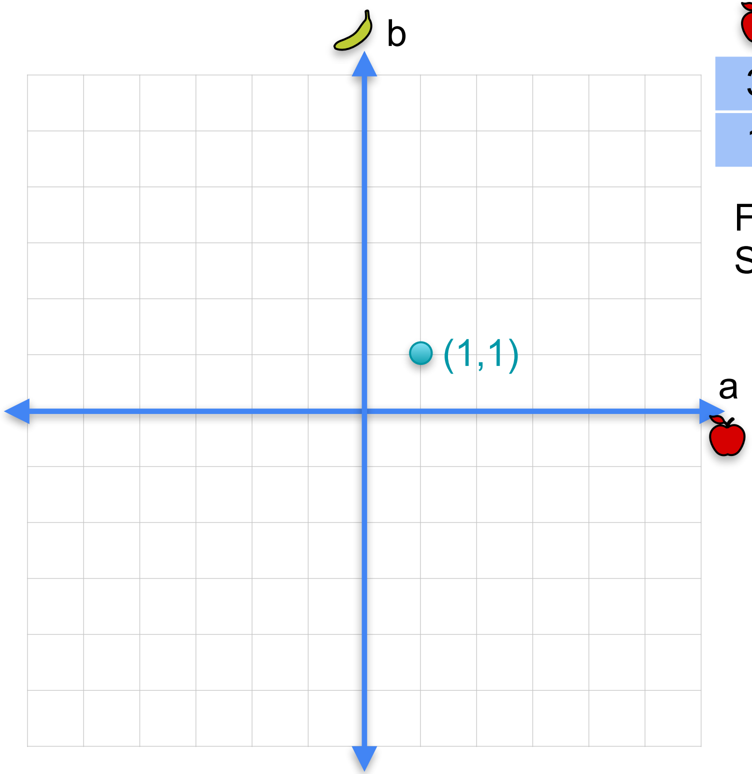
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: 3a + b
Second day: a + 2b

(1,1) → (4,3)



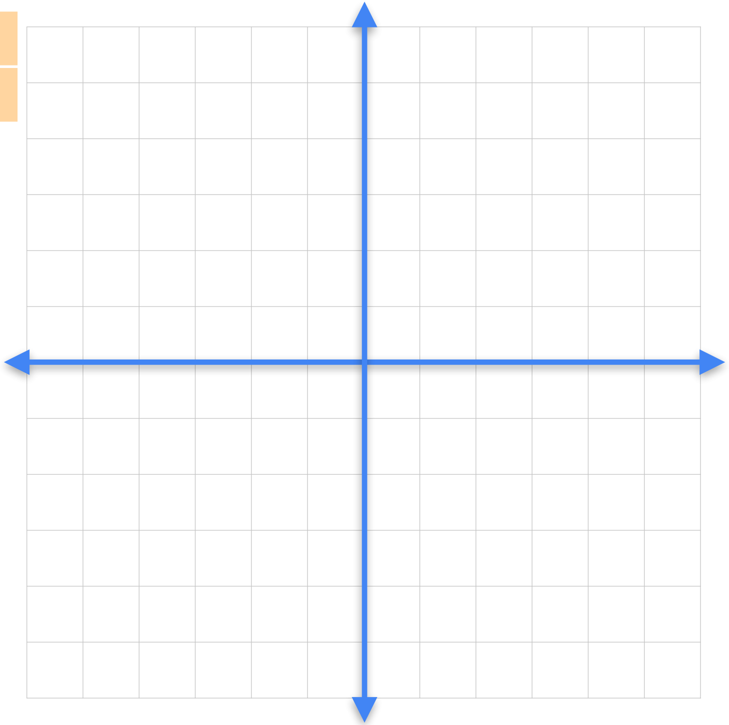
Systems of equations as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: 3a + b
Second day: a + 2b

(1,1) → (4,3)



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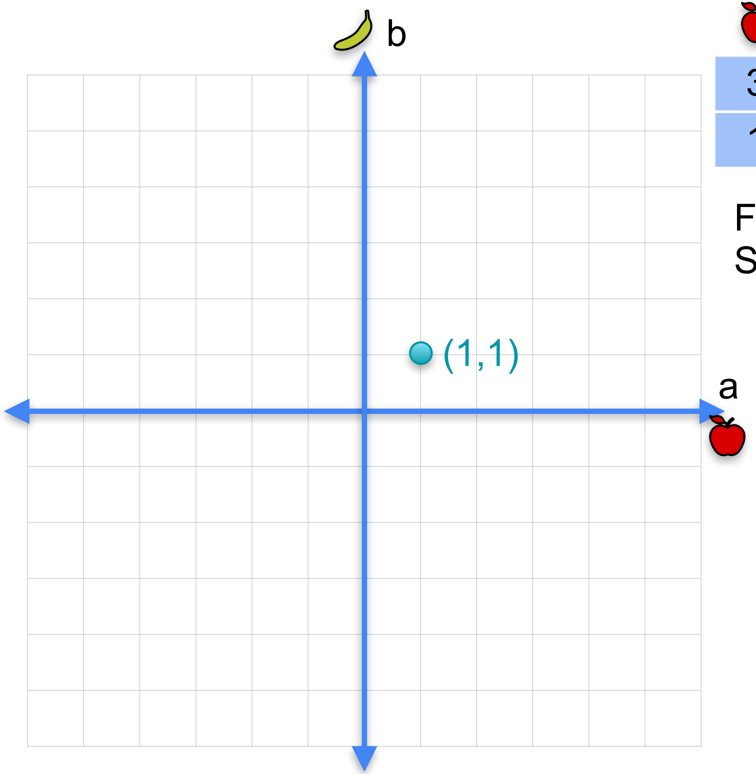
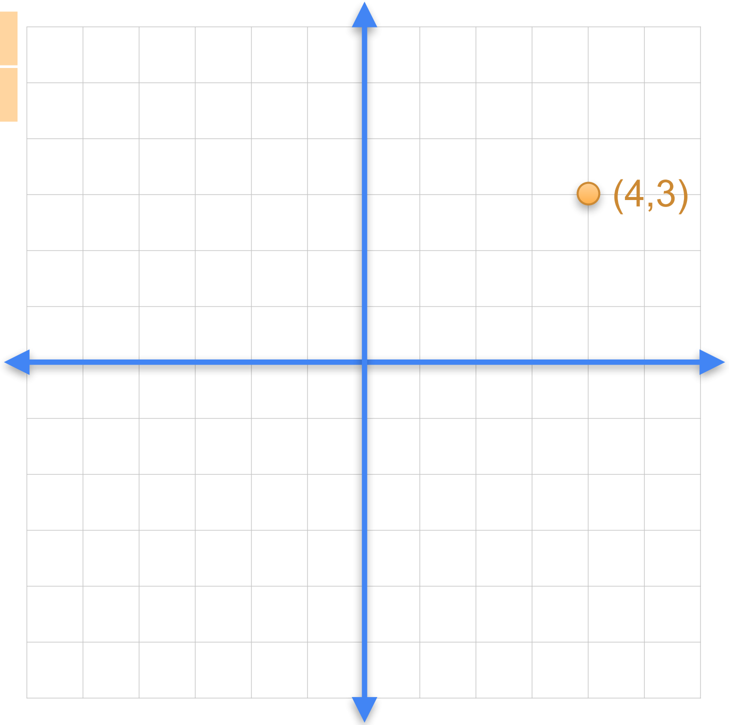
Systems of equations as linear transformations



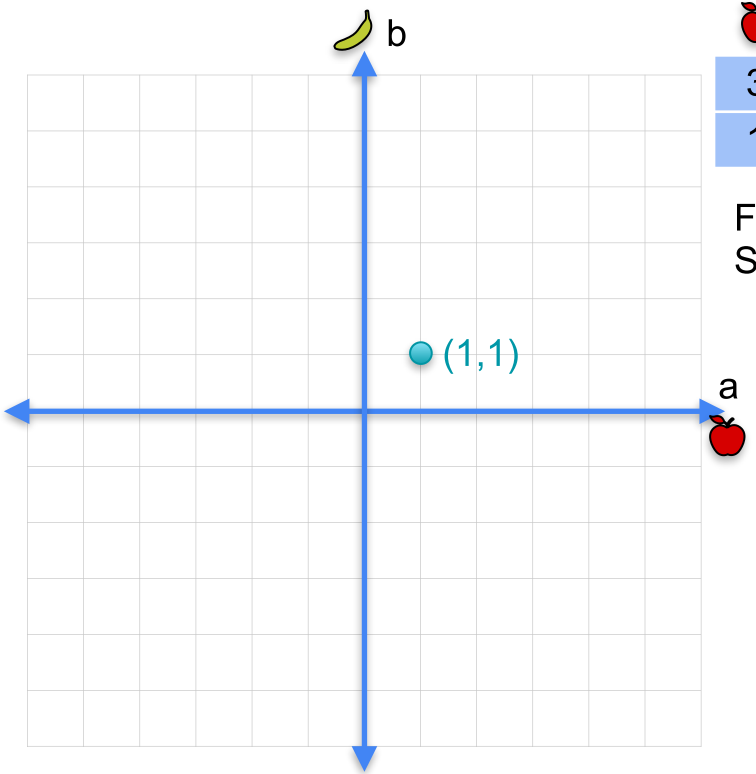
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: $3a + b$
Second day: $a + 2b$

(1,1) → (4,3)



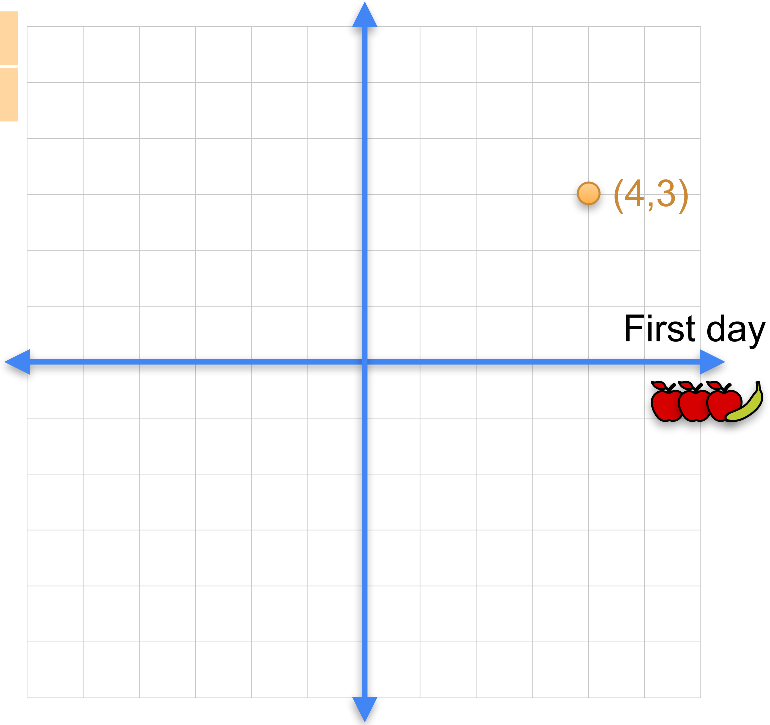
Systems of equations as linear transformations



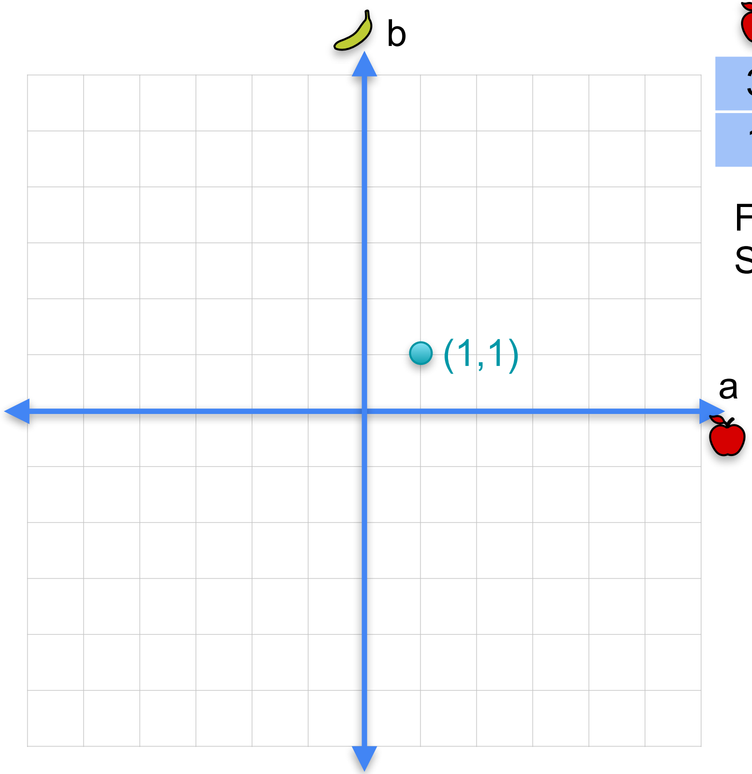
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: 3a + b
Second day: a + 2b

(1,1) → (4,3)



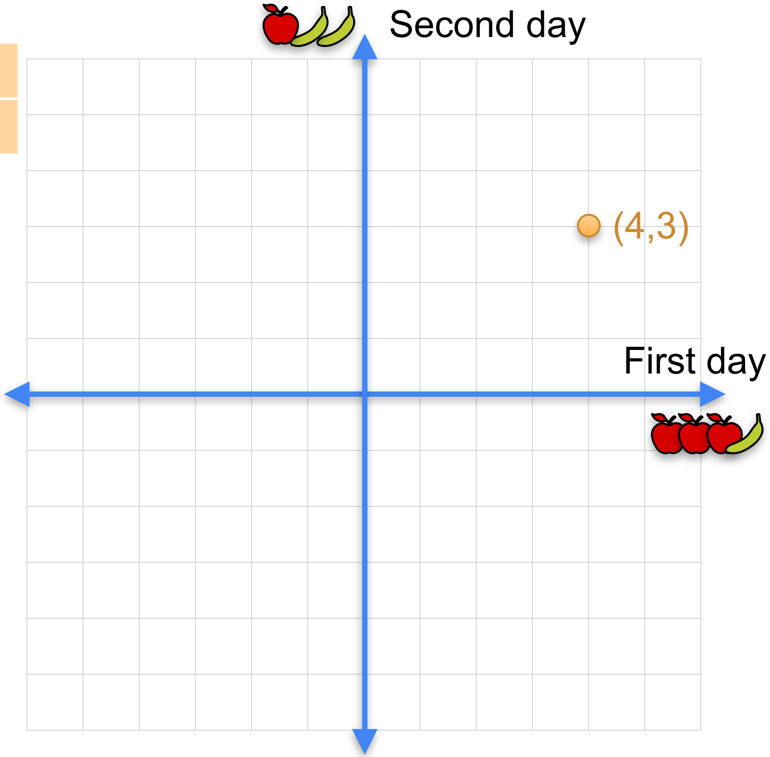
Systems of equations as linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

First day: 3a + b
Second day: a + 2b

(1,1) → (4,3)



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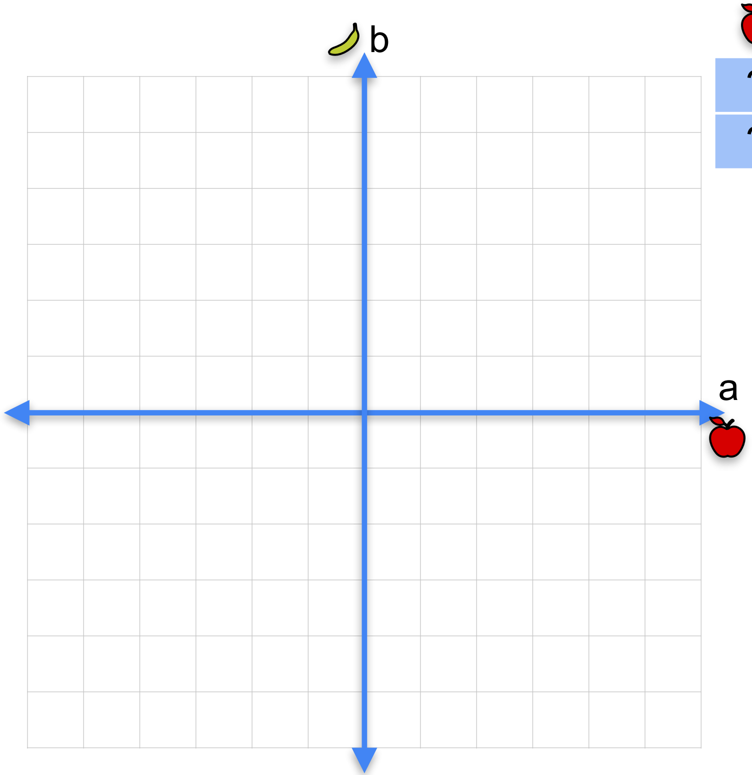
Vectors and Linear Transformations

Linear transformations as matrices

Linear transformations as matrices



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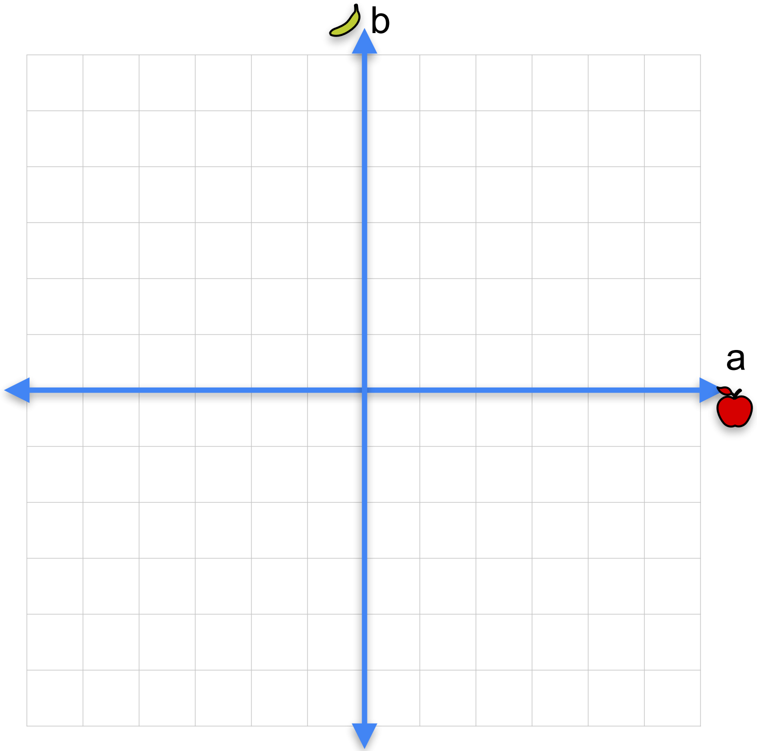
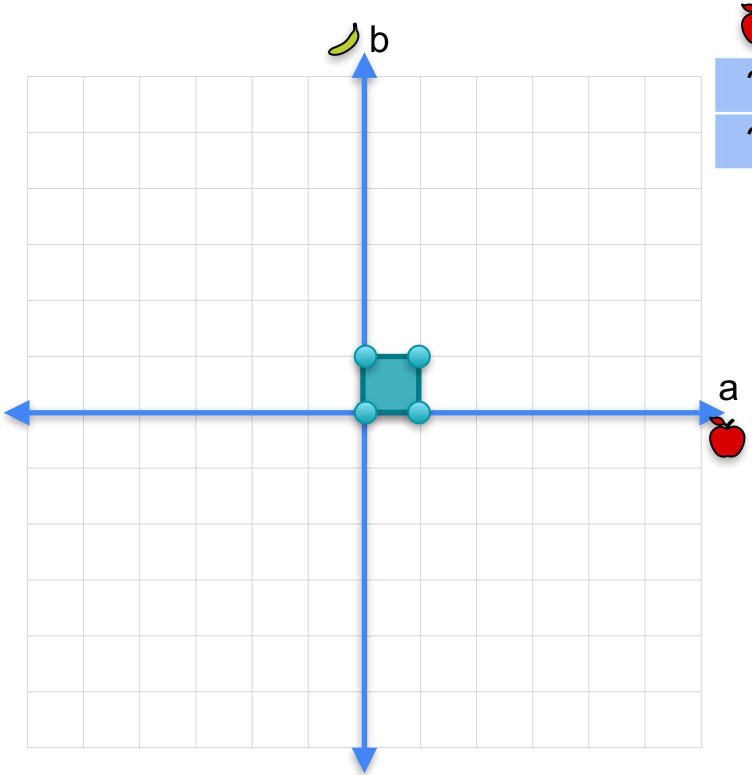
?	?



Linear transformations as matrices



?	?
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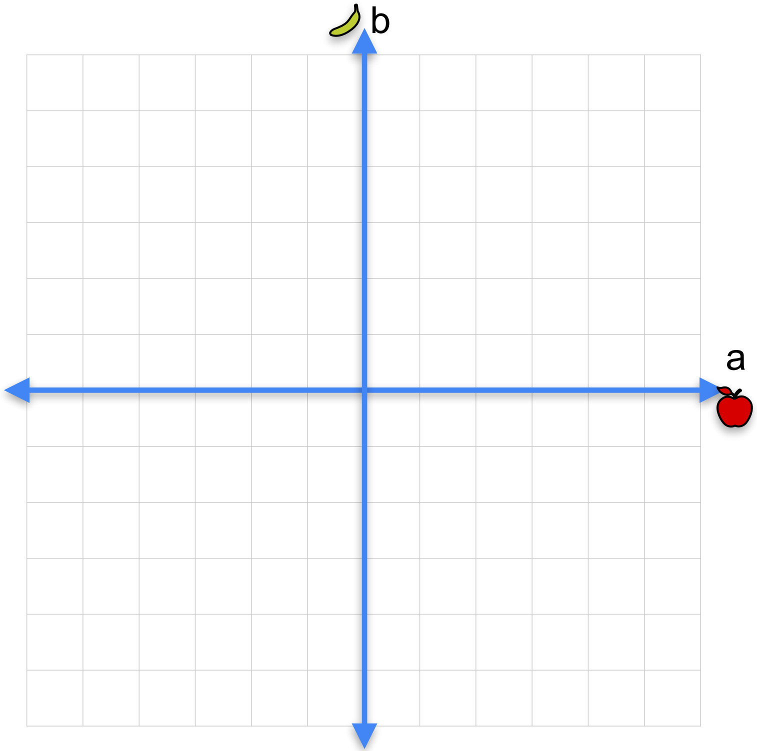
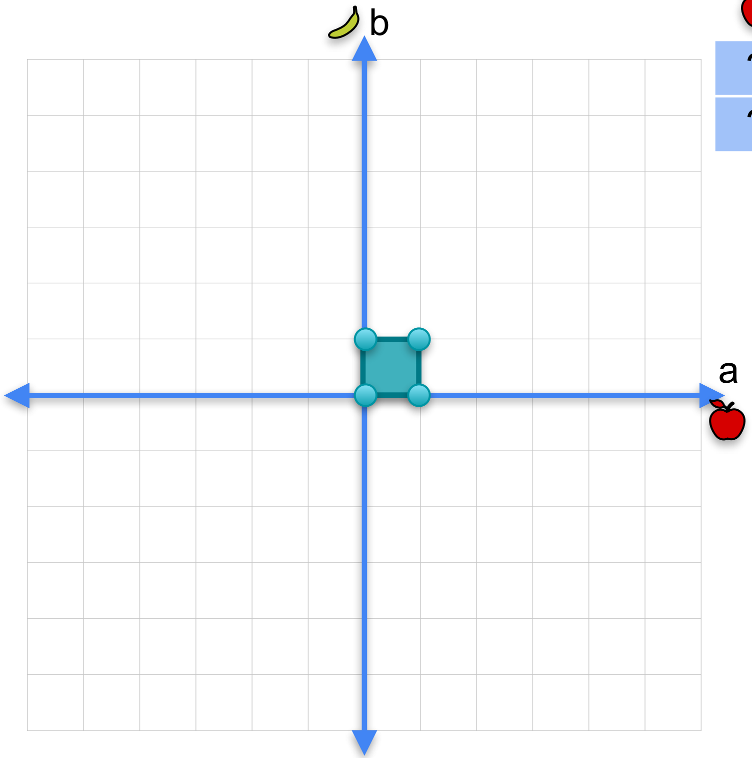


Linear transformations as matrices



🍎	🍌
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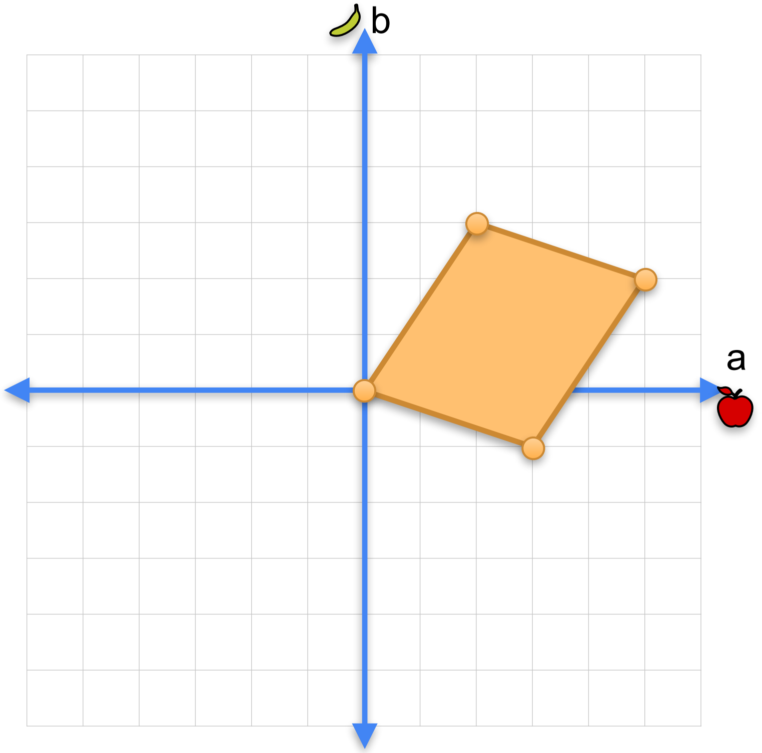
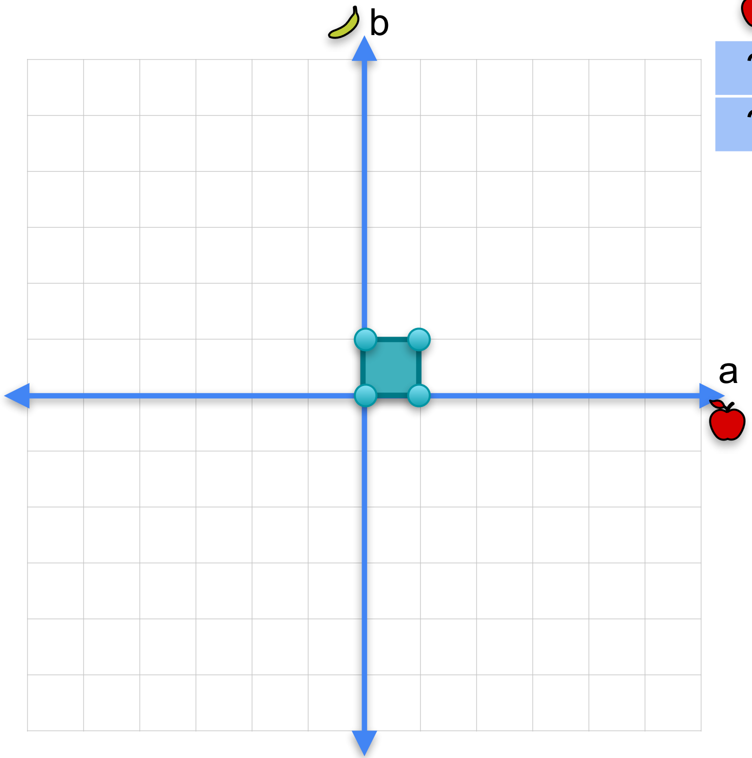
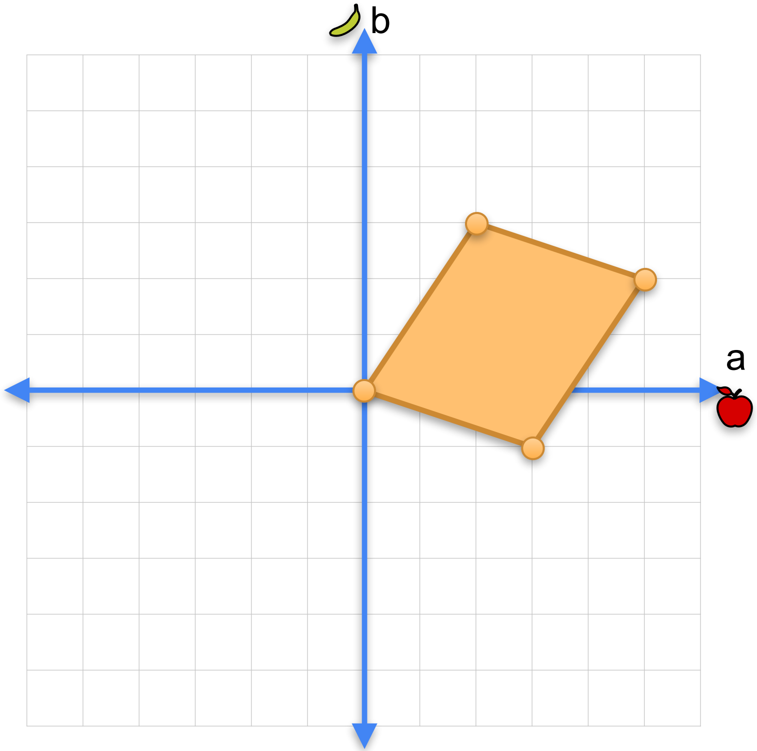


Linear transformations as matrices

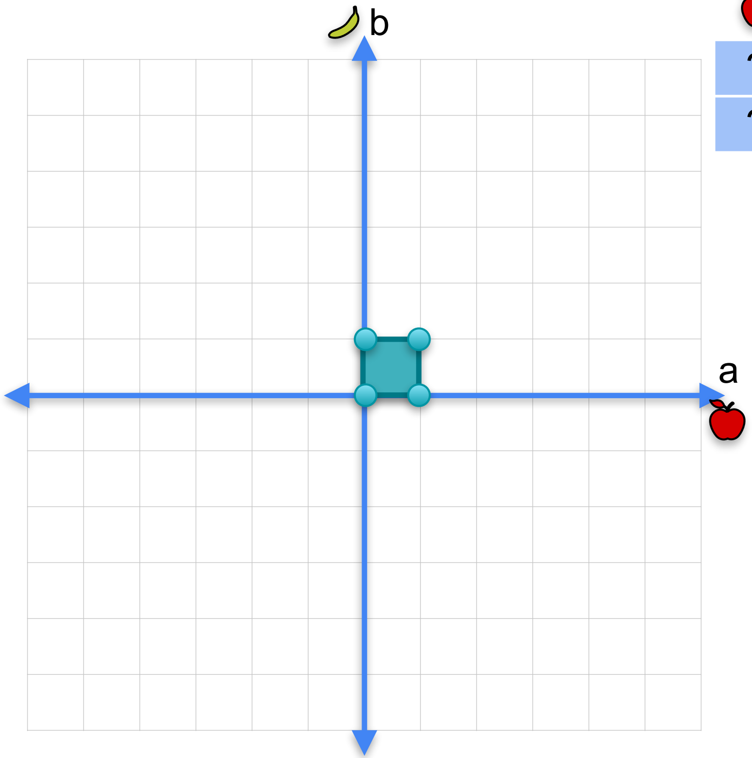


🍎	🍌
?	?
?	?

(0,0) → (0,0)



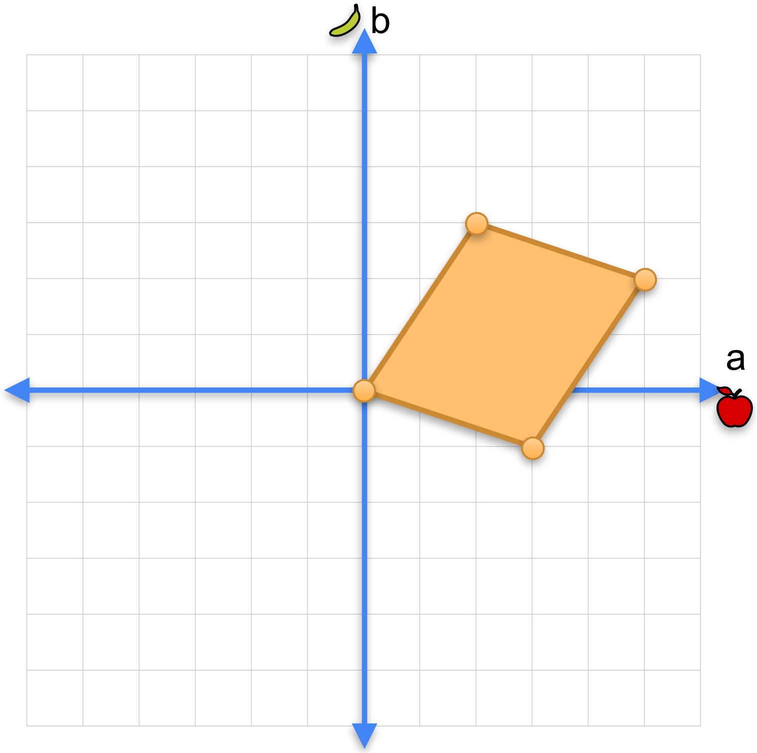
Linear transformations as matrices



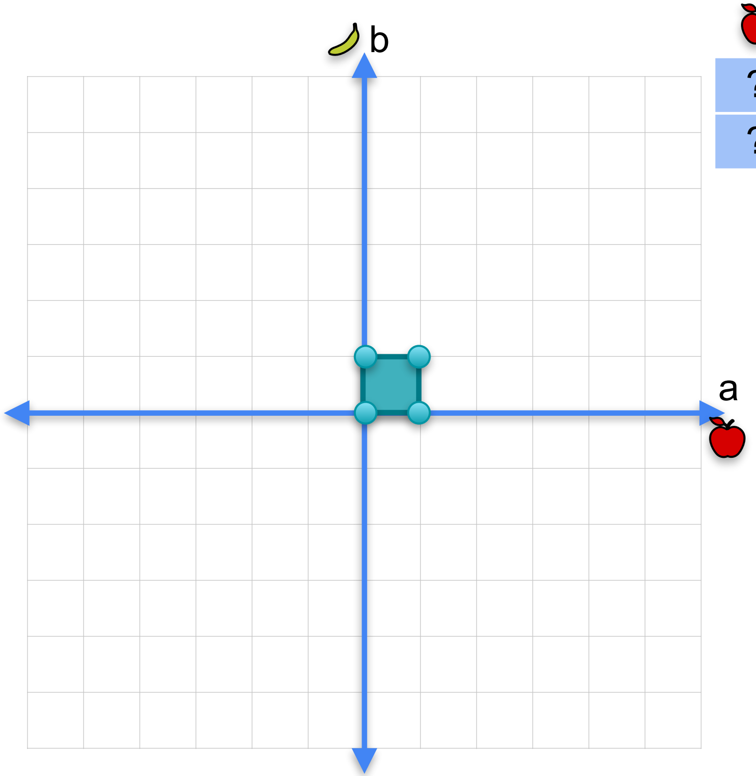
🍎	🍌
?	?
?	?

(0,0) → (0,0)

(1,0) → (3,-1)

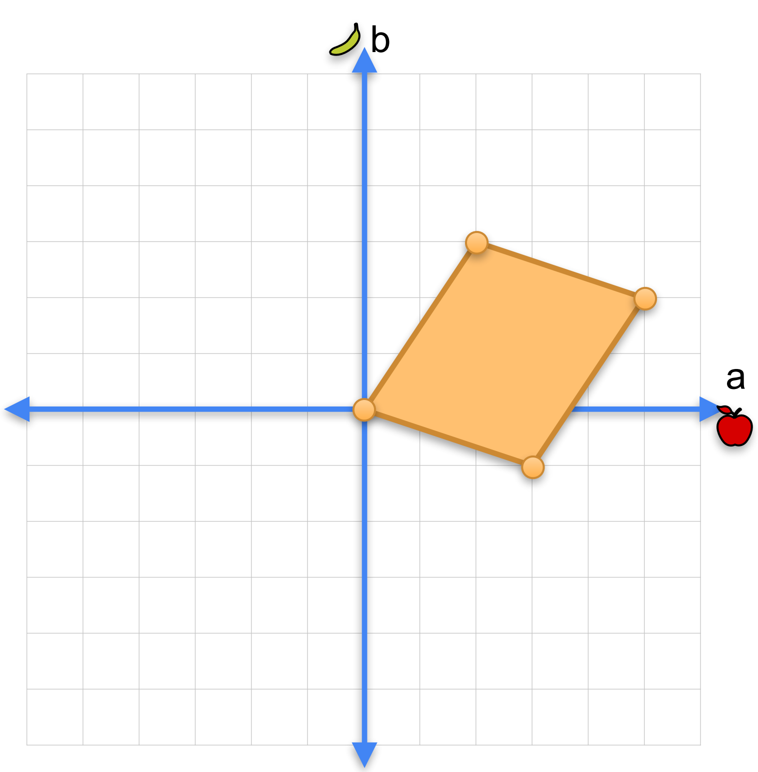


Linear transformations as matrices

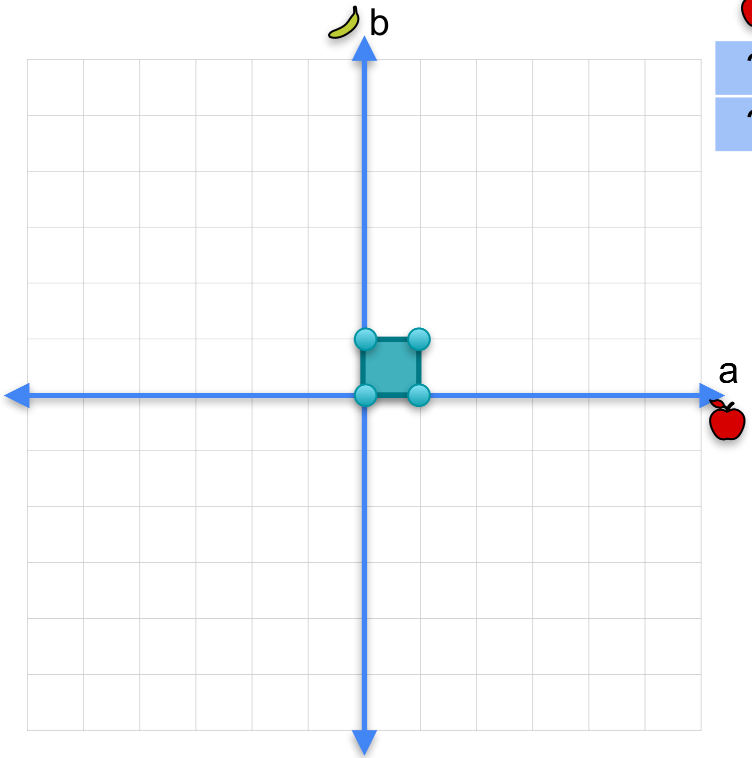


?	?
?	?

(0,0) → (0,0)
(1,0) → (3,-1)
(0,1) → (2,3)



Linear transformations as matrices



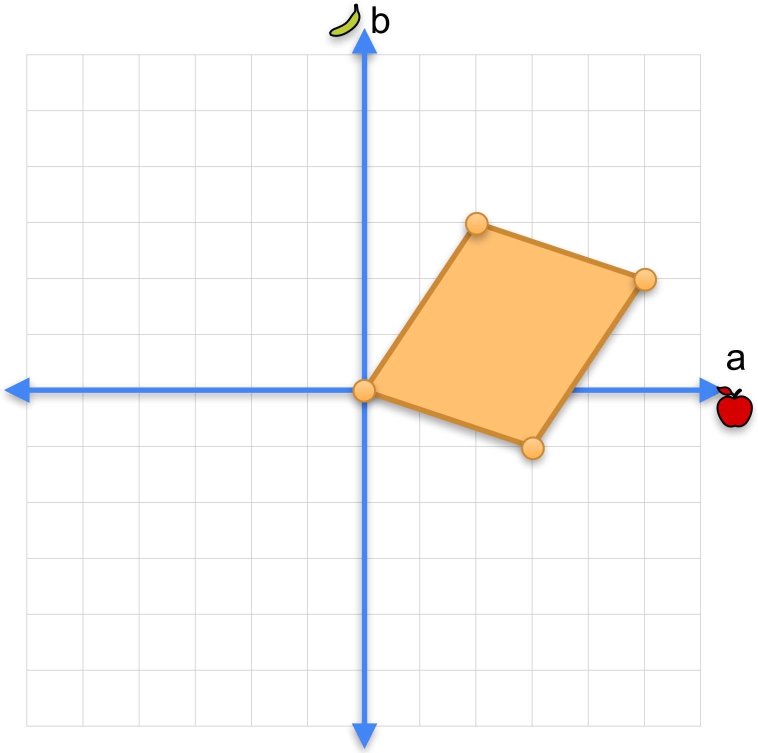
🍎	🍌
?	?
?	?

(0,0) → (0,0)

(1,0) → (3,-1)

(0,1) → (2,3)

(1,1) → (5,2)



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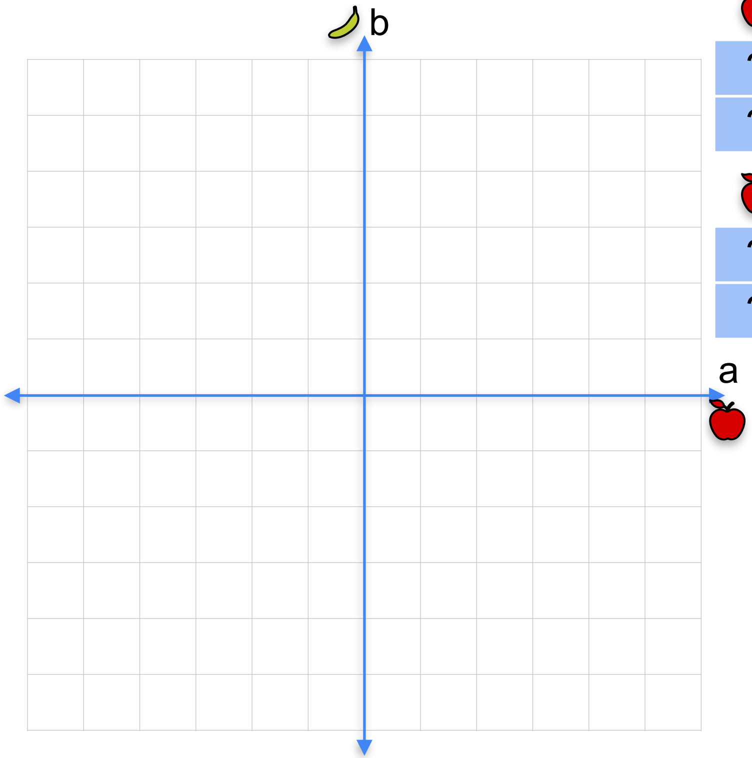
Linear transformations as matrices

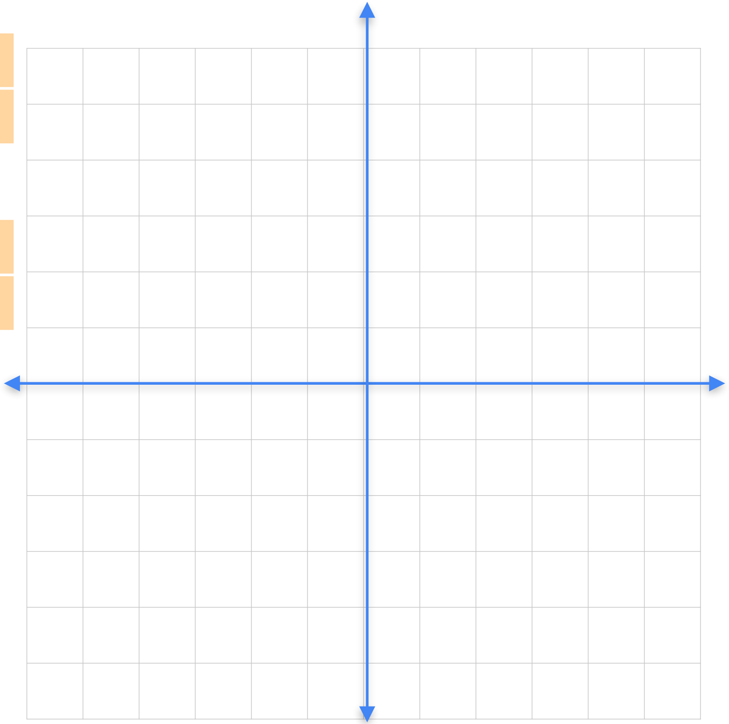


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

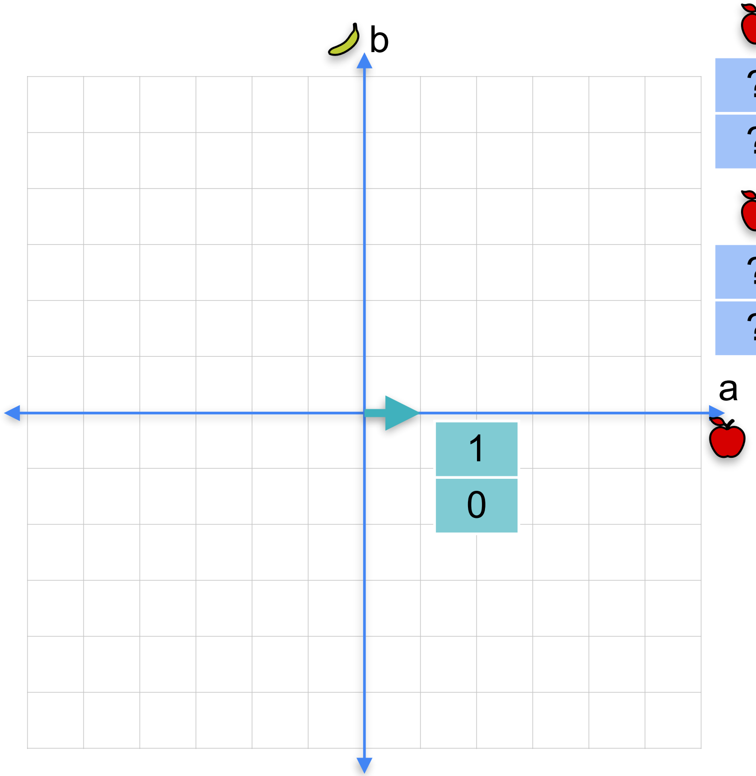
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



Linear transformations as matrices

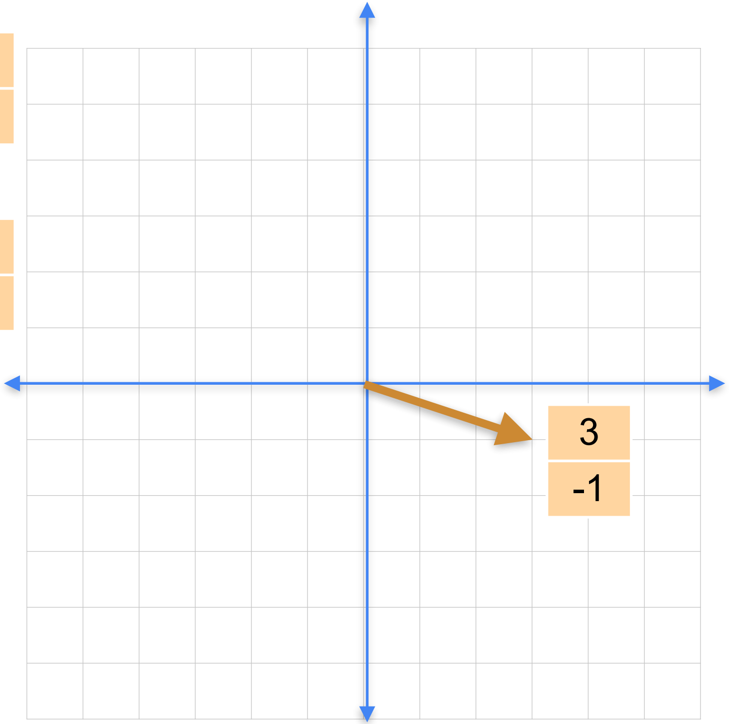


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

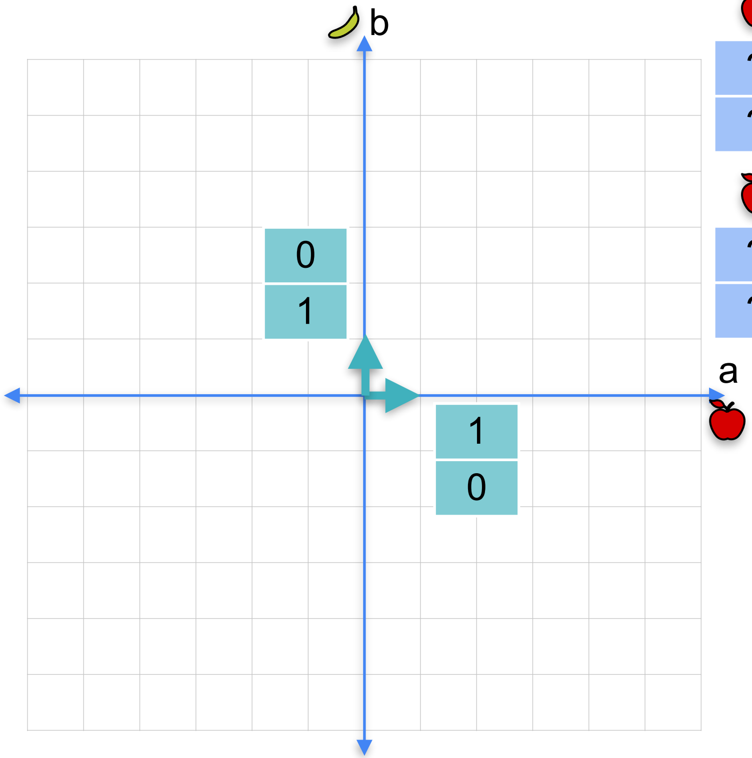
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



Linear transformations as matrices

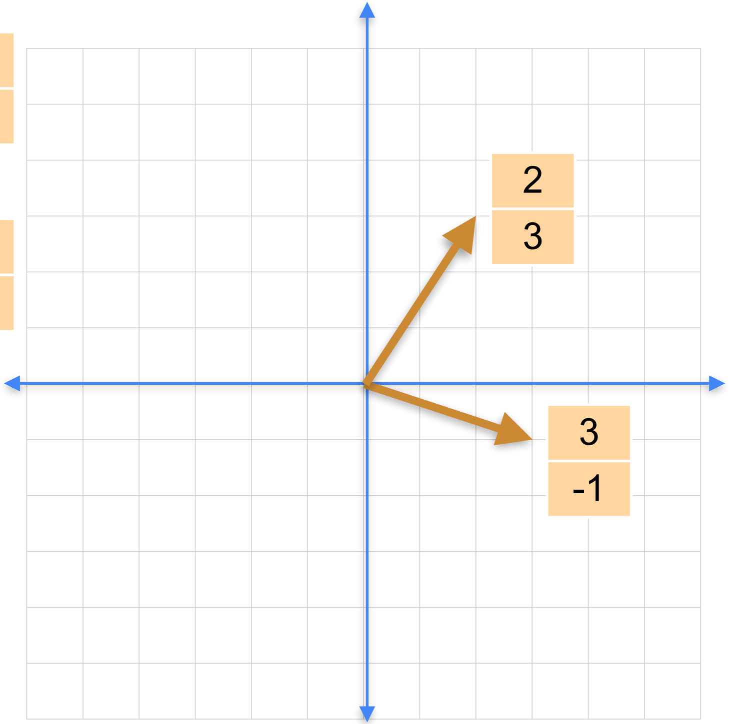


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

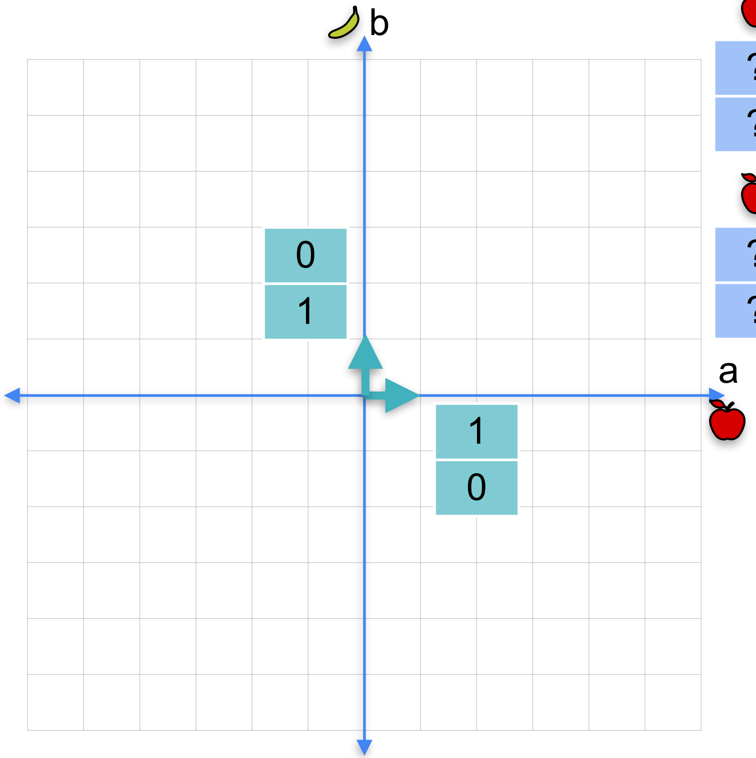
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



Linear transformations as matrices

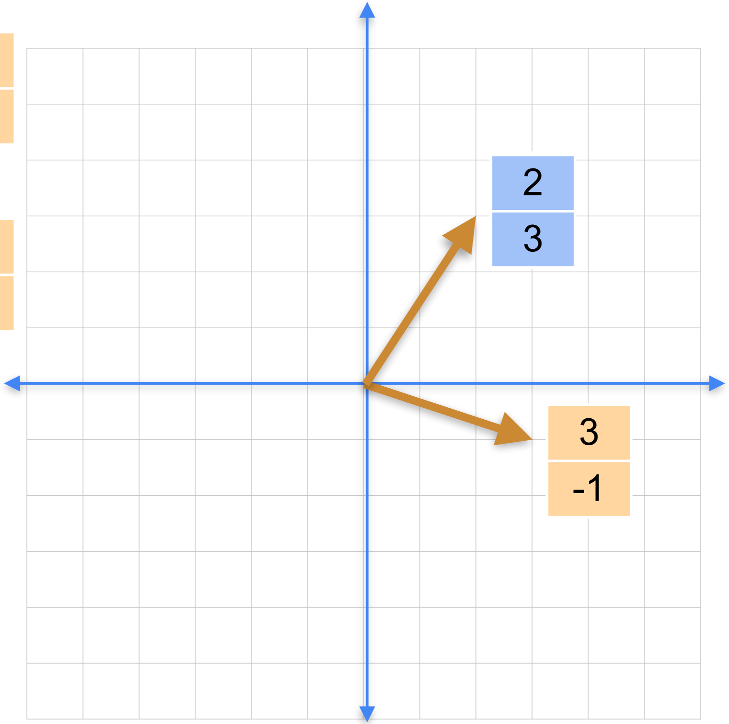


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

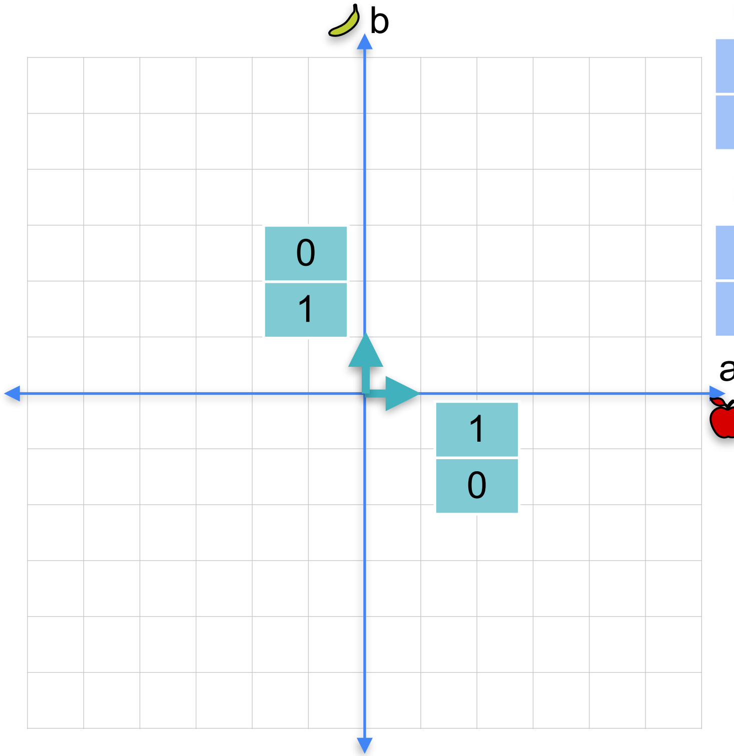
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



Linear transformations as matrices

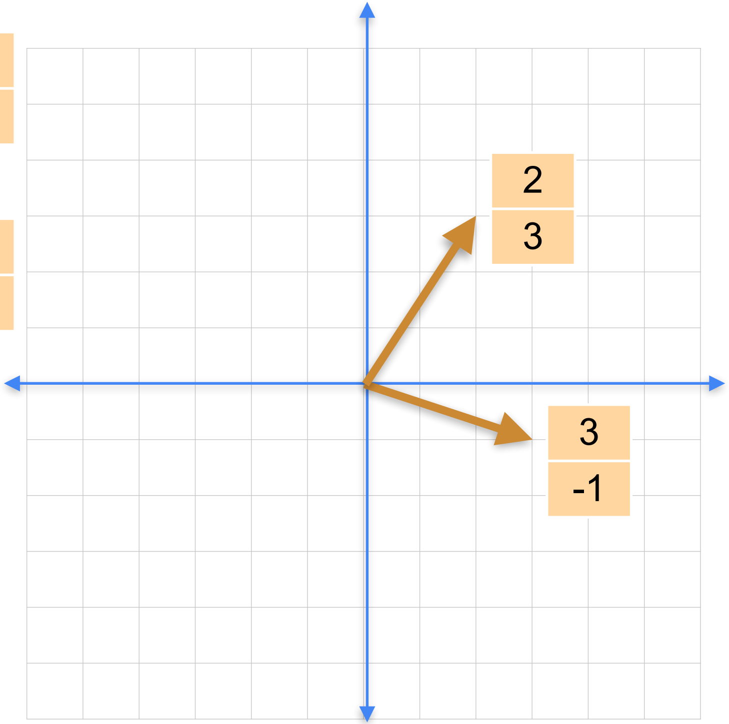


$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

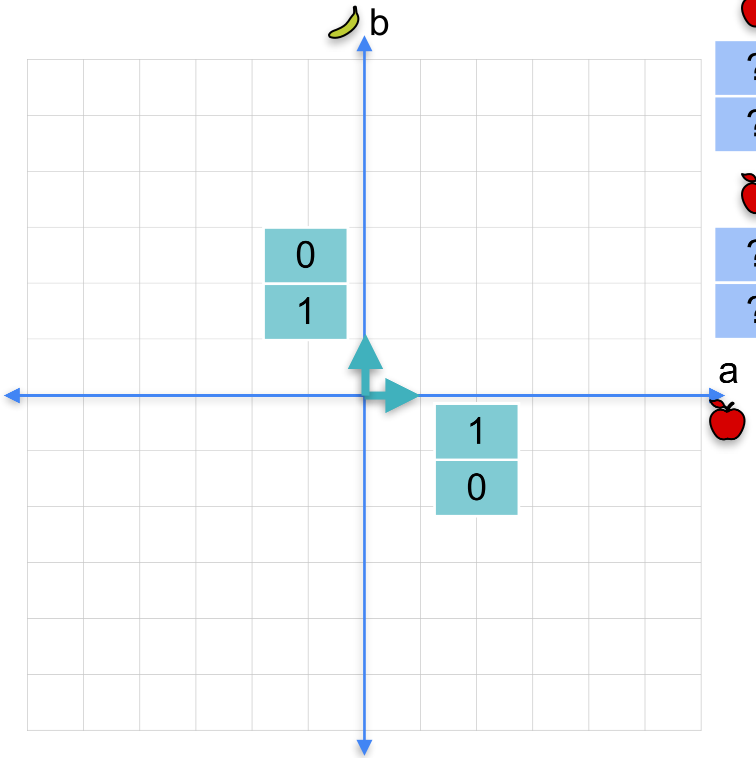
$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



Linear transformations as matrices

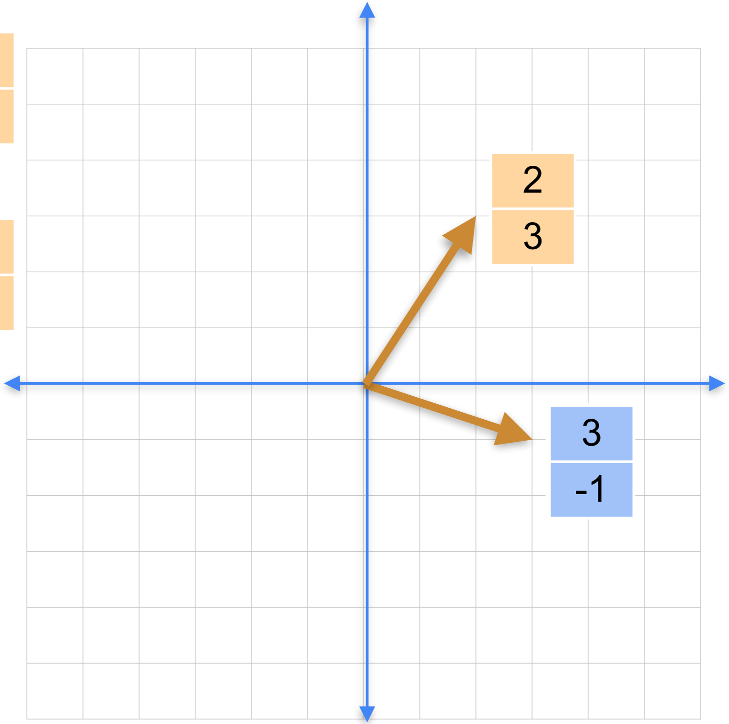


$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

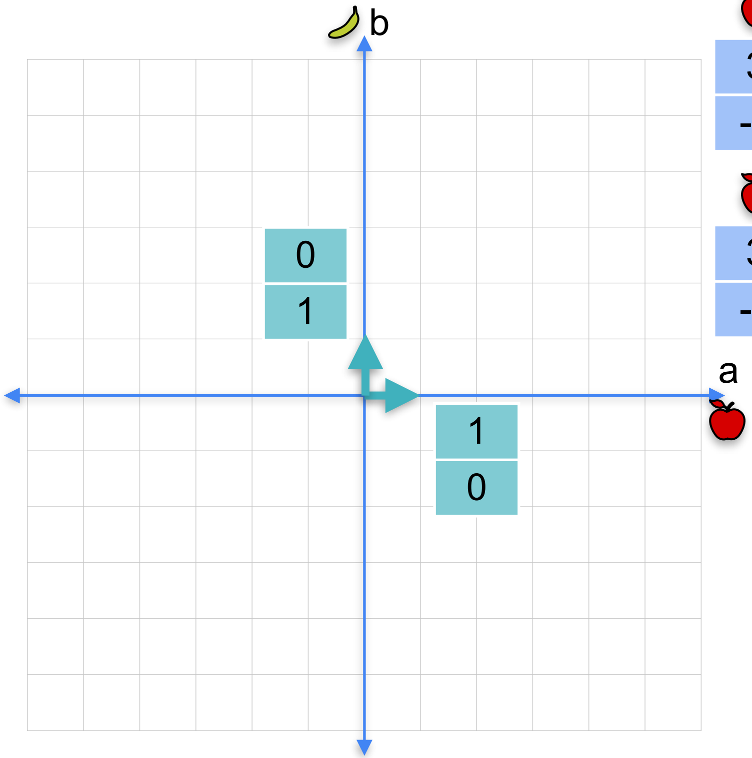
$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



Linear transformations as matrices

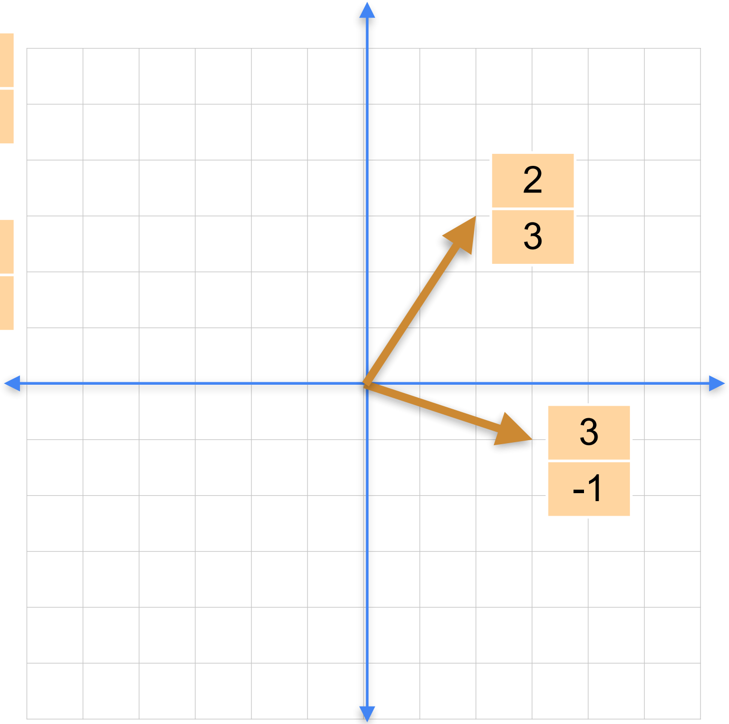


$$\begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



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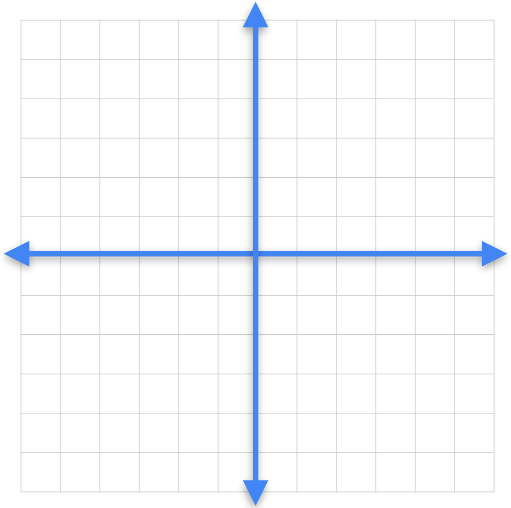
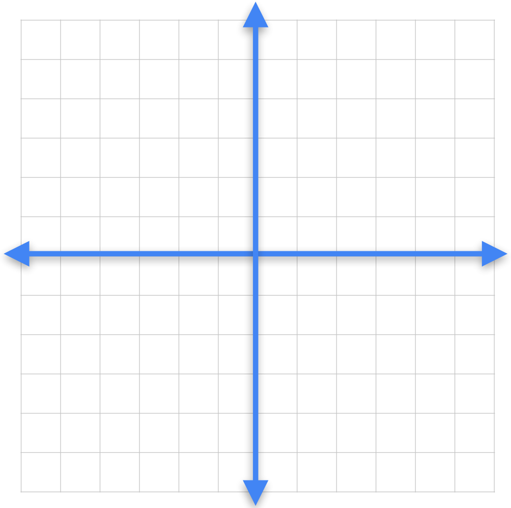
Vectors and Linear Transformations

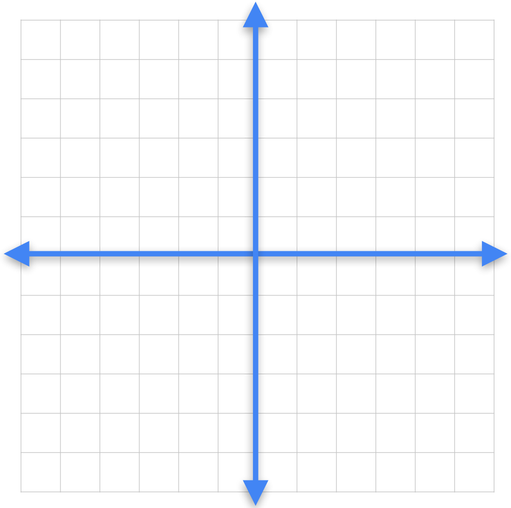
Matrix multiplication

Combining linear transformations

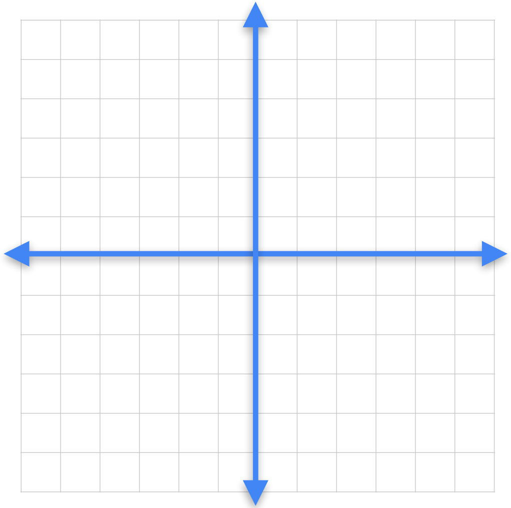




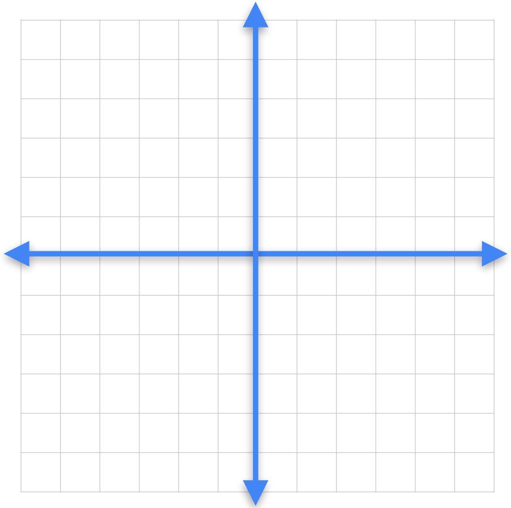
Combining linear transformations



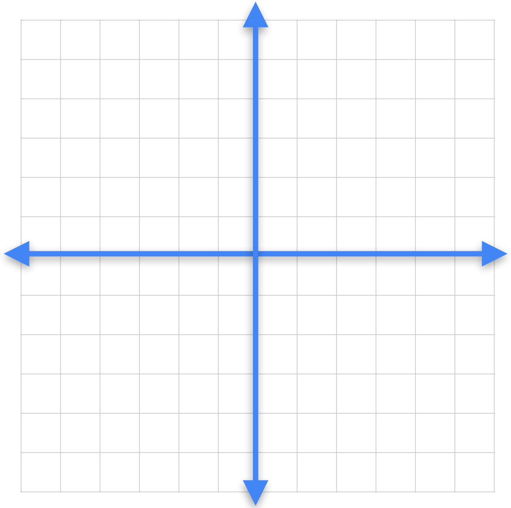
3	1
1	2



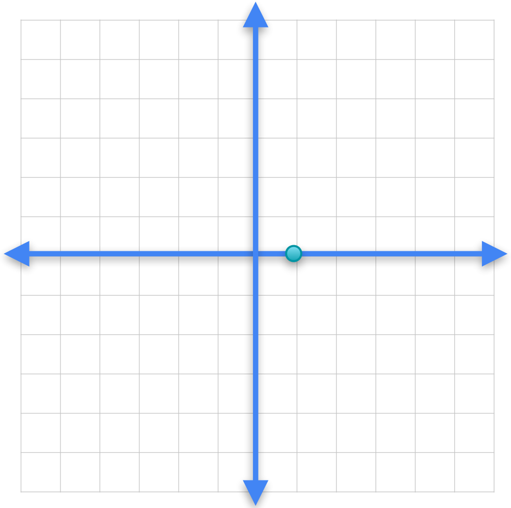
Combining linear transformations



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



Combining linear transformations



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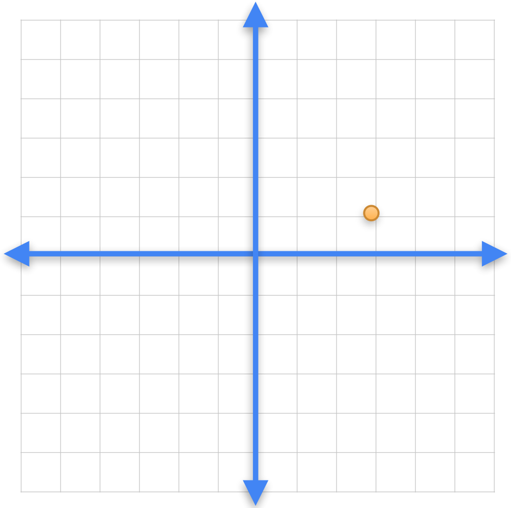
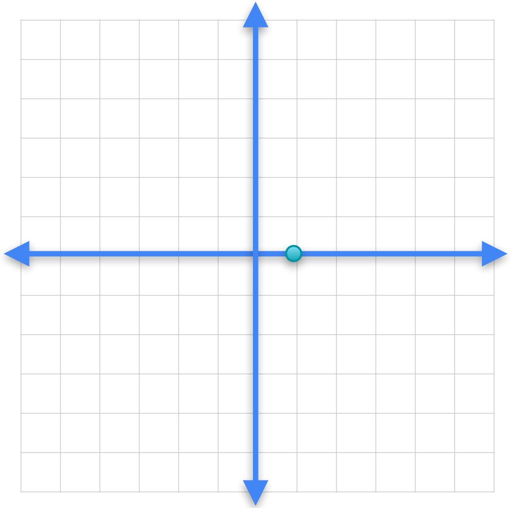


Combining linear transformations



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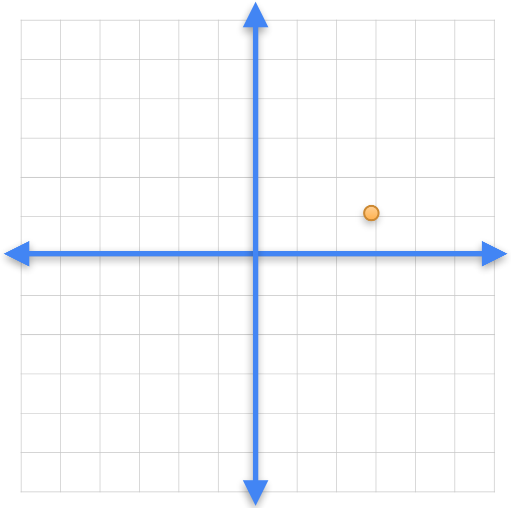

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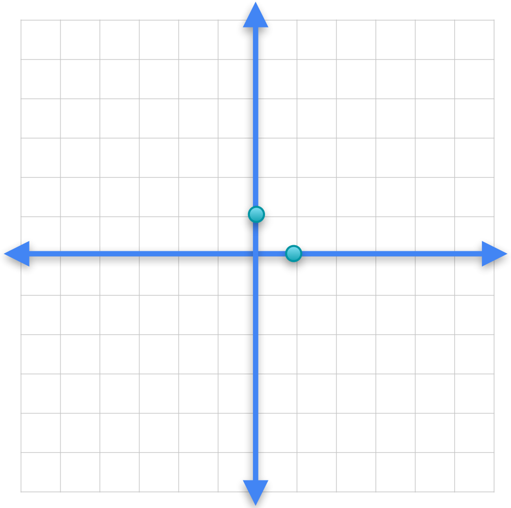
Combining linear transformations



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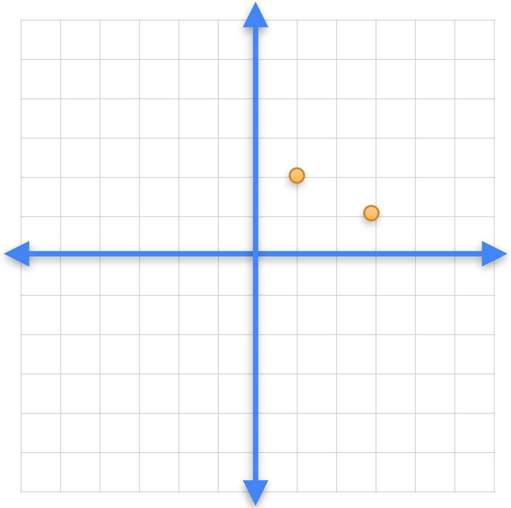
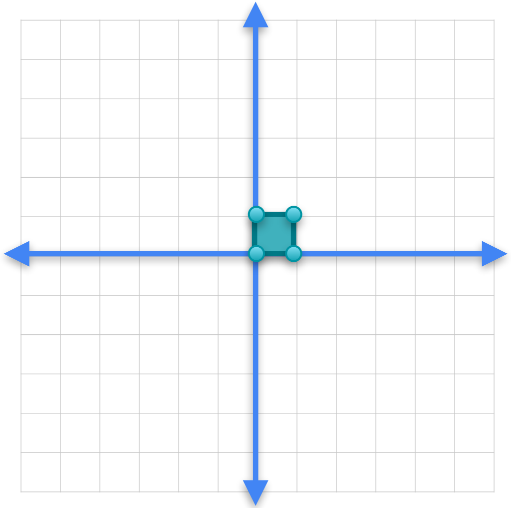


Combining linear transformations



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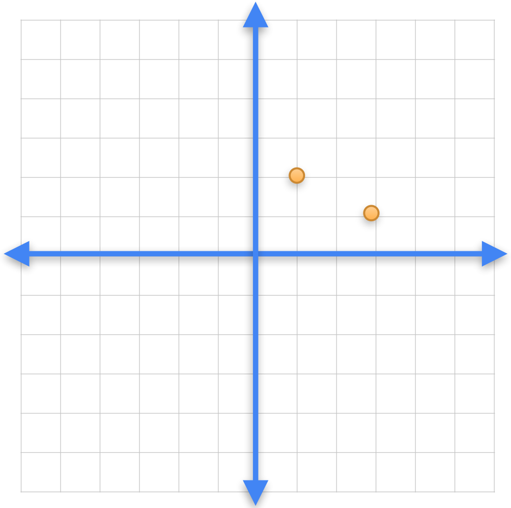

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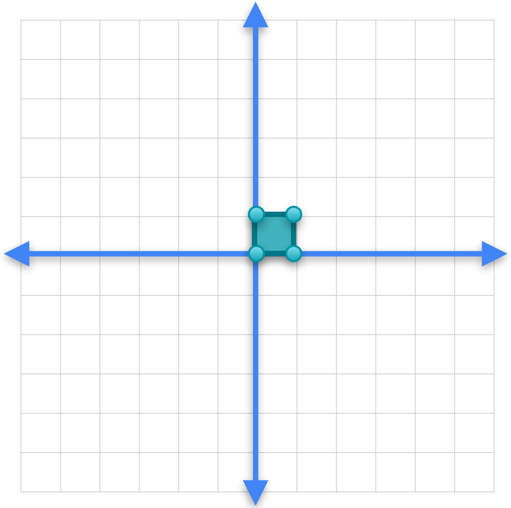
Combining linear transformations



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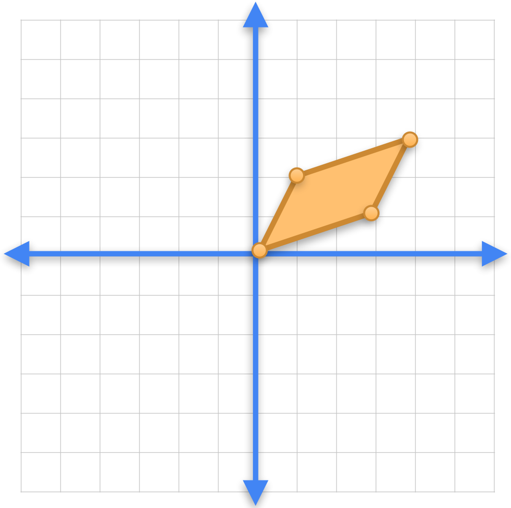
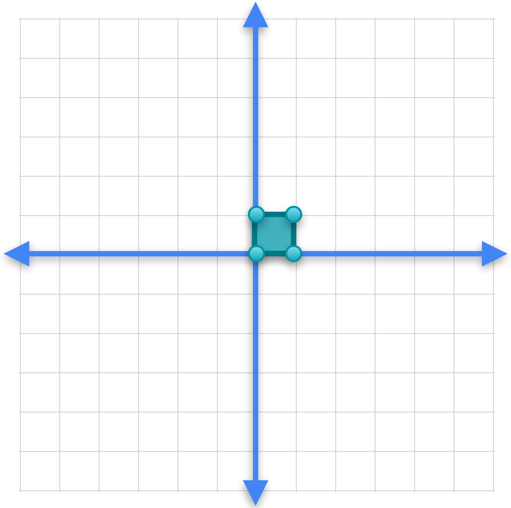


Combining linear transformations



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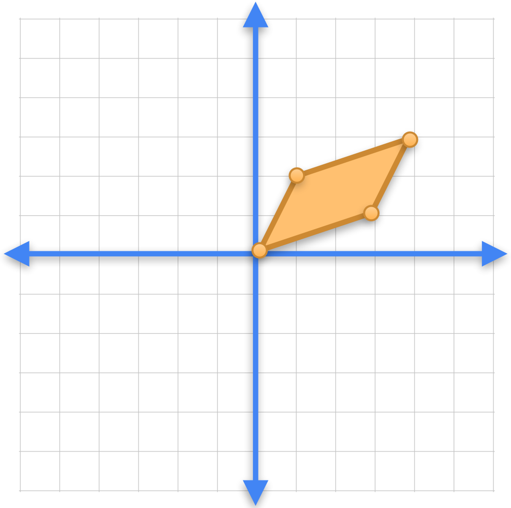

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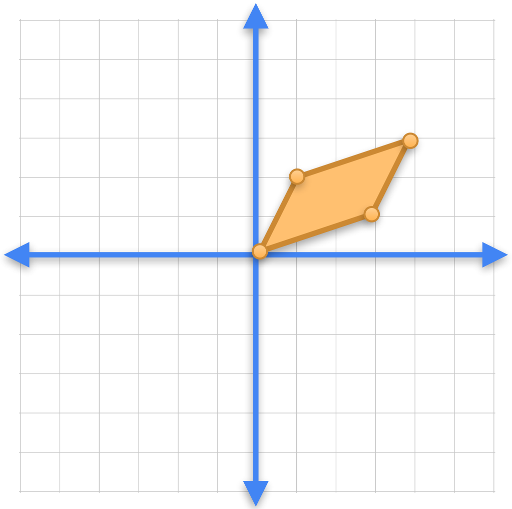
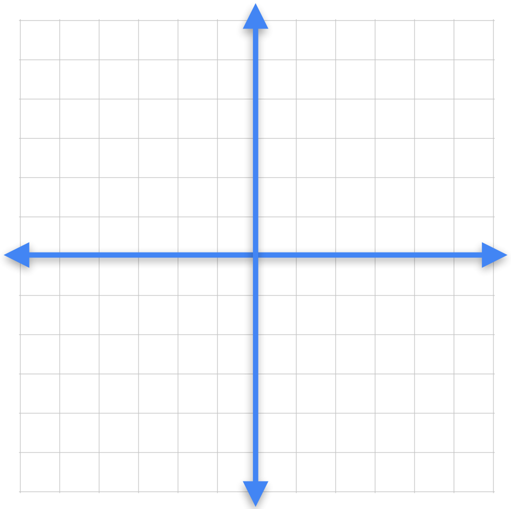
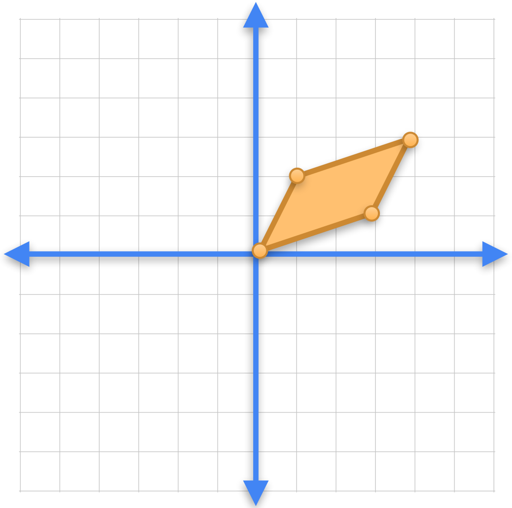
Combining linear transformations

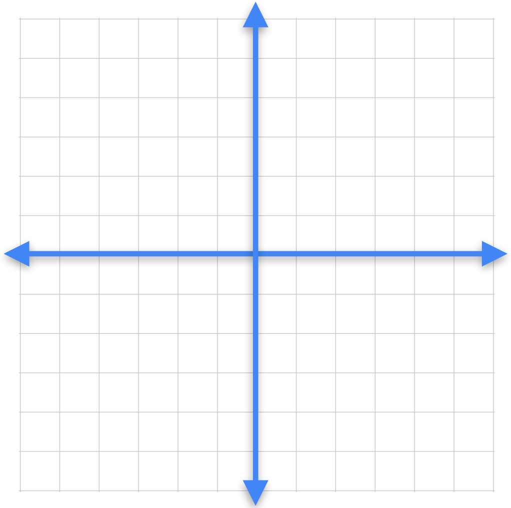




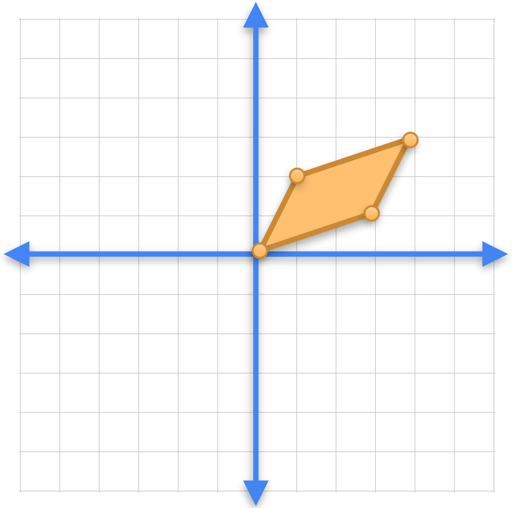
Combining linear transformations



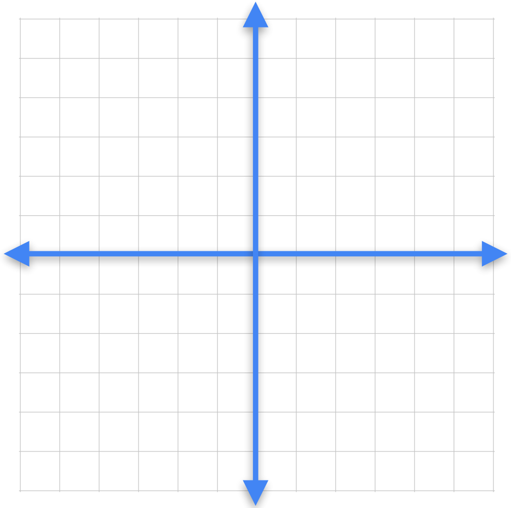
2	-1
0	2



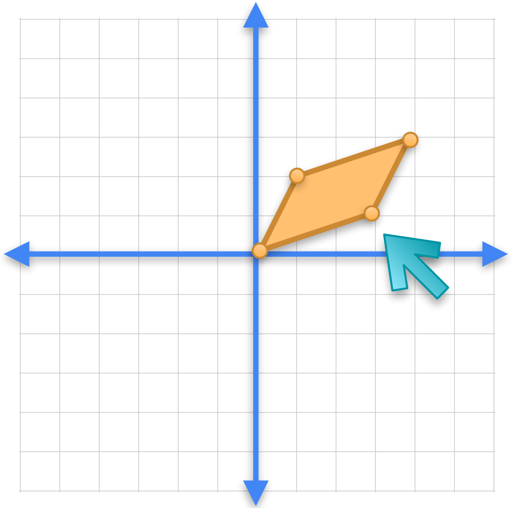
Combining linear transformations



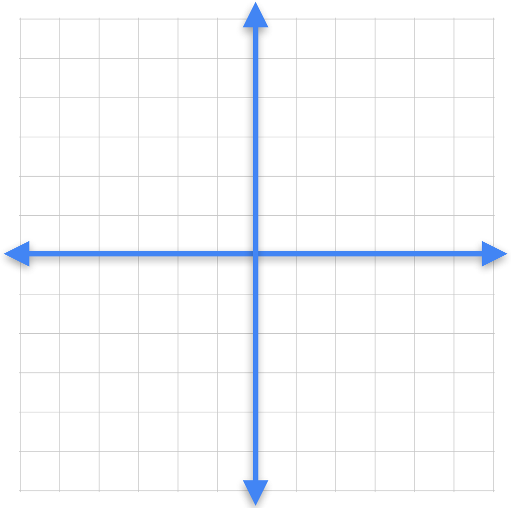
$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



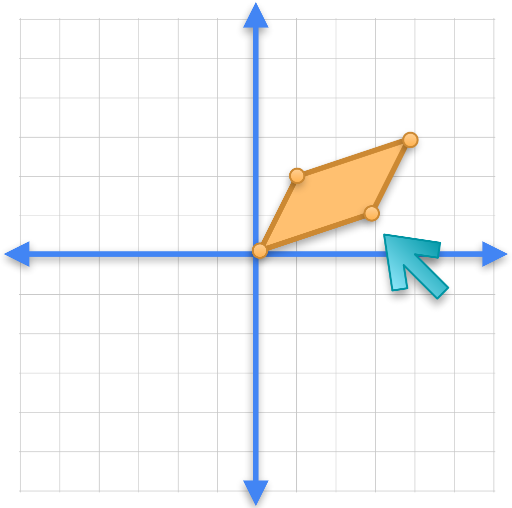
Combining linear transformations



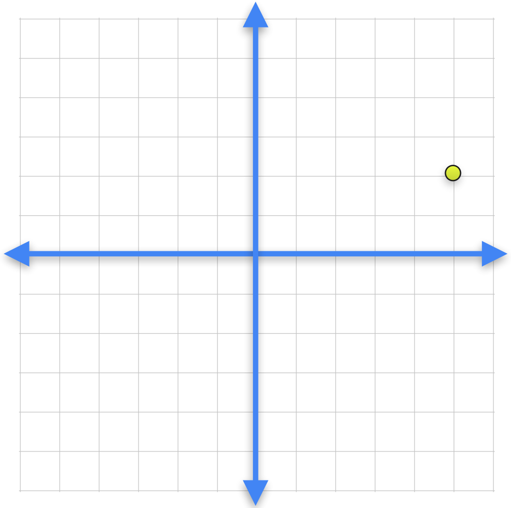
2	-1	3	=	5
0	2	1		2



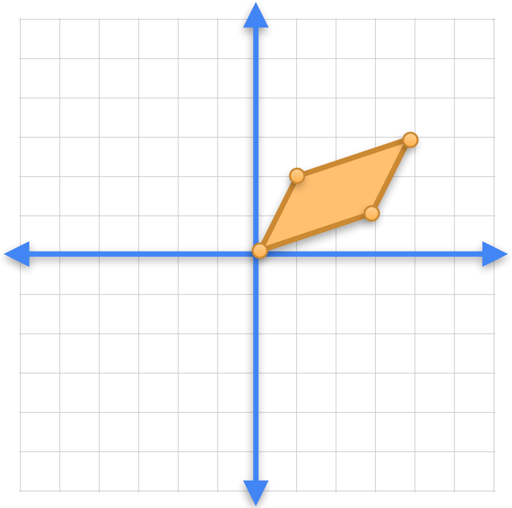
Combining linear transformations



2	-1	3	=	5
0	2	1		2

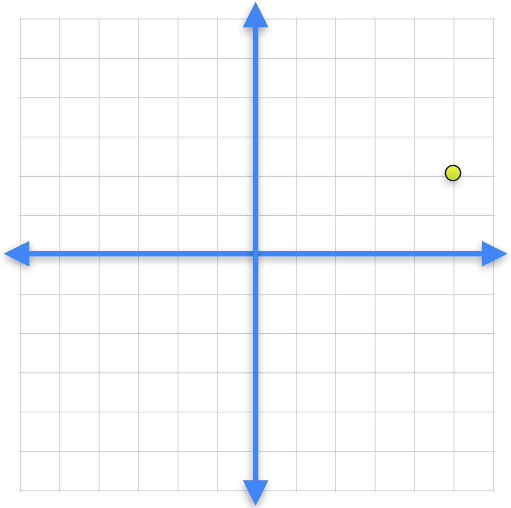


Combining linear transformations

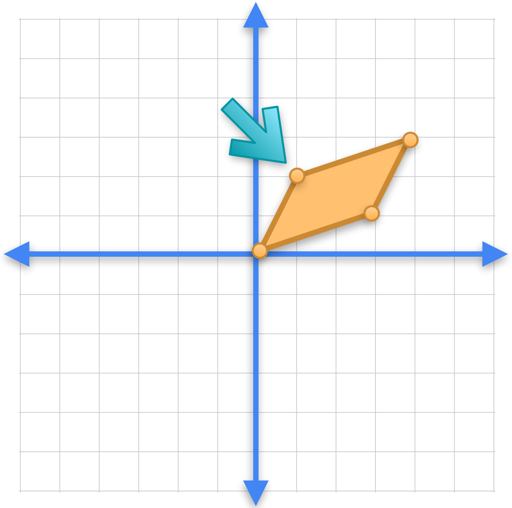


$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$



Combining linear transformations



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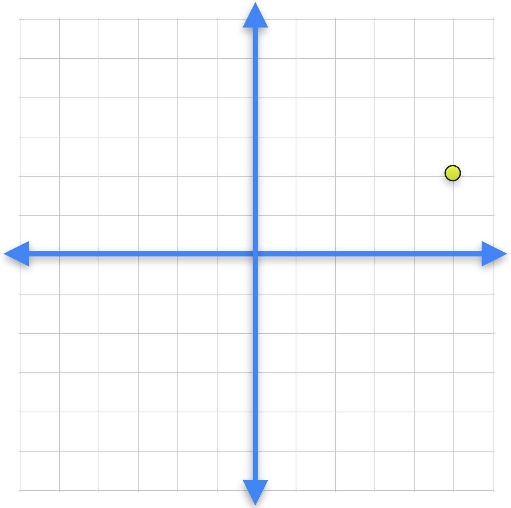
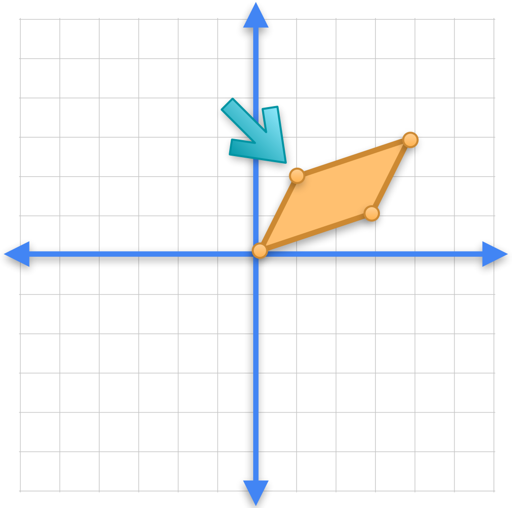


Combining linear transformations



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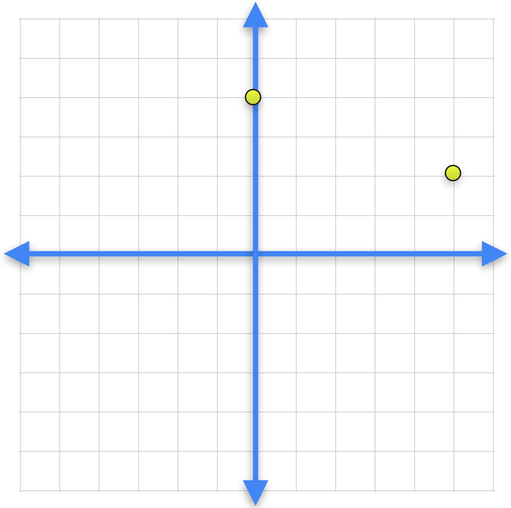

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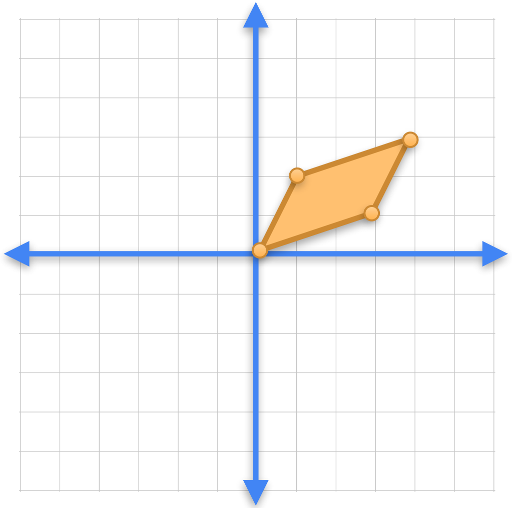
Combining linear transformations



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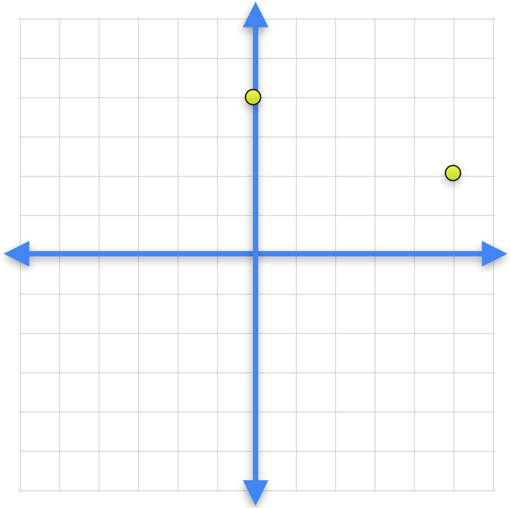

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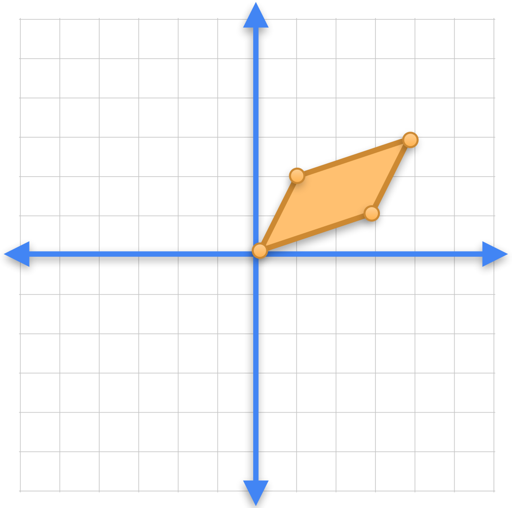
Combining linear transformations



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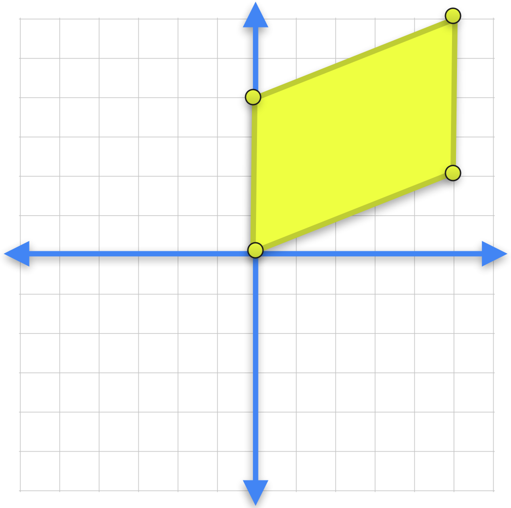
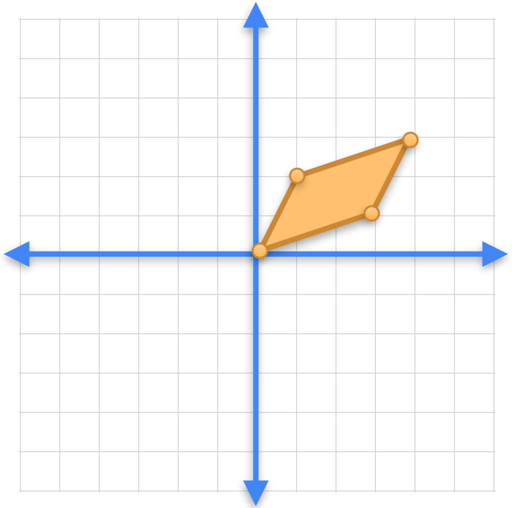


Combining linear transformations



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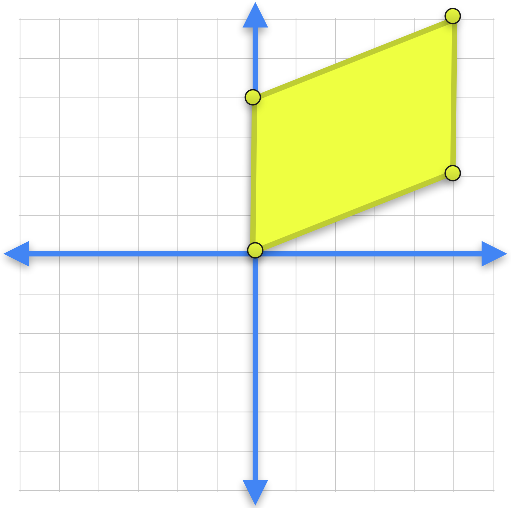
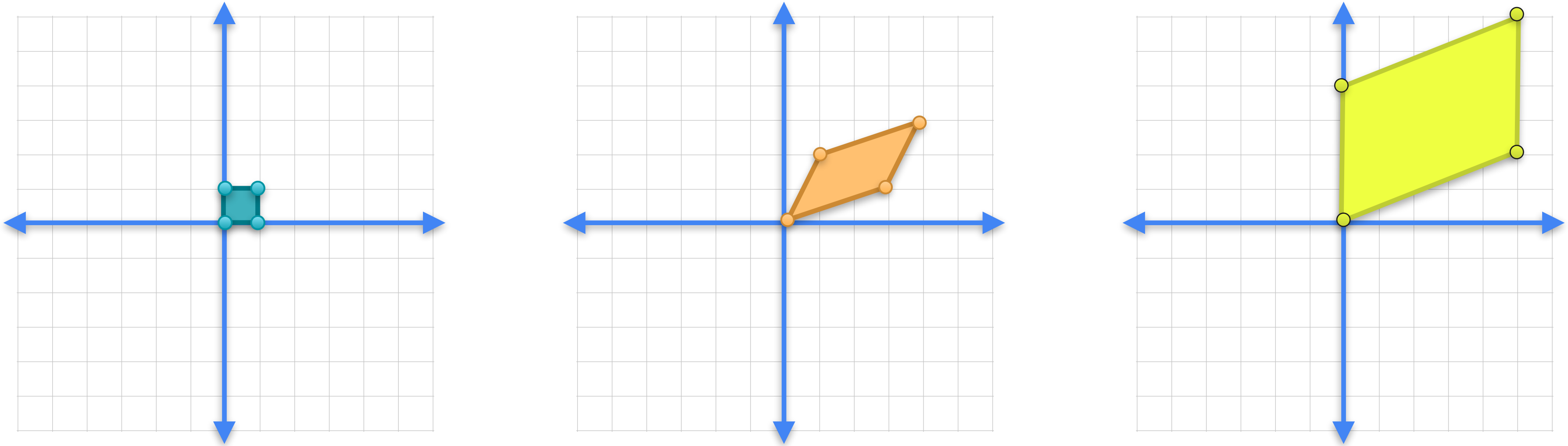
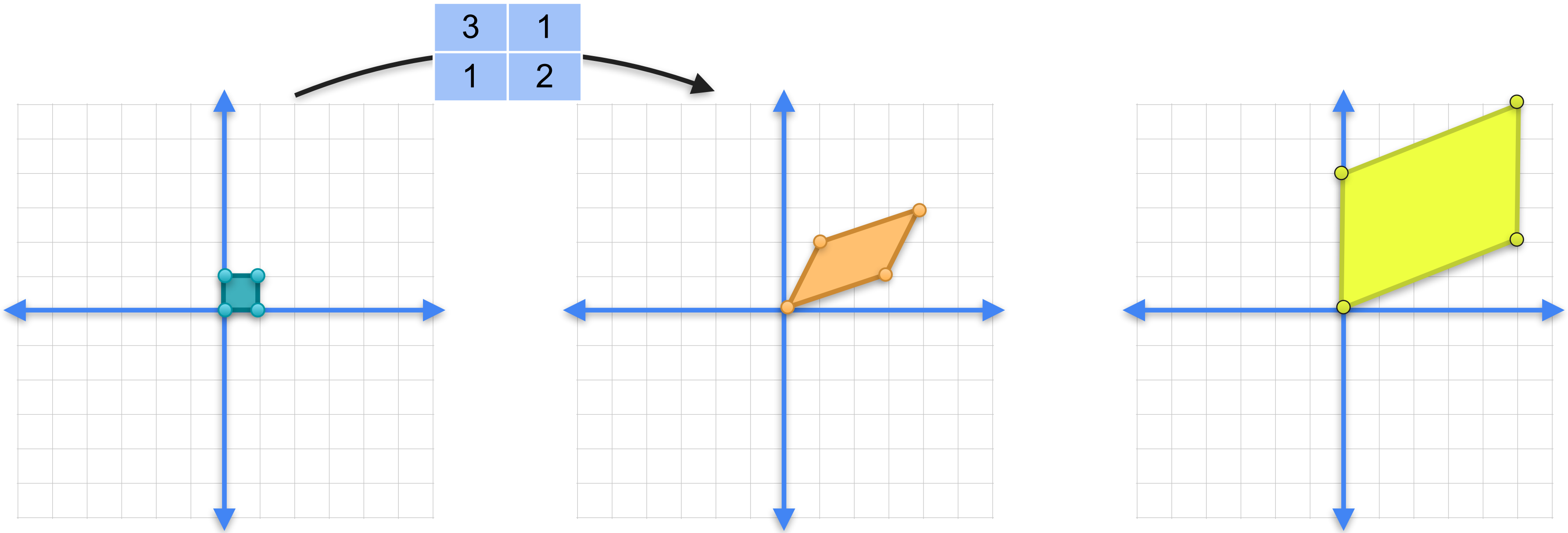


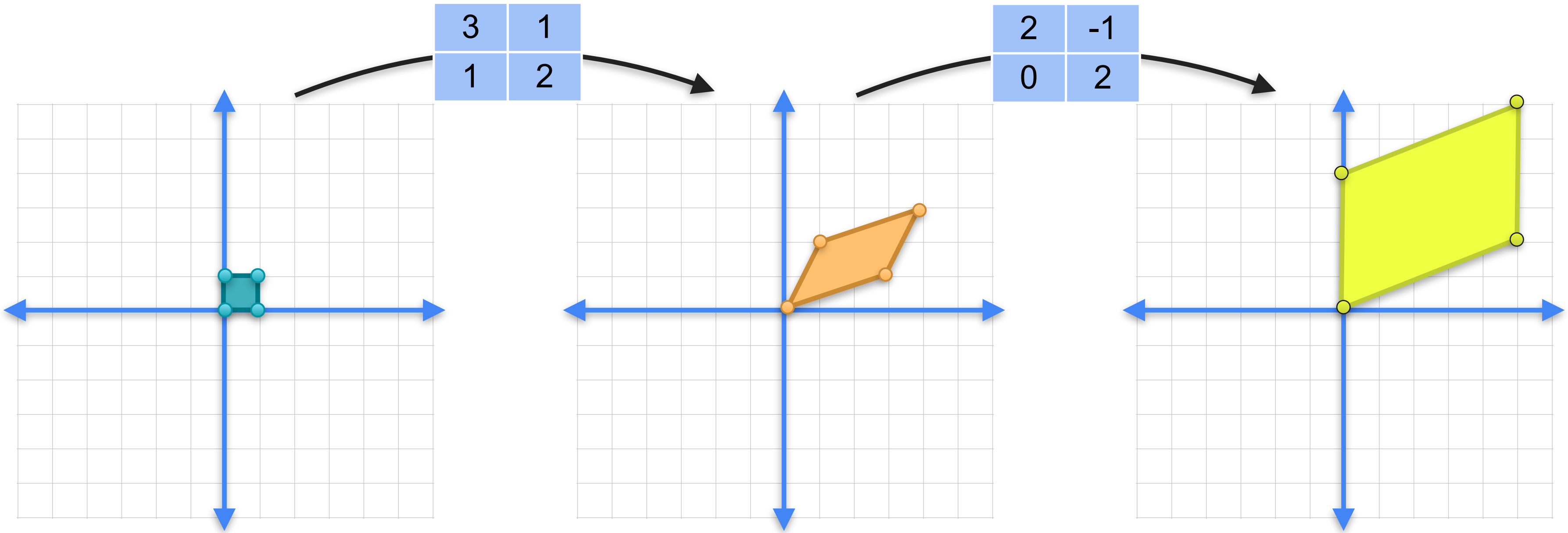
Combining linear transformations



Combining linear transformations



Combining linear transformations



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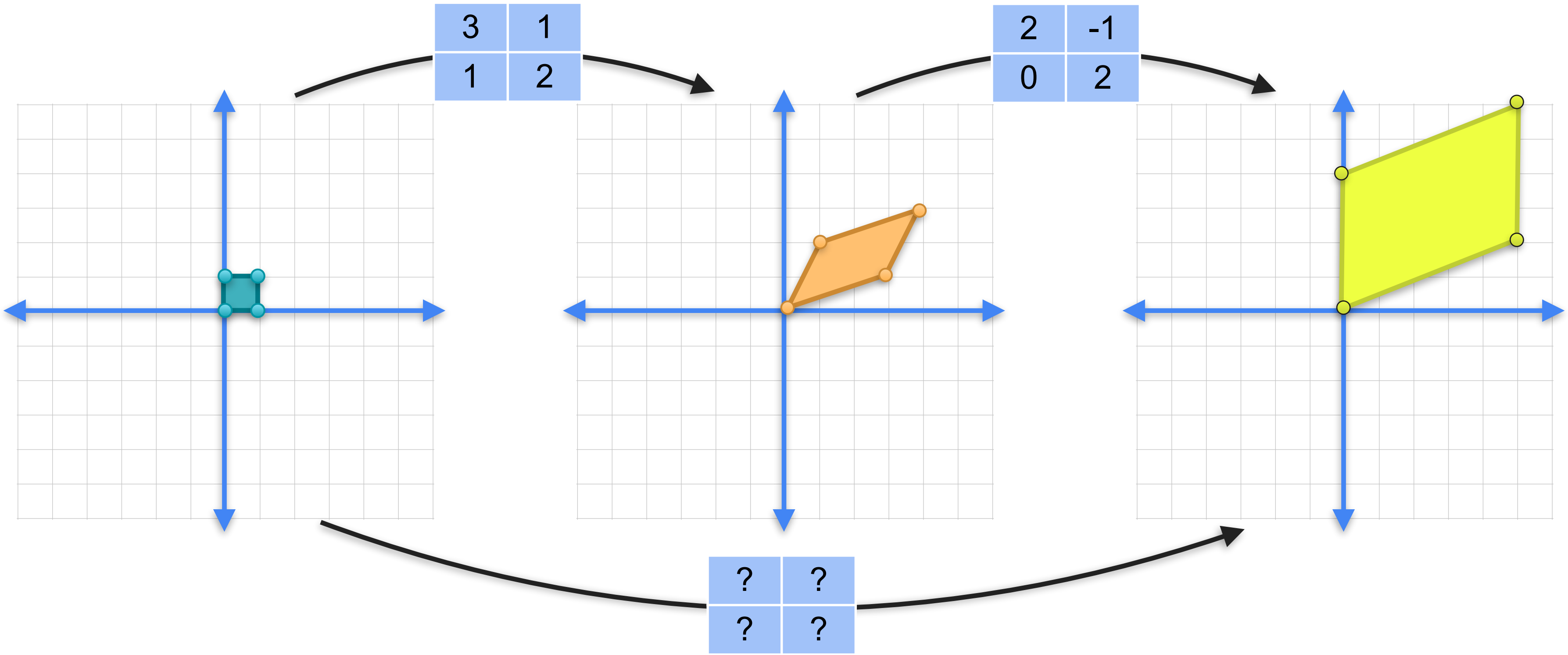
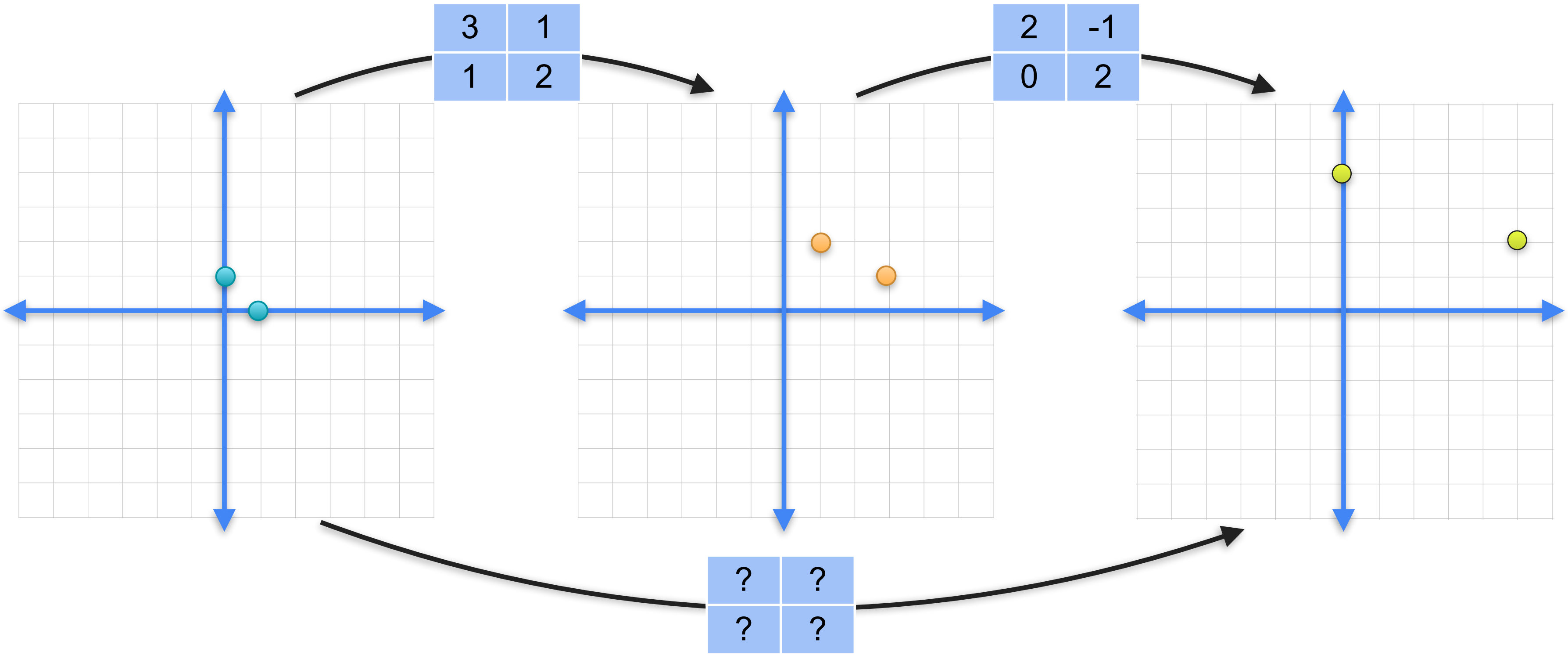
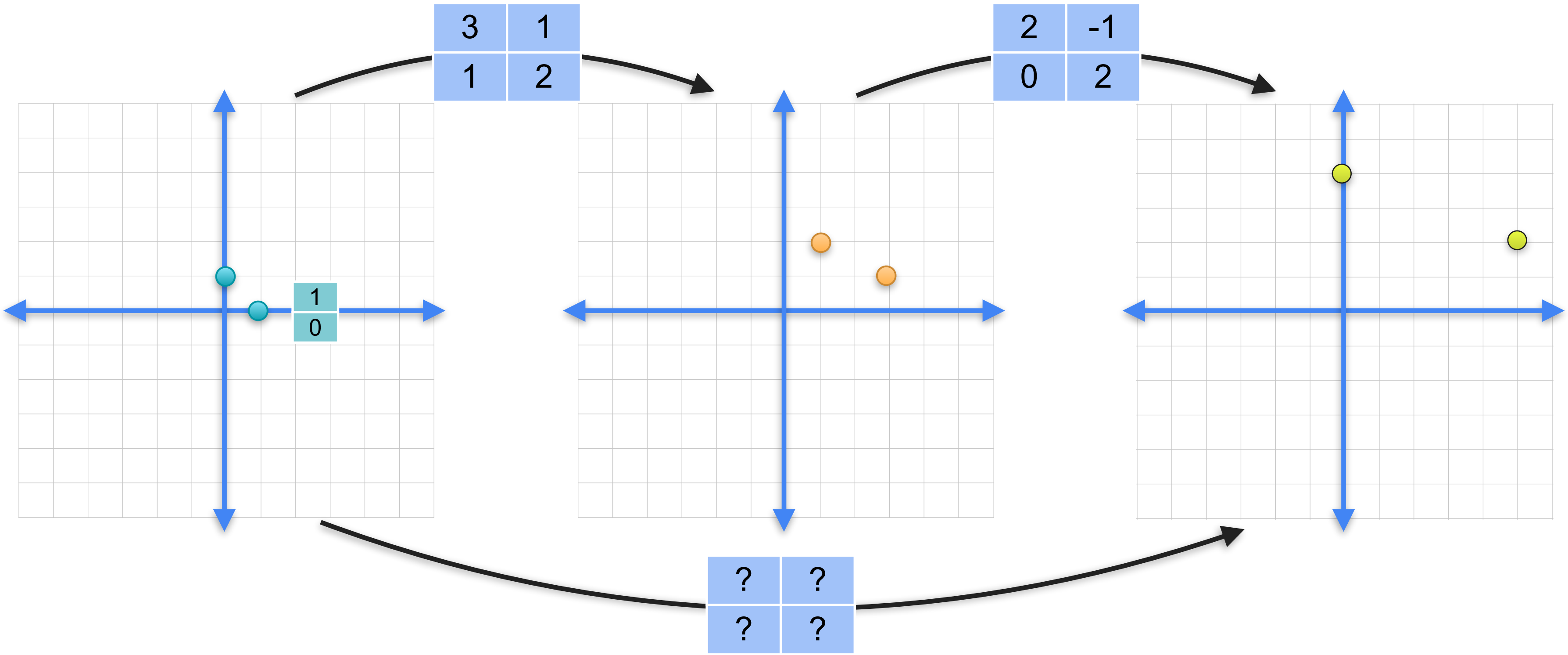
Combining linear transformations



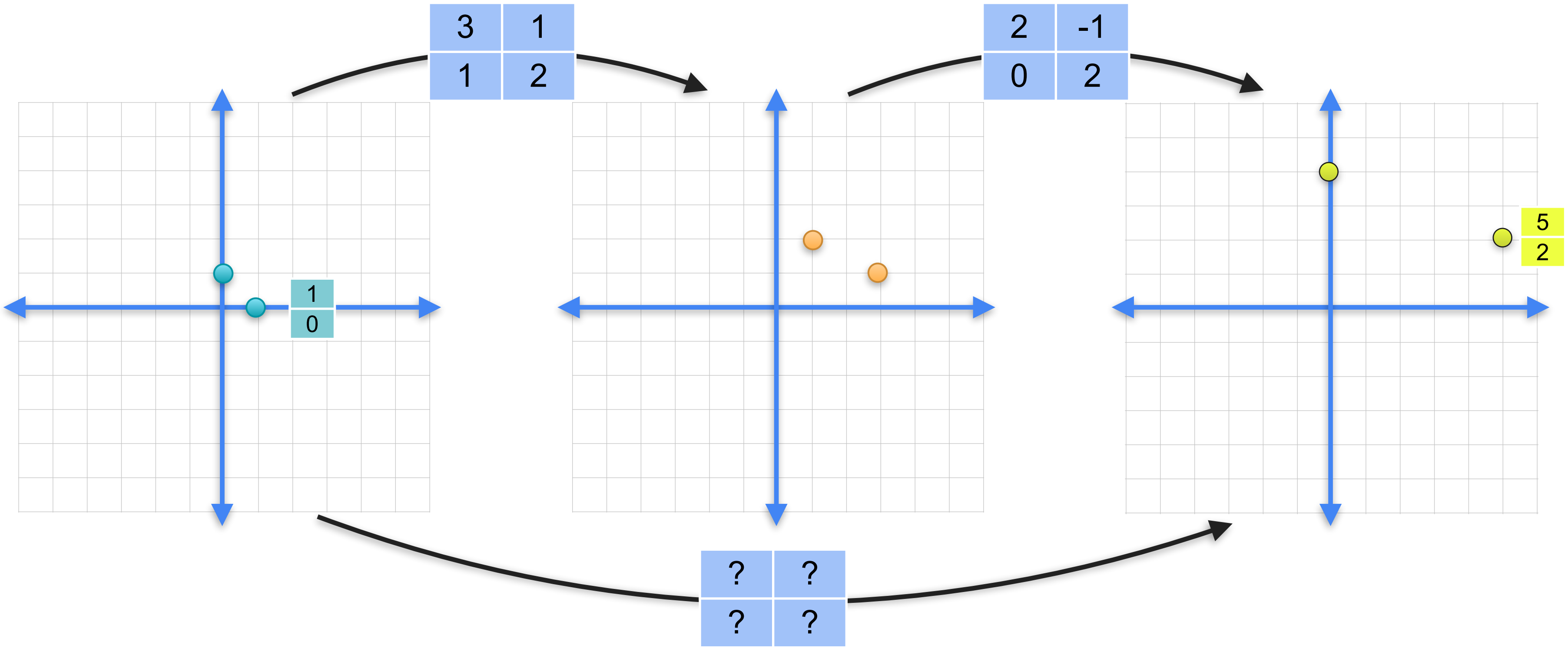
Combining linear transformations



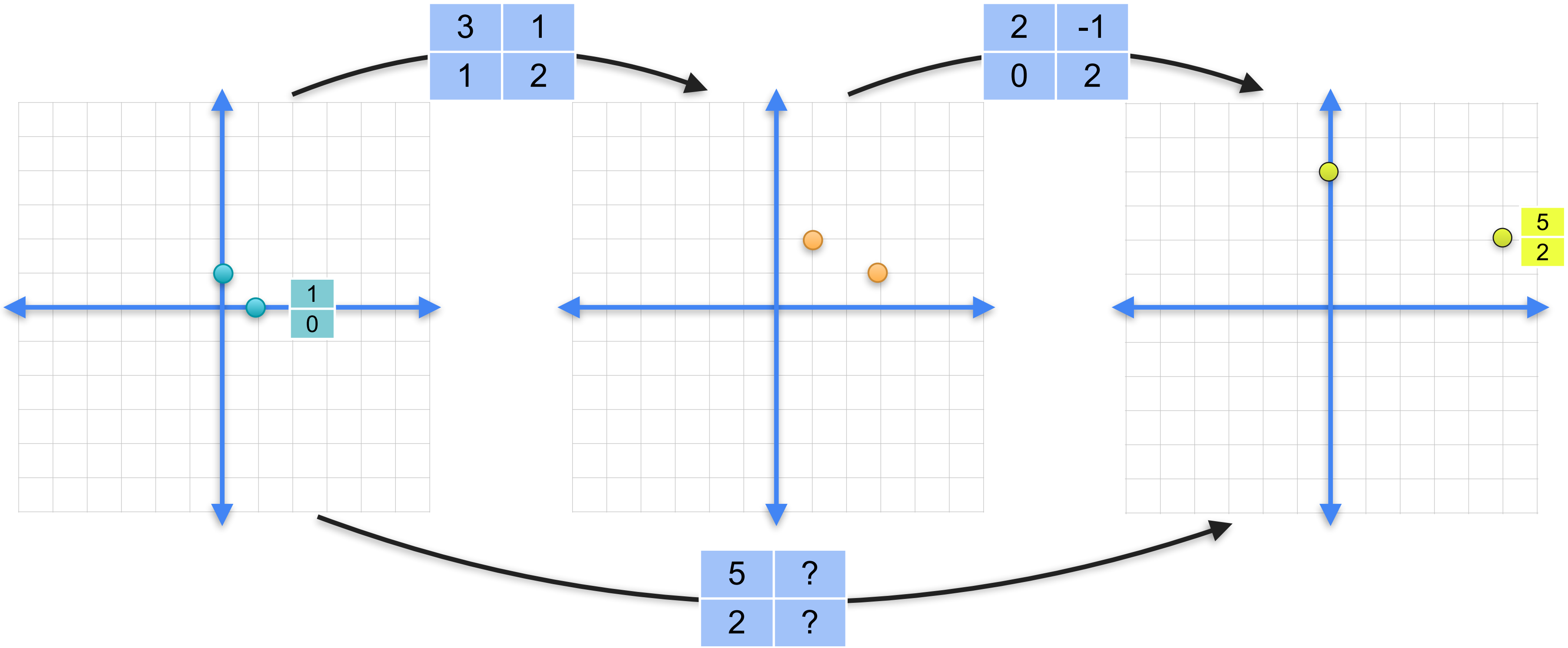
Combining linear transformations



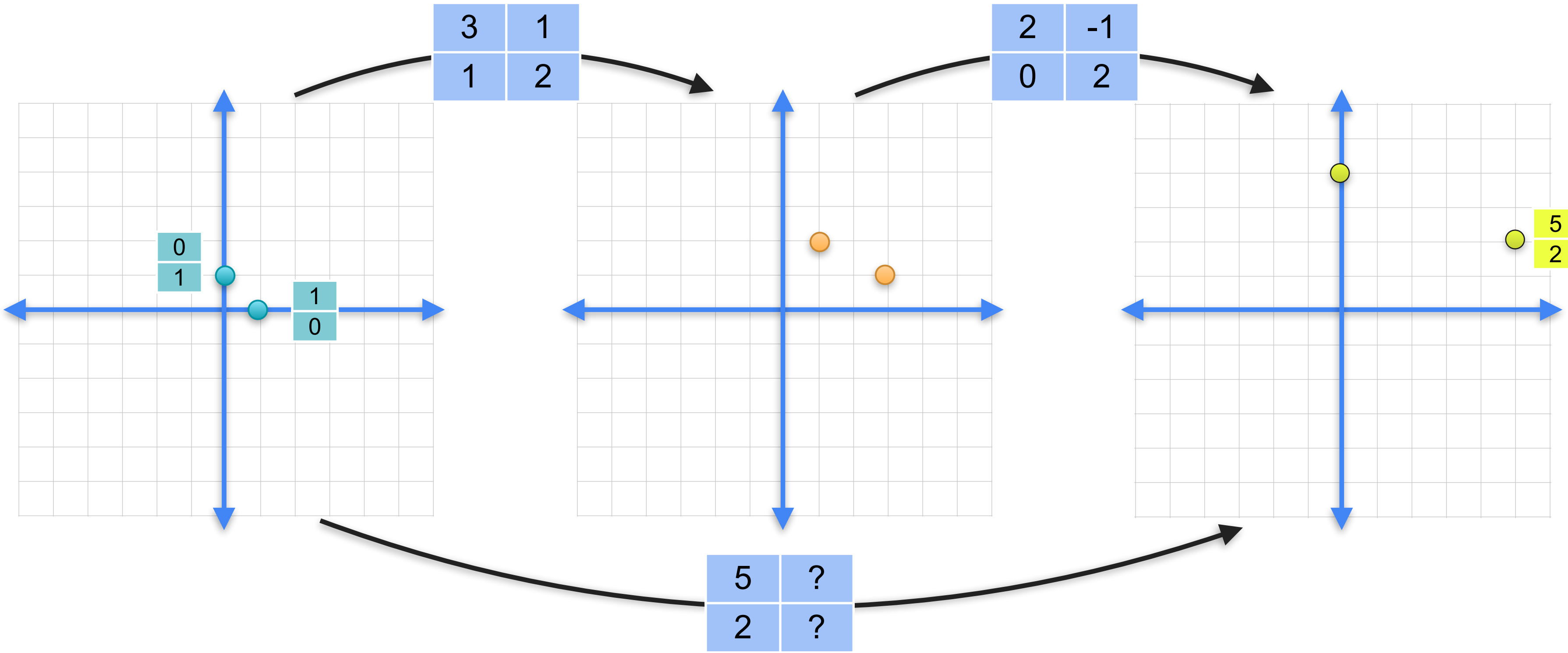
Combining linear transformations



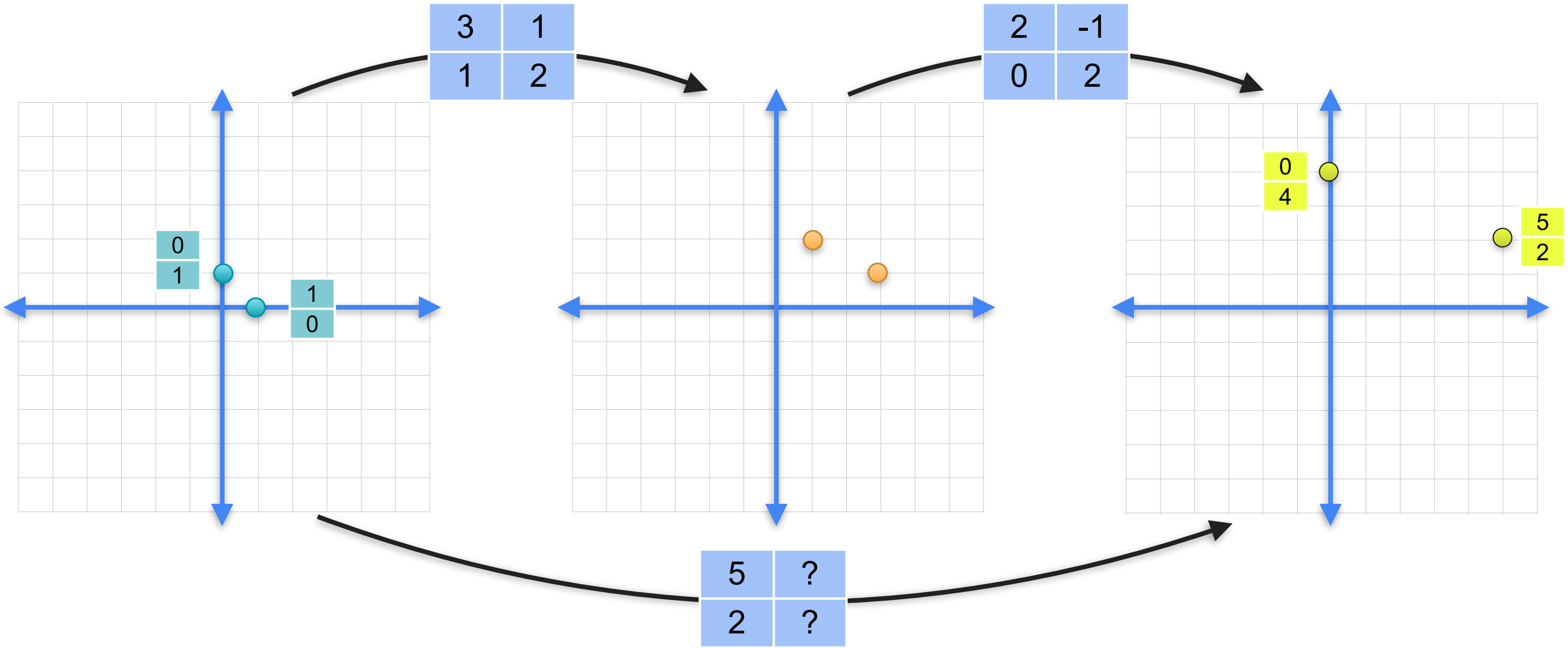
Combining linear transformations



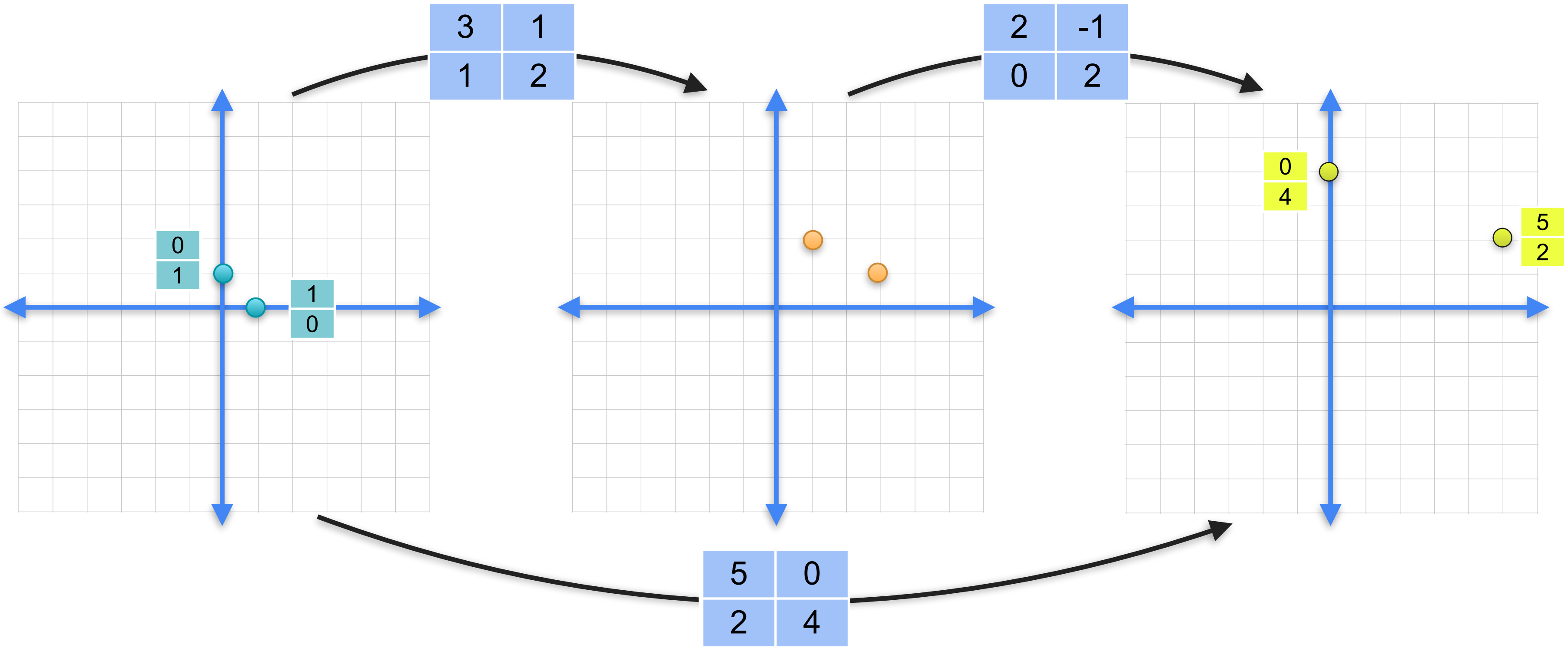
Combining linear transformations



Combining linear transformations



Combining linear transformations



Combining linear transformations

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

First

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

Combining linear transformations

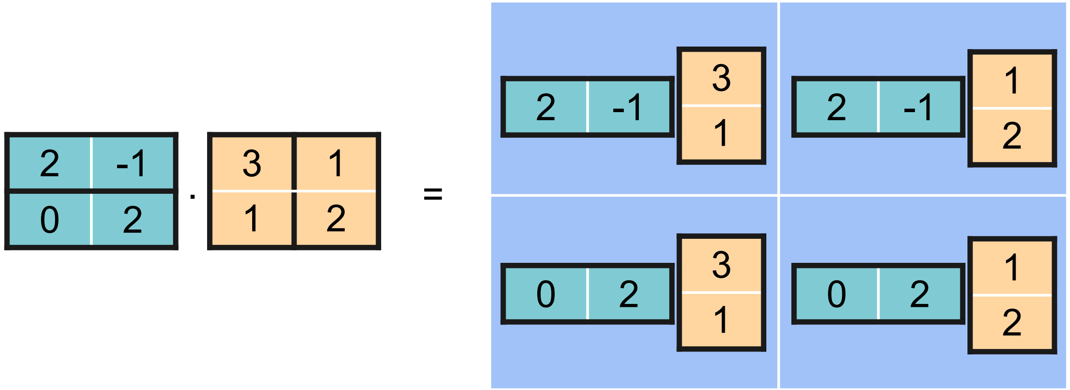
Second → $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ · First → $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

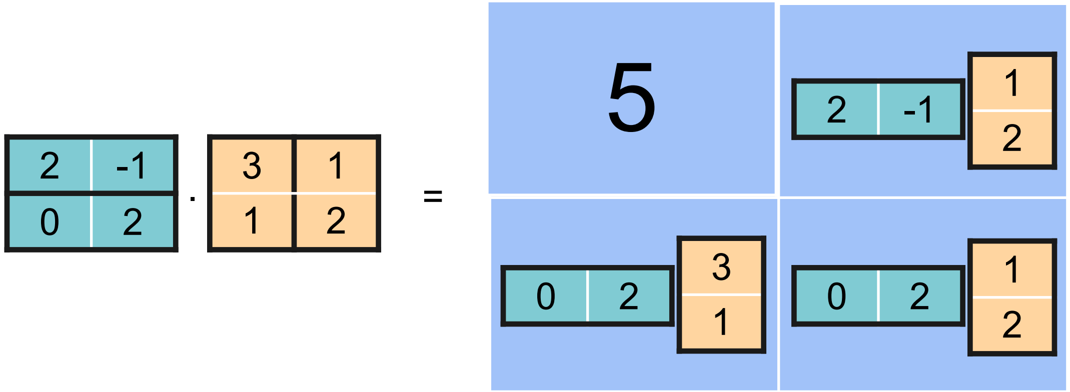
Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} =$$

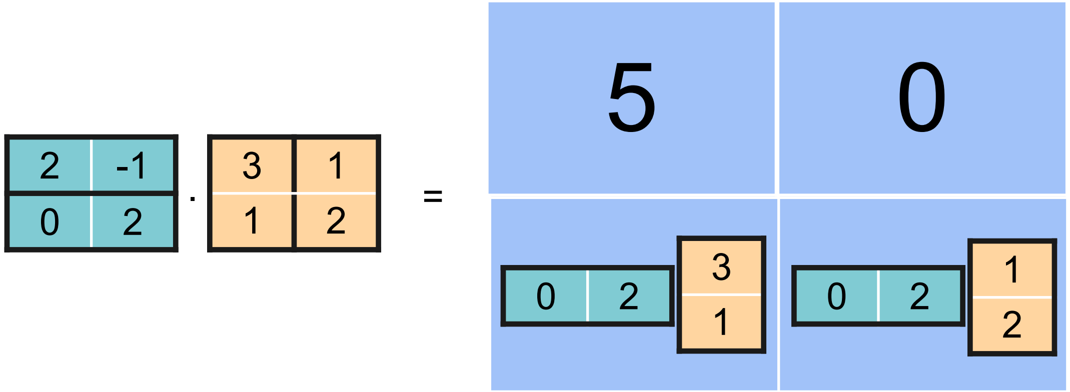
Multiplying matrices



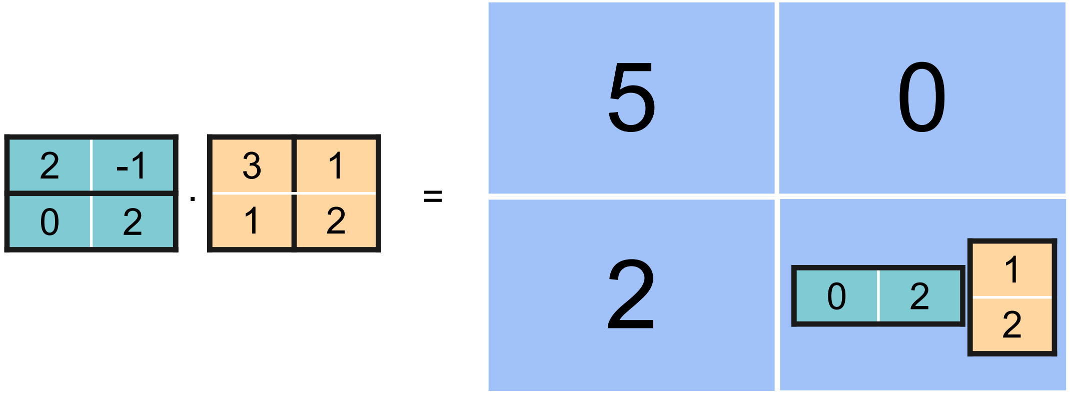
Multiplying matrices



Multiplying matrices



Multiplying matrices



Multiplying matrices

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

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Vectors and Linear Transformations

The identity matrix

The identity matrix

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

The identity matrix

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

The identity matrix

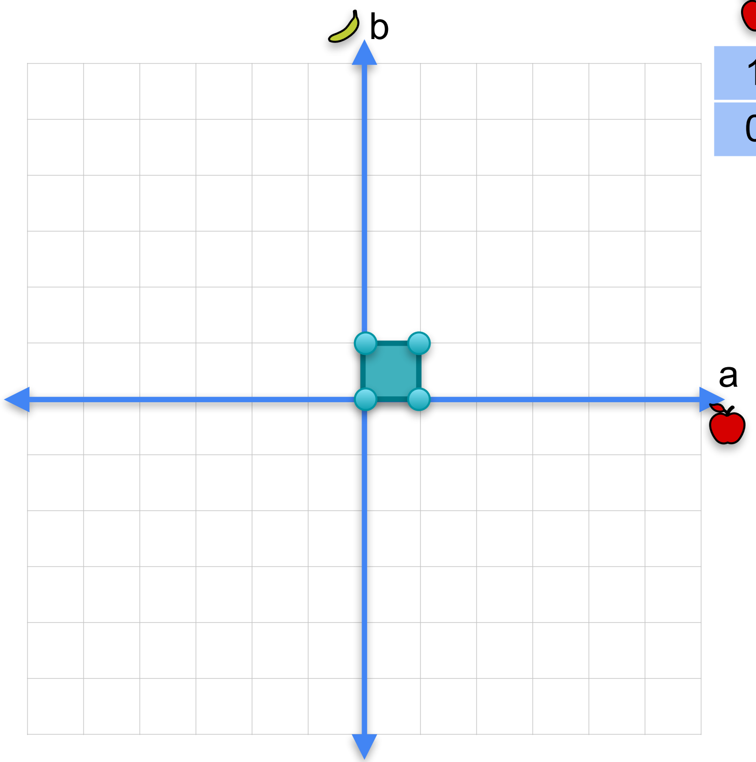
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

The identity matrix

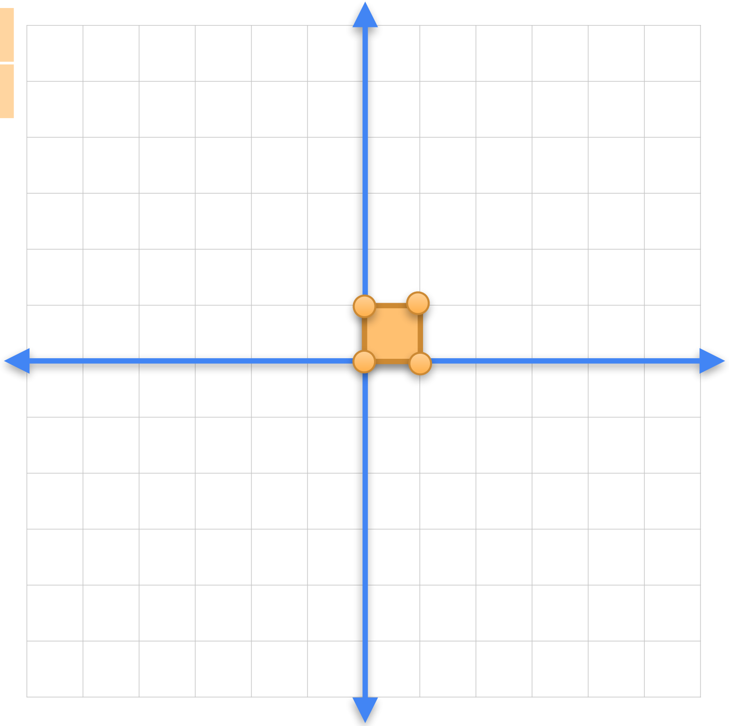
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = a$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

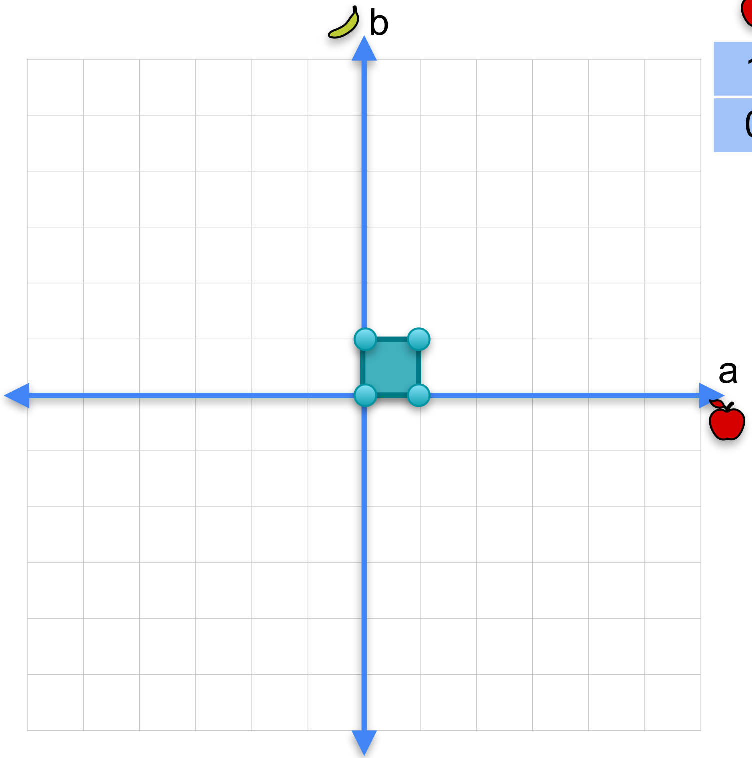
The identity matrix



$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$



The identity matrix



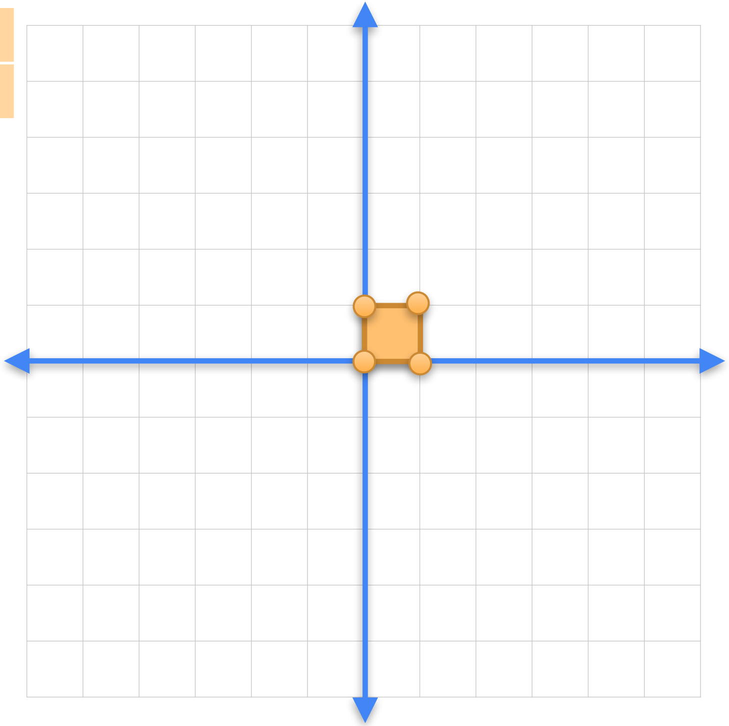
$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}x\\ y\end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (1,0)

(0,1) → (0,1)

(1,1) → (1,1)



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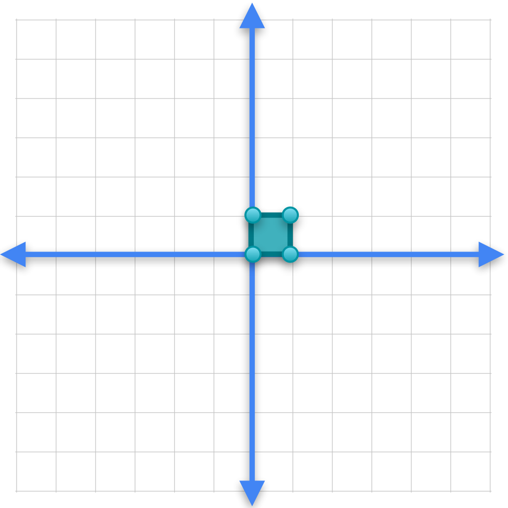
Vectors and Linear Transformations

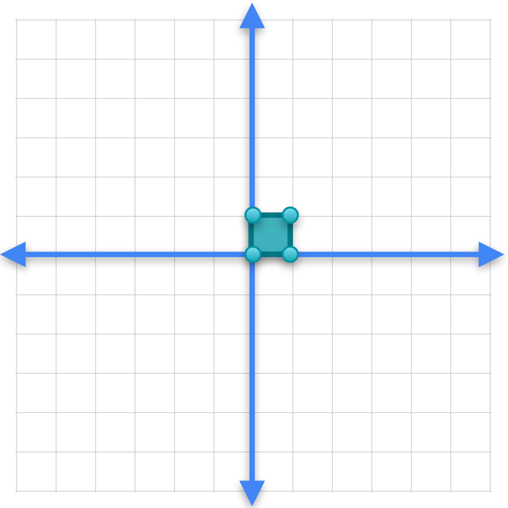
Matrix inverse

Matrix inverses

Matrix inverses

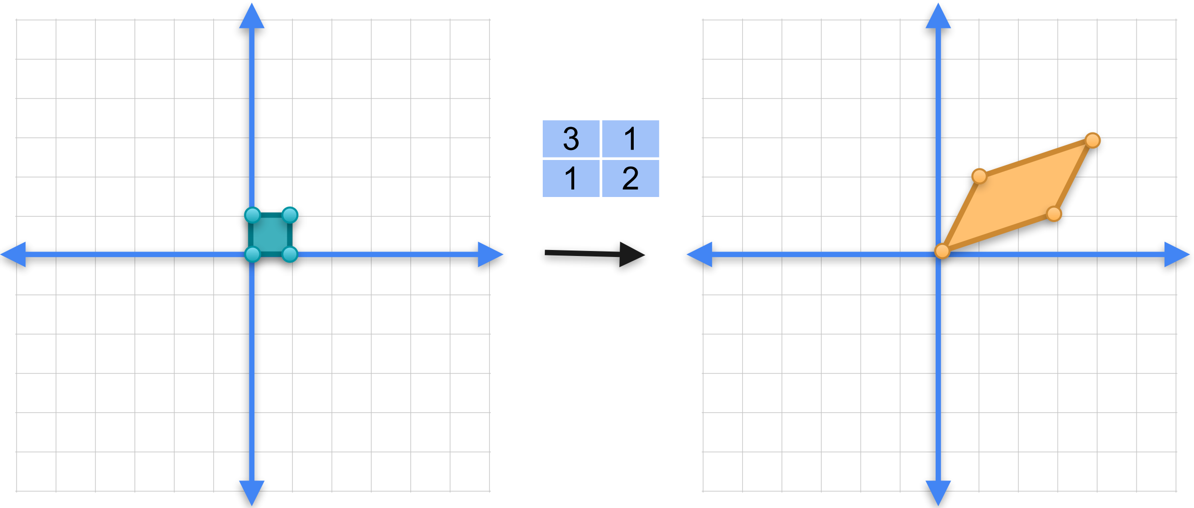


Matrix inverses

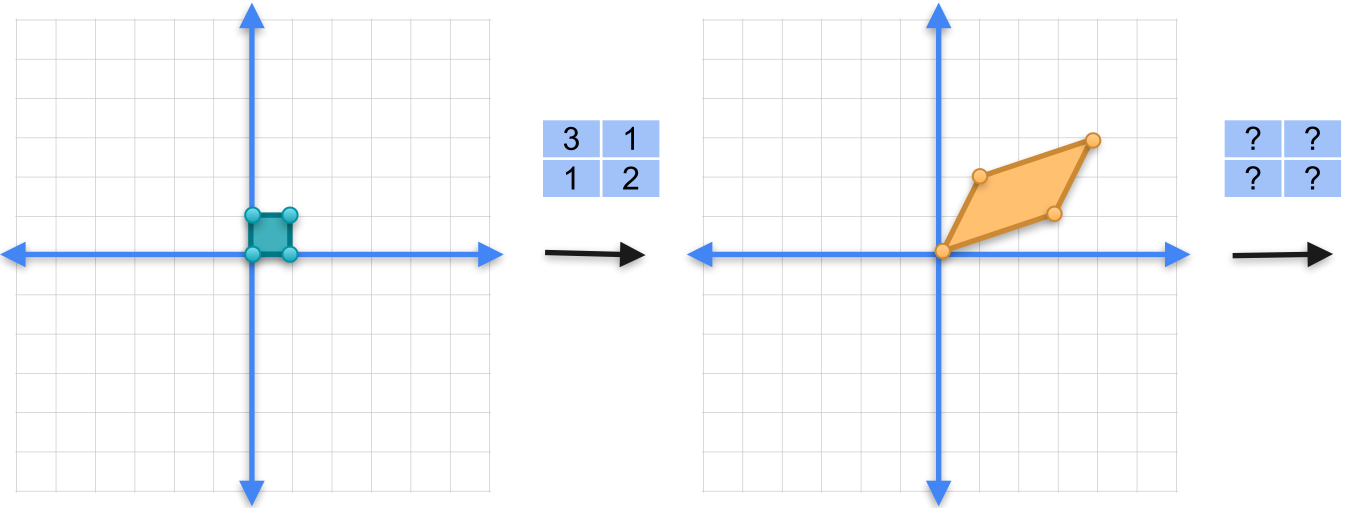


3	1
1	2

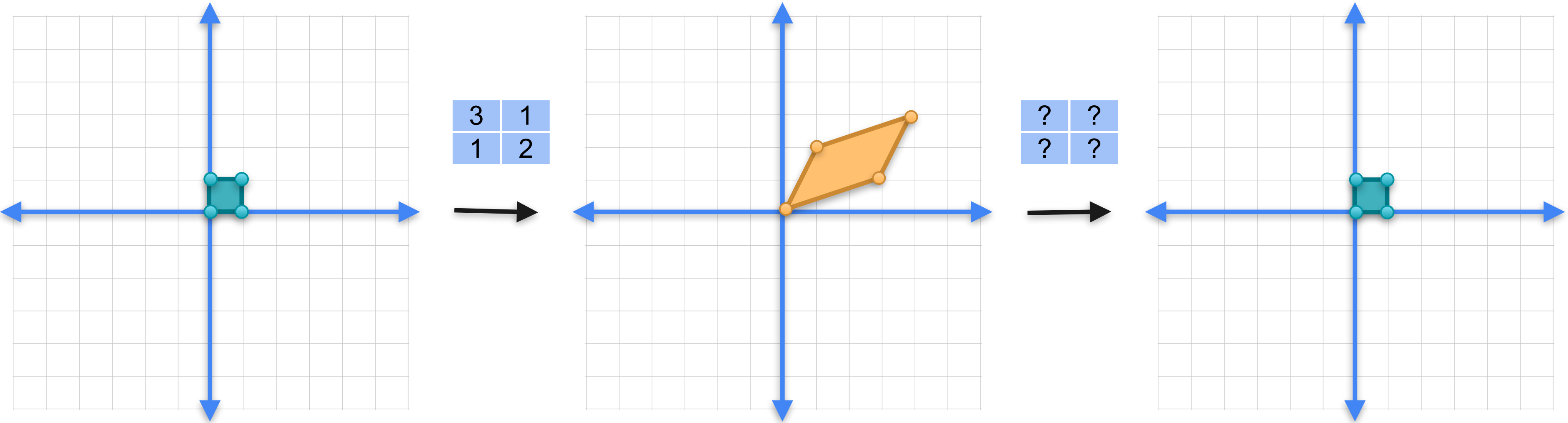
Matrix inverses



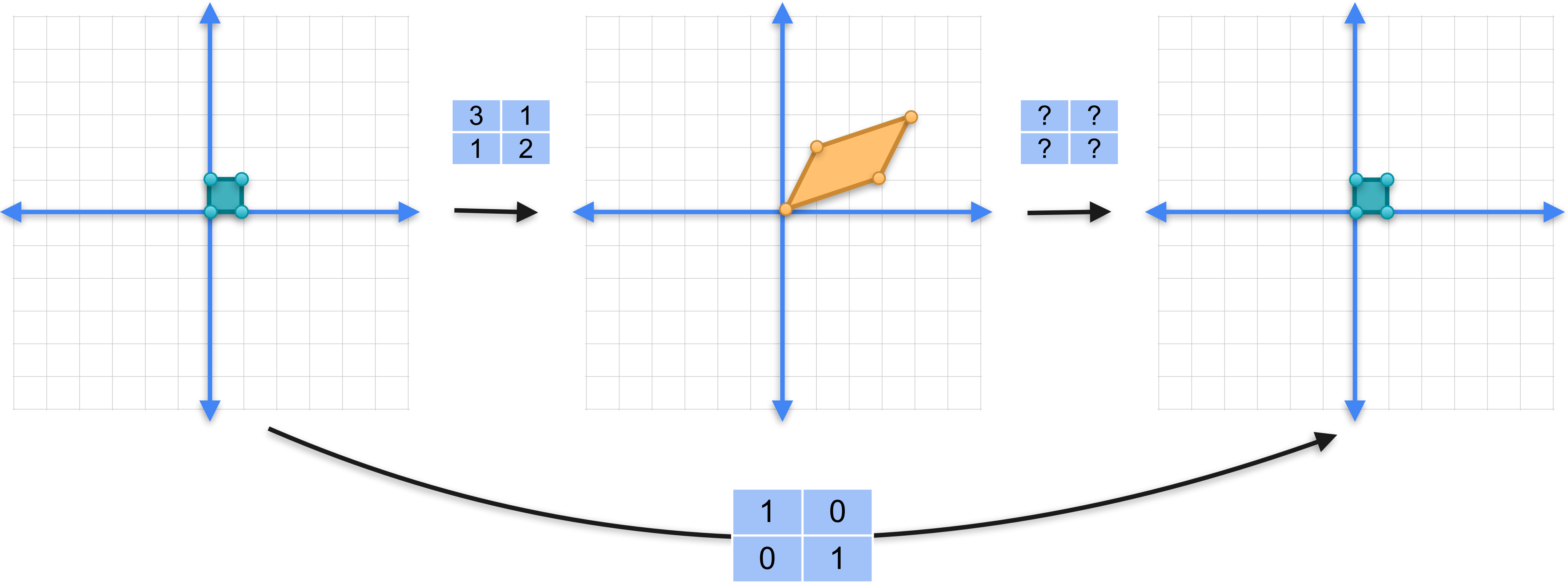
Matrix inverses



Matrix inverses



Matrix inverses



Multiplying matrices

Multiplying matrices

a	b
c	d

Multiplying matrices

a	b
c	d

·

3	1
1	2

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \leftarrow \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

Multiplying matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1$$

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How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1 \qquad 3a + 1b = 1$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0 \qquad 1a + 2b = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 \qquad 3c + 1d = 0$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \qquad 1c + 2d = 1$$

How to find an inverse?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1 \qquad 3a + 1b = 1 \qquad a = \frac{2}{5}$$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0 \qquad 1a + 2b = 0 \qquad b = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 0 \qquad 3c + 1d = 0 \qquad c = -\frac{1}{5}$$

$$\begin{bmatrix} c & d \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \qquad 1c + 2d = 1 \qquad d = \frac{3}{5}$$

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

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Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

- **$5a + 2c = 1$**
- **$5b + 2d = 0$**
- **$a + 2c = 0$**
- **$b + 2d = 1$**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

- **5a + 2c = 1**
- **5b + 2d = 0**
- **a + 2c = 0**
- **b + 2d = 1**

- **a = 1/4**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

- **5a + 2c = 1**
- **5b + 2d = 0**
- **a + 2c = 0**
- **b + 2d = 1**

- **a = 1/4**
- **b = -1/4**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

- **5a + 2c = 1**
- **5b + 2d = 0**
- **a + 2c = 0**
- **b + 2d = 1**

- **a = 1/4**
- **b = -1/4**
- **c = -1/8**

Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

- **5a + 2c = 1**
- **5b + 2d = 0**
- **a + 2c = 0**
- **b + 2d = 1**

- **a = 1/4**
- **b = -1/4**
- **c = -1/8**
- **d = 5/8**

Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a + c = 1$

$2b + 2d = 1$

$2a + 2c = 0$

$b + d = 0$

This is clearly a contradiction, since equation 1 says $a+c=1$, and equation 3 says $2a+2c=0$.

Vectors and Linear Transformations

Which matrices have an inverse?

Which matrices have inverses?

Which matrices have inverses?

$$5^{-1} = 0.2$$

Which matrices have inverses?

$5^{-1} = 0.2$ $8^{-1} = 0.125$

Which matrices have inverses?

$5^{-1} = 0.2$ $8^{-1} = 0.125$ $0^{-1} = ???$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \qquad \begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Which matrices have inverses?

$5^{-1} = 0.2$

$8^{-1} = 0.125$

$0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$5^{-1} = 0.2$ $8^{-1} = 0.125$ $0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix

Which matrices have inverses?

$5^{-1} = 0.2$

$8^{-1} = 0.125$

$0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$5^{-1} = 0.2$ $8^{-1} = 0.125$ $0^{-1} = ???$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 5

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

Det = 5

$$8^{-1} = 0.125$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

Det = 8

$$0^{-1} = ???$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

Which matrices have inverses?

$$5^{-1} = 0.2$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

Det = 5

$$8^{-1} = 0.125$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

Det = 8

$$0^{-1} = ???$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

Det = 0

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 5

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix

Invertible

Det = 8

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix

Non-invertible

Det = 0

Non-zero determinants

Which matrices have inverses?

$$5^{-1} = 0.2 \qquad 8^{-1} = 0.125 \qquad 0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

Non-singular matrix
Invertible

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Singular matrix
Non-invertible

Det = 5

Det = 8

Det = 0

Non-zero determinants

Zero determinant

DeepLearning.AI

Vectors and Linear Transformations

Neural networks and matrices


AI , ML , DL , RL
MATRIX
MULTIPLICATION

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
“Win, win the lottery!” : 7points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
"Win, win the lottery!" : 7points

Rule:
If the number of points of the sentence is bigger than ____, then the email is spam.

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Scores:

Lottery: ___ points

Win: ___ points

Examples

Lottery: 3 point
Win: 2 points
“Win, win the lottery!” : 7points

Rule:
If the number of points of the sentence is bigger than ____, then the email is spam.

Goal: Find the best points and threshold
Lottery: ___ point
Win: ___ point
Threshold: ___ points

Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

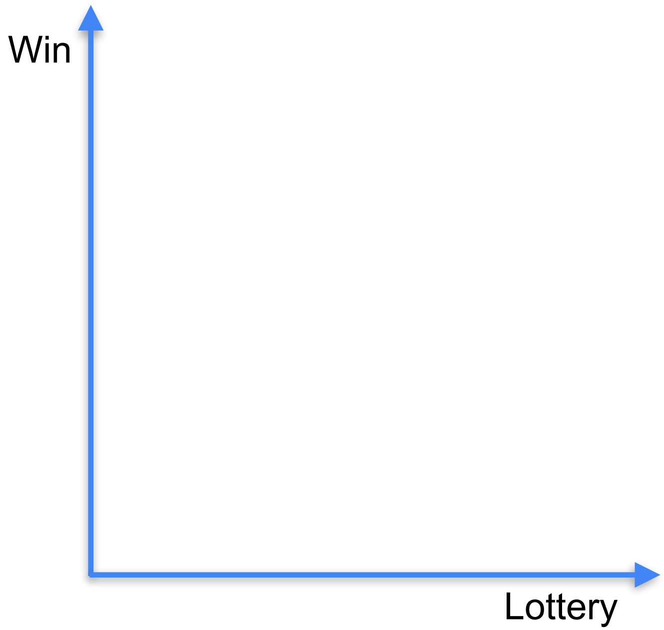
Solution:
Lottery: 1 point
Win: 1 point
Threshold: 1.5 points

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

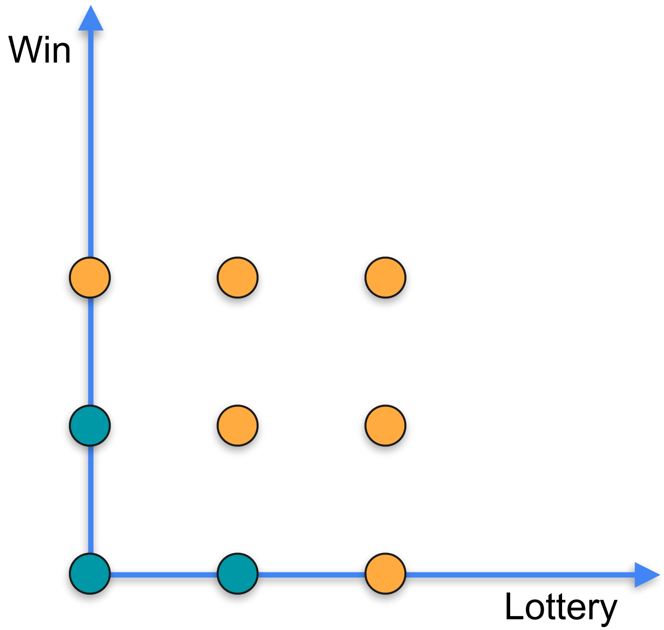
Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

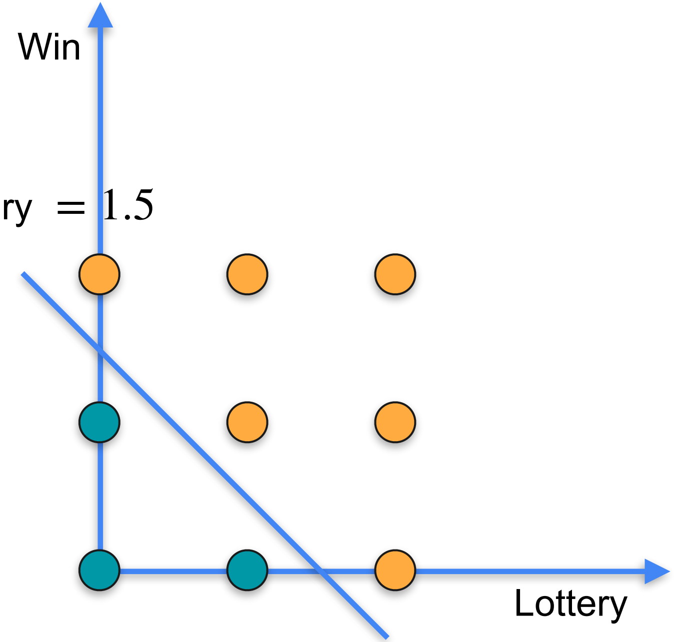
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Line:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$

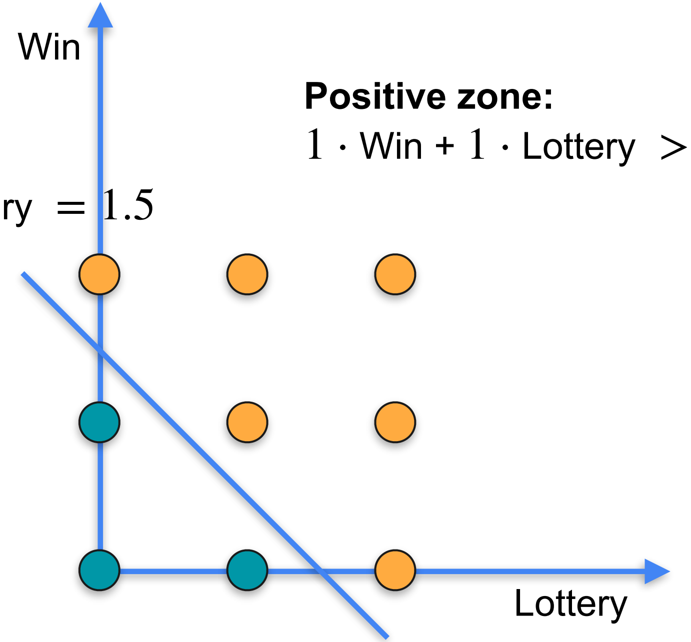


Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

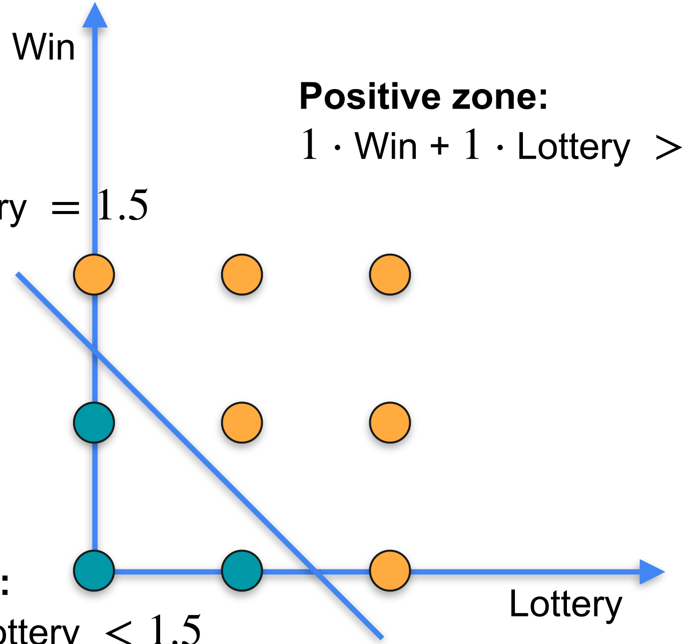
Line:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$

Positive zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$



Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



Line:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} = 1.5$

Positive zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} > 1.5$

Negative zone:
$1 \cdot \text{Win} + 1 \cdot \text{Lottery} < 1.5$

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1

Model
1
1

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1

Model
1
1

= 3

Check: > 1.5?

Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

2	1

Model
1
1

= 3

Check: > 1.5?

Spam

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1

Model
1
1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1

Model
1
1

= 1

Check: > 1.5?

Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

0	1

Model
1
1

= 1

Check: > 1.5?

✗

Not spam

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?

Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Perceptrons

Spam	Word1	Word2	…	WordN
Yes				
Yes				
No				
Yes				
No				
No				
Yes				
Yes				
Yes				

Model
…

=

Prod

Check:

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Model
1
1

Check: > 1.5?

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Threshold and bias

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

Threshold

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Bias

Model
1
1

Check: > 1.5?

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Threshold

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$

Bias

Check: > 1.5?

Model
1
1
-1.5

Bias

Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$ ← Threshold

$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$ ← Bias

Model
1
1
-1.5

-1.5 ← Bias

Check: > 0?

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

Dot prod
0
1
1
2

Check: >1.5?

The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

Model
1
1

=

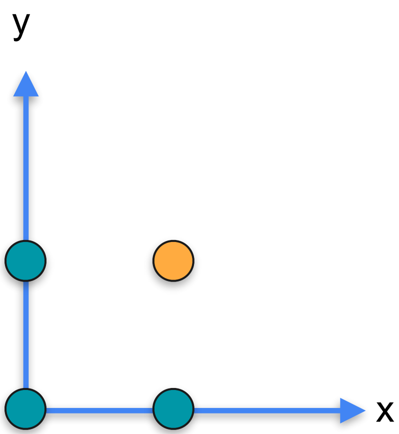
Dot prod
0
1
1
2

Check: >1.5?

Check
No
No
No
Yes

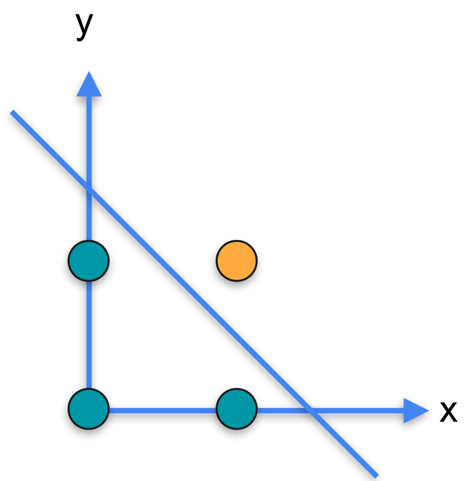
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1



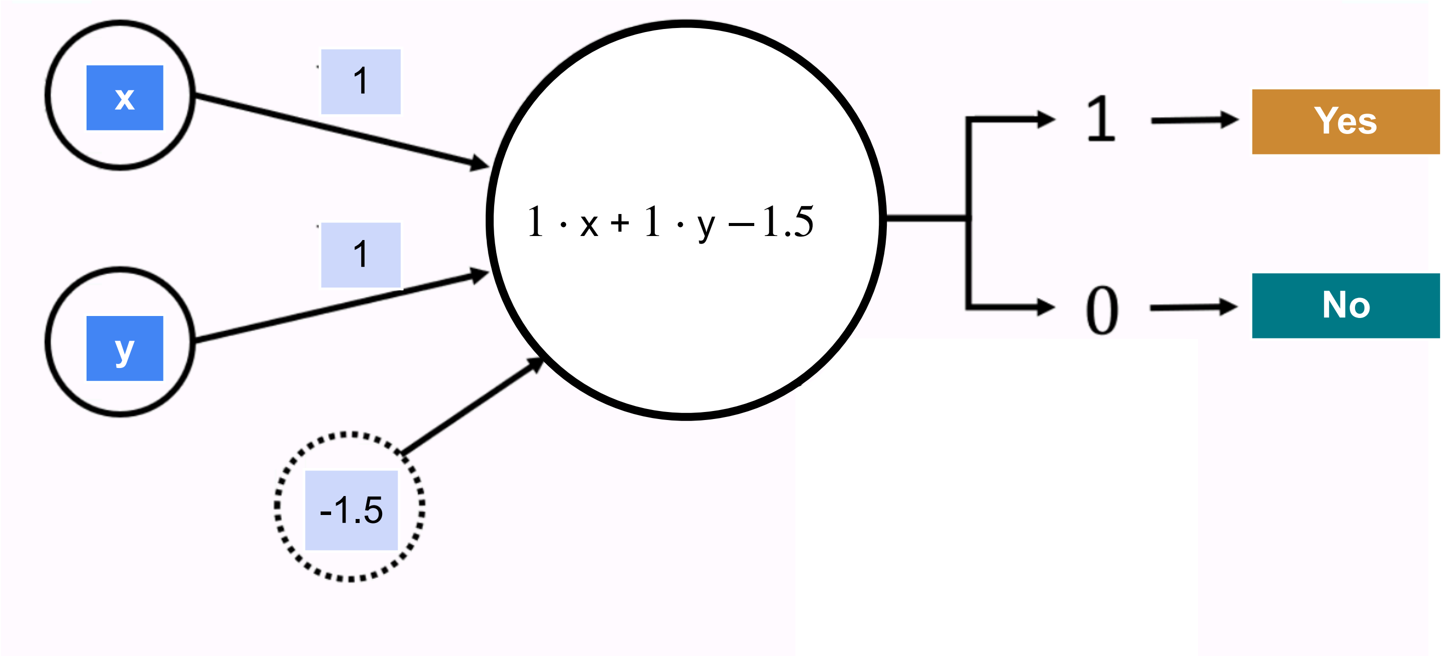
The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

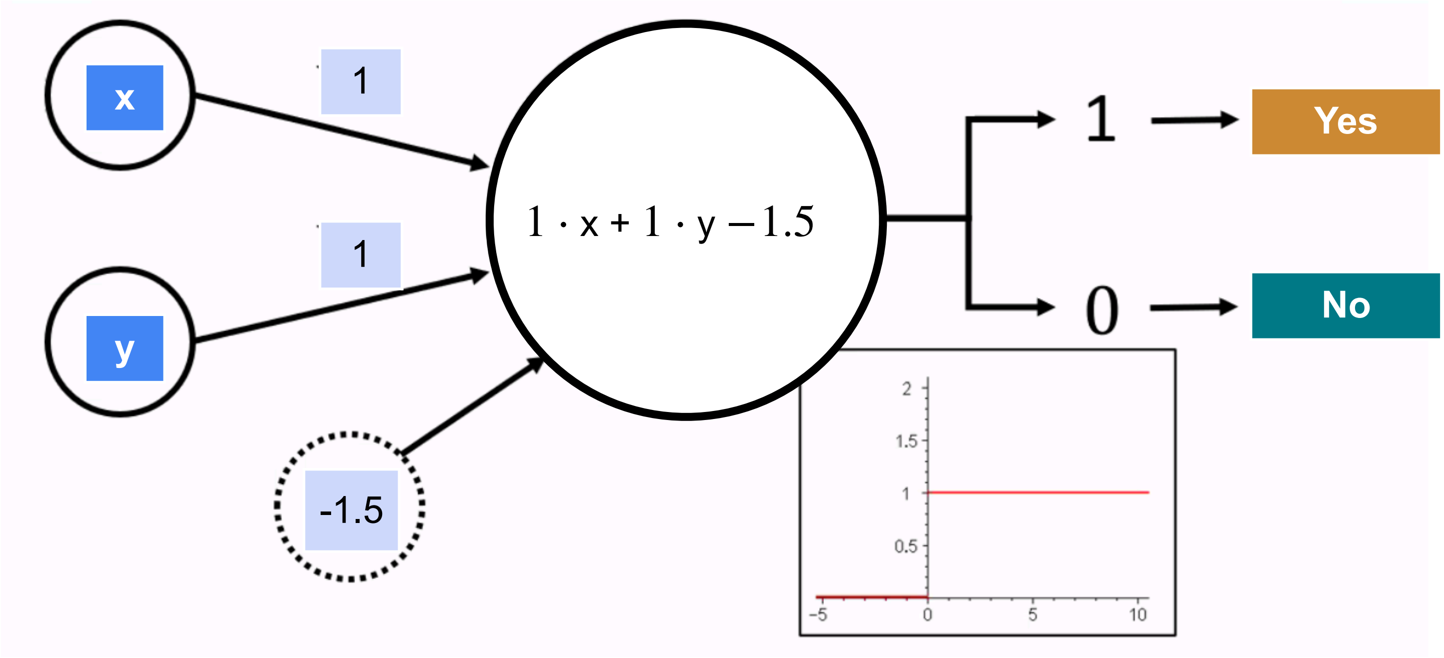


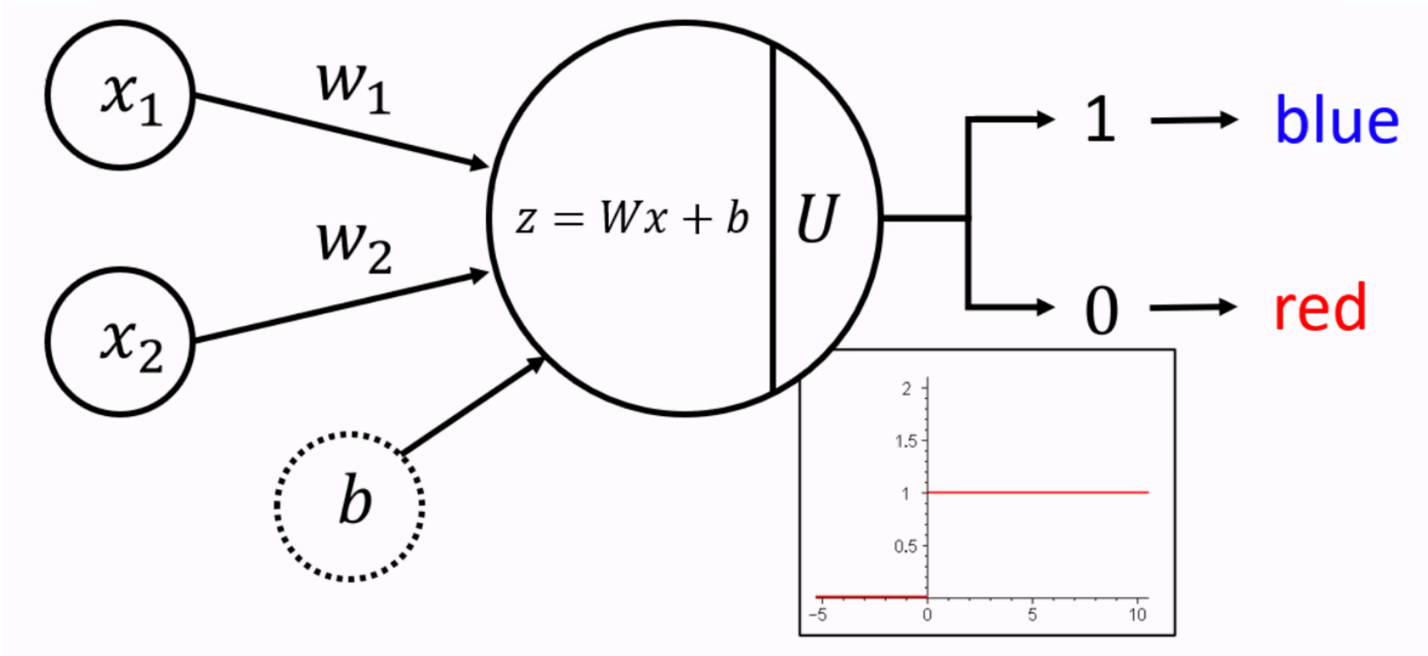
$$1 \cdot x + 1 \cdot y = 1.5$$

The perceptron



The perceptron





x_1
x_2
w_1
w_2
b
$z = Wx + b$
U
1
0
blue
red
2
1.5
1
0.5
−5
0
5
10

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Vectors and Linear Transformations

Conclusion